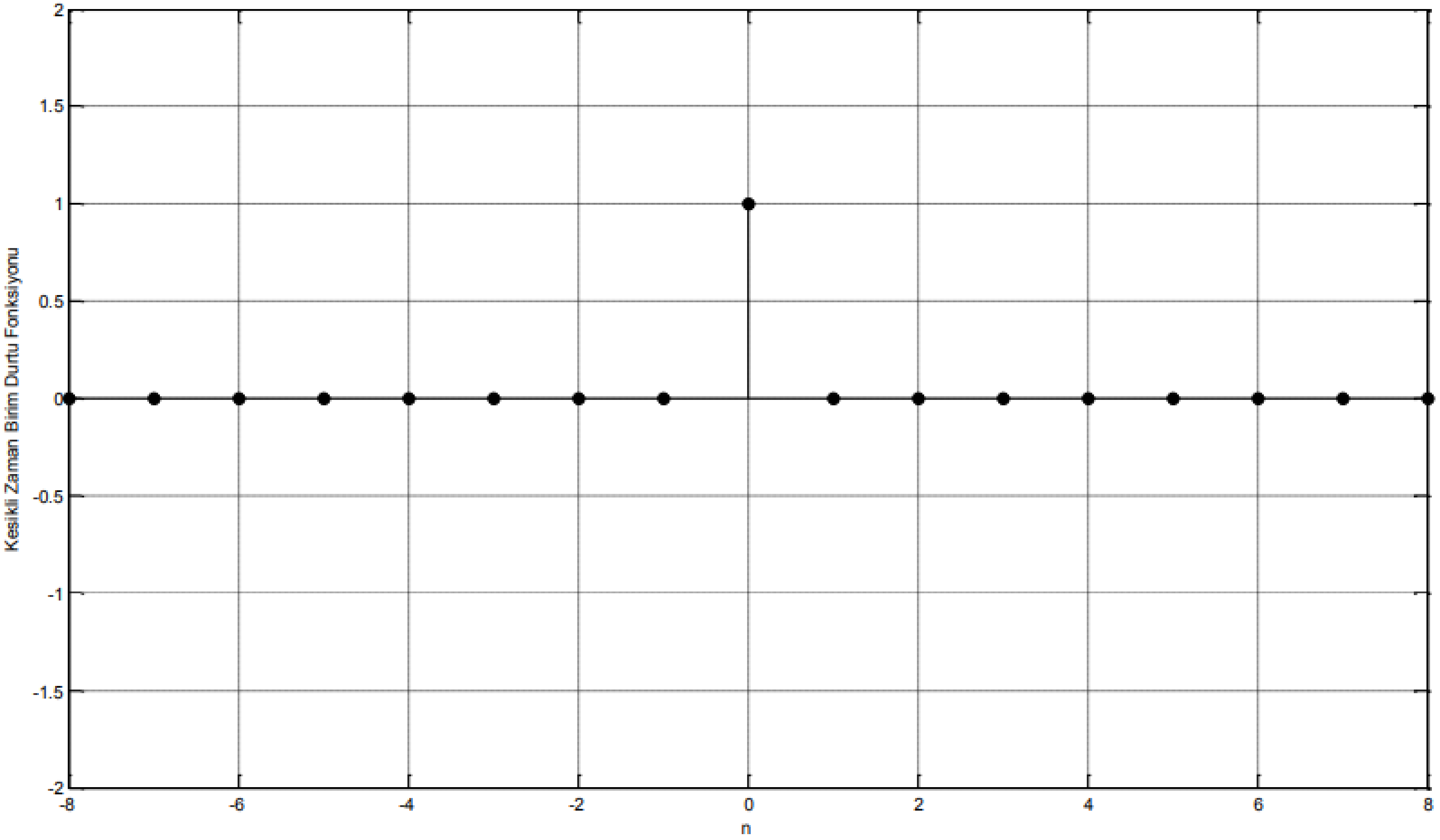
Sinyaller ve sistemler

Sinyallerin matematiksel fonksiyon olarak gösterilmesi

- Sinyaller bilgi taşıyan fiziksel niceliklerdir. Bunlar voltaj, akım, veya elektromanyetik dalga gibi şeyler olabilir.
- Bu değerlerin zamana göre değişimlerini çizersek sinyali zamana göre değişimini görmüş oluruz.
- Diğer bir deyişler sinayalleri zamana bağlı fonksiyonlar olarak ifade edebiliriz.

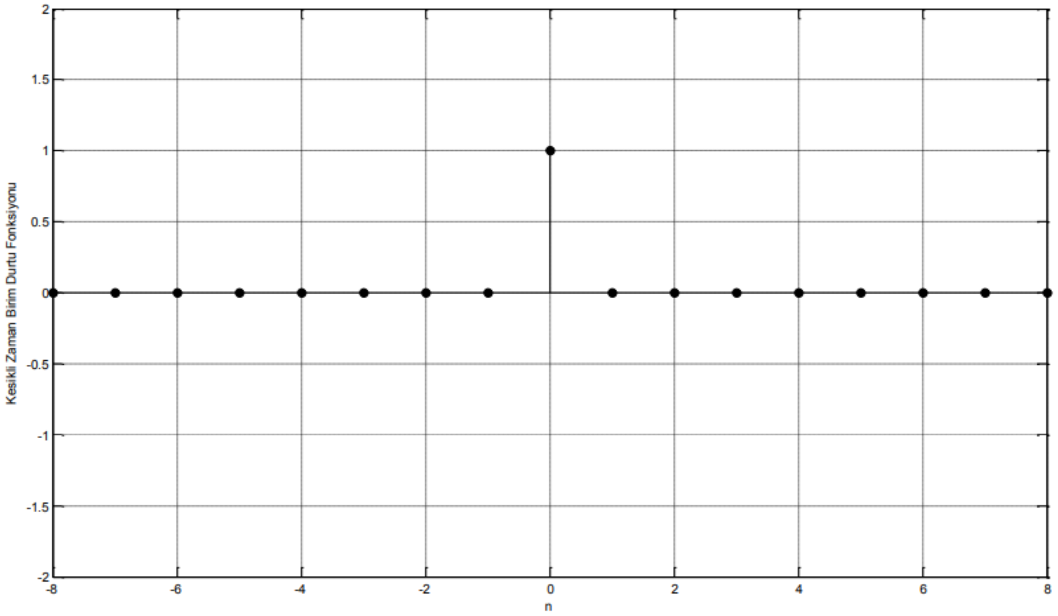
Birim adım(dürtü) fonksiyon (Unit Step Function) – Ayrık işaret

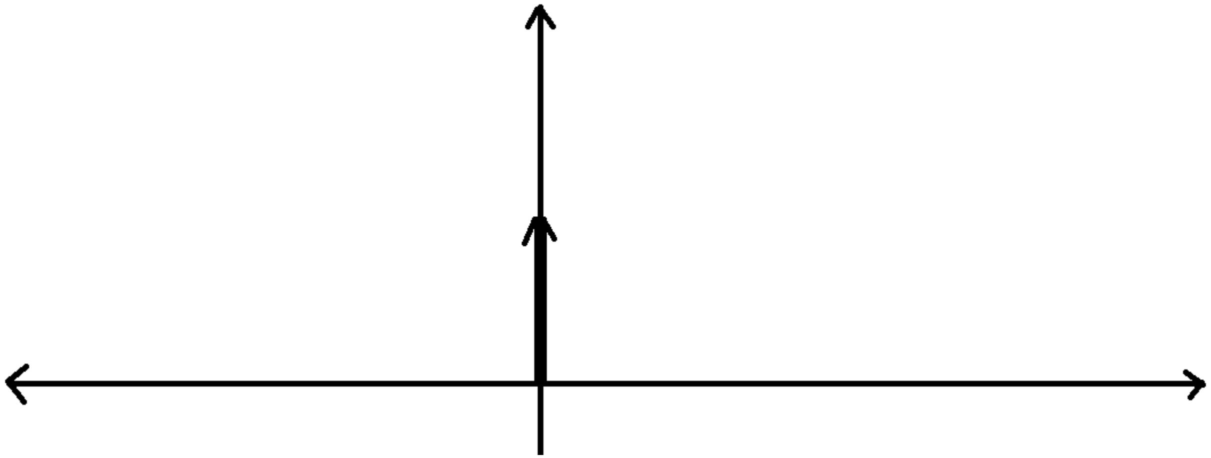
$$\delta[n] = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases}$$



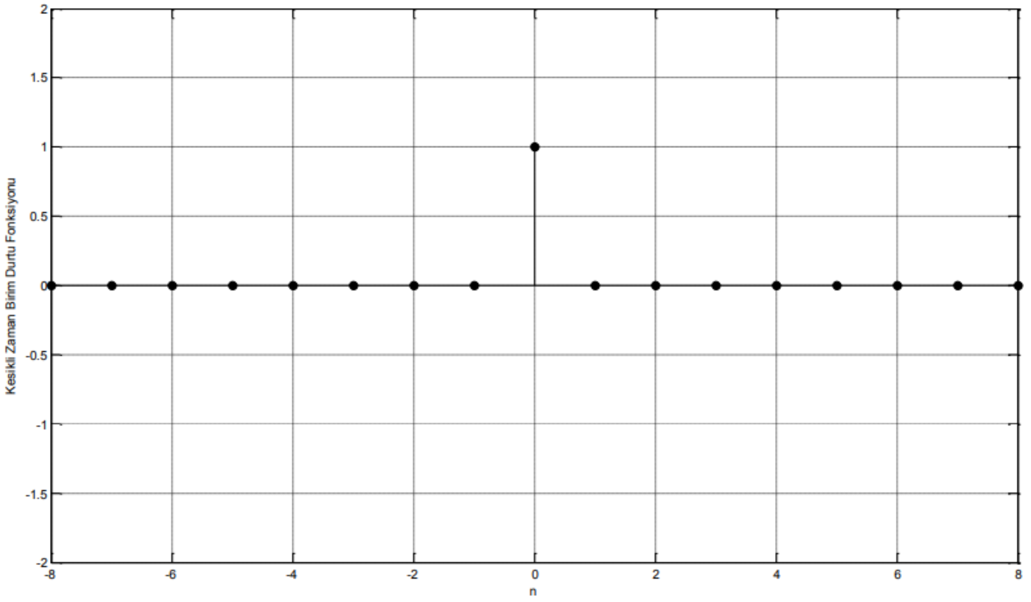
Birim adım(dürtü) fonksiyon (Unit Step Function) – Ayrık işaret

$$\delta[n] = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases}$$





Birim adım(dürtü) fonksiyon (Unit Step Function)

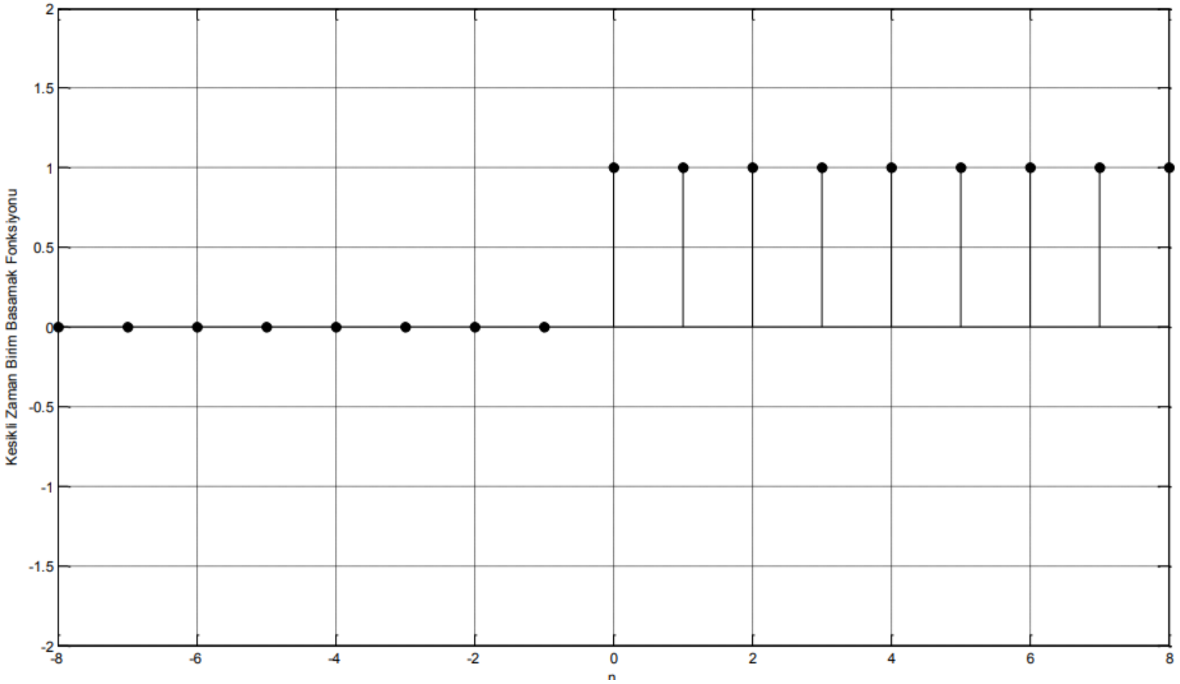


$$\delta[n] = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases}$$

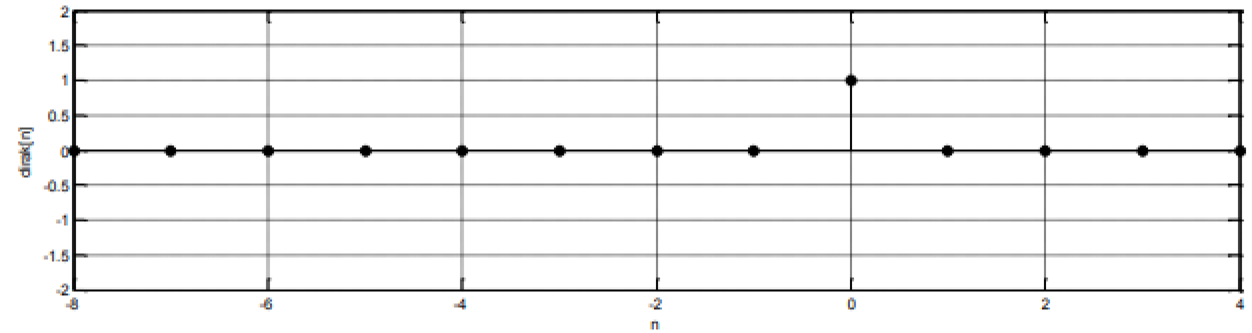
- Zaman eksenini kaydırırsak sadece zamanın o "anında" değeri olan, diğer anlarda değeri 0 olan bir dizi elde etmiş oluruz.
- Akıp giden zaman içinde, zamanın sadece o anını incelemek için benzersiz bir fonksiyona sahip olmuş oluruz.
- Diğer bir deyişle, herhangi bir diziyi birim dürtü dizisi cinsinden ifade etmek mümkündür.

