CS 3530: Assignment 7b

Fall 2023

Problem 7.6 (10 points)

Problem

Show that P is closed under complement.

Hint: As we discussed in class, construct the language (e.g. $PCAT = \{\langle P_1, P_2, w\rangle | P_1, P_2 \in P$ and w is a string, $w = x \cdot y$, $x \in P_1$ and $y \in P_2\}$), then provide a deterministic machine that decides the language in polynomial time.

Solution to P closed under complement.

assume a language P1 ∈ P in which P the the class of languages which are decidable in polynomial time

the TM M which decides $\overline{P1}$ is as follows

M = "On input w:

1. check if w ∈ P1

2. if w ∈ P1, reject

3. if w ! ∈ P1, accept"

Since w must be finite this is decidable in polynomial time

Problem 7.7 (10 points)

Problem

Show that NP is closed under union and concatenation.

Hint: As we discussed in class, construct the language (e.g. $NPCAT = \{\langle NP_1, NP_2, w\rangle | NP_1, NP_2 \in NP$ and w is a string, $w = x \cdot y$, $x \in NP_1$ and $y \in NP_2\}$), then describe a certificate and a verifier that deterministically verifies the certificate in polynomial time.

Solution to NP closed under union.

$NPUN = \{\langle NP_1, NP_2, w\rangle | NP_1, NP_2 \in NP$ and w is a string, where either w ∈ NP1 or w ∈ NP2)

let T_{NP1} and T_{NP2} be the machines that decide NP1 and NP2

we will construct a machine T_{NPu} that decides the union of NP1 and NP2

T_{NPu} = "On input w:

1. run T_{NP1} on w, if T_{NP1} accepts, accept

2. run T_{NP2} on w, if T_{NP2} accepts, accept

3. else reject"

Since T_{NP1} and T_{NP2} are both polynomial the union of the two will also be polynomial

Solution to NP closed under concatenation.

$NPCAT = \{\langle NP_1, NP_2, w\rangle | NP_1, NP_2 \in NP$ and w is a string, $w = x \cdot y$, $x \in NP_1$ and $y \in NP_2\})$

let T_{NP1} and T_{NP2} be the machines that decide NP1 and NP2

we will construct a machine T_{NPc} that decides the concatenation of NP1 and NP2

T_{NPc} = "On input w:

1. split w into w1 and w2 so that w1w2 = w
2. run T_{NP1} on w1, if T_{NP1} rejects, reject
3. run T_{NP2} on w2, if T_{NP2} rejects, reject
4. else, accept