AMS 394 handout 6 two sample test

***two non-parametric tests*：Wilcoxon signed rank & rank sum tests**

how to do inference on two population means

1. ***Review of Theory：Comparing Two Population Means***

1. **The samples are paired** $\Rightarrow$

   Paired samples t-test (if the paired differences are normal)

   OR

   Wilcoxon signed rank test (if the paired differences are not normal)

2. **The samples are independent** $\Rightarrow$

   Independent samples t-test (if both populations are normal)

   (Then we use the F-test to check if $\sigma_1^2 = \sigma_2^2$)

   a) $\sigma_1^2 = \sigma_2^2 \Rightarrow$ **pooled-variance t-test**

   b) $\sigma_1^2 \neq \sigma_2^2 \Rightarrow$ **unpooled-variance t-test**

   OR

   Wilcoxon Rank Sum Test (if at least one population is NOT normal)

1. Inference on two population means: Paired samples:

e.g. Compare the IQ between adult male and female

Pair	IQ Male	IQ Female	Diff
1	200	199	1
2	150	120	30
…	…	…	…
n	109	112	-3

***** After taking the paired differences – make sure you use one variable to subtract the other, consistently – the two population/sample problem is then reduced to a one population/one sample problem: on the population/sample of the paired differences**

$$\begin{cases} H_0 : \mu_{male} = \mu_{female} \\ H_a : \mu_{male} > \mu_{female} \end{cases} \Leftrightarrow \begin{cases} H_0 : \mu_{diff} = 0 \\ H_a : \mu_{diff} > 0 \end{cases}$$

$$\begin{cases} H_0 : \mu_{male} = \mu_{female} \\ H_a : \mu_{male} < \mu_{female} \end{cases} \Leftrightarrow \begin{cases} H_0 : \mu_{diff} = 0 \\ H_a : \mu_{diff} < 0 \end{cases}$$

$$\begin{cases} H_0 : \mu_{male} = \mu_{female} \\ H_a : \mu_{male} \neq \mu_{female} \end{cases} \Leftrightarrow \begin{cases} H_0 : \mu_{diff} = 0 \\ H_a : \mu_{diff} \neq 0 \end{cases}$$

Example 0:

```
data(intake)
attach(intake)
intake
t.test(pre,post,paired=T)
#t.test(pre,post) is wrong

 wilcox.test(pre,post,paired=T)
```

#the result of Wilcox test does not show any difference from that of the t test

Example 1. Paired Samples T-test and the Wilcoxon Signed Rank Test. To study the effectiveness of wall insulation in saving energy for home heating, the energy consumption (in MWh) for 5 houses in Bristol, England, was recorded for two winters; the first winter was before insulation and the second winter was after insulation:

House	1	2	3	4	5
Before	12.1	10.6	13.4	13.8	15.5
After	12.0	11.0	14.1	11.2	15.3

(a) Please provide a 95% confidence interval for the difference between the mean energy consumption before and after the wall insulation is installed. What assumptions are necessary for your inference?

(b) Can you conclude that there is a difference in mean energy consumption before and after the wall insulation is installed at the significance level 0.05? Please test it and evaluate the p-value of your test. What assumptions are necessary for your inference?

(c) Please write the R code to perform the test and examine the necessary assumptions given in (b).

SOLUTION: This is inference on two population means, paired samples.

(a). $\bar{d} = 0.36$, $S_d = 1.30$

CI: $0.36 \pm 2.776 \cdot \dfrac{1.30}{\sqrt{5}} = (-1.25, 1.97)$

(b). $H_0 : \mu_d = 0$, $H_a : \mu_d \neq 0$

(1) $t_0 = \dfrac{\bar{d} - 0}{s_d / \sqrt{n}} = \dfrac{0.36 - 0}{1.30 / \sqrt{5}} \approx 0.619$

$t_{n-1, \alpha/2} = t_{4, 0.025} = 2.776$

Since $|t_0| \approx 0.619$ is smaller than $t_{4,0.025} = 2.776$, we cannot reject H_0.

(2) $p - value = 2 \cdot P(T \geq 0.619) \approx 0.57$

Assumptions for (a) and (b): the paired differences follow a normal distribution.