Birim basamak fonksiyonu

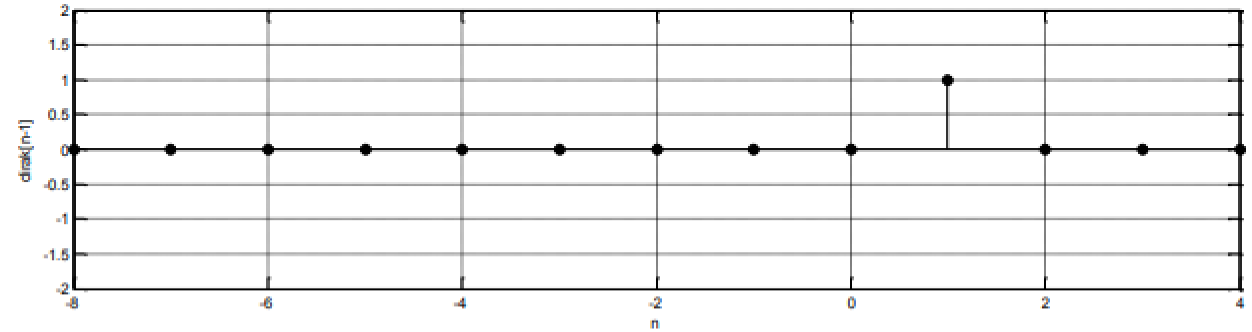
$$u[n] = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases}$$



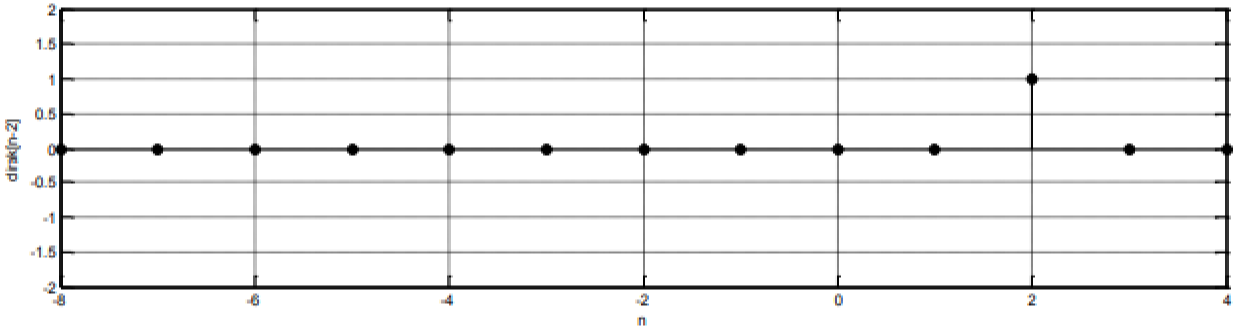
Birim dürtüden birim basamak elde etme



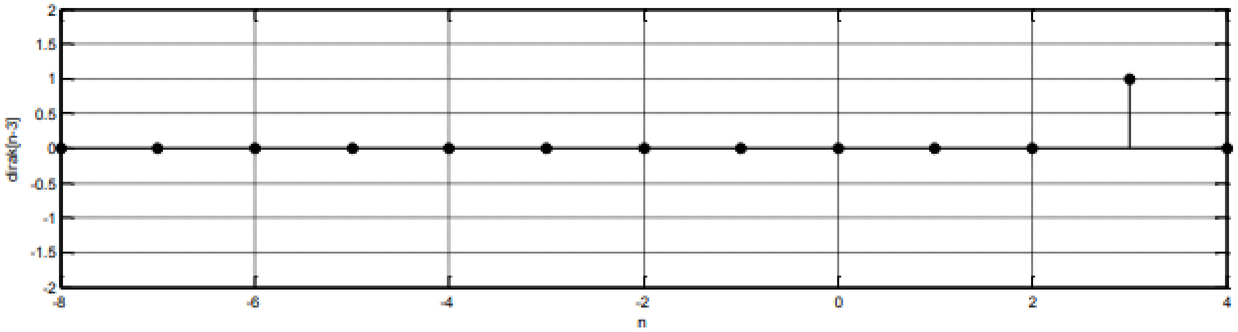
a.



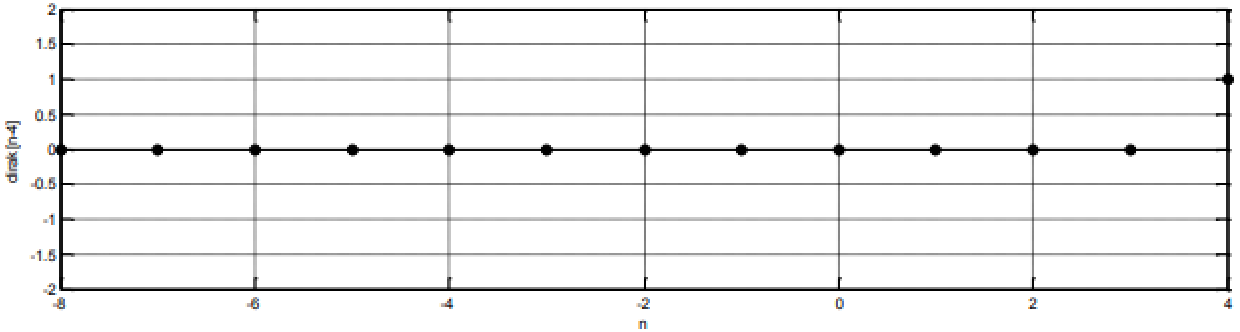
b.


2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
dirak[n-2]
-8
-6
-4
-2
0
2
4
n

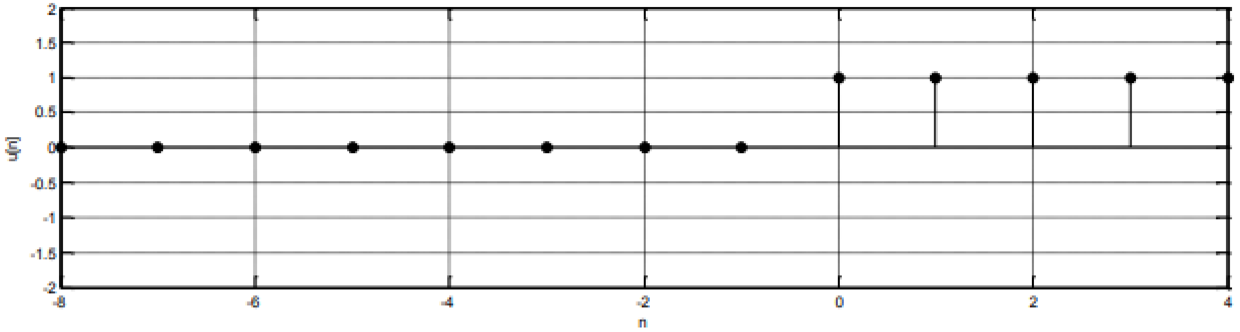
c.


2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
dirak[n-3]
-8
-6
-4
-2
0
2
4
n

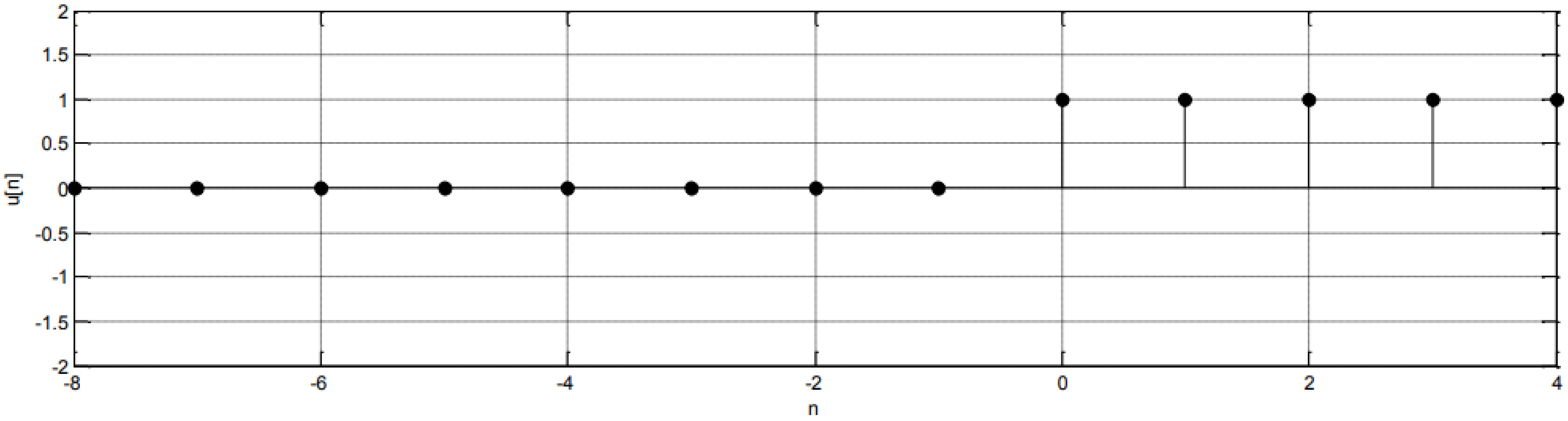
d.



e.



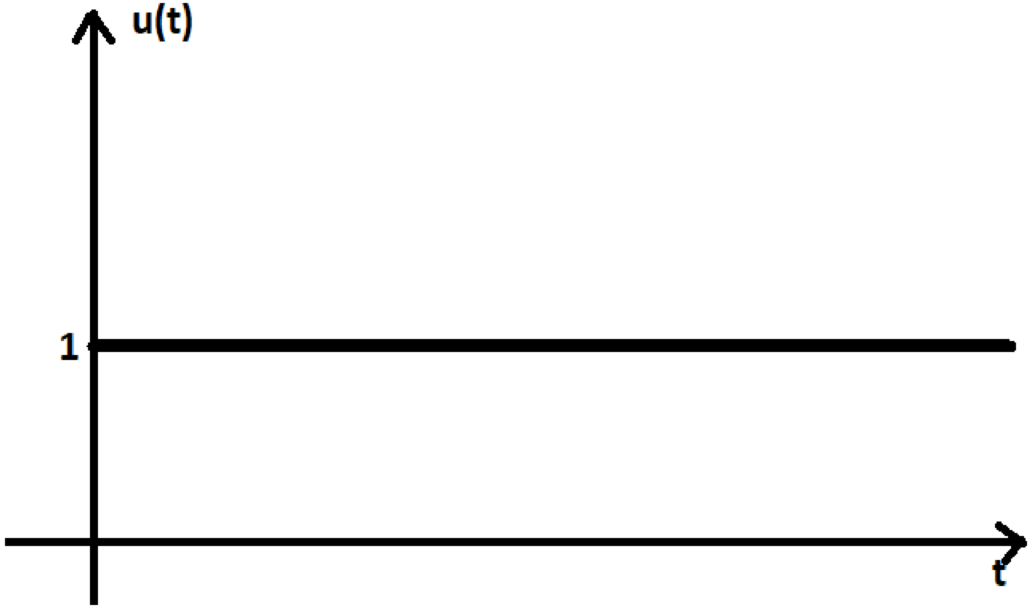
f.



