PUMA560 kinematics And Dynamics Analysis

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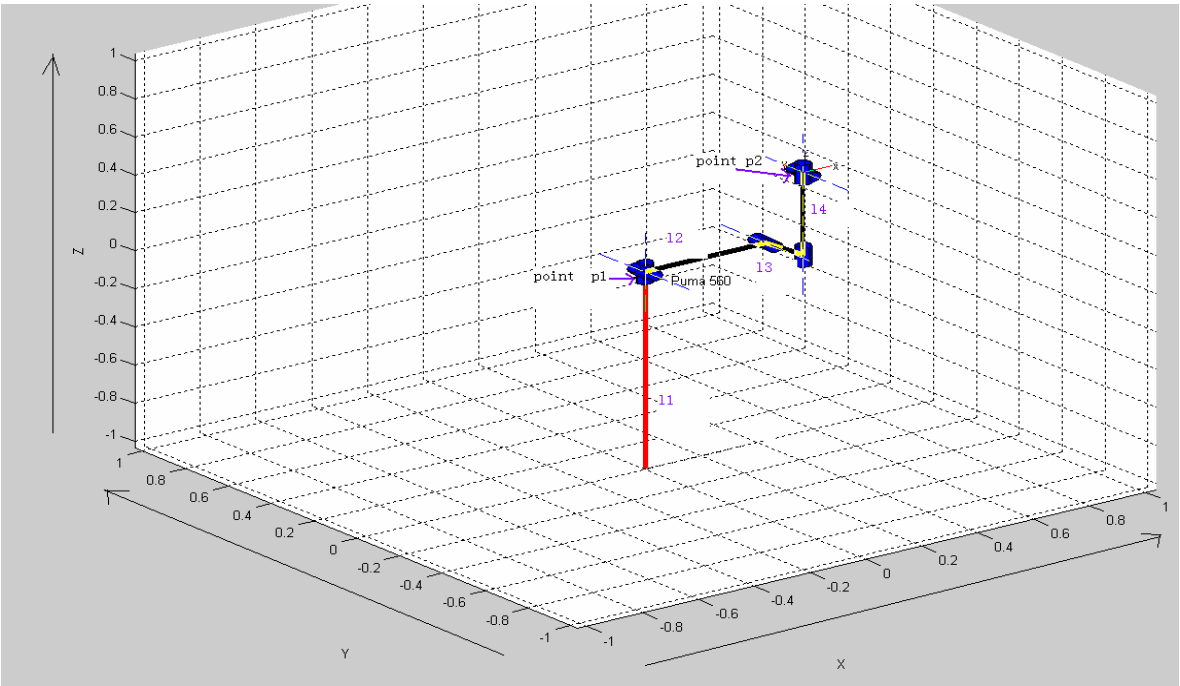
1 PUMA560 forward kinematics



Source code:

```
Syms l1 l2 l3 l4 s1 s2 s3 s4 s5 s6
q=[0 0 l1],w=[0;0;1]; joint1= q_w_rot_joint(q,w,s1)
q=[0 0 l1],w=[0;1;0]; joint2= q_w_rot_joint(q,w,s2)
q=[l2 0 l1], w=[0;1;0]; joint3= q_w_rot_joint(q,w,s3)
q=[l2 -l3 l1], w=[0;0;1]; joint4= q_w_rot_joint(q,w,s4)
q=[l2 -l3 l1+l4], w=[0;1;0]; joint5= q_w_rot_joint(q,w,s5)
q=[l2 -l3 l1+l4], w=[0;0;1]; joint6= q_w_rot_joint(q,w,s6)
gst0=rot_tran_expmap4(eye(3), [l2;-l3;l1+l4])      %%%consider point p2 as the end
```

Rst=joint1*joint2*joint3*joint4* joint5*joint6*gst0

Rst=simple(Rst)

Result:

joint1 =

[cos(s1), -sin(s1), 0, 0]

[sin(s1), cos(s1), 0, 0]

[0, 0, 1, 0]

[0, 0, 0, 1]

joint2 =

[cos(s2), 0, sin(s2), -sin(s2)*l1]

[0, 1, 0, 0]

[-sin(s2), 0, cos(s2), l1-l1*cos(s2)]