(c).

```
Before=c(12.1,10.6,13.4,13.8,15.5)
After=c(12.0,11.0,14.1,11.2,15.3)
#create the paired differences, Diff#
Diff=Before-After
shapiro.test(Diff)
#We can barely conclude Diff follows normal distribution since
p=0.08496 is small but still > 0.05, while < 0.10 #

#One-sample t- test for Diff#
```

t.test(Diff)
#p-value = 0.5707; 95% CI is [-1.26036,1.98036]#

#Alternatively, we can perform the paired-samples t- test directly use the original Before and After data as follows#
t.test(Before, After, paired=TRUE)
p-value = 0.5707#

Review: Paired Samples T-test
t.test(a, b, paired=TRUE)

Note : As you can see from the variable "Diff" is barely normal. In this situation, one can either use the t-test or the signed-rank test(*although not the best practice, but $\alpha = 0.05$ can be used as the significance level to judge the normality; although $\alpha = 0.1$ will be more secure.)

#Wilcoxon Signed-Rank Test, based on the original data#
wilcox.test(Before, After, paired = TRUE)

#Wilcoxon Signed-Rank Test, based on the original data, alternative form#
wilcox.test(Before-After)

#Wilcoxon Signed-Rank Test, based on the paired differences#
wilcox.test(Diff)

Review:
Wilcoxon One-Sample Test (Wilcoxon Signed Rank Test)
wilcox.test(x, y, paired = TRUE, alternative = "greater")
wilcox.test(y - x, alternative = "less")

2._The samples are independent:

a) If both populations are normal and $\sigma_1^2 = \sigma_2^2$

$\Rightarrow$ **Pooled-variance t-test**

b) If both populations are normal and $\sigma_1^2 \neq \sigma_2^2$

$\Rightarrow$ **Unspooled-variance t-test**

c) If at least one population is not normal

$\Rightarrow$ **Wilcoxon rank sum test**

2a.Inference on 2 population means, when both populations are normal. We have 2 independent samples, the population variances are unknown but equal ($\sigma_1^2 = \sigma_2^2 = \sigma^2$)$\Rightarrow$ pooled-variance t-test.

Data: $X_1, \ldots, X_{n_1} \overset{iid}{\sim} N(\mu_1, \sigma_1^2)$

$Y_1, \ldots, Y_{n_2} \overset{iid}{\sim} N(\mu_2, \sigma_2^2)$

Goal: Compare μ_1 and μ_2

1) Point estimator:

$$\widehat{\mu_1 - \mu_2} = \bar{X} - \bar{Y} \sim N\left(\mu_1 - \mu_2, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)$$
$$= N\left(\mu_1 - \mu_2, \left(\frac{1}{n_1} + \frac{1}{n_2}\right)\sigma^2\right)$$

2) Pivotal quantity:

$$Z = \frac{(\bar{X} - \bar{Y}) - (\mu_1 - \mu_2)}{\sigma \cdot \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim N(0,1)$$ not the PQ, since we don't know σ^2

$\because\quad \dfrac{(n_1-1)S_1^2}{\sigma^2}\sim\chi^2_{n_1-1}$, $\dfrac{(n_2-1)S_2^2}{\sigma^2}\sim\chi^2_{n_2-1}$, and they are independent (S_1^2 & S_2^2 are independent because these two samples are independent to each other)

$$\therefore W=\frac{(n_1-1)S_1^2}{\sigma^2}+\frac{(n_2-1)S_2^2}{\sigma^2}\sim\chi^2_{n_1+n_2-2}$$

Definition: $W=Z_1^2+Z_2^2+\cdots+Z_k^2$, where $Z_i\overset{i.i.d.}{\sim}N(0,1)$, then $W\sim\chi_k^2$.

Definition: **t-distribution:**

$Z\sim N(0,1)$, $W\sim\chi_k^2$, and Z & W are independent, then $T=\dfrac{Z}{\sqrt{\dfrac{W}{k}}}\sim t_k$