$$u[n] = \sum_{k=0}^{\infty} \delta[n-k] = \delta[n] + \delta[n-1] + \delta[n-2] + \cdots$$

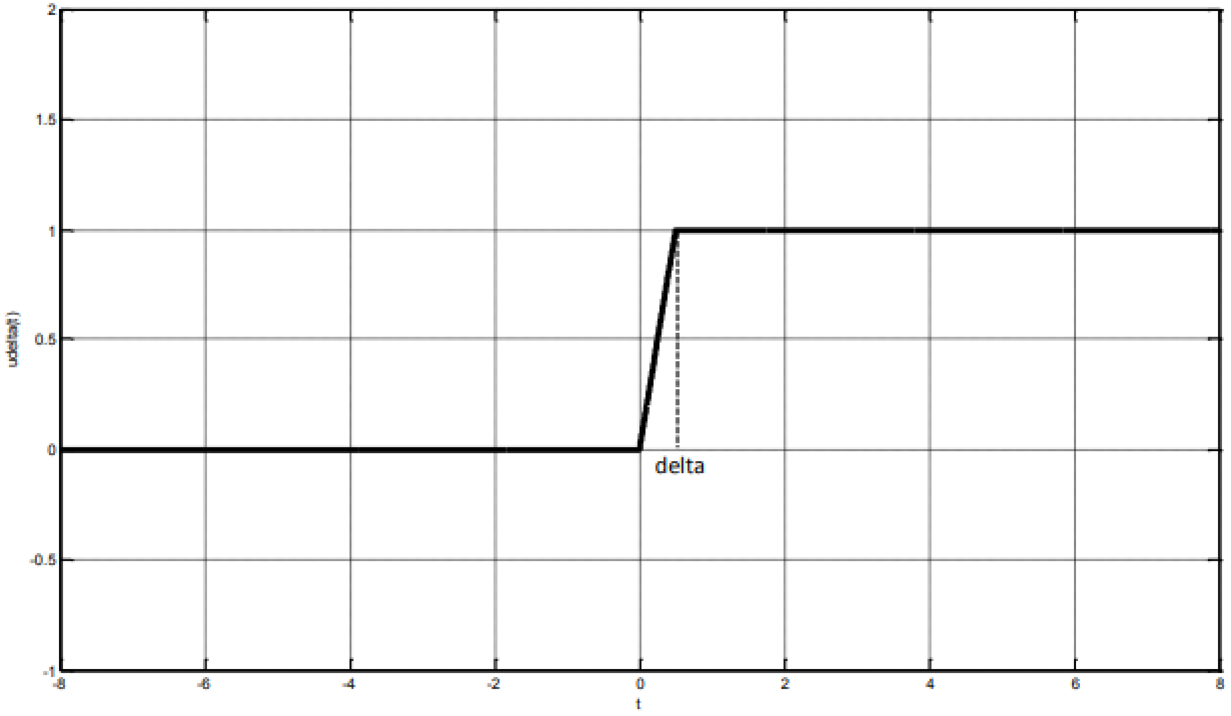
Birim adım fonksiyon

- $u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$



Birim delta adım fonksiyon (Unit delta step function)

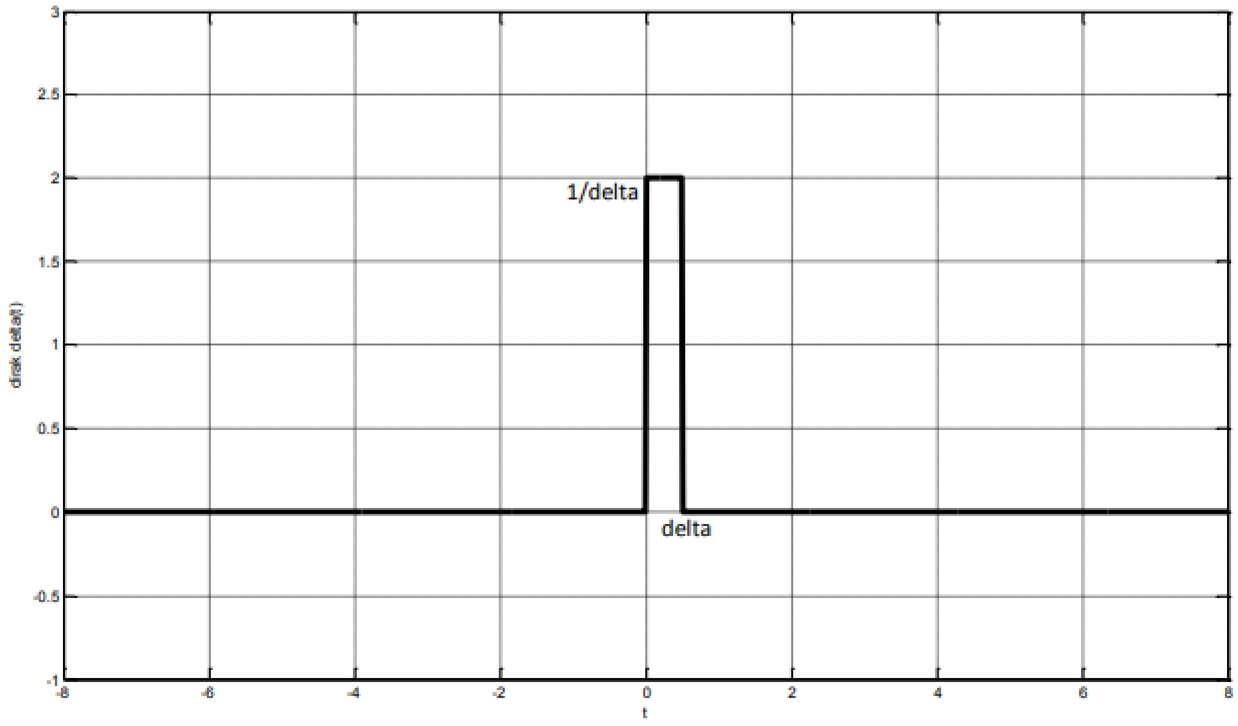
- $\begin{cases} \frac{t}{\Delta} & 0 \leq t \leq \Delta \\ 1 & t > 0 \\ 0 & diğer \end{cases}$



Birim delta dürtü fonksiyon

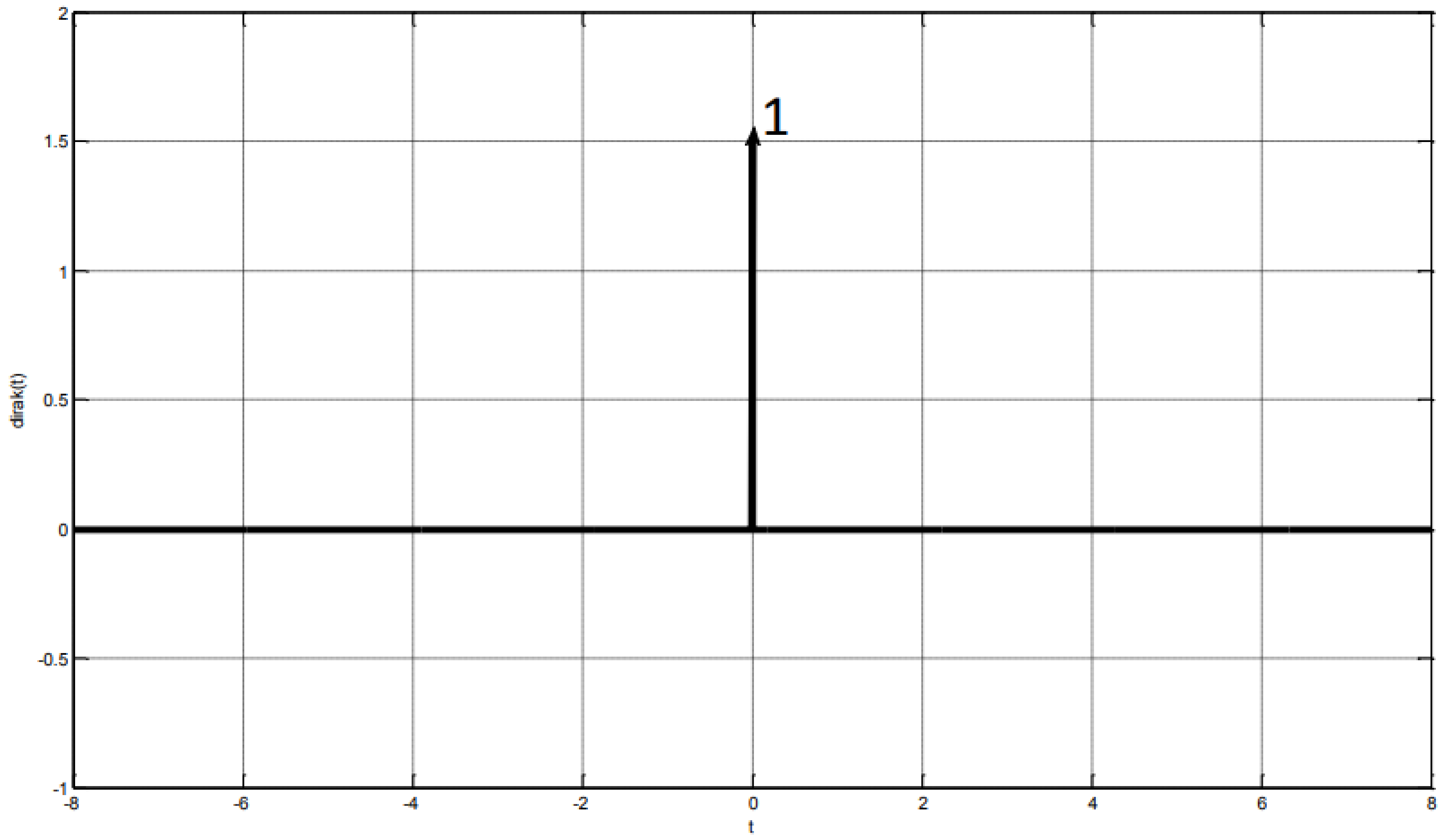
- Birim delta adım fonksiyonun türevi alınarak bulunur.

$$\delta(t) = \frac{du(t)}{dt}$$



Birim dürtü fonksiyonu (Sürekli işaret)

$$\lim_{\Delta \to 0} \delta_{\Delta}(t)$$



Dürtü fonksiyon özellikleri

- Sinyal işleme ve kominikasyonda sık kullanılan bir fonksiyondur

1) δ(t-t_0) f(t)= δ(t-t_0) f(t_0) burada t_0 reel bir sayı olup t ise bir değişkendir.

2) δ(t-t_0)f(t-t_1)= δ(t-t_0)f(t_0-t_1) (1) daha detaylı kullanımı

3) $\int_{-\infty}^{\infty} \delta(t)dt = 1$ veya $\int_{-\infty}^{\infty} \delta(t - t_0)dt = 1$

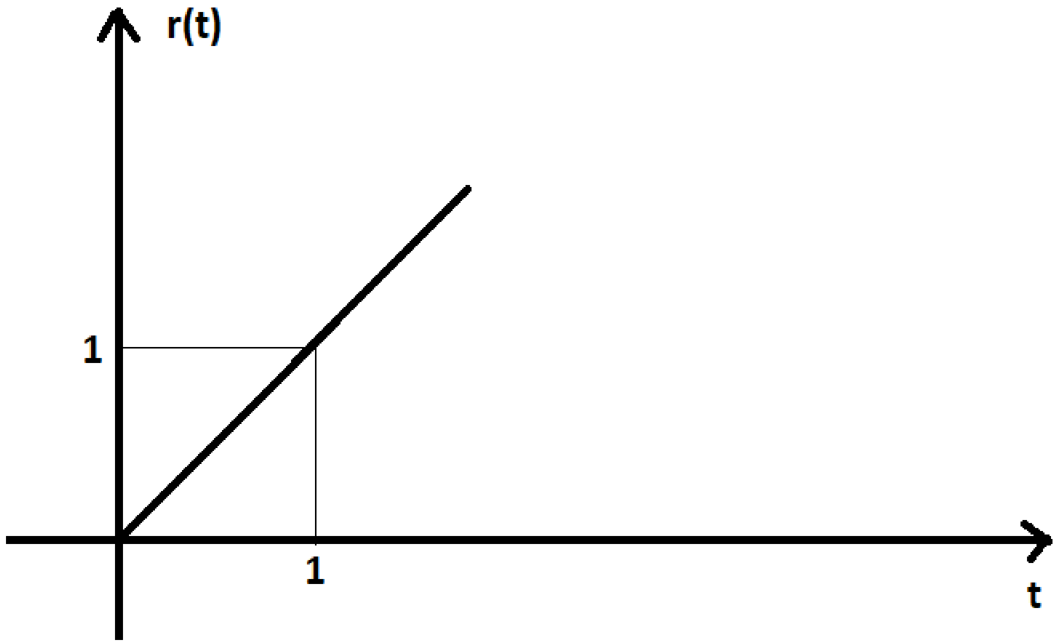
4) $\delta(at) = \frac{1}{|a|}\delta(t) \qquad \delta(a(t - t_0)) = \frac{1}{|a|}\delta(t - t_0)$

