

# CS280 Fall 2022 Assignment 1

## Part A

Basics & MLP

February 25, 2023

**Name: Bingnan Li**

**Student ID: 2020533092**

**1. Gradient descent for fitting GMM (10 points).**

Consider the Gaussian mixture model

$$p(\mathbf{x}|\theta) = \sum_{k=1}^K \pi_k \mathcal{N}(\mathbf{x}|\boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k)$$

where  $\pi_j \geq 0, \sum_{j=1}^K \pi_j = 1$ . (Assume  $\mathbf{x}, \boldsymbol{\mu}_k \in \mathbb{R}^d, \boldsymbol{\Sigma}_k \in \mathbb{R}^{d \times d}$ )

Define the log likelihood as

$$l(\theta) = \sum_{n=1}^N \log p(\mathbf{x}_n|\theta)$$

Denote the posterior responsibility that cluster  $k$  has for datapoint  $n$  as follows:

$$r_{nk} := p(z_n = k|\mathbf{x}_n, \theta) = \frac{\pi_k \mathcal{N}(\mathbf{x}_n|\boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k)}{\sum_{k'} \pi_{k'} \mathcal{N}(\mathbf{x}_n|\boldsymbol{\mu}_{k'}, \boldsymbol{\Sigma}_{k'})}$$

(a) Show that the gradient of the log-likelihood wrt  $\boldsymbol{\mu}_k$  is

$$\frac{d}{d\boldsymbol{\mu}_k} l(\theta) = \sum_n r_{nk} \boldsymbol{\Sigma}_k^{-1} (\mathbf{x}_n - \boldsymbol{\mu}_k)$$

**Proof.** By the *chain rule*, we have

$$\begin{aligned} \frac{\partial l(\theta)}{\partial \boldsymbol{\mu}_k} &= \sum_{n=1}^N \frac{1}{p(\mathbf{x}_n|\theta)} \frac{\partial p(\mathbf{x}_n|\theta)}{\partial \boldsymbol{\mu}_k} \\ &= \sum_{n=1}^N \frac{1}{p(\mathbf{x}_n|\theta)} \frac{\partial \pi_k \mathcal{N}(\mathbf{x}_n|\boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k)}{\partial \boldsymbol{\mu}_k} \\ &= \sum_{n=1}^N \frac{\pi_k \mathcal{N}(\mathbf{x}_n|\boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k)}{p(\mathbf{x}_n|\theta)} \frac{\partial (-\frac{1}{2}(\mathbf{x}_n - \boldsymbol{\mu}_k)^T \boldsymbol{\Sigma}^{-1} (\mathbf{x}_n - \boldsymbol{\mu}_k))}{\partial \boldsymbol{\mu}_k} \\ &= \sum_{n=1}^N r_{nk} \boldsymbol{\Sigma}^{-1} (\mathbf{x}_n - \boldsymbol{\mu}_k) \end{aligned}$$

■

(b) Derive the gradient of the log-likelihood wrt  $\pi_k$  without considering any constraint on  $\pi_k$ . (bonus 2 points: with constraint  $\sum_k \pi_k = 1$ .)

**Proof.** For the case without any constraint on  $\pi_k$ , by the *chain rule*, we have:

$$\begin{aligned} \frac{\partial l(\theta)}{\partial \pi_k} &= \sum_{n=1}^N \frac{1}{p(\mathbf{x}_n|\theta)} \frac{\partial p(\mathbf{x}_n|\theta)}{\partial \pi_k} \\ &= \sum_{n=1}^N \frac{1}{p(\mathbf{x}_n|\theta)} \mathcal{N}(\mathbf{x}_n|\boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k) \\ &= \sum_{n=1}^N \frac{r_{nk}}{\pi_k} \end{aligned}$$

■

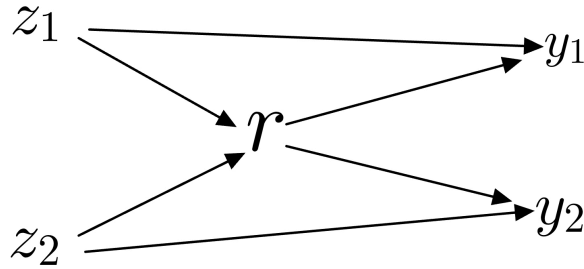
## 2. Softmax & Computation Graph (10 points).