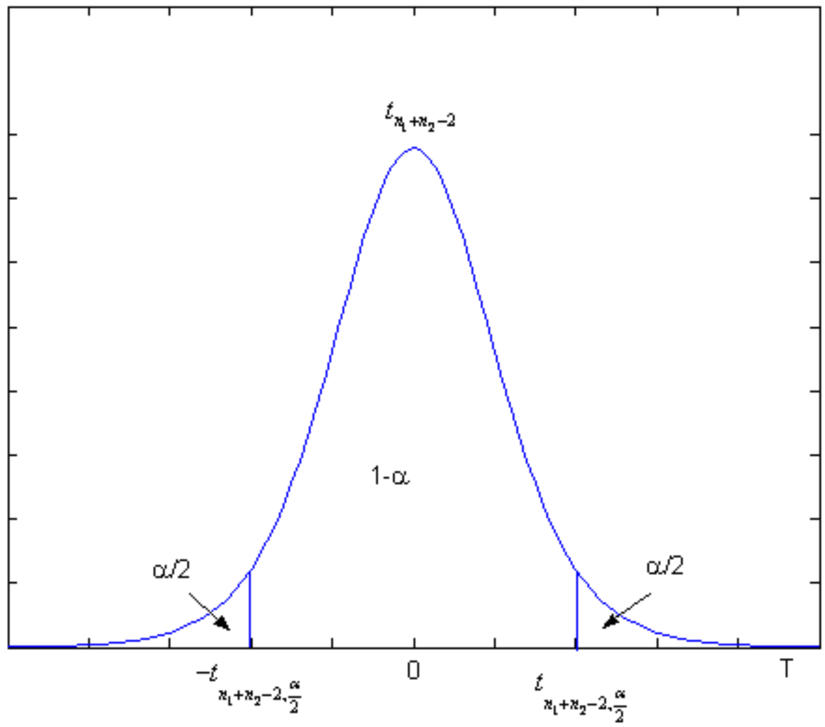
$\therefore$

$$T=\frac{Z}{\sqrt{\dfrac{W}{n_1+n_2-2}}}=\frac{\dfrac{(\bar{X}-\bar{Y})-(\mu_1-\mu_2)}{\sigma\cdot\sqrt{\dfrac{1}{n_1}+\dfrac{1}{n_2}}}}{\sqrt{\dfrac{\dfrac{(n_1-1)S_1^2}{\sigma^2}+\dfrac{(n_2-1)S_2^2}{\sigma^2}}{n_1+n_2-2}}}=\frac{(\bar{X}-\bar{Y})-(\mu_1-\mu_2)}{S_p\sqrt{\dfrac{1}{n_1}+\dfrac{1}{n_2}}}\sim t_{n_1+n_2-2}$$

where $S_p^2=\dfrac{(n_1-1)S_1^2+(n_2-1)S_2^2}{n_1+n_2-2}$ is the pooled variance.

This is the PQ of the inference on the parameter of interest $(\mu_1-\mu_2)$

3) Confidence Interval for $(\mu_1-\mu_2)$



$$1-\alpha = P(-t_{n_1+n_2-2,\frac{\alpha}{2}} \le T \le t_{n_1+n_2-2,\frac{\alpha}{2}})$$

$$1-\alpha = P\left(-t_{n_1+n_2-2,\frac{\alpha}{2}} \le \frac{(\bar{X}-\bar{Y})-(\mu_1-\mu_2)}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} \le t_{n_1+n_2-2,\frac{\alpha}{2}}\right)$$

$$1-\alpha = P\left(-t_{n_1+n_2-2,\frac{\alpha}{2}}\cdot S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} \le (\bar{X}-\bar{Y})-(\mu_1-\mu_2) \le t_{n_1+n_2-2,\frac{\alpha}{2}}\cdot S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}\right)$$

$$1-\alpha = P\left(\bar{X}-\bar{Y}-t_{n_1+n_2-2,\frac{\alpha}{2}}\cdot S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} \le \mu_1-\mu_2 \le \bar{X}-\bar{Y}+t_{n_1+n_2-2,\frac{\alpha}{2}}\cdot S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}\right)$$

$\therefore$ This is the $100(1-\alpha)\%$ C.I for $(\mu_1-\mu_2)$

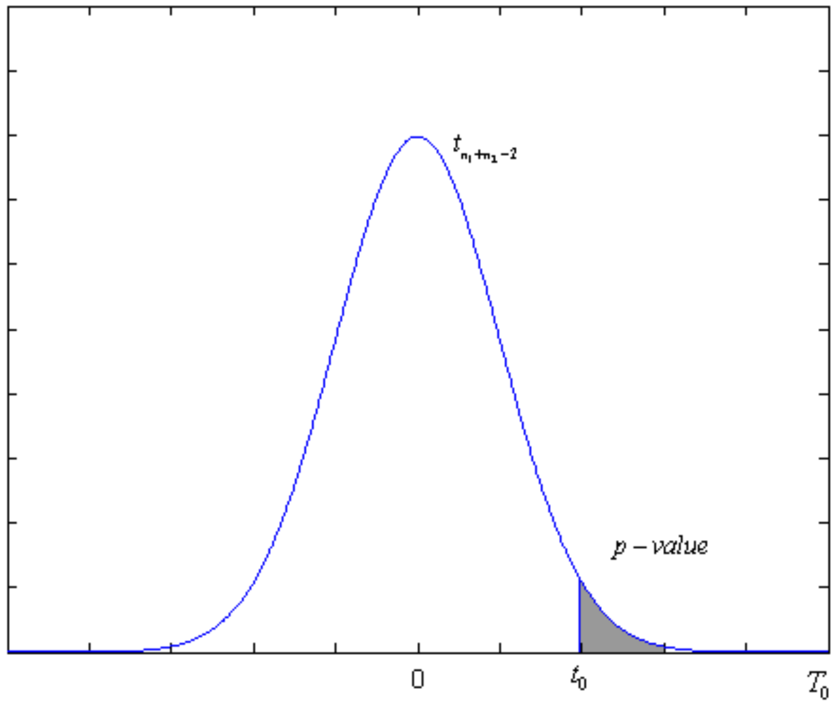
4) Test:

