CS 3530: Assignment 0d

Fall 2023

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Problem 0.10 (7 points)

Problem

Find the error in the following proof that $2 = 1$.

Consider the equation $a = b$. Multiply both sides by a to obtain $a^2 = ab$. Subtract b^2 from both sides to get $a^2 - b^2 = ab - b^2$. Now factor each side, $(a+b)(a-b) = b(a-b)$, and divide each side by $(a-b)$ to get $a + b = b$. Finally, let a and b equal 1, which shows that $2 = 1$.

Solution

Since a = b, when you divide by (a-b) that would be the same as dividing by 0 which would break the proof.

Exercise 0.11 (13 points)

Problem

Let $S(n) = 1 + 2 + \cdots + n$ be the sum of the first n natural numbers and let $C(n) = 1^3 + 2^3 + \cdots + n^3$ be the sum of the first n cubes. Prove the following equalities by induction on n, to arrive at the curious conclusion that $C(n) = S^2(n)$ for every n.

a. $S(n) = \frac{1}{2}n(n+1)$.

   Solution

   n = 1: $1 = \frac{1}{2}1(1+1)$

   $1 = 1$

   n = n + 1: $S(n+1) = 1 + 2 + ...n + n + 1$

   $= S(n) + n + 1$

   $= \frac{n(n+1)}{2} + n + 1$

   $= \frac{(n+1)(n+2)}{2}$

b. $C(n) = \frac{1}{4}(n^4 + 2n^3 + n^2) = \frac{1}{4}n^2(n+1)^2$.

   Solution

   n=1: $C(1) or 1^3 = \frac{1}{4}1^2(1+1)^2$

   $1 = 1$

   n=n+1: $C(n+1) = \frac{1}{4}n^2(n+1)^2 + (n+1)^3$

   $= \frac{(n+1)^2(n+2)^2}{4}$

   So when looking at the proofs for S(n+1) and C(n+1) you can see that $C(n+1) = (S(n+1))^2$