Recall that the softmax function takes in a vector  $(z_1, \dots, z_D)$  and returns a vector  $(y_1, \dots, y_D)$ . We can express it in the following form:

$$r = \sum_j e^{z_j} \quad y = \frac{e^{z_j}}{r}$$

(a) Consider  $D = 2$ , i.e. just two inputs and outputs to the softmax. Draw the computation graph relating  $z_1$ ,  $z_2$ ,  $r$ ,  $y_1$ , and  $y_2$ .

**Solution:**



(b) Determine the backprop updates for computing the  $\bar{z}_j$  when given the  $\bar{y}_i$ . You need to justify your answer. (You may give your answer either for  $D = 2$  or for the more general case.)

**Solution:**

By the *chain rule* and the forward precess, we have the backprop updates as follows:

$$\begin{aligned} \bar{r} &= \sum_{i=1}^D \bar{y}_i \frac{\partial y_i}{\partial r} = - \sum_{i=1}^D \bar{y}_i \frac{e^{z_i}}{r^2} \\ \bar{z}_i &= \bar{y}_i \frac{\partial y_i}{\partial z_i} + \bar{r} \frac{\partial r}{\partial z_i} = \bar{y}_i \frac{e^{z_i}}{r} + \bar{r} e^{z_i} \end{aligned}$$

(c) Write a function to implement the vector-Jacobian product (VJP) for the softmax function based on your answer from part (b). For efficiency, it should operate on a mini-batch. The inputs are:

- a matrix  $\mathbf{Z}$  of size  $N \times D$  giving a batch of input vectors.  $N$  is the batch size and  $D$  is the number of dimensions. Each row gives one input vector  $z = (z_1, \dots, z_D)$ .
- A matrix  $\mathbf{Y}_{\text{bar}}$  giving the output error signals. It is also  $N \times D$

The output should be the error signal  $\mathbf{Z}_{\text{bar}}$ . Do not use a for loop.

# perceptron

February 24, 2023

## 0.1 Perceptron Learning Algorithm

The perceptron is a simple supervised machine learning algorithm and one of the earliest neural network architectures. It was introduced by Rosenblatt in the late 1950s. A perceptron represents a binary linear classifier that maps a set of training examples (of  $d$  dimensional input vectors) onto binary output values using a  $d - 1$  dimensional hyperplane. But Today, we will implement **Multi-Class Perceptron Learning Algorithm** Given: \* dataset  $\{(x^i, y^i)\}$ ,  $i \in (1, M)$  \*  $x^i$  is  $d$  dimension vector,  $x^i = (x_1^i, \dots, x_d^i)$  \*  $y^i$  is multi-class target variable  $y^i \in \{0, 1, 2\}$

A perceptron is trained using gradient descent. The training algorithm has different steps. In the beginning (step 0) the model parameters are initialized. The other steps (see below) are repeated for a specified number of training iterations or until the parameters have converged.

**Step0:** Initial the weight vector and bias with zeros

**Step1:** Compute the linear combination of the input features and weight.  $y_{pred}^i = \arg \max_k W_k * x^i + b$

**Step2:** Compute the gradients for parameters  $W_k$ ,  $b$ . **Derive the parameter update equation Here (5 points)**

TODO: Derive you answer hear #####

$$\Delta W_k = \begin{cases} 0, & k = y_{pred}^i = y^i \\ x^i, & k = y_{pred}^i \neq y^i \end{cases} \text{ and } W_k^{new} = W_k^{old} - \eta \Delta W_k$$

$$\Delta b = \begin{cases} 0, & k = y_{pred}^i = y^i \\ 1, & k = y_{pred}^i \neq y^i \end{cases} \text{ and } b^{new} = b^{old} - \eta \Delta b$$

```
[106]: from sklearn import datasets
import numpy as np
# from sklearn.cross_validation import train_test_split
from sklearn.model_selection import train_test_split
import matplotlib.pyplot as plt
import random

np.random.seed(0)
random.seed(0)
```

```
[107]: iris = datasets.load_iris()
X = iris.data
print(type(X))
```

```

y = iris.target
y = np.array(y)
print('X_Shape:', X.shape)
print('y_Shape:', y.shape)
print('Label Space:', np.unique(y))

```

```

<class 'numpy.ndarray'>
X_Shape: (150, 4)
y_Shape: (150,)
Label Space: [0 1 2]

```

```

[108]: ## split the training set and test set
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.3,
↳ random_state=0)
print('X_train_Shape:', X_train.shape)
print('X_test_Shape:', X_test.shape)
print('y_train_Shape:', y_train.shape)
print('y_test_Shape:', y_test.shape)

print(type(y_train))