Test statistic: $T_0 = \dfrac{(\bar{X}-\bar{Y})-c_0}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} \overset{H_0}{\sim} t_{n_1+n_2-2}$

a) $\begin{cases} H_0: \mu_1-\mu_2 = c_0 \\ H_a: \mu_1-\mu_2 > c_0 \end{cases}$ (The most common situation is $c_0 = 0 \Leftrightarrow H_0: \mu_1 = \mu_2$)

At the significance level α, we reject H_0 in favor of H_a iff $T_0 \geq t_{n_1+n_2-2,\alpha}$

If $P-value = P(T_0 \geq t_0 \mid H_0) < \alpha$, reject H_0



b) $\begin{cases} H_0: \mu_1 - \mu_2 = c_0 \\ H_a: \mu_1 - \mu_2 < c_0 \end{cases}$

At significance level α, reject H_0 in favor of H_a iff $T_0 \leq -t_{n_1+n_2-2,\alpha}$

If $P-value = P(T_0 \leq t_0 \mid H_0) < \alpha$, reject H_0

c) $\begin{cases} H_0: \mu_1 - \mu_2 = c_0 \\ H_a: \mu_1 - \mu_2 \neq c_0 \end{cases}$

At α=0.05, reject H_0 in favor of H_a iff

$|T_0| \geq t_{n_1+n_2-2,\frac{\alpha}{2}}$

If $P-value = 2P(T_0 \geq |t_0| \mid H_0) < \alpha$, reject H_0

2b. Inference on 2 population means, when both populations are normal and the population variances are not equal ($\sigma_1^2 \neq \sigma_2^2$). We have two independent samples. $\Rightarrow$ unpooled-variance t-test.

1) **P.Q:**

$$T = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} \overset{\bullet}{\sim} t_{df}$$

Formula for the df:

Walch Satterthwaite method:

$$df = \frac{(W_1 + W_2)^2}{W_1^2/(n_1 - 1) + W_2^2/(n_2 - 1)} \quad \text{where } W_1 = \frac{S_1^2}{n_1}, W_2 = \frac{S_2^2}{n_2}$$

or another way to find df (less accurate and more conservative)

$$df = \min(n_1 - 1, n_2 - 1)$$

2) **100(1-α)% C.I. for ($\mu_1 - \mu_2$)**

$$\bar{X} - \bar{Y} \pm t_{df,\frac{\alpha}{2}} \cdot \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$$

3) **Test:**

Test statistic: $$T_0 = \frac{(\bar{X} - \bar{Y}) - c_0}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} \overset{H_0}{\overset{\bullet}{\sim}} t_{df}$$

a) $$\begin{cases} H_0 : \mu_1 - \mu_2 = c_0 \\ H_a : \mu_1 - \mu_2 > c_0 \end{cases}$$

At α=0.05, reject H_0 in favor of H_a iff

$$T_0 \geq t_{df,\alpha}$$

If $P-value = P(T_0 \geq t_0 \mid H_0) < \alpha$**, reject** H_0

b) $\begin{cases} H_0 : \mu_1 - \mu_2 = c_0 \\ H_a : \mu_1 - \mu_2 < c_0 \end{cases}$

At α=0.05, reject H_0 **in favor of** H_a **iff**

$T_0 \leq -t_{df,\alpha}$

If $P-value = P(T_0 \leq t_0 \mid H_0) < \alpha$**, reject** H_0

c) $\begin{cases} H_0 : \mu_1 - \mu_2 = c_0 \\ H_a : \mu_1 - \mu_2 \neq c_0 \end{cases}$

At α=0.05, reject H_0 **in favor of** H_a **iff**

$|T_0| \geq t_{df,\frac{\alpha}{2}}$

If $P-value = 2P(T_0 \geq |t_0| \mid H_0) < \alpha$**, reject** H_0

4) F-test:

