AMS 394

how to do inference on two population means in SAS.

1. ***Review of Theory and Introduction to SAS for Comparing Two Population Means***

Overview: Inference on two population means:

1. **The samples are paired** $\Rightarrow$

   Paired samples t-test (if the paired diff are normal)

   OR

   Wilcoxon signed rank test (if the diff not normal)

2. **The samples are independent** $\Rightarrow$

   Independent samples t-test (if both populations are normal)(Then we use the F-test to check if $\sigma_1^2 = \sigma_2^2$)

   a) $\sigma_1^2 = \sigma_2^2 \Rightarrow$ **pooled-variance t-test**

   b) $\sigma_1^2 \neq \sigma_2^2 \Rightarrow$ **unpooled-variance t-test**

   OR

   Wilcoxon Rank Sum Test (if at least one population is NOT normal)

2._The samples are independent:

a) If both populations are normal and $\sigma_1^2 = \sigma_2^2$

$\Rightarrow$ **Pooled-variance t-test**

b) If both populations are normal and $\sigma_1^2 \neq \sigma_2^2$

$\Rightarrow$ **Unspooled-variance t-test**

c) If at least one population is not normal

$\Rightarrow$ **Wilcoxon rank sum test**

Example 2. An experiment was conducted to compare the mean number of tapeworms in the stomachs of sheep that had been treated for worms against the mean number in those that were untreated. A sample of 14 worm-infected lambs was randomly divided into 2 groups. Seven were injected with the drug and the remainders were left untreated. After a 6-month period, the lambs were slaughtered and the following worm counts were recorded:

Drug-treated sheep	18 43 28 50 16 32 13
Untreated sheep	40 54 26 63 21 37 39

(a). Test at $\alpha = 0.05$ whether the treatment is effective or not.
(b) What assumptions do you need for the inference in part (a)?
(c). Please write up the entire SAS program necessary to answer questions raised in (a) and (b).

SOLUTION: Inference on two population means. Two small and independent samples.
Drug-treated sheep: $\overline{X}_1 = 28.57$, $s_1^2 = 198.62$, $n_1 = 7$
Untreated sheep: $\overline{X}_2 = 40.0$, $s_2^2 = 215.33$, $n_2 = 7$

(a) Under the normality assumption, we first test if the two population variances are equal. That is, $H_0 : \sigma_1^2 = \sigma_2^2$ versus $H_a : \sigma_1^2 \neq \sigma_2^2$. The test statistic is

$$F_0 = \frac{s_1^2}{s_2^2} = \frac{198.62}{215.33} \approx 0.92, \quad F_{6,6,0.05,U} = 4.28 \text{ and}$$

$$F_{6,6,0.05,L} = 1/4.28 \approx 0.23.$$

Since F_0 is between 0.23 and 4.28, we cannot reject H_0. Therefore it is reasonable to assume that $\sigma_1^2 = \sigma_2^2$.
Next we perform the pooled-variance t-test with hypotheses
$H_0 : \mu_1 - \mu_2 = 0$ versus $H_a : \mu_1 - \mu_2 < 0$

$$t_0 = \frac{\overline{X}_1 - \overline{X}_2 - 0}{s_p\sqrt{\frac{1}{n} + \frac{1}{n_2}}} = \frac{(28.57 - 40.0) - 0}{14.39\sqrt{\frac{1}{7} + \frac{1}{7}}} \approx -1.49$$

Since $t_0 \approx -1.49$ is greater than $-t_{12,0.05} = -1.782$, we cannot reject H_0. We have insufficient evidence to reject the hypothesis that there is no difference in the mean number of worms in treated and untreated lambs.

