CS280 Spring 2023 Assignment 2
Part A

Convolutional Neural Network

March 14, 2023

Name: Bingnan Li

Student ID: 2020533092

1. Convolution Cost (10 points)

Assume an input of shape $c_i \times h \times w$ and a convolution kernel of shape $c_o \times c_i \times k_h \times k_w$, padding of (p_h, p_w) , and stride of (s_h, s_w) .

- What is the computational cost (multiplications and additions) for the forward propagation?
  Solution:

  The output size has the formula as follows:

$$c_{out} = \#\{kernels\}$$

$$h_{out} = \left\lfloor \frac{h_{in} + 2 \times padding\_size_h - (kernel\_size_h - 1) - 1}{stride\_size_h} + 1 \right\rfloor$$

$$w_{out} = \left\lfloor \frac{w_{in} + 2 \times padding\_size_w - (kernel\_size_w - 1) - 1}{stride\_size_w} + 1 \right\rfloor$$

  Substitute $h_{in}, w_{in}, padding\_size_h, padding\_size_w, stride\_size_h, stride\_size_w, kernel\_size_h$ and $kernel\_size_w$ with $h, w, p_h, p_w, s_h, s_w, k_h$ and k_w respectively, the output size is

$$c_{out} = c_0$$

$$h_{out} = \left\lfloor \frac{h + 2p_h - k_h}{s_h} + 1 \right\rfloor$$

$$w_{out} = \left\lfloor \frac{w + 2p_w - k_w}{s_w} + 1 \right\rfloor$$

  Then for each element in output, it is calculated by $c_i \times k_h \times k_w$ times multiplications and $c_i \times k_h \times k_w - 1$ times additions. Thus, the computational cost is

$$cost = c_0 \left\lfloor \frac{h + 2p_h - k_h}{s_h} + 1 \right\rfloor \left\lfloor \frac{w + 2p_w - k_w}{s_w} + 1 \right\rfloor (2c_i k_h k_w - 1)$$

- What is the memory footprint?

2. Convolution Kernel (10 points)

Assume there are two convolution kernels of size k_1 and k_2 respectively (with no nonlinear activation function in-between).

- Prove that the results of the two convolution operations can be expressed by a single convolution operation.
- What is the dimensionality of the equivalent single convolution?
- Is the converse true, i.e., Can a convolution operation be decomposed into two smaller convolution operations?