

CS 3530: Assignment 4d

Fall 2023

Problem 4.18 (20 points)

Problem

Let A and B be two disjoint languages. Say that language C **separates** A and B if $A \subseteq C$ and $B \subseteq \overline{C}$. Show that any two disjoint co-Turing-recognizable languages are separable by some decidable language.

Notes: A and B are disjoint, means $A \cap B = \emptyset$.

Solution

Two DFA M and N for languages A and B . DFA Y will be a separator for C . Language A is assumed to be a subset of C and B is assumed to be a subset of \overline{C} . C is separating A and B because A is decided by C and B is decided by \overline{C} , but not both.

F = a Turing machine which runs on input where A and B are two DFAs

Construct DFA O for C . This will decide languages A and B

Running C on Turing Machine T will identify whether O is a decider

If T accepts then string is recognized, and vice versa if it rejects

If T accepts then O is decider for A , if it rejects it is the decider for B

This shows that two disjoint Co-Turing recognizable languages are separable by using a decidable language