```

```

X_train_Shape: (105, 4)
X_test_Shape: (45, 4)
y_train_Shape: (105,)
y_test_Shape: (105,)
<class 'numpy.ndarray'>

```

```

[109]: class MultiClsPLA(object):

    ## We recommend to absorb the bias into weight. W = [w, b]

    def __init__(self, X_train, y_train, X_test, y_test, lr, num_epoch,
↳ weight_dimension, num_cls):
        super(MultiClsPLA, self).__init__()
        self.X_train = X_train # N x (D + 1)
        self.y_train = y_train # N x 1
        self.X_test = X_test
        self.y_test = y_test
        self.weight = self.initial_weight(weight_dimension, num_cls) # C x (D
↳ + 1)

        self.sample_mean = np.mean(self.X_train, 0)
        self.sample_std = np.std(self.X_train, 0)
        self.num_epoch = num_epoch
        self.lr = lr
        self.total_acc_train = []
        self.total_acc_tst = []

```

```

def initial_weight(self, weight_dimension, num_cls):
    weight = None
    #####
    ## TODO: Initialize the weight with ##
    ## small std and zero mean gaussian ##
    #####
    weight = np.random.normal(0, 0.01, (num_cls, weight_dimension))

    return weight

def data_preprocessing(self, data):
    #####
    ## TODO: Normalize the data ##
    #####
    norm_data = (data - self.sample_mean) / self.sample_std
    return norm_data

def train_step(self, X_train, y_train, shuffle_idx):
    np.random.shuffle(shuffle_idx)
    X_train: np.ndarray = X_train[shuffle_idx]
    y_train: np.ndarray = y_train[shuffle_idx]
    train_acc = None
    #####
    ## TODO: to implement the training process ##
    ## and update the weights ##
    #####
    y_pred = np.argmax(X_train @ self.weight.T, axis=0)

    train_acc = np.sum(y_pred == y_train) / y_train.size()
    self.total_acc_train.append(train_acc)
    diff_one_hot = np.eye(X_train.shape[0], self.weight.shape[0])[y !=
↪y_pred] # N x C
    self.weight -= self.lr * diff_one_hot.T @ X_train

    return train_acc

def test_step(self, X_test, y_test):
    X_test = self.data_preprocessing(data=X_test)
    num_sample = X_test.shape[0]
    test_acc = None

    #####
    ## TODO: Evaluate the test set and ##
    ## return the test acc ##
    #####

    y_pred = np.argmax(X_test @ self.weight.T, axis=0)

```

```

        test_acc = np.sum(y_pred == y_test) / y_test.size()
        self.total_acc_tst.append(test_acc)

    return test_acc

def train(self):
    self.X_train = self.data_preprocessing(data=self.X_train)
    num_sample = self.X_train.shape[0]

    #####
    ### TODO: In order to absorb the bias into weights ###
    ### we need to modify the input data. ###
    ### So You need to transform the input data ###
    #####

    self.X_train = np.insert(self.X_train, self.X_train.shape[1], 1)
    self.X_test = np.insert(self.X_test, self.X_test.shape[1], 1)

    shuffle_index = np.array(range(0, num_sample))
    for epoch in range(self.num_epoch):
        training_acc = self.train_step(X_train=self.X_train, y_train=self.
→y_train, shuffle_idx=shuffle_index)
        tst_acc = self.test_step(X_test=self.X_test, y_test=self.y_test)
        self.total_acc_train.append(training_acc)
        self.total_acc_tst.append(tst_acc)
        print('epoch:', epoch, 'traing_acc:%.3f' % training_acc, 'tst_acc:%.
→3f' % tst_acc)

    def vis_acc_curve(self):
        train_acc = np.array(self.total_acc_train)
        tst_acc = np.array(self.total_acc_tst)
        plt.plot(train_acc)
        plt.plot(tst_acc)
        plt.legend(['train_acc', 'tst_acc'])
        plt.show()

```

```

[110]: np.random.seed(0)
        random.seed(0)
        #####
        ### TODO:
        ### 1. You need to import the model and pass some parameters.
        ### 2. Then training the model with some epoches.
        ### 3. Visualize the training acc and test acc versus epoches
        perceptronModel = MultiClsPLA(X_train, y_train, X_test, y_test, lr=1e-3,
→num_epoch=1000,
                                     weight_dimension=X_train.shape[1] + 1, num_cls=3)
        perceptronModel.train()

