Problem 2.30d (10 points)

Problem

Use the pumping lemma to show that the following languages are not context free.

    d. $\{t_1 \# t_2 \# \cdots \# t_k | k \geq 2,$ each $t_i \in \{a, b\}^*,$ and $t_i = t_j$ for some $i \neq j\}$

    Solution

    $S = a^P b^P \# a^P b^P$

    members of this language always have the same sub string repeating, separated by $\#$'s

    neither v or y can contain $\#$ because then uv^0xy^0 does not contain $\#$, so it cannot be a member

    if either v or y is empty we can treat them as if they both happened on the same side of $\#$ which is not a member

    if v and y are both nonempty, and are on the left side of $\#$, uv^2xy^2z is longer on the left side of $\#$ and cannot be a member

    same for the right hand side when using uv^0xy^0

Problem 2.31 (10 points)

Problem

Let B be the language of all palindromes over $\{0, 1\}$ containing an equal number of 0s and 1s. Show that B is not context free.

Solution

$s = 0^n 1^{2n} 0^n$

n = 2

s = 00111100

split 1: u = 00, v = 11, x = 1, y = 10, z = 0

split 2: u = 0, v = 01, x = 11, y = 1, z = 00

for i = 2

1: 00 1111 1 1010 1 is not a member of B

2: 0 0101 11 11 00 is not a member of B