CS 3530: Assignment 3b

Fall 2023

Exercise 3.5ab (5 points)

Problem

Examine the formal definition of a Turing machine to answer the following questions, and explain your reasoning.

Note: Refer to the definition 3.3 in the textbook.

a. Can a Turing machine ever write the blank symbol $\sqcup$ on its tape?

b. Can the tape alphabet Γ be the same as the input alphabet Σ?

Solution

a. Yes, the tape alphabet, or all possible options to write to the tape, is always going to be Σ + the blank symbol.

b. No, the tape alphabet will always have the blank added while the input alphabet will not. Σ is always a subset of Γ, but not the other way around.

Exercise 3.7 (5 points)

Problem

Explain why the following is not a description of a legitimate Turing machine.

Let M_{bad} = "The input is a polynomial p over variables $x_1, \cdots, x_k$.

1. Try all possible settings of $x_1, \cdots, x_k$ to integer values.
2. Evaluate p on all of these settings.
3. If any of these settings evaluates to 0, *accept*; otherwise, *reject*."

Note: Look at the subsection *Hilbert's Problems* for more information on polynomials and finding integral roots.

Solution

Step 1 can require an infinite amount of time, and step 2 can require an infinite amount of time. If these things happen step 3 will never resolve making this an illegitamate turing machine.

Exercise 3.15ab (10 points)

Problem

Show that the collection of decidable languages is closed under the operation of

a. union.

b. concatenation.

Solution

a. If 2 languages exist and are known to be decidable, then the machines can be combined into 1 machine in a way that will accept if either accepts, and will reject only if they both reject. For a given input run it through both machines if either accepts then accept, else reject.

b. Must account for any possible split of the input (2 parts). Run these splits on their respective machines, split 1 to machine 1 and vice versa. If either of these rejects then reject, else accept.