```

```
fig = plt.figure()
plt.plot(perceptronModel.total_acc_train)
plt.plot(perceptronModel.total_acc_tst)
```

```
epoch: 0 traing_acc:0.029 tst_acc:0.022
epoch: 1 traing_acc:0.029 tst_acc:0.022
epoch: 2 traing_acc:0.029 tst_acc:0.022
epoch: 3 traing_acc:0.029 tst_acc:0.022
epoch: 4 traing_acc:0.029 tst_acc:0.022
epoch: 5 traing_acc:0.029 tst_acc:0.022
epoch: 6 traing_acc:0.029 tst_acc:0.022
epoch: 7 traing_acc:0.029 tst_acc:0.022
epoch: 8 traing_acc:0.029 tst_acc:0.022
epoch: 9 traing_acc:0.029 tst_acc:0.022
epoch: 10 traing_acc:0.029 tst_acc:0.022
epoch: 11 traing_acc:0.029 tst_acc:0.044
epoch: 12 traing_acc:0.038 tst_acc:0.044
epoch: 13 traing_acc:0.038 tst_acc:0.044
epoch: 14 traing_acc:0.038 tst_acc:0.044
epoch: 15 traing_acc:0.038 tst_acc:0.044
epoch: 16 traing_acc:0.038 tst_acc:0.044
epoch: 17 traing_acc:0.038 tst_acc:0.044
epoch: 18 traing_acc:0.038 tst_acc:0.044
epoch: 19 traing_acc:0.038 tst_acc:0.044
epoch: 20 traing_acc:0.038 tst_acc:0.044
epoch: 21 traing_acc:0.048 tst_acc:0.044
epoch: 22 traing_acc:0.048 tst_acc:0.044
epoch: 23 traing_acc:0.048 tst_acc:0.067
epoch: 24 traing_acc:0.048 tst_acc:0.067
epoch: 25 traing_acc:0.048 tst_acc:0.067
epoch: 26 traing_acc:0.048 tst_acc:0.067
epoch: 27 traing_acc:0.048 tst_acc:0.067
epoch: 28 traing_acc:0.067 tst_acc:0.067
epoch: 29 traing_acc:0.076 tst_acc:0.089
epoch: 30 traing_acc:0.076 tst_acc:0.089
epoch: 31 traing_acc:0.076 tst_acc:0.089
epoch: 32 traing_acc:0.076 tst_acc:0.089
epoch: 33 traing_acc:0.105 tst_acc:0.133
epoch: 34 traing_acc:0.152 tst_acc:0.244
epoch: 35 traing_acc:0.229 tst_acc:0.333
epoch: 36 traing_acc:0.381 tst_acc:0.378
epoch: 37 traing_acc:0.410 tst_acc:0.400
epoch: 38 traing_acc:0.495 tst_acc:0.467
epoch: 39 traing_acc:0.524 tst_acc:0.489
epoch: 40 traing_acc:0.562 tst_acc:0.489
epoch: 41 traing_acc:0.590 tst_acc:0.511
```