5) $\int_{-\infty}^{\infty} (t - t_0)f(t)dt = f(t_0)$

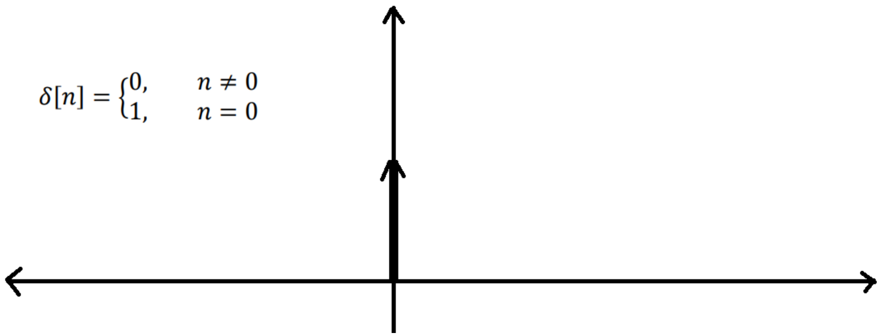
$$\int_{-\infty}^{\infty} (t - t_0)f(t - t_1)dt = f(t_0 - t_1)$$

Ramp Fonksiyonu

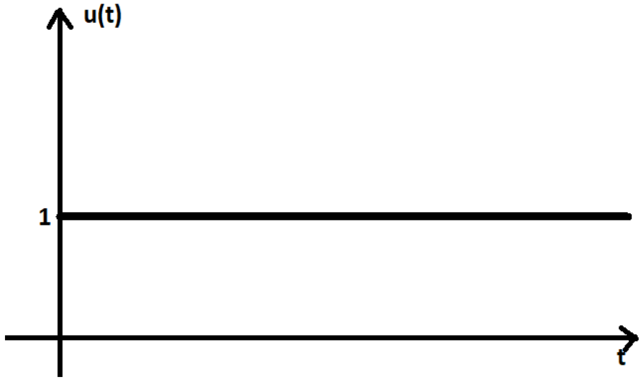
- $r(t) = \begin{cases} t & t \geq 0 \\ 0 & Diğer \end{cases}$



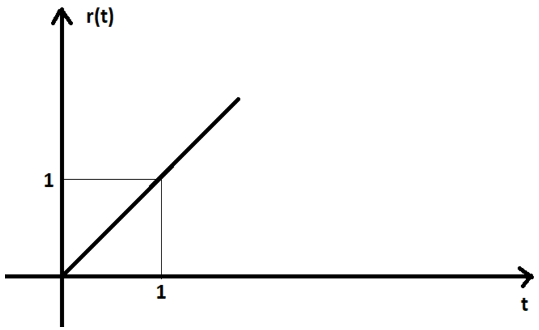
- $\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & Diğer \end{cases}$



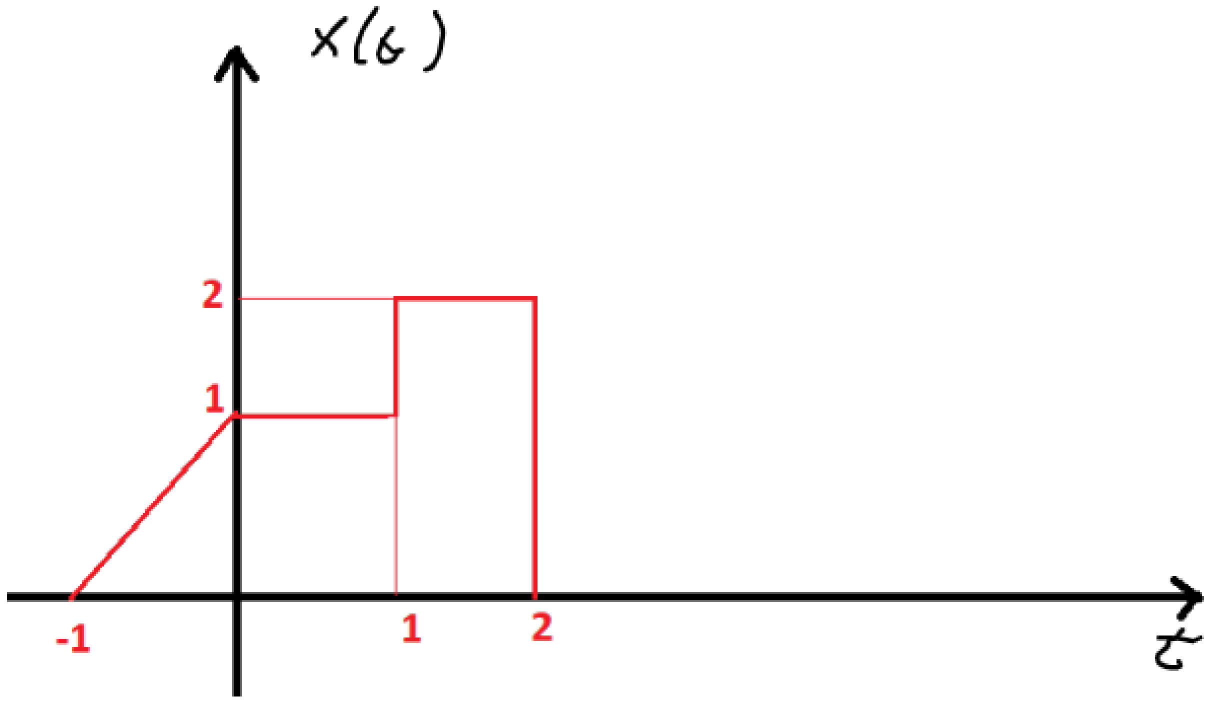
- $u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$



- $r(t) = \begin{cases} t & t \geq 0 \\ 0 & Diğer \end{cases}$

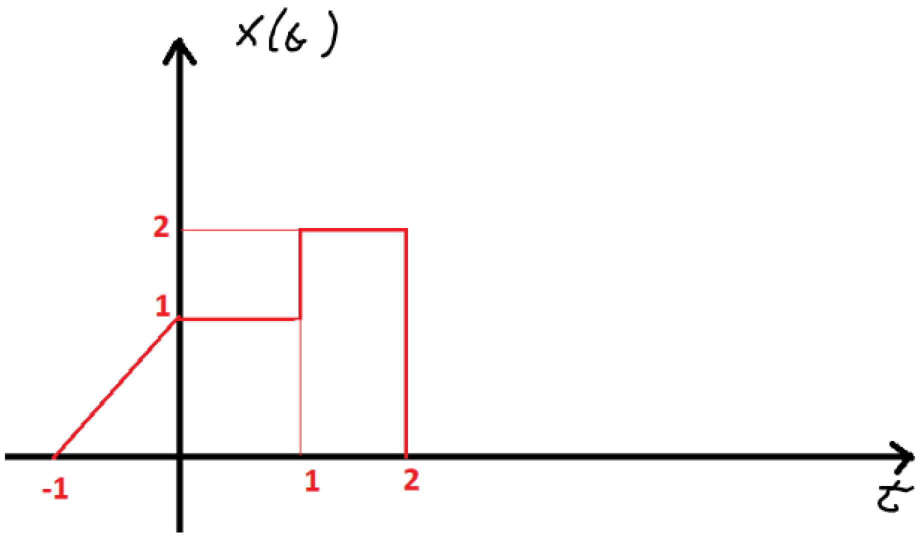


Örnek



$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$

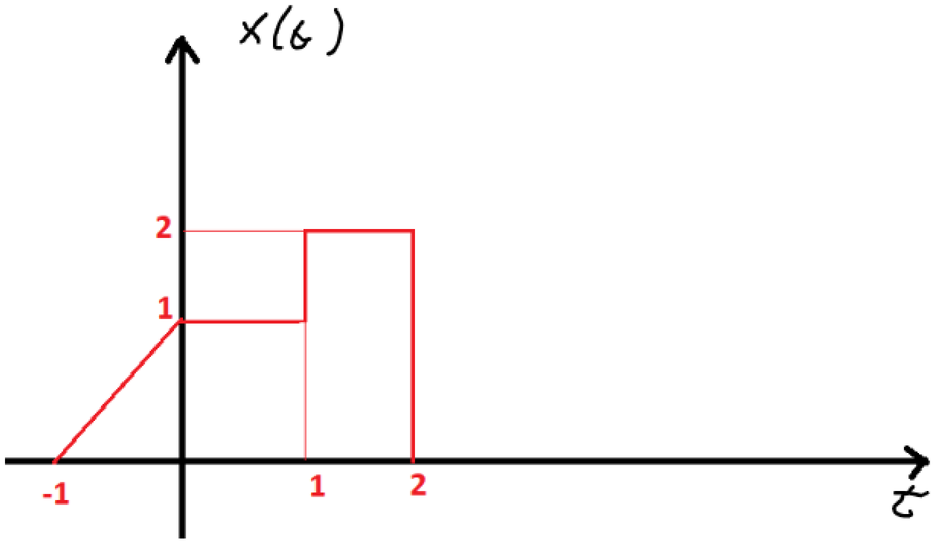
$$x(t)\,u(1-t) = ?$$



$$u(t) = \begin{cases} 0 & t<0 \\ 1 & t \geq 0 \end{cases}$$

$$x(t)\,u(1-t) = ?$$

$$u(-t) = \begin{cases} 0 & t>0 \\ 1 & t \leq 0 \end{cases} \Rightarrow u(1-t) = \begin{cases} 0 & t>1 \\ 1 & t \leq 1 \end{cases}$$

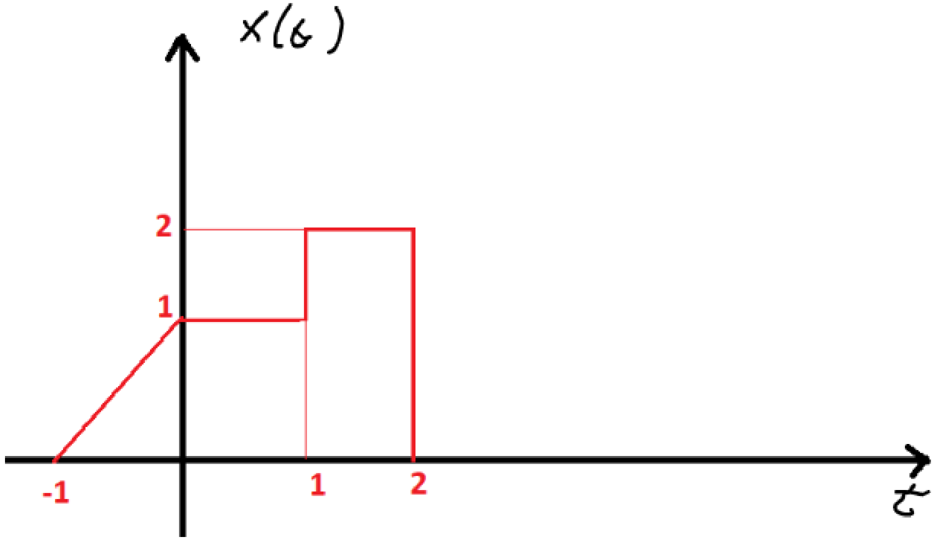


$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$

$$x(t)\,u(1-t) = ?$$

$$u(-t) = \begin{cases} 0 & t > 0 \\ 1 & t \leq 0 \end{cases} \Rightarrow u(1-t) = \begin{cases} 0 & t > 1 \\ 1 & t \leq 1 \end{cases}$$