[0, 0, 0, 1]

joint3 =

[cos(s3), 0, sin(s3), l2-l2*cos(s3)-sin(s3)*l1]

[0, 1, 0, 0]

[-sin(s3), 0, cos(s3), sin(s3)*l2+l1-l1*cos(s3)]

[0, 0, 0, 1]

joint4 =

[cos(s4), -sin(s4), 0, l2-l2*cos(s4)-sin(s4)*l3]

[sin(s4), cos(s4), 0, -sin(s4)*l2-l3+l3*cos(s4)]

[0, 0, 1, 0]

[0, 0, 0, 1]

joint5 =

[cos(s5), 0, sin(s5), (1-cos(s5))*l2-sin(s5)*(l1+l4)]

[0, 1, 0, 0]

[-sin(s5), 0, cos(s5), sin(s5)*l2+(1-cos(s5))*(l1+l4)]

[0, 0, 0, 1]

joint6 =

[cos(s6), -sin(s6), 0, l2-l2*cos(s6)-sin(s6)*l3]

[sin(s6), cos(s6), 0, -sin(s6)*l2-l3+l3*cos(s6)]

[0, 0, 1, 0]

[0, 0, 0, 1]

gst0 =

[1, 0, 0, l2]

[0, 1, 0, -l3]

[0, 0, 1, l1+l4]

[0, 0, 0, 1]

Rst =

[cos(s6)*cos(s5)*cos(s4)*cos(s1)*cos(s2)*cos(s3)-cos(s6)*cos(s5)*cos(s4)*cos(s1)*sin(s2)*sin(s3)-cos(s6)*cos(s5)*sin(s1)*sin(s4)-cos(s6)*sin(s5)*cos(s1)*cos(s2)*sin(s3)-cos(s6)*sin(s5)*cos(s1)*sin(s2)*cos(s3)-sin(s6)*sin(s4)*cos(s1)*cos(s2)*cos(s3)+sin(s6)*sin(s4)*cos(s1)*sin(s2)*sin(s3)-sin(s6)*sin(s1)*cos(s4),
-sin(s6)*cos(s5)*cos(s4)*cos(s1)*cos(s2)*cos(s3)+sin(s6)*cos(s5)*cos(s4)*cos(s1)*sin(s2)*sin(s3)+sin(s6)*cos(s5)*sin(s1)*sin(s4)+sin(s6)*sin(s5)*cos(s1)*cos(s2)*sin(s3)+sin(s6)*sin(s5)*cos(s1)*sin(s2)*cos(s3)-cos(s6)*sin(s4)*cos(s1)*cos(s2)*cos(s3)+cos(s6)*sin(s4)*cos(s1)*sin(s2)*sin(s3)-cos(s6)*sin(s1)*cos(s4),
sin(s5)*cos(s4)*cos(s1)*cos(s2)*cos(s3)-sin(s5)*cos(s4)*cos(s1)*sin(s2)*sin(s3)-sin(s5)*sin(s1)*sin(s4)+cos(s5)*cos(s1)*cos(s2)*sin(s3)+cos(s5)*cos(s1)*sin(s2)*cos(s3),
cos(s1)*cos(s2)*sin(s3)*l4+cos(s1)*sin(s2)*cos(s3)*l4+sin(s1)*l3+cos(s1)*cos(s2)*l2]