(b) (1) Both populations are normally distributed
(2) $\sigma_1^2 = \sigma_2^2$

#Example 2 (c) in R#
sheep1<-Drug.treated.sheep<-c(18,43,28,50,16,32,13)
sheep2<-Untreated.sheep<-c(40,54,26,63,21,37,39)
#Normality test for each population#
shapiro.test(sheep1) #p=0.5142#
shapiro.test(sheep2) #p=0.7515#
#F-test for equal variances#
var.test(sheep1,sheep2)
#F-test shows the ratio of two variances should equal to 1#
#var.qual=TRUE:two variances are equal, p=0.163#
t.test(sheep1,sheep2,var.equal=TRUE)
wilcox.test(sheep1,sheep2,conf.int=TRUE)
#P=0.2086, and hence we think there is#
#Both the t.test and Wilcoxon rank sum test show that we have insufficient evidence to reject the hypothesis that there is no difference in the mean number of worms in treated and untreated lambs#

(c)
```
/*Problem #2B*/
data sheep;
input group worms;
datalines;
1 18
1 43
1 28
1 50
1 16
1 32
1 13
2 40
2 54
2 26
2 63
2 21
2 37
2 39
;
run;

proc univariate data=sheep normal;
class group;
var worms;
title 'Check for normality';
```

```
run;

proc ttest data=sheep;
class group;
var worms;
title 'Independent samples t-test';
run;

proc ttest data=sheep sides=lower;
 class group;
 var worms;
 title 'Independent samples t-test';
 run;


/*******
proc npar1way data=sheep wilcoxon;
 class group;
 var worms;
 title 'Nonparametric test for two-mean
 comparisons';
  *EXACT WILCOXON;
 run;

 ********/
```

Check for normality

The UNIVARIATE Procedure

Variable: worms

group = 1

Tests for Normality

Test	Statistic		p Value	
Shapiro-Wilk	**W**	0.925604	**Pr < W**	0.5142
Kolmogorov-Smirnov	**D**	0.201976	**Pr > D**	>0.1500
Cramer-von Mises	**W-Sq**	0.039988	**Pr > W-Sq**	>0.2500
Anderson-Darling	**A-Sq**	0.271024	**Pr > A-Sq**	>0.2500

Variable: worms

group = 2

Tests for Normality

Test	Statistic		p Value	
Shapiro-Wilk	**W**	0.952397	**Pr < W**	0.7515
Kolmogorov-Smirnov	**D**	0.214286	**Pr > D**	>0.1500
Cramer-von Mises	**W-Sq**	0.041652	**Pr > W-Sq**	>0.2500
Anderson-Darling	**A-Sq**	0.240103	**Pr > A-Sq**	>0.2500

Independent samples t-test

The TTEST Procedure

Variable: worms

Method	Variances	DF	t Value	Pr > |t|
Pooled	Equal	12	-1.49	0.1630
Satterthwaite	Unequal	11.98	-1.49	0.1631

Equality of Variances

Method	Num DF	Den DF	F Value	Pr > F
Folded F	6	6	1.08	0.9244

Nonparametric test for two-mean comparisons

The NPAR1WAY Procedure

Wilcoxon Two-Sample Test (Wilcoxon Rank Sum Test)

Statistic	42.0000
Normal Approximation	
Z	-1.2778
One-Sided Pr < Z	0.1007
Two-Sided Pr > |Z|	0.2013
t Approximation	
One-Sided Pr < Z	0.1118
Two-Sided Pr > |Z|	0.2237

Z includes a continuity correction of 0.5.

