

CS 3530: Assignment 2a

Fall 2023

Exercise 2.1abcd (10 points)

Problem

Recall the CFG G_4 that we gave in example 2.4. For convenience, let's rename its variables with single letters as follows.

$$E \rightarrow E + T \mid T$$

$$T \rightarrow T \times F \mid F$$

$$F \rightarrow (E) \mid a$$

Give derivations for each string.

a. a

b. $a + a$

c. $a + a + a$

d. $((a))$

Solution

a.

$$E \rightarrow T$$

$$E \rightarrow F$$

$$E \rightarrow a$$

b.

$$E \rightarrow E + T$$

$$E \rightarrow T + T$$

$$E \rightarrow F + T$$

$$E \rightarrow a + T$$

$$E \rightarrow a + F$$

$$E \rightarrow a + a$$

c.

$$E \rightarrow E + T$$

$$E \rightarrow E + T + T$$

$$E \rightarrow T + T + T$$

$$E \rightarrow F + T + T$$

$$E \rightarrow a + T + T$$

$$E \rightarrow a + F + T$$

$$E \rightarrow a + a + T$$

$$E \rightarrow a + a + F$$

$$E \rightarrow a + a + a$$

d.

$$E \rightarrow T$$

$$E \rightarrow F$$

$$E \rightarrow (E)$$

$$E \rightarrow (T)$$

$$E \rightarrow (F)$$

$$E \rightarrow ((E))$$

$$E \rightarrow ((T))$$

$$E \rightarrow ((F))$$

$$E \rightarrow ((a))$$

Exercise 2.4bc (10 points)

Problem

Give context-free grammars that generate the following languages. In all parts, the alphabet Σ is $\{0, 1\}$.

b. $\{w \mid w \text{ starts and ends with the same symbol} \}$

c. $\{w \mid \text{the length of } w \text{ is odd} \}$

Solution

b.

$$S \rightarrow 0P0 \mid 1P1 \mid 0 \mid 1$$

$$P \rightarrow 0P \mid 1P \mid \varepsilon$$

c.

$$S \rightarrow 0 \mid 1 \mid 00S \mid 01S \mid 10S \mid 11S$$