CS 3530: Assignment 7a

Fall 2023

Problem 7.6 (10 points)

Problem

Show that P is closed under union and concatenation.

Hint: As we discussed in class, construct the language (e.g. $PCAT = \{\langle P_1, P_2, w\rangle | P_1, P_2 \in P$ and w is a string, $w = x \cdot y$, $x \in P_1$ and $y \in P_2\}$), then provide a deterministic machine that decides the language in polynomial time.

Solution to P closed under union.

$PUN = \{\langle P_1, P_2, w\rangle | P_1, P_2 \in P$ and w is a string, where either w $\in$ P1, or w $\in$ P2$\}$

M = "On input w:

1. Check if w $\in$ P1, if accept, then accept w

2. If not check if w $\in$ P2, if accept then accept w

3. If both reject, reject input w"

Since each check is requires polynomial time the overall time is polynomial

Solution to P closed under concatenation.

$PCAT = \{\langle P_1, P_2, w\rangle | P_1, P_2 \in P$ and w is a string, $w = x \cdot y$, $x \in P_1$ and $y \in P_2\}$

M = "On input w on length n:

1. w can be split into two strings in n different ways

2. for each split w1, and w2

a. check if w1 $\in$ P1

b. check if w2 $\in$ P2

3. If any split succeeds we accept, else we reject if all splits have been tried

Problem 7.5 (10 points)

Is the following formula satisfiable? (Give your reasoning.)

$$(x \vee y) \wedge (x \vee \overline{y}) \wedge (\overline{x} \vee y) \wedge (\overline{x} \vee \overline{y})$$

Solution

1. x = T, y = F

$$(T \vee F) \wedge (T \vee \overline{F}) \wedge (\overline{T} \vee F) \wedge (\overline{T} \vee \overline{F})$$

$$T \wedge (T \vee \overline{F}) \wedge (\overline{T} \vee F) \wedge (\overline{T} \vee \overline{T})$$

$$T \wedge T \wedge F \wedge T$$

$$T \wedge F$$

$$F$$

$$!Satisfiable$$

2. x = F, y = T

$$(F \vee T) \wedge (F \vee \overline{T}) \wedge (\overline{F} \vee T) \wedge (\overline{F} \vee \overline{T})$$

$$T \wedge (F \vee \overline{T}) \wedge (\overline{F} \vee T) \wedge (\overline{F} \vee \overline{T})$$

$$T \wedge F \wedge T \wedge T$$

$$F \wedge T$$

$$F$$

$$!Satisfiable$$