[cos(s6)*cos(s5)*cos(s4)*sin(s1)*cos(s2)*cos(s3)-cos(s6)*cos(s5)*cos(s4)*sin(s1)*sin(s2)*sin(s3)+cos(s6)*cos(s5)*cos(s1)*sin(s4)-cos(s6)*sin(s5)*sin(s1)*cos(s2)*sin(s3)-cos(s6)*sin(s5)*sin(s1)*sin(s2)*cos(s3)-sin(s6)*sin(s4)*sin(s1)*cos(s2)*cos(s3)+sin(s6)*sin(s4)*sin(s1)*sin(s2)*sin(s3)+sin(s6)*cos(s1)*cos(s4),
-sin(s6)*cos(s5)*cos(s4)*sin(s1)*cos(s2)*cos(s3)+sin(s6)*cos(s5)*cos(s4)*sin(s1)*sin(s2)*sin(s3)-sin(s6)*cos(s5)*cos(s1)*sin(s4)+sin(s6)*sin(s5)*sin(s1)*cos(s2)*sin(s3)+sin(s6)*sin(s5)*sin(s1)*sin(s2)*cos(s3)-cos(s6)*sin(s4)*sin(s1)*cos(s2)*cos(s3)+cos(s6)*sin(s4)*sin(s1)*sin(s2)*sin(s3)+cos(s6)*cos(s1)*cos(s4),
sin(s5)*cos(s4)*sin(s1)*cos(s2)*cos(s3)-sin(s5)*cos(s4)*sin(s1)*sin(s2)*sin(s3)+sin(s5)*cos(s1)*sin(s4)+cos(s5)*sin(s1)*cos(s2)*sin(s3)+cos(s5)*sin(s1)*sin(s2)*cos(s3),
sin(s1)*cos(s2)*sin(s3)*l4+sin(s1)*sin(s2)*cos(s3)*l4-cos(s1)*l3+sin(s1)*cos(s2)*l2]

[

-cos(s6)*cos(s4)*cos(s5)*sin(s2)*cos(s3)-cos(s6)*cos(s4)*cos(s5)*cos(s2)*sin(s3)+cos(s6)*sin(s5)*sin(s2)*sin(s3)-cos(s6)*sin(s5)*cos(s2)*cos(s3)+sin(s4)*sin(s6)*sin(s2)*cos(s3)+sin(s4)*sin(s6)*cos(s2)*sin(s3),
sin(s6)*cos(s4)*cos(s5)*sin(s2)*cos(s3)+sin(s6)*cos(s4)*cos(s5)*cos(s2)*sin(s3)-sin(s6)*sin(s5)*sin(s2)*sin(s3)+sin(s6)*sin(s5)*cos(s2)*cos(s3)+sin(s4)*cos(s6)*sin(s2)*cos(s3)+sin(s4)*cos(s6)*cos(s2)*sin(s3),
-cos(s4)*sin(s5)*sin(s2)*cos(s3)-cos(s4)*sin(s5)*cos(s2)*sin(s3)-cos(s5)*sin(s2)*sin(s3)+cos(s5)*cos(s2)*cos(s3),
l1+cos(s2)*cos(s3)*l4-sin(s2)*sin(s3)*l4-sin(s2)*l2]

[0,0,0,1]

2 PUMA560 inverse kinematics

Suppose $e^{\hat{\xi}_1\theta_1}e^{\hat{\xi}_2\theta_2}e^{\hat{\xi}_3\theta_3}e^{\hat{\xi}_4\theta_4}e^{\hat{\xi}_5\theta_5}e^{\hat{\xi}_6\theta_6}gst(0) = g_d$

$$e^{\hat{\xi}_1\theta_1}e^{\hat{\xi}_2\theta_2}e^{\hat{\xi}_3\theta_3}e^{\hat{\xi}_4\theta_4}e^{\hat{\xi}_5\theta_5}e^{\hat{\xi}_6\theta_6} = g_d gst^{-1}(0) := g_1$$

$$\left\|g_1q_2 - q_1\right\| = \left\|e^{\hat{\xi}_1\theta_1}e^{\hat{\xi}_2\theta_2}e^{\hat{\xi}_3\theta_3}e^{\hat{\xi}_4\theta_4}e^{\hat{\xi}_5\theta_5}e^{\hat{\xi}_6\theta_6}q_2 - q_1\right\| = \left\|e^{\hat{\xi}_1\theta_1}e^{\hat{\xi}_2\theta_2}e^{\hat{\xi}_3\theta_3}q_2 - q_1\right\|$$

$$= \left\|e^{\hat{\xi}_1\theta_1}e^{\hat{\xi}_2\theta_2}e^{\hat{\xi}_3\theta_3}q_2 - e^{\hat{\xi}_1\theta_1}e^{\hat{\xi}_2\theta_2}q_1\right\| = \left\|e^{\hat{\xi}_3\theta_3}q_2 - q_1\right\|$$

Using sub problem 3, we can get θ_3 ,

$$g_1q_2 = e^{\hat{\xi}_1\theta_1}e^{\hat{\xi}_2\theta_2}(e^{\hat{\xi}_3\theta_3}q_2)$$

