例 6.4

$E(X_i)=\mu$, $V(X_i)=\sigma^2=E(X_i^2)=\mu^2$

$\rightarrow E(\bar{X})=\mu$, $V(\bar{X})=\frac{\sigma^2}{n}=E(\bar{X}^2)-\mu^2$

$$E(\hat{\theta}_1)=E\left(\frac{\sum_{i=1}^{n}(X_i-\bar{X})^2}{n}\right)=\frac{1}{n}E\left(\sum_{i=1}^{n}X_i^2-n\bar{X}^2\right)$$

$$=\frac{1}{n}(n\sigma^2+n\mu^2-\sigma^2-n\mu^2)=\frac{n-1}{n}\sigma^2$$

$$E(\hat{\theta}_2)=E\left(\frac{\sum_{i=1}^{n}(X_i-\bar{X})^2}{n-1}\right)=\frac{1}{n-1}E\left(\sum_{i=1}^{n}X_i^2-n\bar{X}^2\right)$$

$$=\frac{1}{n-1}(n\sigma^2+n\mu^2-\sigma^2-n\mu^2)=\sigma^2$$

$\Rightarrow \hat{\theta}_2=\sum_{i=1}^{n}(X_i-\bar{X})^2/(n-1)$ 為母体变異数 θ^2 之不偏估計量，而 $\hat{\theta}_1=\sum_{i=1}^{n}(X_i-\bar{X}^2)/n$ 為母体变異数 θ^2 之偏误估計量。