Data: $X_1, \ldots, X_{n_1} \overset{iid}{\sim} N(\mu_1, \sigma_1^{\,2})$

$Y_1, \ldots, Y_{n_2} \overset{iid}{\sim} N(\mu_2, \sigma_2^{\,2})$

$$\begin{cases} H_0 : \sigma_1^2 = \sigma_2^2 \\ H_a : \sigma_1^2 \neq \sigma_2^2 \end{cases} \Leftrightarrow \begin{cases} H_0 : \dfrac{\sigma_1^2}{\sigma_2^2} = 1 \\ H_a : \dfrac{\sigma_1^2}{\sigma_2^2} \neq 1 \end{cases}$$

① **Point estimator:** $\dfrac{\hat{\sigma}_1^{\,2}}{\hat{\sigma}_2^{\,2}} = \dfrac{S_1^{\,2}}{S_2^{\,2}}$

② **P.Q:**

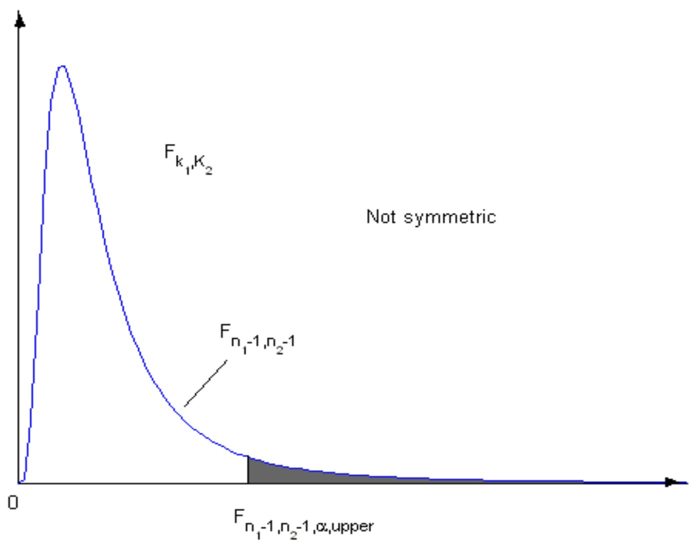
> ***Definition*****:** F-Distribution
>
> Let $W_1 \sim \chi^2_{k_1}, W_2 \sim \chi^2_{k_2}$, and W_1, W_2 are independent. Then
>
> $F = \dfrac{W_1 / k_1}{W_2 / k_2} \sim F_{k_1, k_2}$

$\because \dfrac{(n_1 - 1)S_1^2}{\sigma_1^2} \sim \chi^2_{n_1 - 1}$

$$\frac{(n_2-1)S_2^2}{\sigma_2^2} \sim \chi^2_{n_2-1}$$

$$\therefore\ F = \frac{\dfrac{(n_1-1)S_1^2}{\sigma_1^2}\Big/(n_1-1)}{\dfrac{(n_2-1)S_2^2}{\sigma_2^2}\Big/(n_2-1)} = \frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} \sim F_{n_1-1,n_2-1} \quad \rightarrow$$

Pivotal Quantity



③ **Conference Interval for** $\dfrac{\sigma_1^2}{\sigma_2^2}$

$$1-\alpha = P(F_{n_1-1,n_2-1,L} \le F \le F_{n_1-1,n_2-1,U}) = P\left(F_L \le \frac{\dfrac{S_1^2}{S_2^2}}{\dfrac{\sigma_1^2}{\sigma_2^2}} \le F_U\right)$$

$$= P\left(\frac{\dfrac{S_1^2}{S_2^2}}{F_U} \le \frac{\sigma_1^2}{\sigma_2^2} \le \frac{\dfrac{S_1^2}{S_2^2}}{F_L}\right)$$

④ **F-test: $H_0: \sigma_1^2 = \sigma_2^2$ (or equivalently: $H_0: \frac{\sigma_1^2}{\sigma_2^2} = 1$)**

Test Statistic:

$$F_0 = \frac{S_1^2}{S_2^2} = \overset{H_0}{\sim} F_{n_1-1, n_2-1}$$

a) $\begin{cases} H_0 : \dfrac{\sigma_1^2}{\sigma_2^2} = 1 \\ H_a : \dfrac{\sigma_1^2}{\sigma_2^2} \neq 1 \end{cases}$

