CS 3530: Assignment 7f

Fall 2023

Problem 7.22 (20 points)

Problem

Let DOUBLE-SAT = $\{\phi | \phi$ has at least two satisfying assignments $\}$. Show that DOUBLE-SAT is NP-complete.

Hint: Consider a reduction from SAT or 3SAT.

Note: In order to receive credit for this assignment, you must complete the full NP-completeness proof process outlined in previous assignments.

1. DOUBLE-SAT $\in$ NP

A certificate for DOUBLE-SAT will be two Boolean values t1, and t2.

The verifier will check if both t1, and t2 are satisfactory. If they are accept, else reject

2. Showing reduction from SAT to DOUBLE-SAT

the function f which maps an instance of SAT to DOUBLE-SAT works as follows: $\emptyset' = \emptyset \bigwedge (x1 \bigvee x2)$

x1 and x2 are new variables they do not appear in $\emptyset$

if $\emptyset$ is unsatisfiable then $\emptyset'$ is as well because we have only conducted an additional item.

but if $\emptyset$ has some satisfying assignment t, then $\emptyset$ has at least three satisfying assignments corresponding to the three different ways of extending t to the new variables x1, and x2

Since we know SAT is NP-hard, we can conclude DOUBLE-SAT is also NP-hard because we provided a reduction between the two.

Since DOUBLE-SAT is NP, and NP-hard it must also be NP-complete