CS 3530: Assignment 7d

Fall 2023

Problem 7.33 (20 points)

Problem

In the following solitaire game, you are given an $m \times m$ board. On each of its m^2 positions lies either a blue stone, a red stone, or nothing at all. You play by removing stones from the board until each column contains only stones of a single color and each row contains at least one stone. You win if you achieve this objective. Winning may or may not be possible, depending upon the initial configuration. Let $\textsc{Solitaire} = \{G|G$ is a winnable game configuration $\}$. Prove that $\textsc{Solitaire}$ is NP-complete.

Hint: This is an "A" problem from the text. They have provided a solution, with a reduction from 3SAT.

Note: In order to receive credit for this assignment, you must complete the full NP-completeness proof process outlined here.

Prove Solitaire $\in$ NP

Describe a certificate for Solitaire

let certificate = C where C is a set of moves that will lead to a winning state

Provide a polynomial verifier for Solitaire

V = "On input C:

1. Make way through set executing each move in order.
2. If game is in winning state accept, else reject

Prove Solitaire is NP-hard

Given that 3SAT is NP-complete, show that 3SAT $\leq_P$ $\textsc{Solitaire}$ with the following steps.

Provide reduction from 3SAT to Solitaire

given ∅ with m variables V1....Vm and k clauses C1....Ck

construct gameboard g with $k * m$ board

assume ∅ has no clauses that contain both Vi and $\overline{Vi}$

if the variable Vi is in clause Ci then put a blue stone in row Ci column Vi

if the variable $\overline{Vi}$ is in clause Ci then put a red stone in row Ci column Vi

We can make the board square by repeating a row or adding a blank column w out affecting solvability

Prove reduction from 3SAT to Solitaire is polynomial

Time is polynomial because we are checking to see if each variable is or is not in each clause, and placing a colored stone based on the results. This will leave us looking through k clauses to check if Vi is in the clause. Leaving us doing k*m amount of work

Show 3SAT has a satisfying assignment if Solitaire is winnable

A satisfying assignment is taken

if Vi is true remove the red stone from the corresponding column

if Vi is false remove the blue stone from the corresponding column

So the only the stones that correspond to true literals remain

Because every clause has a true literal, every row has a stone

Therefore G has a solution

Show Solitaire is winnable if 3SAT has a satisfying assignment

Take a game solution

if the red stone removed from a column, set the corresponding variable true

if the blue stone removed from a column, set the corresponding variable false

Every row has a stone remaining, so each clause has a true literal

Therefore $\varnothing$ is satisfied

Conclude that Solitaire is NP-hard

Conclude that Solitaire is NP-complete

Given that we know 3SAT is NP-complete, and because of this must also be NP-hard. We can conclude that Solitaire is also NP-complete because we were able to provide a reduction from 3SAT to Solitaire. And since we know Solitaire is NP-complete it must also be NP-hard