At the significance level α, we will reject H_0 in favor of H_a iff $F_0 \geq F_{n_1-1, n_2-1, \alpha/2, upper}$ or $F_0 \leq F_{n_1-1, n_2-1, \alpha/2, lower}$

b) $\begin{cases} H_0 : \dfrac{\sigma_1^2}{\sigma_2^2} = 1 \\ H_a : \dfrac{\sigma_1^2}{\sigma_2^2} > 1 \end{cases}$ Reject H_0 iff

$F_0 \geq F_{n_1-1, n_2-1, \alpha, upper}$

c) $\begin{cases} H_0 : \dfrac{\sigma_1^2}{\sigma_2^2} = 1 \\ H_a : \dfrac{\sigma_1^2}{\sigma_2^2} < 1 \end{cases}$ Reject H_0 iff

$F_0 \leq F_{n_1-1, n_2-1, \alpha, lower}$

5) A trick for the F distribution

If $F \sim F_{k_1, k_2}$**, then** $\frac{1}{F} \sim F_{k_2, k_1}$

Some F-table only gives the upper bound. If we know $F_{k_1, k_2, \alpha, U}$ **, how to find** $F_{k_1, k_2, \alpha, L}$**?**

$$\alpha = P(F \leq F_{k_1, k_2, \alpha, L}) = P(\frac{1}{F} \geq F_{k_2, k_1, \alpha, U}) = P(\frac{1}{F} \geq \frac{1}{F_{k_1, k_2, \alpha, L}})$$

$$\because \frac{1}{F_{k_1, k_2, \alpha, L}} = F_{k_2, k_1, \alpha, U}$$

Example 2. **Independent Samples T-Test.** An experiment was conducted to compare the mean number of tapeworms in the stomachs of sheep that had been treated for worms against the mean number in those that were untreated. A sample of 14 worm-infected lambs was randomly divided into 2 groups. Seven were injected with the drug and the remainders were left untreated. After a 6-month period, the lambs were slaughtered and the following worm counts were recorded:

Drug-treated sheep	18 43 28 50 16 32 13
Untreated sheep	40 54 26 63 21 37 39

(a). Test at $\alpha = 0.05$ whether the treatment is effective or not.
(b) What assumptions do you need for the inference in part (a)?
(c). Please write up the entire R program necessary to answer questions raised in (a) and (b).
SOLUTION: Inference on two population means. Two small and independent samples.
Drug-treated sheep: $\overline{X}_1 = 28.57$, $s_1^2 = 198.62$, $n_1 = 7$
Untreated sheep: $\overline{X}_2 = 40.0$, $s_2^2 = 215.33$, $n_2 = 7$

(a) Under the normality assumption, we first test if the two population variances are equal. That is, $H_0 : \sigma_1^2 = \sigma_2^2$ versus $H_a : \sigma_1^2 \neq \sigma_2^2$. The test statistic is

$$F_0 = \frac{s_1^2}{s_2^2} = \frac{198.62}{215.33} \approx 0.92, \quad F_{6,6,0.05,U} = 4.28 \text{ and}$$

$$F_{6,6,0.05,L} = 1/4.28 \approx 0.23.$$

Since F_0 is between 0.23 and 4.28, we cannot reject H_0. Therefore it is reasonable to assume that $\sigma_1^2 = \sigma_2^2$.
Next we perform the pooled-variance t-test with hypotheses
$H_0 : \mu_1 - \mu_2 = 0$ versus $H_a : \mu_1 - \mu_2 < 0$

$$t_0 = \frac{\overline{X}_1 - \overline{X}_2 - 0}{s_p\sqrt{\frac{1}{n} + \frac{1}{n_2}}} = \frac{(28.57 - 40.0) - 0}{14.39\sqrt{\frac{1}{7} + \frac{1}{7}}} \approx -1.49$$

Since $t_0 \approx -1.49$ is greater than $-t_{12,0.05} = -1.782$, we cannot reject H_0. We have insufficient evidence to reject the hypothesis that there is no difference in the mean number of worms in treated and untreated lambs.
(b) (1) Both populations are normally distributed
(2) $\sigma_1^2 = \sigma_2^2$
#Example 2 (c) in R#

```
sheep1<-Drug.treated.sheep<-c(18,43,28,50,16,32,13)
sheep2<-Untreated.sheep<-c(40,54,26,63,21,37,39)
#Normality test for each population#
shapiro.test(sheep1) #p=0.5142#
shapiro.test(sheep2) #p=0.7515#
#F-test for equal variances#
var.test(sheep1,sheep2)
#F-test shows the ratio of two variances should equal to 1#
t.test(sheep1,sheep2,var.equal=TRUE,a="l")
#var.qual=TRUE:two variances are equal, p=0.08152#
#wilcox.test(sheep1,sheep2,conf.int=TRUE ,a="l")
#P= 0.1043, and hence we think there is#
#Both the t.test and Wilcoxon rank sum test show that we have insufficient evidence to reject the hypothesis that there is no difference in the mean number of worms in treated and untreated lambs#
```

