CS 3530: Assignment 5b

Fall 2023

Problem 5.24

Let $J = \{w|$ either $w = 0x$ for some $x \in A_{\mathrm{TM}}$, or $w = 1y$ for some $y \in \overline{A_{\mathrm{TM}}}\}$. Show that neither J nor $\overline{J}$ is Turing-recognizable.

You must use reductions to get credit on this problem. This means you should not assume J is decidable, nor should you construct a decider for A_{TM}, etc. You may not use Rice's theorem.

Note that like Theorem 5.30 there will be two parts to this proof. In this assignment, you will solve the first part.

Problem (part 1) (20 points)

Show that J is not Turing-recognizable by providing a reduction from A_{TM} to $\overline{J}$. To complete your proof, you'll want to use the Definition 5.20 and Corollary 5.29.

Solution (part 1)

$f : \Sigma^* to \Sigma^* of \overline{A_{\mathrm{TM}}} to J$

assume a string $z \in \Sigma^*$ So that $f(z) = 1z$

By definition of J, $z \in \overline{A_{\mathrm{TM}}}$ iff $1z \in J$

f is a reduction of $\overline{A_{\mathrm{TM}}}$ to J, thus $\overline{A_{\mathrm{TM}}} \leq_{\mathrm{m}} J$

By using the Corollary

if $\overline{A_{\mathrm{TM}}} \leq_{\mathrm{m}} B$ is not Turing-recognizable then B is not Turing-recognizable.

because $\overline{A_{\mathrm{TM}}}$ is not Turing recognizable, J is not Turing-recognizable