Using sub problem 2, we can get θ_1 and θ_2

$$e^{\hat{\xi}_4\theta_4}e^{\hat{\xi}_5\theta_5}e^{\hat{\xi}_6\theta_6} = (e^{\hat{\xi}_1\theta_1}e^{\hat{\xi}_2\theta_2}e^{\hat{\xi}_3\theta_3})^{-1}g_d gst^{-1}(0) := g_2$$

$$g_2q_2 = e^{\hat{\xi}_4\theta_4}e^{\hat{\xi}_5\theta_5}e^{\hat{\xi}_6\theta_6}q_2 = e^{\hat{\xi}_4\theta_4}q_2$$

Using sub problem 1, we can get θ_4

$$e^{\hat{\xi}_5\theta_5}e^{\hat{\xi}_6\theta_6} = (e^{\hat{\xi}_1\theta_1}e^{\hat{\xi}_2\theta_2}e^{\hat{\xi}_3\theta_3}e^{\hat{\xi}_4\theta_4})^{-1}g_d gst^{-1}(0) := g_3$$

$$g_3q_1 = e^{\hat{\xi}_5\theta_5}e^{\hat{\xi}_6\theta_6}q_1$$

Using sub problem 2 we can get θ_5 and θ_6

3 Jacobians in spatial frame

Source code:

```
Syms l1 l2 l3 l4 s1 s2 s3 s4 s5 s6
q=[0 0 l1],w=[0;0;1]; screw1=screw(w,q)
q=[0 0 l1],w=[0;1;0]; screw2=screw(w,q)
q=[l2 0 l1], w=[0;1;0]; screw3=screw(w,q)
q=[l2 -l3 l1], w=[0;0;1]; screw4=screw(w,q)
q=[l2 -l3 l1+l4], w=[0;1;0]; screw5=screw(w,q)
q=[l2 -l3 l1+l4], w=[0;0;1]; screw6=screw(w,q)
twist=[screw1 screw2 screw3 screw4 screw5 screw6], sita=[s1 s2 s3 s4 s5 s6]
Jmatrix=Jacobi(twist,sita)
Jmatrix=simple(Jmatrix)
```

```
Jmatrix =
[

0,
-cos(s1)*l1,
-cos(s1)*(-l2*sin(s2)+l1),
-l3*cos(s1)*cos(s2)*cos(s3)+l3*cos(s1)*sin(s2)*sin(s3)+l2*sin(s1)*cos(s3)-sin(s1)*cos(s2)*sin(s
3)*l1-sin(s1)*sin(s2)*cos(s3)*l1,
sin(s2)*l2*cos(s1)*cos(s4)+sin(s1)*sin(s4)*l4-l1*cos(s1)*cos(s4)-sin(s4)*cos(s1)*l3*cos(s2)*sin
(s3)+l1*sin(s4)*sin(s1)*cos(s2)*cos(s3)-sin(s4)*cos(s1)*l3*sin(s2)*cos(s3)-l1*sin(s4)*sin(s1)*si
n(s2)*sin(s3)-cos(s4)*cos(s1)*cos(s2)*cos(s3)*l4+cos(s4)*cos(s1)*sin(s2)*sin(s3)*l4+l2*sin(s4)
*sin(s1)*sin(s3),
-l1*sin(s5)*cos(s1)*sin(s4)-l2*sin(s5)*cos(s4)*sin(s1)*sin(s3)+l2*cos(s5)*sin(s1)*cos(s3)-l4*sin
(s5)*cos(s4)*sin(s1)-cos(s1)*l3*cos(s5)*cos(s2)*cos(s3)-l1*sin(s5)*cos(s4)*sin(s1)*cos(s2)*cos(
s3)+l1*sin(s5)*cos(s4)*sin(s1)*sin(s2)*sin(s3)-l1*cos(s5)*sin(s1)*cos(s2)*sin(s3)-l1*cos(s5)*sin
(s1)*sin(s2)*cos(s3)+cos(s1)*l3*cos(s4)*sin(s5)*sin(s2)*cos(s3)+cos(s1)*l3*cos(s4)*sin(s5)*cos
(s2)*sin(s3)+cos(s1)*l3*cos(s5)*sin(s2)*sin(s3)+sin(s2)*sin(s3)*l4*sin(s5)*cos(s1)*sin(s4)+sin(s
2)*l2*sin(s5)*cos(s1)*sin(s4)-cos(s2)*cos(s3)*l4*sin(s5)*cos(s1)*sin(s4)]
```

```
[
```