$$x(t) \cdot u(1-t) = \begin{cases} 0 & t > 1 \\ x(t) & t \leq 1 \end{cases}$$

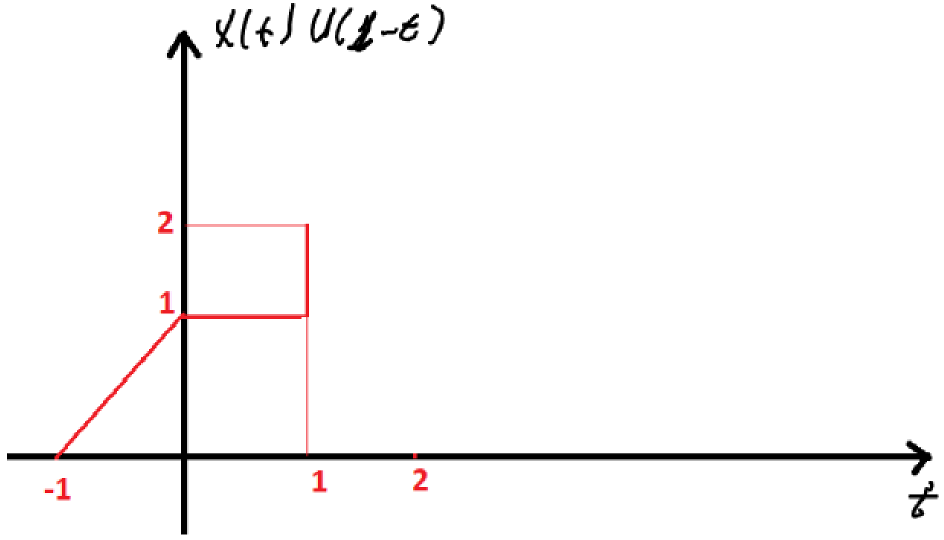


$$u(t) = \begin{cases} 0 & t<0 \\ 1 & t\geq 0 \end{cases}$$

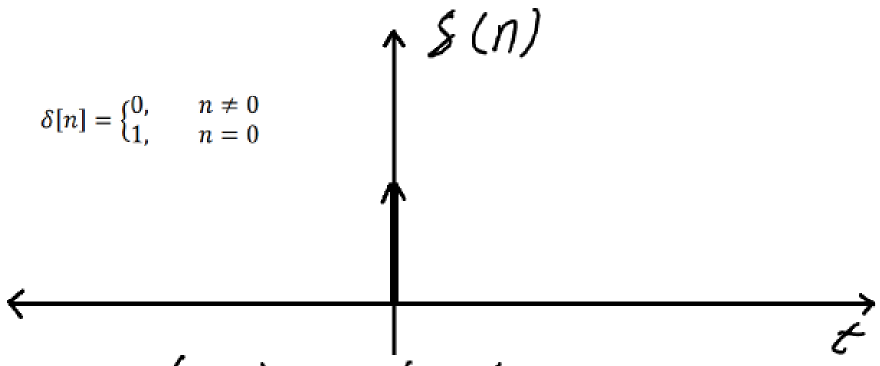
$$x(t)\,u(1-t) = ?$$

$$u(-t) = \begin{cases} 0 & t>0 \\ 1 & t\leq 0 \end{cases} \Rightarrow u(1-t) = \begin{cases} 0 & t>1 \\ 1 & t\leq 1 \end{cases}$$

$$x(t)\cdot u(1-t) = \begin{cases} 0 & t>1 \\ x(t) & t\leq 1 \end{cases}$$



$$\delta[n] = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases}$$



$\delta \rightarrow$ birim dürtü

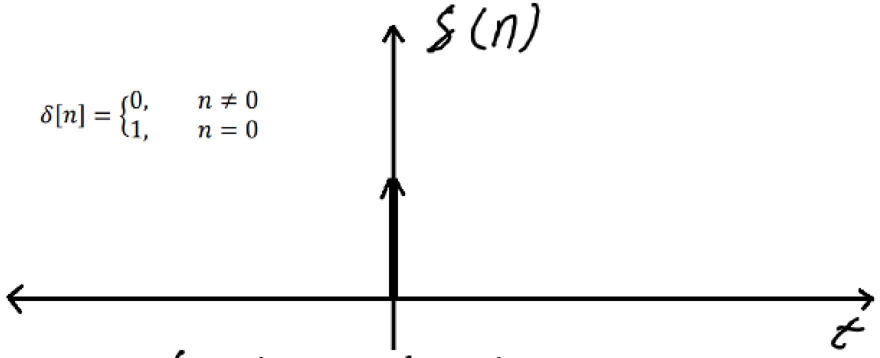
$f(t) = 2t^2 + 1$ fonksiyonu için

a) $f(t)\,\delta(t-1) = ?$

b) $\int_{-\infty}^{\infty} f(t)\,\delta(t)\,dt = ?$

c) $\int_{-\infty}^{\infty} f(t)\,\delta(t-2)\,dt = ?$

$\delta[n] = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases}$



$\delta \rightarrow$ birim dürtü

$f(t) = 2t^2 + 1$ fonksiyonu için

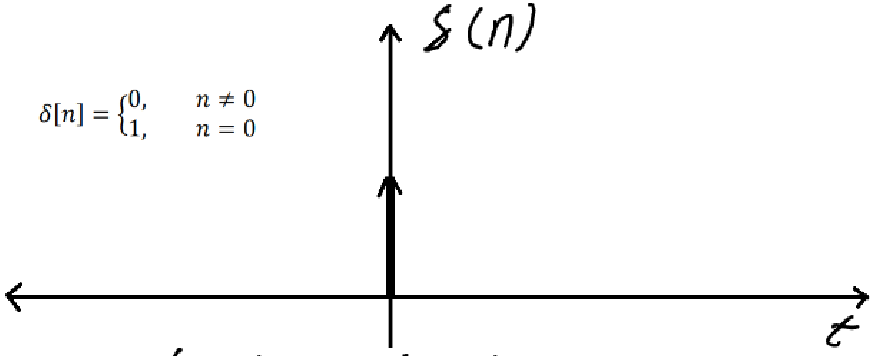
a) $f(t)\,\delta(t-1) = ?$

b) $\int_{-\infty}^{\infty} f(t)\,\delta(t)\,dt = ?$

c) $\int_{-\infty}^{\infty} f(t)\,\delta(t-2)\,dt = ?$

a) $f(t)\,\delta(\underbrace{t-1}_{0 \rightarrow\, t=1}) \Rightarrow (2 \cdot 1^2 + 1)\,\delta(t-1) = 5\,\delta(t-1)$

$\delta[n] = \begin{cases}0, & n \neq 0\\ 1, & n = 0\end{cases}$



δ → birim dürtü

$f(t) = 2t^2 + 1$ fonksiyonu için

a) $f(t)\,\delta(t-1) = ?$

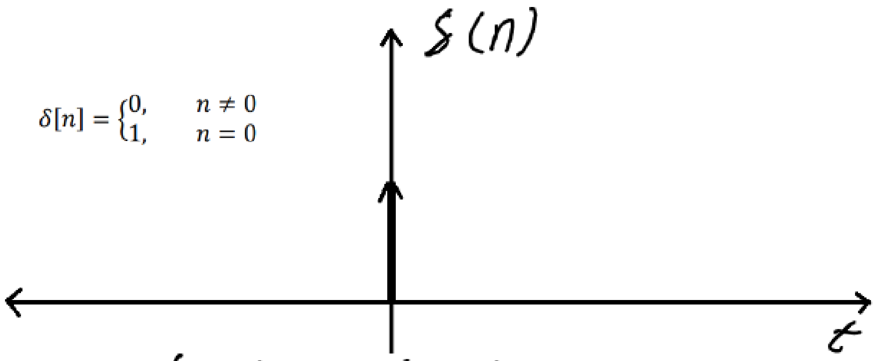
b) $\int_{-\infty}^{\infty} f(t)\,\delta(t)\,dt = ?$

c) $\int_{-\infty}^{\infty} f(t)\,\delta(t-2)\,dt = ?$

a) $f(t)\,\delta\underbrace{(t-1)}_{0 \to t=1} \Rightarrow (2\cdot 1^2+1)\,\delta(t-1) = 5\,\delta(t-1)$

b) $\int_{-\infty}^{\infty} f(t)\cdot\delta\underbrace{(t)}_{0 \to t=0}\,dt \Rightarrow f(0) = 2\cdot 0^2 + 1 = 1$

$$\delta[n] = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases}$$



$\delta \rightarrow$ birim dürtü

$f(t) = 2t^2 + 1$ fonksiyonu için

a) $f(t)\,\delta(t-1) = ?$

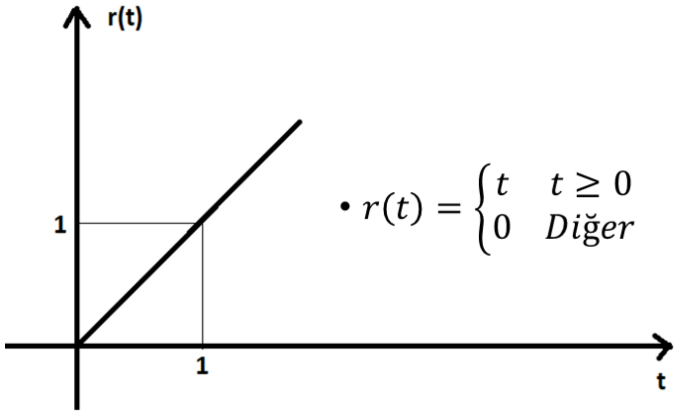
b) $\int_{-\infty}^{\infty} f(t)\,\delta(t)\,dt = ?$

c) $\int_{-\infty}^{\infty} f(t)\,\delta(t-2)\,dt = ?$

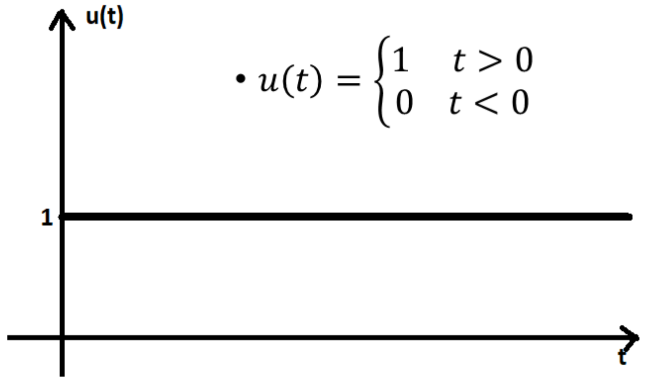
a) $f(t)\,\delta\underbrace{(t-1)}_{0 \rightarrow\ t=1} \Rightarrow (2\cdot 1^2+1)\,\delta(t-1) = 5\,\delta(t-1)$

b) $\int_{-\infty}^{\infty} f(t)\cdot\delta\underbrace{(t)}_{0 \rightarrow t=0}\,dt \Rightarrow f(0) = 2\cdot 0^2 + 1 = 1$

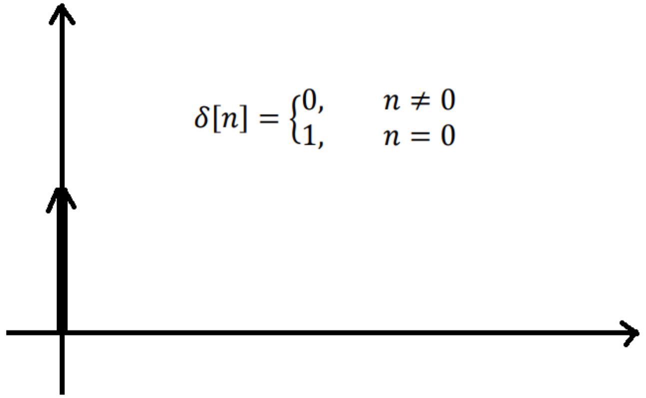
c) $\int_{-\infty}^{\infty} f(t)\,\delta\underbrace{(t-2)}_{0\quad t=2}\,dt \Rightarrow f(2) \rightarrow 2\cdot 2^2+1 = 9$



$$\bullet\ r(t) = \begin{cases} t & t \geq 0 \\ 0 & Diğer \end{cases}$$

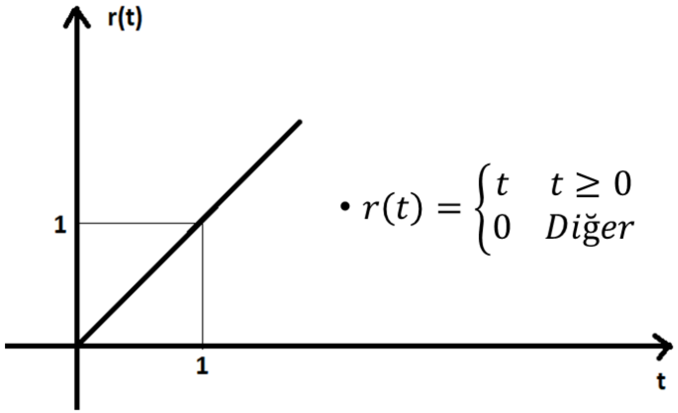


$$\bullet\ u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$

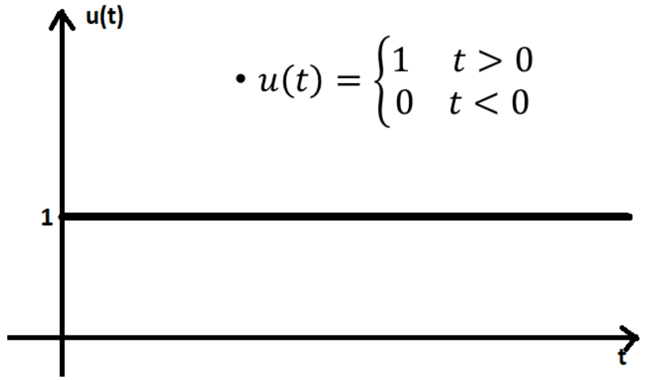


$$\delta[n] = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases}$$

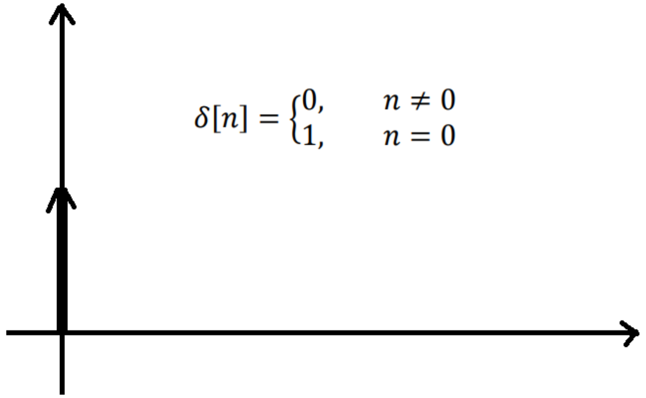
- f(t)=δ(t+2)+ δ(t-3)
- f(t)=-δ(t+2)+ δ(t)
- f(t)=u(t-3)
- f(t)=δ(t+1)+ u(t-1)
- f(t)=r(t-3)
- f(t)=δ(t-3)+ r(t-1)