Kruskal-Wallis Test

Chi-Square	1.8000
DF	1
Pr > Chi-Square	0.1797

Example 3:

A new drug for reducing blood pressure (BP) is compared to an old drug. 20 patients with comparable high BP were recruited and randomized evenly to the 2 drugs. Reductions in BP after 1 month of taking the drugs are as follows:

New drug: 0, 10, -3, 15, 2, 27, 19, 21, 18, 10

Old drug: 8, -4, 7, 5, 10, 11, 9, 12, 7, 8

① **Assume both populations are normal, please test at α = 0.05 whether the new drug is better than the old one.**

② **Run the SAS program to do the above tests (including the normality test)**

<u>Solution:</u>

① n_1=10, $\bar{X}$=11.9, S_1^2=97.43, n_2=10, $\bar{Y}$=7.3, S_2^2 =20.01

F test: $\begin{cases} H_0 : \sigma_1^2 = \sigma_2^2 \\ H_a : \sigma_1^2 \neq \sigma_2^2 \end{cases}$

Test statistic: $F_0 = \dfrac{S_1^2}{S_2^2} = 4.87 > F_{10,10,0.25,U}$

$\therefore$ **At α=0.05, reject H_0 and use unpooled-variance t-test**

T-test: $\begin{cases} H_0 : \mu_1 - \mu_2 = 0 \\ H_a : \mu_1 - \mu_2 > 0 \end{cases}$

Test statistic: T_0 =

$$\frac{\bar{X} - \bar{Y}}{\sqrt{\dfrac{S_1^2}{n_1} + \dfrac{S_2^2}{n_2}}} = 1.34 < t_{df=12.547,0.05} \approx 1.776$$

$\therefore$ **At α =0.05, fail to reject H_0 and we cannot say that the new drug is better than the old one.**

②

```
Data BP;
input drug BPR;
datalines;
1 0
1 10
1 -3
1 15
1 2
1 27
1 19
1 21
1 18
1 10
2 8
2 -4
2 7
2 5
2 10
2 11
2 9
2 12
2 7
2 8
;
run;

PROC UNIVARIATE DATA=BP NORMAL;
CLASS DRUG;
VAR BPR;
RUN;
/*******
PROC TTEST DATA=BP;
CLASS DRUG;
VAR BPR;
RUN;
*******/
```

Selected result:

```
                         Tests for Normality

Test (class=1)         --Statistic---     -----p Value------

Shapiro-Wilk           W     0.955526     Pr < W        0.7339
Kolmogorov-Smirnov     D     0.142059     Pr > D       >0.1500
Cramer-von Mises       W-Sq  0.037679     Pr > W-Sq    >0.2500
Anderson-Darling       A-Sq  0.238555     Pr > A-Sq    >0.2500

 Test (class =2)        --Statistic---     -----p Value------

Shapiro-Wilk           W     0.805147     Pr < W        0.0167
Kolmogorov-Smirnov     D     0.273266     Pr > D        0.0334
```

```
Cramer-von Mises        W-Sq  0.124461    Pr > W-Sq    0.0447
Anderson-Darling        A-Sq  0.776159    Pr > A-Sq    0.0294
```

***The data of old drug is not normal, use nonparametric test.**

```
Method           Variances        DF    t Value    Pr > |t|

Pooled           Equal            18       1.34      0.1962
Satterthwaite    Unequal      12.547       1.34      0.2033

                    Equality of Variances

       Method      Num DF     Den DF     F Value     Pr > F

       Folded F         9          9        4.87     0.0274
```

If the populations are not normal, use nonparametric test for 2 means from 2 independent samples (Wilcoxon Rank Sum test)

```
PROC NPAR1WAY DATA=BP wilcoxon;
CLASS DRUG;
VAR BPR;
*EXACT WILCOXON;
RUN;
```

Selected result:

```
                     Wilcoxon Two-Sample Test

                Statistic               123.0000

                Normal Approximation
                Z                         1.3259
                One-Sided Pr > Z          0.0924
                Two-Sided Pr > |Z|        0.1849

                t Approximation
                One-Sided Pr > Z          0.1003
                Two-Sided Pr > |Z|        0.2006
```

***In summary, at α =0.05, we can NOT conclude that the new drug is better than the old one using the nonparametric Wilcoxon Rank Sum test.**

Note: As you can see from this example, the normality test is very important – indeed we should not have used the unspooled variance t-test because the normality assumption is not satisfied in this problem.