CS 3530: Assignment 5c

Fall 2023

Problem 5.24

Let $J = \{w |$ either $w = 0x$ for some $x \in A_{\mathrm{TM}}$, or $w = 1y$ for some $y \in \overline{A_{\mathrm{TM}}}\}$. Show that neither J nor $\overline{J}$ is Turing-recognizable.

You must use reductions to get credit on this problem. This means you should not assume J is decidable, nor should you construct a decider for A_{TM}, etc. You may not use Rice's theorem.

Note that like Theorem 5.30 there will be two parts to this proof. In this assignment, you will solve the second part.

Problem (part 2) (20 points)

Show that $\overline{J}$ is not Turing-recognizable by providing a reduction from A_{TM} to J. To complete your proof, you'll want to use the Definition 5.20 and Corollary 5.29.

Solution (part 2)

Assume a string: $t \in \Sigma^*$ so that $g(t) = 0t$

By definition of J: $t \in A_{\mathrm{TM}} iff 0t \in J$

g is reduction of A_{TM} to J, so $A_{\mathrm{TM}} \leq_{\mathrm{m}} J$

a function that reduces language L_1 to language L_2 also reduces $\overline{L_1}$ to $\overline{L_2}$ so, g is also the reduction of $\overline{A_{\mathrm{TM}}} \leq_{\mathrm{m}} \overline{J}$

if $\overline{A_{\mathrm{TM}}} \leq_{\mathrm{m}}$ B, A is not Turing recognizable then B is not Turing recognizable

because $\overline{A_{\mathrm{TM}}}$ is not Turing recognizable, by Corollary $\overline{J}$ is also not Turing recognizable

This along with the last part shows that neither J nor $\overline{J}$ is Turing recognizable