- $r(t) = \begin{cases} t & t \geq 0 \\ 0 & Diğer \end{cases}$

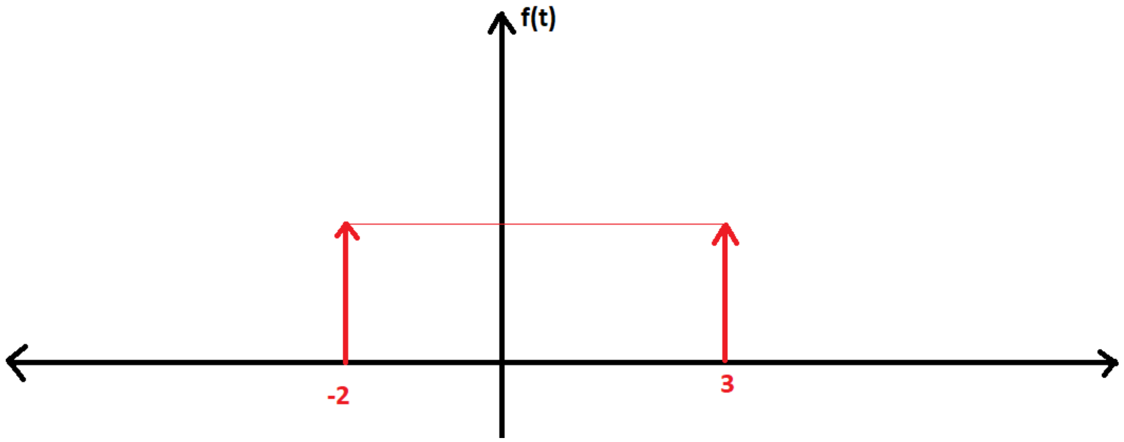


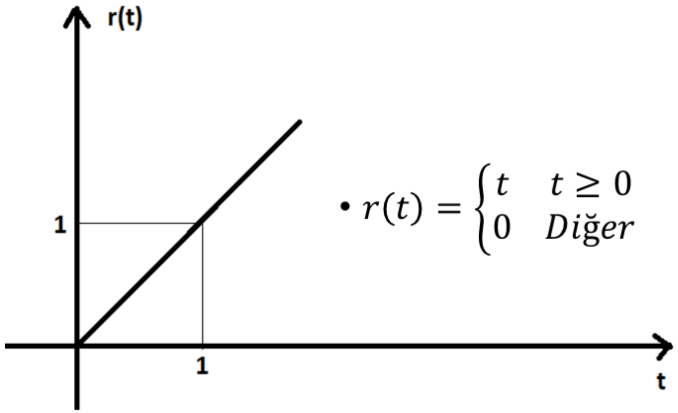
- $u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$



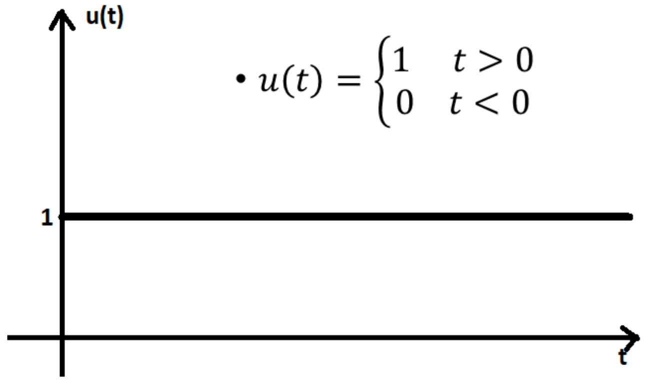
$$\delta[n] = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases}$$

- f(t)=δ(t+2)+ δ(t-3)
- f(t)=-δ(t+2)+ δ(t)
- f(t)=u(t-3)
- f(t)=δ(t+1)+ u(t-1)
- f(t)=r(t-3)
- f(t)=δ(t-3)+ r(t-1)

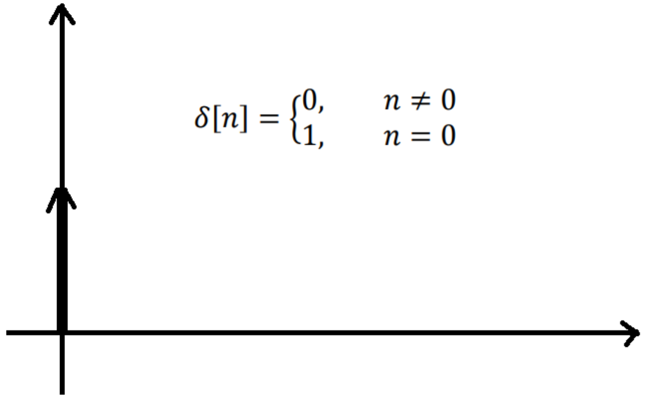




- $r(t) = \begin{cases} t & t \geq 0 \\ 0 & Diğer \end{cases}$

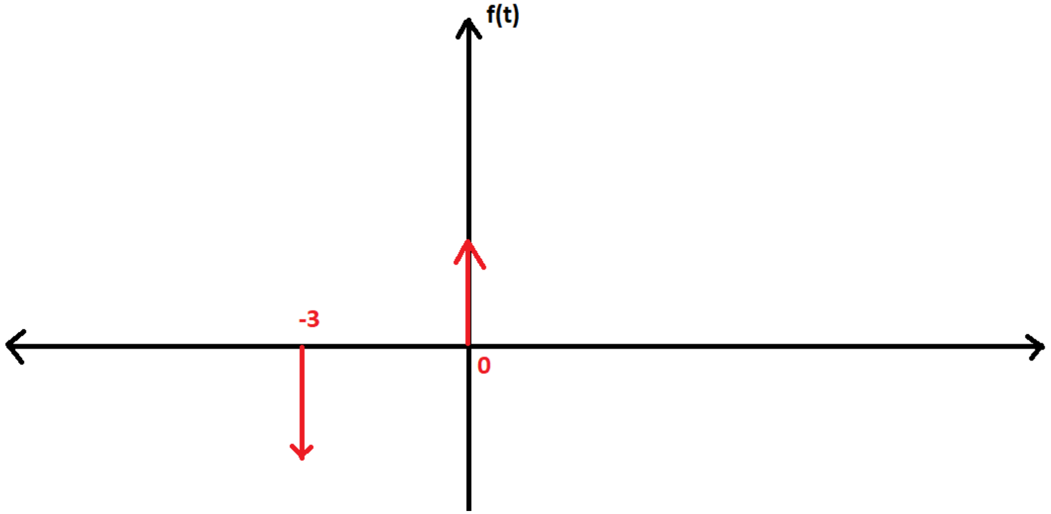


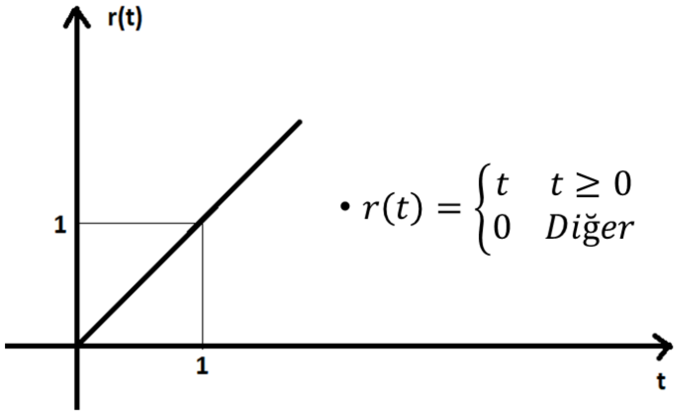
- $u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$



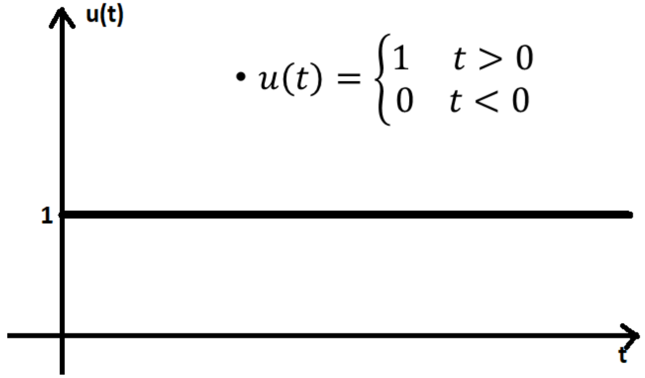
$$\delta[n] = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases}$$

- $f(t)=\delta(t+2)+ \delta(t-3)$
- $f(t)=-\delta(t+2)+ \delta(t)$
- $f(t)=u(t-3)$
- $f(t)=\delta(t+1)+ u(t-1)$
- $f(t)=r(t-3)$
- $f(t)=\delta(t-3)+ r(t-1)$

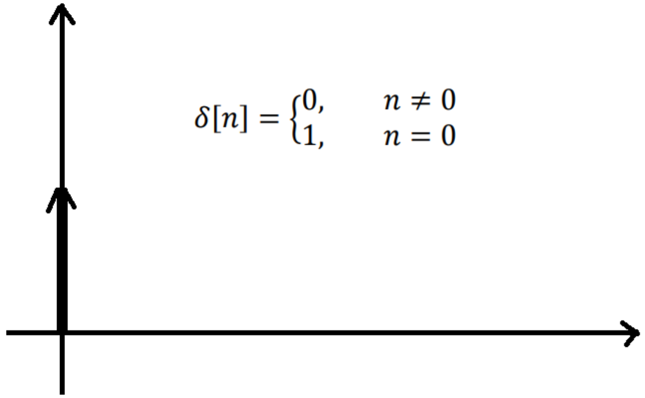




- $r(t) = \begin{cases} t & t \geq 0 \\ 0 & Diğer \end{cases}$

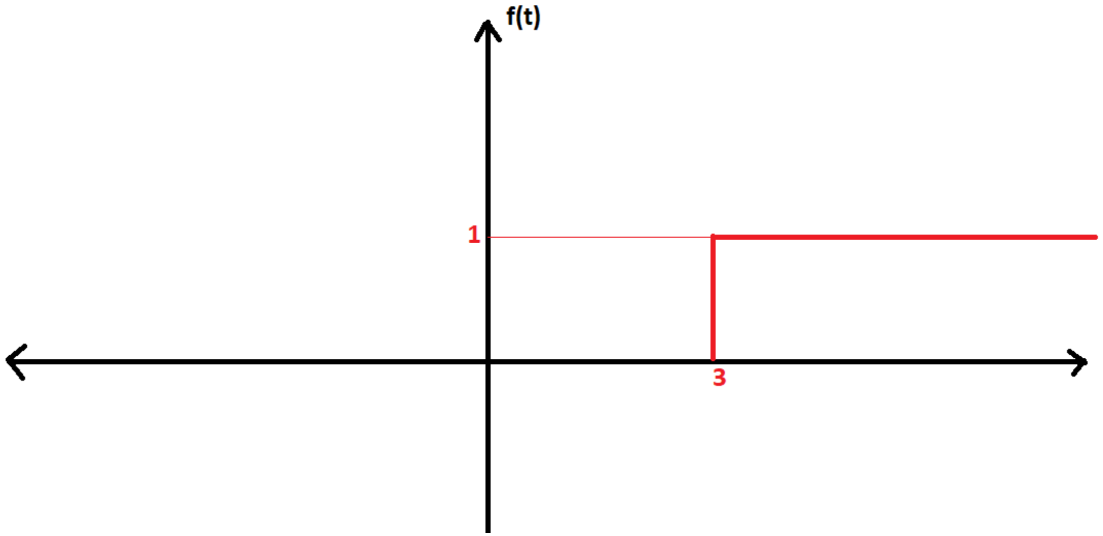


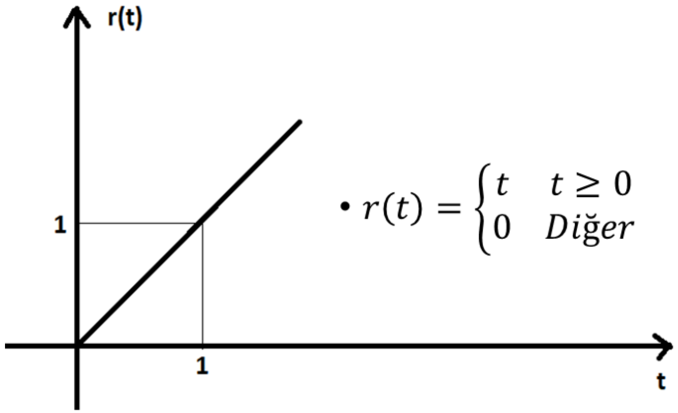
- $u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$



