CS 3530: Assignment 4c

Fall 2023

Problem 4.12 (10 points)

Problem

Let $A = \{\langle R, S\rangle | R$ and S are regular expressions and $L(R) \subseteq L(S)\}$. Show that A is decidable.

Solution

The following turing machine T decides language A

1. Check that w encodes a pair of regular expressions, R and S, if not reject w

2. Translate R and S into equivalent DFAs, DR and DS

3. Build a DFA DS^c that accepts $L(DS)^c$

4. Build a DFA D that is the intersection of DR with DS^c

5. Now run Turing Machine T with input D to determine if L(D) is empty:

If T accepts D, then accept w.

If T rejects D, then reject w.

Now prove machine T decides language A

let w be any word

Check if w codifies a pair of regular expressions, this will take a finite amount of time

if x doesn't codify a pair of regular expressions then T stops rejecting w

otherwise assume w codifies a pair of regular expressions

Since regular languages are closed under intersection and complimentation one can construct DFAs by following steps 2-4 above in finite time

In all cases T halts on w after some finite amount of time, which makes it decidable

Problem 4.15 (10 points)

Problem

Let $A = \{\langle R\rangle | R$ is a regular expression describing a language containing at least one string w that has 111 as a substring (i.e., $w = x111y$ for some x and y) $\}$. Show that A is decidable.

Solution

S = $w \varepsilon \Sigma^*$ | w contains 111 as a substring

M = ”on input $\langle R\rangle$| R is a regular expression”

construct a DFA Ds for language S

Transform R into a DFA Dr by using Kleene's Theorem

build a DFA $D^{S\ \&\ L(R)}$ for the language $S\ \ \&\ \ L(R)$

build a TM T that decides E^{DFA} on input $\langle D^{S\ \&\ L(R)} \rangle$

if T accepts, reject. if T rejects, accept.