CS 3530: Assignment 3c

Fall 2023

Exercise 3.5cd (5 points)

Problem

Examine the formal definition of a Turing machine to answer the following questions, and explain your reasoning.

Note: Refer to the definition 3.3 in the textbook.

c. Can a Turing machine's head *ever* be in the same location in two successive steps?

d. Can a Turing machine contain just a single state?

Solution

c. Yes. If the head is at the far left position and tries to move left it has nowhere to go so it will stay at the same spot.

d. No. If Q is the set of states then there is a q^{accept} state that is a member of Q. There is also a q^{reject} that is a member of Q, and q^{accept} != q^{reject} so these two states must be distinct from eachother. A Turing machine must have at least 2 states

Exercise 3.15cde (10 points)

Problem

Show that the collection of decidable languages is closed under the operation of

c. star.

d. complementation.

e. intersection.

Solution

c. we split the input into every possible split that can be made. If we accept each of these splits then we accept, else if all splits have been tried w/ out success we reject. Since there are a finite amount of splits for the input the machine will halt after a finite amount of steps

d. M' is the complement of machine M. It will always do the opposite of whatever M does. It decides the complement of the language

e. We take the two languages and machines and evaluate them one after another. We run the first and if it rejects we rejects the entire thing. Repeat for second language and machine. If both machines accept then we accept, else we reject.