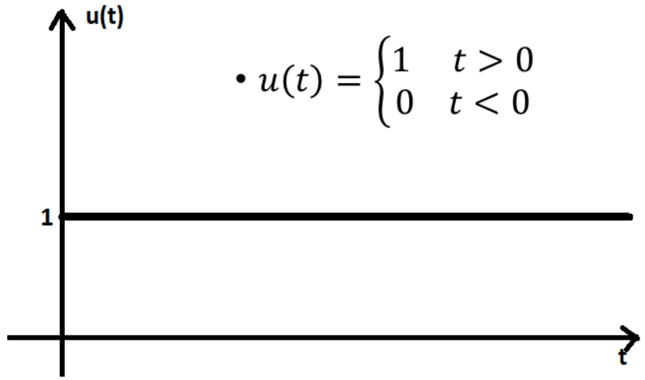
$$\delta[n] = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases}$$

- f(t)=δ(t+2)+ δ(t-3)
- f(t)=-δ(t+2)+ δ(t)
- f(t)=u(t-3)
- f(t)=δ(t+1)+ u(t-1)
- f(t)=r(t-3)
- f(t)=δ(t-3)+ r(t-1)

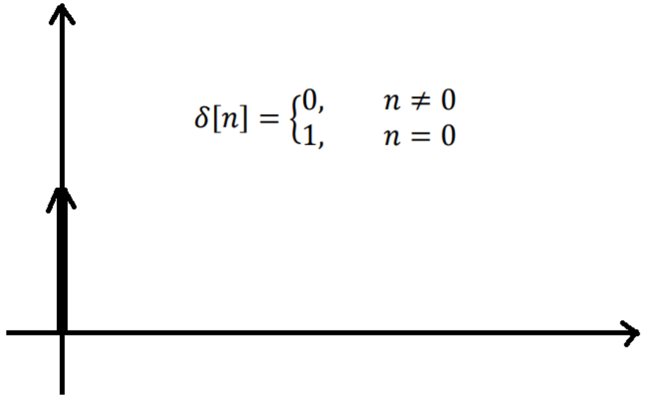




- $r(t) = \begin{cases} t & t \geq 0 \\ 0 & Diğer \end{cases}$

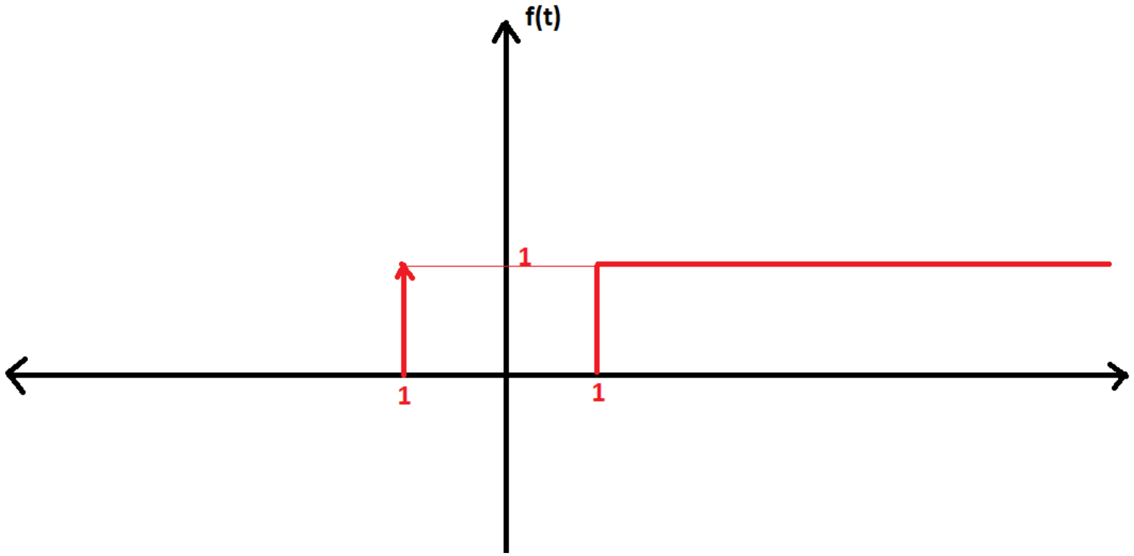


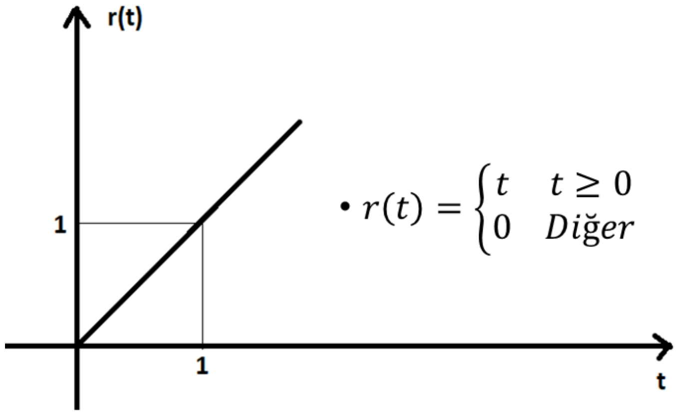
- $u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$



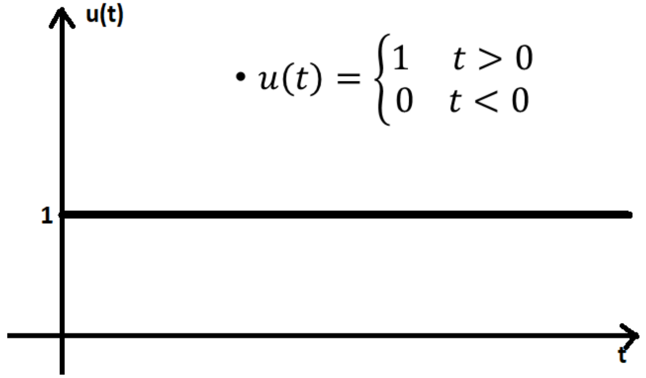
$\delta[n] = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases}$

- f(t)=δ(t+2)+ δ(t-3)
- f(t)=-δ(t+2)+ δ(t)
- f(t)=u(t-3)
- f(t)=δ(t+1)+ u(t-1)
- f(t)=r(t-3)
- f(t)=δ(t-3)+ r(t-1)

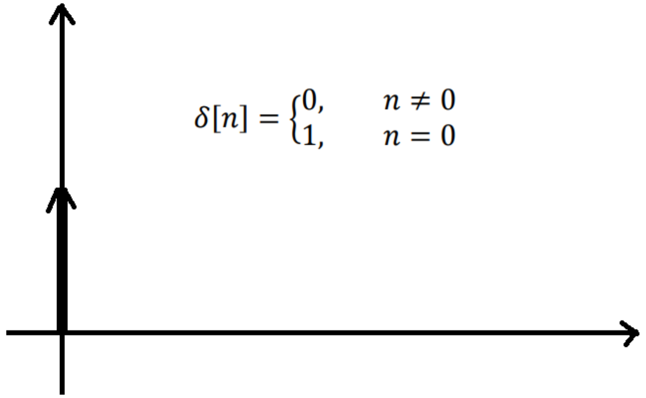




- $r(t) = \begin{cases} t & t \geq 0 \\ 0 & Diğer \end{cases}$

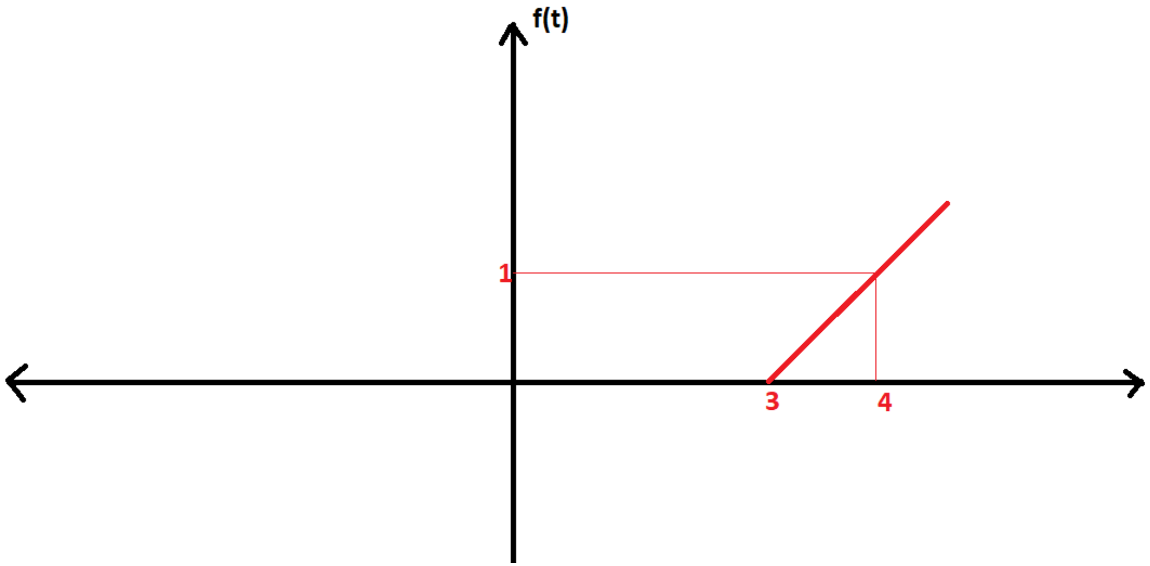


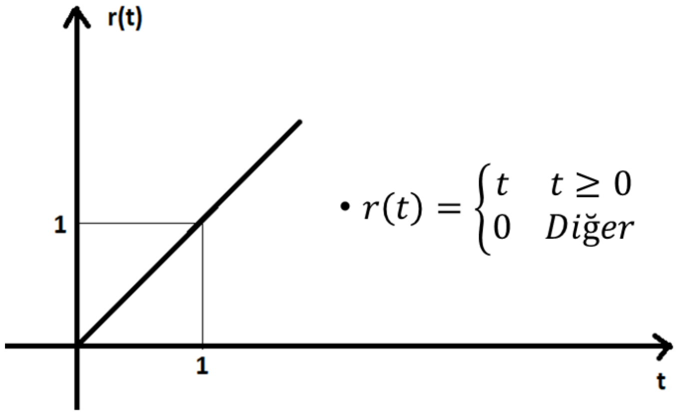
- $u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$



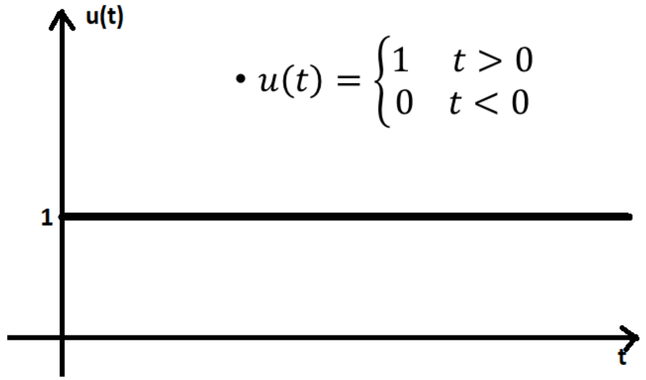
$$\delta[n] = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases}$$

- f(t)=δ(t+2)+ δ(t-3)
- f(t)=-δ(t+2)+ δ(t)
- f(t)=u(t-3)
- f(t)=δ(t+1)+ u(t-1)
- f(t)=r(t-3)
- f(t)=δ(t-3)+ r(t-1)

