CS 3530: Assignment 1c

Fall 2023

Your Name Here

Exercise 1.15 (6 points)

Problem

Give a counterexample to show that the following construction fails to prove Theorem 1.49, the closure of the class of regular languages under the star operation.

Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognize A_1. Construct $N = (Q_1, \Sigma, \delta, q_1, F)$ as follows. N is supposed to recognize A_1^*.

a The states of N are the states of N_1.

b The start state of N is the same as the start state of N_1.

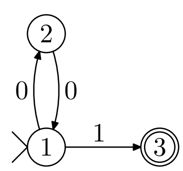
c $F = \{q_1\} \cup F_1$.
The accept states F are the old accept states plus its start state.

d Define δ so that for any $q \in Q$ and any $a \in \Sigma_\varepsilon$,

$$\delta(q,a) = \begin{cases} \delta_1(q,a) & q \notin F_1 \text{ or } a \neq \varepsilon \\ \delta_1(q,a) \cup \{q_1\} & q \in F_1 \text{ and } a = \varepsilon. \end{cases}$$

(Suggestion: Show this construction graphically, as in Figure 1.50.)

Solution



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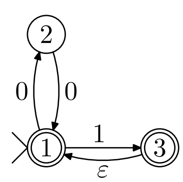
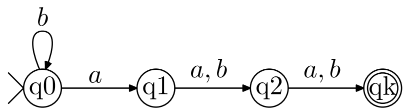


Problem 1.60 (7 points)

Problem

Let $\Sigma = \{a, b\}$. For each $k \geq 1$, let C_k be the language consisting of all strings that contain an a exactly k places from the right-hand end. Thus $C_k = \Sigma^* a \Sigma^{k-1}$. Describe an NFA with $k+1$ states that recognizes C_k, both in terms of a state diagram and a formal description.

Solution



$N = (Q, \Sigma, \delta, q_0, F)$

$Q = setofstates\{q_0, q_1, q_2, ..., q_k\}$

$\Sigma = alphabet\{a, b\}$

$q_0 = startstate$

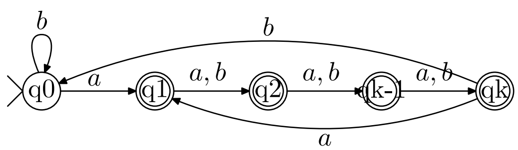
$F = setoffinalstates\{q_k\}$

Problem 1.62 (7 points)

Problem

Let $\Sigma = \{a, b\}$. For each $k \geq 1$, let D_k be the language consisting of all strings that have at least one a among the last k symbols. Thus $D_k = \Sigma^* a(\Sigma \cup \varepsilon)^{k-1}$. Describe a DFA with at most $k + 1$ states that recognizes D_k, both in terms of a state diagram and a formal description.

Solution



$Q_k = \{q_0, q_1, ..., q_k\} setofallstates$

$\{q_0\} = startstate$

$\Sigma = alphabet\{a, b\}$

$F = \{q_1, q_2, ..., q_k\} setoffinalstates$

$$\delta_k(q, 1) = \begin{cases} q_1 & i = 0^1 = a \\ q_0 & i = 0^1 = b \\ q_1 & i \neq 0^1 = a \\ q_(i+1) mod k & i \neq 0^1 = b \end{cases}$$