Review: Independent samples t-test

http://ww2.coastal.edu/kingw/statistics/R-tutorials/independent-t.html

As part of his senior research project in the Fall semester of 2001, Scott Keats looked for a possible relationship between marijuana smoking and a deficit in performance on a task measuring short term memory--the digit span task from the Wechsler Adult Intelligence Scale. Two groups of ten subjects were tested. One group, the "nonsmokers," claimed not to smoke marijuana. A second group, the "smokers," claimed to smoke marijuana regularly. The data set is small and easily entered by hand...

```
#enter data#
nonsmokers = c(18,22,21,17,20,17,23,20,22,21)
smokers = c(16,20,14,21,20,18,13,15,17,21)
#It is nice to learn how to draw the side-by-side box plots#
boxplot(nonsmokers, smokers,
ylab="Scores on Digit Span Task",
names=c("nonsmokers","smokers"),
main="Digit Span Performance by\n Smoking Status")
shapiro.test(nonsmokers)
shapiro.test(smokers)
var.test(nonsmokers, smokers)

#By default, R will perform the unspooled variance t-test, also called the Welch Two Sample t-test#
t.test(nonsmokers,smokers)

#One has to specify equal variances in order to perform the pooled-variance t-test#
t.test(nonsmokers,smokers,alternative="greater",var.equal=T)
```

Example 3: Wilcoxon Rank Sum Test

A new drug for reducing blood pressure (BP) is compared to an old drug. 20 patients with comparable high BP were recruited and randomized evenly to the 2 drugs. Reductions in BP after 1 month of taking the drugs are as follows:

New drug: 0, 10, -3, 15, 2, 27, 19, 21, 18, 10

Old drug: 8, -4, 7, 5, 10, 11, 9, 12, 7, 8

Please test at α = 0.05 whether the new drug is better than the old one.

Solution:

① n_1=10, $\bar{X}$=11.9, S_1^2=97.43, n_2=10, $\bar{Y}$=7.3, S_2^2 =20.01

F test: $\begin{cases} H_0: \sigma_1^2 = \sigma_2^2 \\ H_a: \sigma_1^2 \neq \sigma_2^2 \end{cases}$

Test statistic: $F_0 = \dfrac{S_1^2}{S_2^2} = 4.87 > F_{10,10,0.25,U}$

∴ At α=0.05, reject H_0 and use unpooled-variance t-test

T-test: $\begin{cases} H_0: \mu_1 - \mu_2 = 0 \\ H_a: \mu_1 - \mu_2 > 0 \end{cases}$

Test statistic: T_0 =

$$\frac{\bar{X} - \bar{Y}}{\sqrt{\dfrac{S_1^2}{n_1} + \dfrac{S_2^2}{n_2}}} = 1.34 < t_{df=12.547,0.05} \approx 1.776$$

∴ At α =0.05, fail to reject H_0 and we cannot say that the new drug is better than the old one.

```
#Example3#
Nd<-New.drug<-c(0,10,-3,15,2,27,19,21,18,10)
Od<-Old.drug<-c(8,-4,7,5,10,11,9,12,7,8)
shapiro.test(Nd) #p=0.7339#
```

shapiro.test(Od) #p=0.01674#
#Hence we conclude that the population distribution of old drugs (Od) is not a normal distribution#
wilcox.test(Nd,Od)
wilcox.test(Nd,Od,alternative="g")
#p=0.1849/2>0.05, so we cannot conclude that the new drug is better than the old one using the nonparametric Wilcoxon Rank Sum test#

Note: As you can see from this example, the normality test is very important – indeed we should not have used the unspooled variance t-test because the normality assumption is not satisfied in this problem

Two sample *t*-test

The two-sample t test is used to test the hypothesis that two samples may be assumed to come from distributions with the same mean.
The theory for the two-sample t test is not very different in principle from that of the one-sample test. Data are now from two groups, x11,,x12,, …,x1n and x21,x22, …, x2n, which we assume are sampled from the normal distributions N(u1,sigma1) and N(u2,sigma2), and it is desired to test the null hypothesis H0: u1=u2. You then calculate t=(x2-x1)/SEM
where SEM is the standard error of the mean.