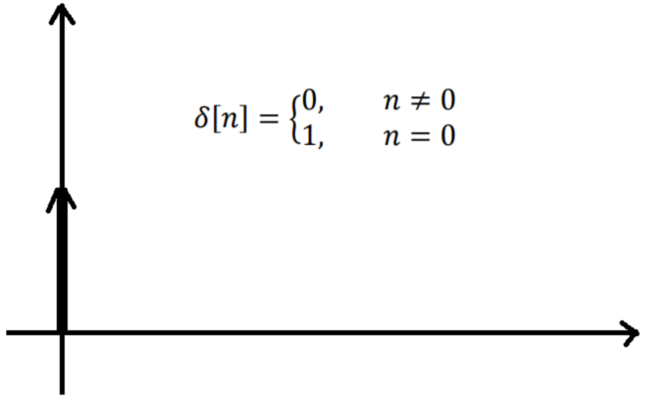




- $r(t) = \begin{cases} t & t \geq 0 \\ 0 & Diğer \end{cases}$

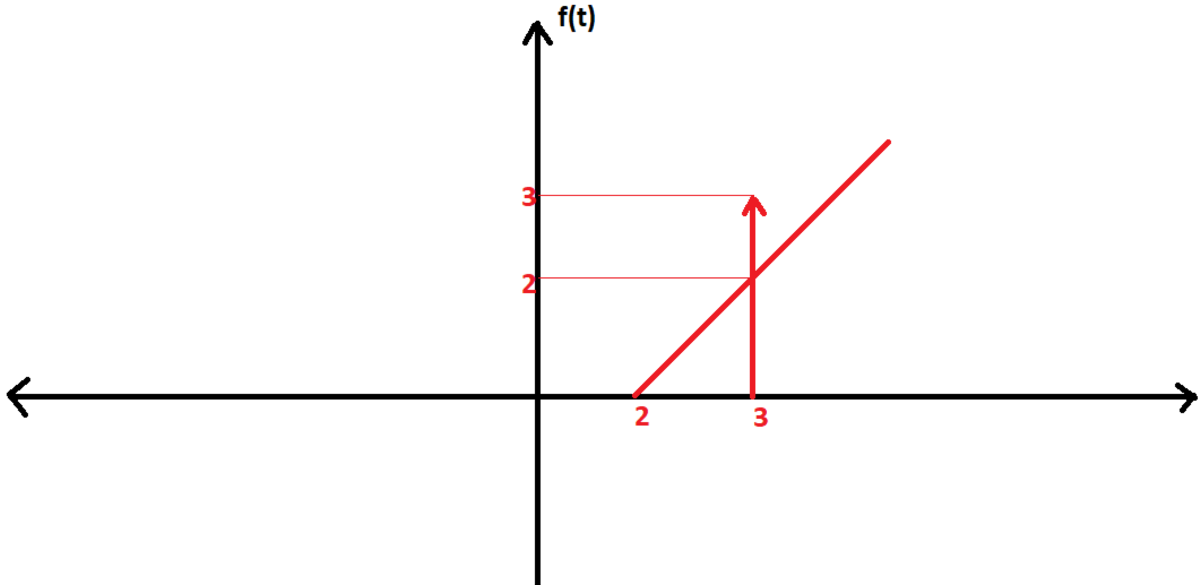


- $u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$

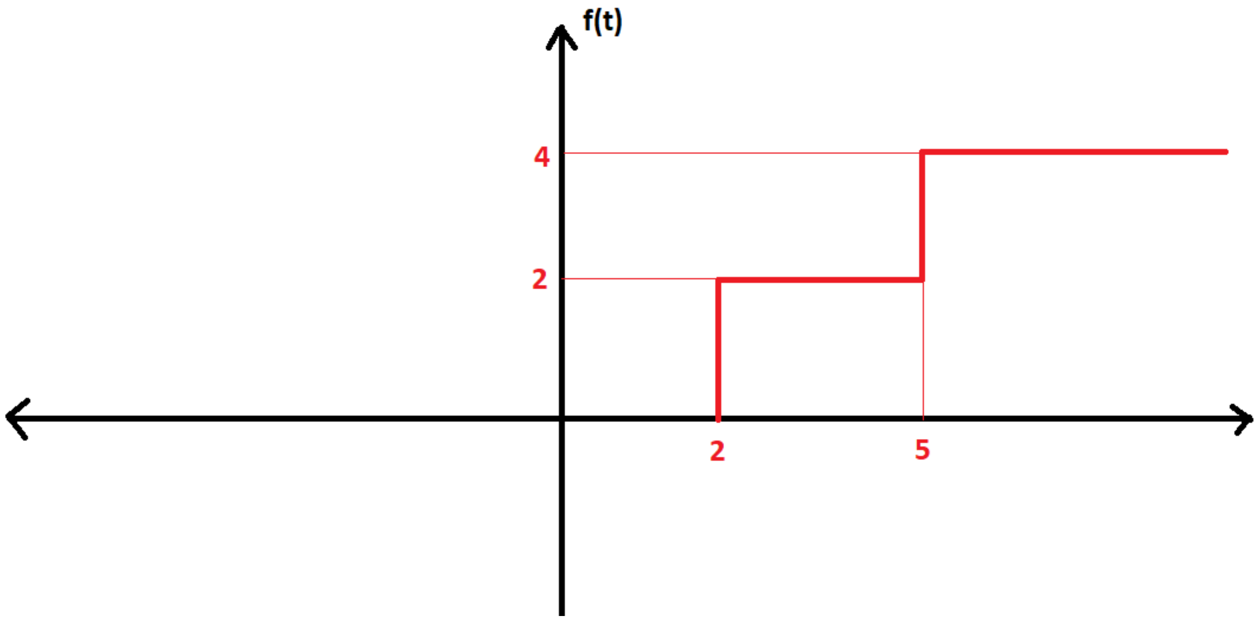


$\delta[n] = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases}$

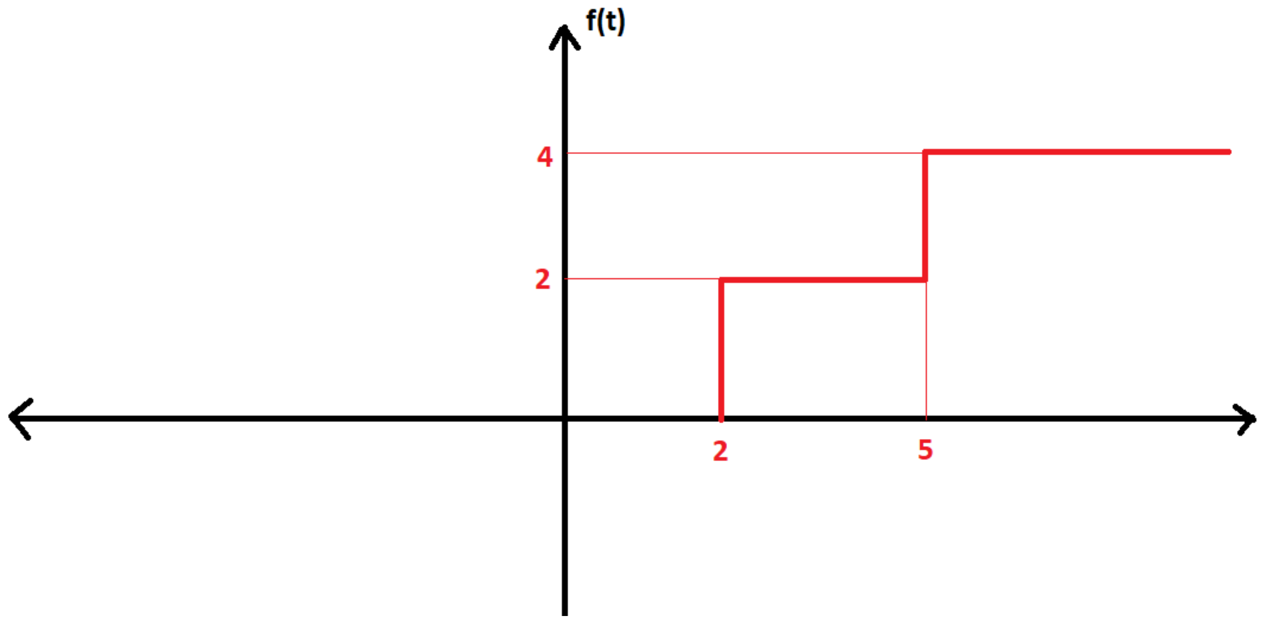
- f(t)=δ(t+2)+ δ(t-3)
- f(t)=-δ(t+2)+ δ(t)
- f(t)=u(t-3)
- f(t)=δ(t+1)+ u(t-1)
- f(t)=r(t-3)
- f(t)=δ(t-3)+ r(t-1)



Aşağıda grafiği verilen fonksiyonu birim adım fonksiyon cinsinden yazınız.

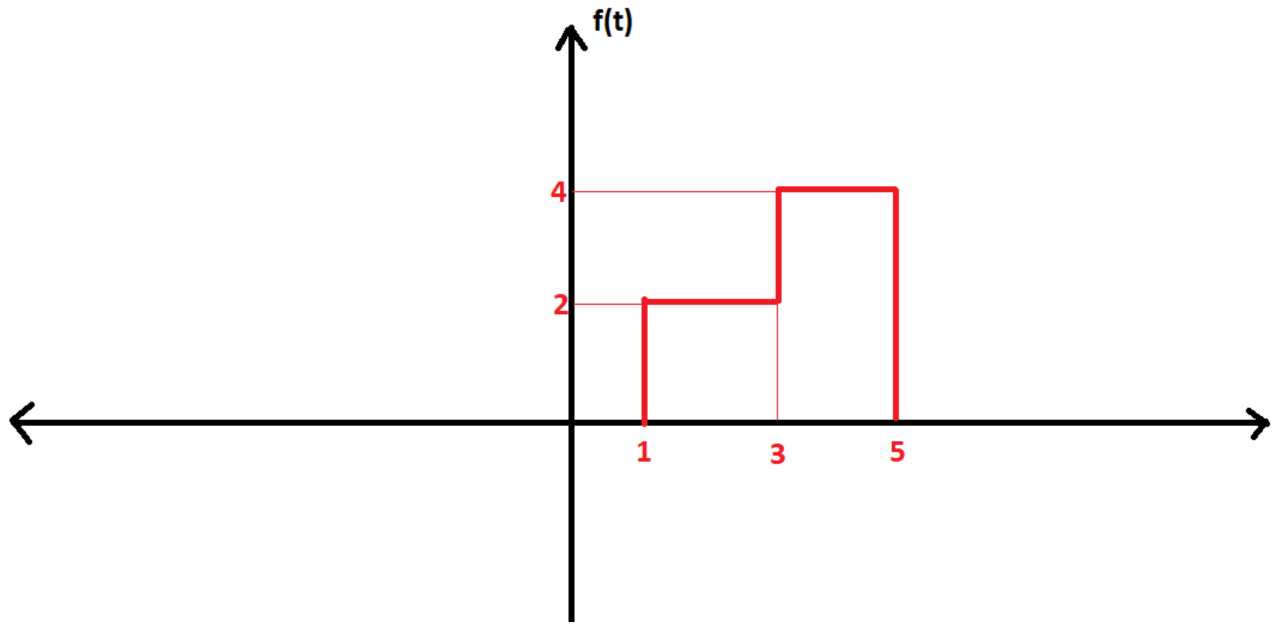


Aşağıda grafiği verilen fonksiyonu birim adım fonksiyon cinsinden yazınız.

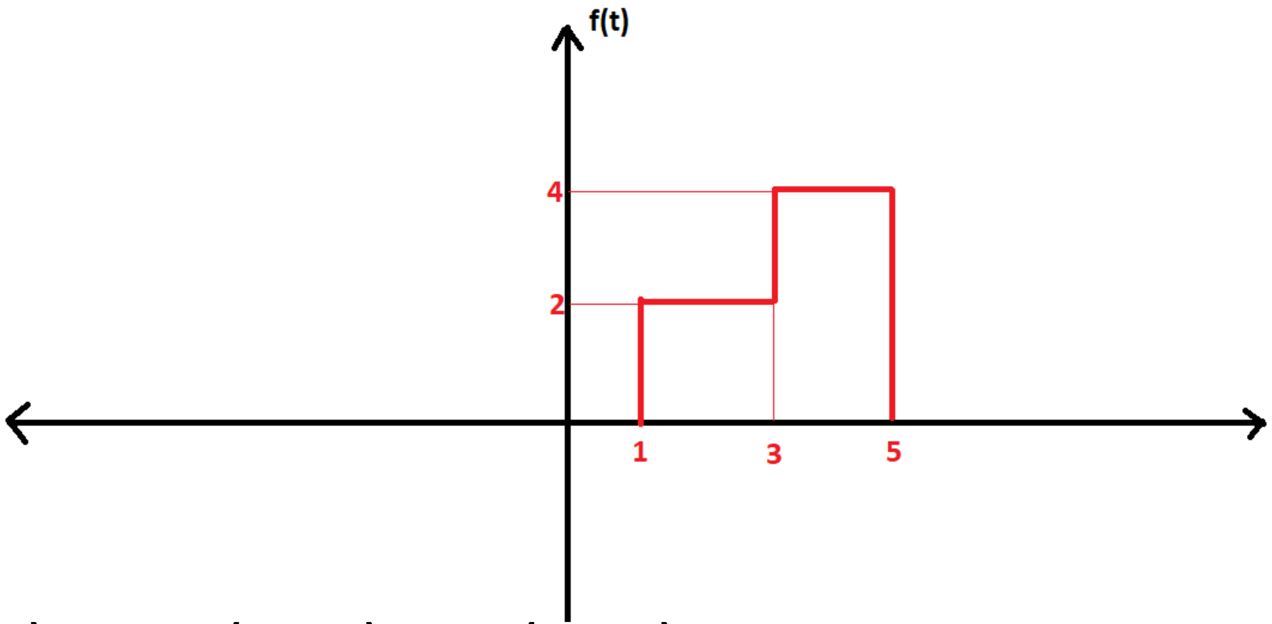


- $f(t)=2u(t-2)+2u(t-5)$

Aşağıda grafiği verilen fonksiyonu birim adım fonksiyon cinsinden yazınız.



Aşağıda grafiği verilen fonksiyonu birim adım fonksiyon cinsinden yazınız.



- $f(t)=2u(t-1)+2u(t-3)-4u(t-5)$