0,
-sin(s1)*l1,
-sin(s1)*(-l2*sin(s2)+l1),
-l3*sin(s1)*cos(s2)*cos(s3)+l3*sin(s1)*sin(s2)*sin(s3)-l2*cos(s1)*cos(s3)+cos(s1)*cos(s2)*sin(s3)*l1+cos(s1)*sin(s2)*l1*cos(s3),
l1*sin(s4)*cos(s1)*sin(s2)*sin(s3)-l1*sin(s4)*cos(s1)*cos(s2)*cos(s3)-cos(s4)*sin(s1)*cos(s2)*cos(s3)*l4+cos(s4)*sin(s1)*sin(s2)*sin(s3)*l4-sin(s4)*sin(s1)*l3*cos(s2)*sin(s3)-sin(s4)*sin(s1)*l3*sin(s2)*cos(s3)+sin(s2)*l2*sin(s1)*cos(s4)-l1*sin(s1)*cos(s4)-cos(s1)*sin(s4)*l4-l2*sin(s4)*cos(s1)*sin(s3),
sin(s2)*sin(s3)*l4*sin(s5)*sin(s1)*sin(s4)-cos(s2)*cos(s3)*l4*sin(s5)*sin(s1)*sin(s4)+sin(s2)*l2*sin(s5)*sin(s1)*sin(s4)+sin(s1)*l3*cos(s4)*sin(s5)*sin(s2)*cos(s3)+sin(s1)*l3*cos(s4)*sin(s5)*cos(s2)*sin(s3)+sin(s1)*l3*cos(s5)*sin(s2)*sin(s3)-sin(s1)*l3*cos(s5)*cos(s2)*cos(s3)-l1*sin(s5)*cos(s4)*cos(s1)*sin(s2)*sin(s3)+l1*cos(s5)*cos(s1)*cos(s2)*sin(s3)+l1*sin(s5)*cos(s4)*cos(s1)*cos(s2)*cos(s3)+l1*cos(s5)*cos(s1)*sin(s2)*cos(s3)-l1*sin(s5)*sin(s1)*sin(s4)+l2*sin(s5)*cos(s4)*cos(s1)*sin(s3)+l4*sin(s5)*cos(s4)*cos(s1)-l2*cos(s5)*cos(s1)*cos(s3)]

[

0,
0,
cos(s2)*l2,
l3*(cos(s2)*sin(s3)+sin(s2)*cos(s3)),
cos(s2)*l2*cos(s4)+cos(s4)*sin(s2)*cos(s3)*l4+cos(s4)*cos(s2)*sin(s3)*l4+l3*sin(s4)*sin(s2)*sin(s3)-l3*sin(s4)*cos(s2)*cos(s3),
l3*cos(s5)*sin(s2)*cos(s3)-l3*sin(s5)*cos(s4)*sin(s2)*sin(s3)+l3*sin(s5)*cos(s4)*cos(s2)*cos(s3)+l3*cos(s5)*cos(s2)*sin(s3)+sin(s2)*cos(s3)*l4*sin(s5)*sin(s4)+cos(s2)*sin(s3)*l4*sin(s5)*sin(s4)+cos(s2)*l2*sin(s5)*sin(s4)]

[

0,
-sin(s1),
-sin(s1),
cos(s1)*(cos(s2)*sin(s3)+sin(s2)*cos(s3)),
-sin(s4)*cos(s1)*cos(s2)*cos(s3)+sin(s4)*cos(s1)*sin(s2)*sin(s3)-sin(s1)*cos(s4),
sin(s5)*cos(s4)*cos(s1)*cos(s2)*cos(s3)-sin(s5)*cos(s4)*cos(s1)*sin(s2)*sin(s3)-sin(s5)*sin(s1)*sin(s4)+cos(s5)*cos(s1)*cos(s2)*sin(s3)+cos(s5)*cos(s1)*sin(s2)*cos(s3)]