Review:
Wilcoxon Two-Sample Test (Wilcoxon Rank Sum Test), also referred to as the Mann-Whitney U Test

```
# Wilcoxon Rank Sum Test
wilcox.test(y~A)
# where y is numeric and A is A binary factor

# Wilcoxon Rank Sum Test
wilcox.test(y,x)
# where y and x are numeric
```

How to comment in R:

A hash (#) anywhere on a line will comment out the rest of the line.
Try!

```
data(energy)
attach(energy)
energy
t.test(expend~stature)
t.test(expend~stature,var.equal=T)
```

Comparison of Variances:

```
var.test(expend~stature)
```

Two-sample Wilcoxon test:

```
wilcox.test(expend~stature)
```

Example 0:

```
> ret<-
read.table("~/Desktop/d_logret_6stocks.txt",
header=T)
 > attach(ret)
 > res3<-t.test(Pfizer, Intel)
 > names(res3)
 > res3$stat
 > t.test(Pfizer, Intel, var.equal=T)
```

Exercise:
(1) Perform for 'Citigroup' one sample test with the null hypothesis that the mean is zero. (2) Perform the Wilcoxon signed-rank test for 'Citigroup'.
(3) Perform the two-sample test for 'Pfizer' and 'Citigroup'.

Comparison of Variances:

```
 > var.test(Pfizer, Intel)
```

Two-sample Wilcoxon test:

```
 > wilcox.test(Pfizer, Intel)
```

<u>Exercise 1.</u> An agricultural experiment station was interested in comparing the yields for two new varieties of corn. Because the investigators thought that there might be a great deal of variability in yield from one field to another, each variety was randomly assigned to a different 1-acre plot on each of seven farms. The 1-acre plots were planted; the corn was harvested at maturity. The results of the experiment (in bushels of corn) are listed here. Use these data to test the null hypothesis that there is no difference in mean yields for the two varieties of corn. Use $\alpha = .05$.

Farm	1	2	3	4	5	6	7
Variety A	48.2	44.6	49.7	40.5	54.6	47.1	51.4
Variety B	41.5	40.1	44.0	41.2	49.8	41.7	46.8

```
#Exercise1#
df<-data.frame(
Farm=c(1,2,3,4,5,6,7),VarietyA=c(48.2,44.6,49.7,40.5,54.6,47.1,5
1.4),VarietyB=c(41.5,40.1,44.0,41.2,49.8,41.7,46.8)
);
t(df)
#transpose the data frame#
diff=df[,2]-df[,3];diff
#Get the data ready for tests#
shapiro.test(diff)
#p=0.01498<0.05, which means diff does not follow normal
distribution#
#Hence, we cannot use t-test#
wilcox.test(diff)
#p=0.03125, which suggests wo should reject null hypothesis#
#and think there is differences in mean yields for the two varieties
of corn#
stem(diff,scale=2)
boxplot(diff)
qqnorm(diff)
qqline(diff)
#ususally we use qqnorm and qqline together#
#Also by q-qplot, we cannot think diff follows normal
distribution#
```

Exercise 2. A pollution-control inspector suspected that a riverside community was releasing semi-treated sewage into a river and this, as a consequence, was changing the level of dissolved oxygen of the river. To check this, he drew 5 randomly selected specimens of river water at a location above the town and another 5 specimens below. The dissolved oxygen readings, in parts per million, are given in the accompanying table. Do the data provide sufficient evidence to indicate a difference in mean oxygen content between locations above and below the town? Use $\alpha = .05$.

Above town	4.8	5.2	5.0	4.9	5.1
Below town	5.0	4.7	4.9	4.8	4.9

```
#Exercise2#
At<-Above.town<-c(4.8,5.2,5.0,4.9,5.1)
Bt<-Below.town<-c(5.0,4.7,4.9,4.8,4.9)
shapiro.test(At)
shapiro.test(Bt)
stem(At,scale=2)
boxplot(At)
qqnorm(At)
qqline(At)
stem(Bt,scale=2)
boxplot(Bt)
qqnorm(Bt)
qqline(Bt)
#The results from shapiro.test and output images show At and Bt
follow normal distribution#
#Hence, we can use t.test#
var.test(At,Bt) #F-test#
#p=0.5421, so the variances of At and Bt are equal#
t.test(At,Bt,var.qual=TRUE)
#p=0.1507>0.05#
#Hence, we should accept null hypothesis that there is no
differences between At and Bt#
```