CS 3530: Assignment 4b

Fall 2023

Exercise 4.4 (10 points)

Problem

Let $A_{\varepsilon\text{CFG}} = \{\langle G\rangle | G \text{ is a CFG that generates } \varepsilon\}$. Show that $A_{\varepsilon\text{CFG}}$ is decidable.

Solution

Build a Turing machine T that recognizes $A_{\varepsilon\text{CFG}}$ for all context grammars G

if G derives ε then $T(\langle G\rangle$ accepts

if G does not derive a member of language then $T(\langle G\rangle$ rejects

Exercise 4.5abcdef (10 points)

Problem

Let $X = \{1, 2, 3, 4, 5\}$ and $Y = \{6, 7, 8, 9, 10\}$. We describe the functions $f : X \to Y$ and $g : X \to Y$ in the following tables. Answer each part and give a reason for each answer.

n	$f(n)$
1	6
2	7
3	6
4	7
5	6

n	$g(n)$
1	10
2	9
3	8
4	7
5	6

a. Is f one-to-one?

b. Is f onto?

c. Is f a correspondence?

d. Is g one-to-one?

e. Is g onto?

f. Is g a correspondence?

Solution

a. no, f(1), f(3), and f(5) all = 6

b. no, 8, 9, and 10 are never outputs

c. no, it would have to be both one-to-one and onto

d. yes, every x value produces a unique y output

e. yes, every member of y is an output at some point

f. yes, it is both one-to-one and onto at the same time