[

0,

```
cos(s1),
cos(s1),
sin(s1)*(cos(s2)*sin(s3)+sin(s2)*cos(s3)),
-sin(s4)*sin(s1)*cos(s2)*cos(s3)+sin(s4)*sin(s1)*sin(s2)*sin(s3)+cos(s1)*cos(s4),
sin(s5)*cos(s4)*sin(s1)*cos(s2)*cos(s3)-sin(s5)*cos(s4)*sin(s1)*sin(s2)*sin(s3)+sin(s5)*cos(s1)*
sin(s4)+cos(s5)*sin(s1)*cos(s2)*sin(s3)+cos(s5)*sin(s1)*sin(s2)*cos(s3)]


   [

1,
0,
0,
-sin(s2)*sin(s3)+cos(s2)*cos(s3),
sin(s4)*(cos(s2)*sin(s3)+sin(s2)*cos(s3)),
-cos(s4)*sin(s5)*sin(s2)*cos(s3)-cos(s4)*sin(s5)*cos(s2)*sin(s3)-cos(s5)*sin(s2)*sin(s3)+cos(s5)
*cos(s2)*cos(s3)]
```

4 PUMA560 Dynamics Analysis

Source code:

```
      syms l0 l1 l2 l3 l4 s1 s2 s3 s4 s5 s6 r0 r1 r2 r3    r4 r5 r6 m1 m2 m3 m4 m5 m6 Ix1 Ix2 Ix3
Ix4 Ix5 Ix6 Iy1 Iy2 Iy3 Iy4 Iy5 Iy6 Iz1 Iz2 Iz3 Iz4 Iz5 Iz6

   q=[0 0 l1].',w=[0;0;1]; screw1= screw(q,w)
   q=[0 0 l1].',w=[0;1;0]; screw2= screw(q,w)
   q=[l2 0 l1].', w=[0;1;0]; screw3= screw(q,w)
   q=[l2 -l3 l1].', w=[0;0;1]; screw4= screw(q,w)
   q=[l2 -l3 l1+l4].', w=[0;1;0]; screw5= screw(q,w)
   q=[l2 -l3 l1+l4].', w=[0;0;1]; screw6= screw(q,w)

   gst10=[eye(3,3) [0 ;0;r1] ;
   0 0 0 1]
   gst20=[eye(3,3) [r2 ;0;l1] ;
   0 0 0 1]
   gst30=[eye(3,3) [l2 ;-r3;l1] ;
   0 0 0 1]
   gst40=[eye(3,3) [l2 ;-l3;l1+r4] ;
   0 0 0 1]
   gst50=[eye(3,3) [l2 ;-l3;l1+l4] ;
   0 0 0 1]
```

gst60=[eye(3,3) [l2 ;-l3;l1+l4] ;
0 0 0 1]

M1=diag([m1 m1 m1 Ix1 Iy1 Iz1])
M2=diag([m2 m2 m2 Ix2 Iy2 Iz2])
M3=diag([m3 m3 m3 Ix3 Iy3 Iz3])
M4=diag([m4 m4 m4 Ix4 Iy4 Iz4])
M5=diag([m5 m5 m5 Ix5 Iy5 Iz5])
M6=diag([m6 m6 m6 Ix6 Iy6 Iz6])

dyn_getMFN([screw1 screw2 screw3 screw4 screw5 screw6],[gst10 gst20 gst30 gst40 gst50 gst60],[s1 s2 s3 s4 s5 s6],[M1 M2 M3 M4 M5 M6])

[result]
Then we can get M,F and N.
The dynamic equation of the system

$$M(\theta)\ddot{\theta}+C(\theta,\dot{\theta})\dot{\theta}+N(\theta,\dot{\theta})=\tau$$

where

$$M(\theta)=\sum_{i=1}^{n}(J_{sli}^{b})^{T}I_{i}J_{sli}^{b}$$

$$C_{ij}(\theta,\dot{\theta})=\sum_{k=1}^{n}(\Gamma_{ijk})\dot{\theta}_{k}=\frac{1}{2}\sum_{k=1}^{n}(\frac{\partial M_{ij}}{\partial\theta_{k}}+\frac{\partial M_{ik}}{\partial\theta_{j}}-\frac{\partial M_{kj}}{\partial\theta_{i}})\dot{\theta}_{k}$$

The result is too long,so I emit them.