epoch: 42 traing\_acc:0.610 tst\_acc:0.533  
epoch: 43 traing\_acc:0.619 tst\_acc:0.533  
epoch: 44 traing\_acc:0.638 tst\_acc:0.533  
epoch: 45 traing\_acc:0.648 tst\_acc:0.533  
epoch: 46 traing\_acc:0.629 tst\_acc:0.533  
epoch: 47 traing\_acc:0.648 tst\_acc:0.533  
epoch: 48 traing\_acc:0.667 tst\_acc:0.556  
epoch: 49 traing\_acc:0.667 tst\_acc:0.556  
epoch: 50 traing\_acc:0.667 tst\_acc:0.578  
epoch: 51 traing\_acc:0.676 tst\_acc:0.600  
epoch: 52 traing\_acc:0.676 tst\_acc:0.622  
epoch: 53 traing\_acc:0.686 tst\_acc:0.622  
epoch: 54 traing\_acc:0.705 tst\_acc:0.622  
epoch: 55 traing\_acc:0.695 tst\_acc:0.622  
epoch: 56 traing\_acc:0.705 tst\_acc:0.622  
epoch: 57 traing\_acc:0.695 tst\_acc:0.622  
epoch: 58 traing\_acc:0.705 tst\_acc:0.622  
epoch: 59 traing\_acc:0.714 tst\_acc:0.644  
epoch: 60 traing\_acc:0.686 tst\_acc:0.644  
epoch: 61 traing\_acc:0.714 tst\_acc:0.644  
epoch: 62 traing\_acc:0.695 tst\_acc:0.644  
epoch: 63 traing\_acc:0.705 tst\_acc:0.667  
epoch: 64 traing\_acc:0.695 tst\_acc:0.667  
epoch: 65 traing\_acc:0.695 tst\_acc:0.644  
epoch: 66 traing\_acc:0.695 tst\_acc:0.644  
epoch: 67 traing\_acc:0.695 tst\_acc:0.644  
epoch: 68 traing\_acc:0.695 tst\_acc:0.644  
epoch: 69 traing\_acc:0.695 tst\_acc:0.644  
epoch: 70 traing\_acc:0.695 tst\_acc:0.644  
epoch: 71 traing\_acc:0.695 tst\_acc:0.644  
epoch: 72 traing\_acc:0.695 tst\_acc:0.644  
epoch: 73 traing\_acc:0.705 tst\_acc:0.644  
epoch: 74 traing\_acc:0.705 tst\_acc:0.644  
epoch: 75 traing\_acc:0.705 tst\_acc:0.644  
epoch: 76 traing\_acc:0.705 tst\_acc:0.644  
epoch: 77 traing\_acc:0.705 tst\_acc:0.644  
epoch: 78 traing\_acc:0.705 tst\_acc:0.644  
epoch: 79 traing\_acc:0.705 tst\_acc:0.644  
epoch: 80 traing\_acc:0.705 tst\_acc:0.644  
epoch: 81 traing\_acc:0.705 tst\_acc:0.667  
epoch: 82 traing\_acc:0.705 tst\_acc:0.667  
epoch: 83 traing\_acc:0.705 tst\_acc:0.667  
epoch: 84 traing\_acc:0.695 tst\_acc:0.644  
epoch: 85 traing\_acc:0.695 tst\_acc:0.644  
epoch: 86 traing\_acc:0.705 tst\_acc:0.644  
epoch: 87 traing\_acc:0.686 tst\_acc:0.644  
epoch: 88 traing\_acc:0.695 tst\_acc:0.644  
epoch: 89 traing\_acc:0.695 tst\_acc:0.644

epoch: 90 traing\_acc:0.695 tst\_acc:0.644  
epoch: 91 traing\_acc:0.686 tst\_acc:0.644  
epoch: 92 traing\_acc:0.695 tst\_acc:0.644  
epoch: 93 traing\_acc:0.695 tst\_acc:0.644  
epoch: 94 traing\_acc:0.705 tst\_acc:0.644  
epoch: 95 traing\_acc:0.695 tst\_acc:0.644  
epoch: 96 traing\_acc:0.705 tst\_acc:0.644  
epoch: 97 traing\_acc:0.686 tst\_acc:0.644  
epoch: 98 traing\_acc:0.686 tst\_acc:0.644  
epoch: 99 traing\_acc:0.714 tst\_acc:0.644  
epoch: 100 traing\_acc:0.686 tst\_acc:0.644  
epoch: 101 traing\_acc:0.695 tst\_acc:0.644  
epoch: 102 traing\_acc:0.686 tst\_acc:0.622  
epoch: 103 traing\_acc:0.695 tst\_acc:0.622  
epoch: 104 traing\_acc:0.705 tst\_acc:0.622  
epoch: 105 traing\_acc:0.714 tst\_acc:0.622  
epoch: 106 traing\_acc:0.714 tst\_acc:0.622  
epoch: 107 traing\_acc:0.714 tst\_acc:0.622  
epoch: 108 traing\_acc:0.714 tst\_acc:0.622  
epoch: 109 traing\_acc:0.714 tst\_acc:0.622  
epoch: 110 traing\_acc:0.714 tst\_acc:0.622  
epoch: 111 traing\_acc:0.705 tst\_acc:0.622  
epoch: 112 traing\_acc:0.714 tst\_acc:0.622  
epoch: 113 traing\_acc:0.705 tst\_acc:0.644  
epoch: 114 traing\_acc:0.705 tst\_acc:0.644  
epoch: 115 traing\_acc:0.714 tst\_acc:0.644  
epoch: 116 traing\_acc:0.714 tst\_acc:0.644  
epoch: 117 traing\_acc:0.705 tst\_acc:0.644  
epoch: 118 traing\_acc:0.724 tst\_acc:0.644  
epoch: 119 traing\_acc:0.714 tst\_acc:0.644  
epoch: 120 traing\_acc:0.714 tst\_acc:0.644  
epoch: 121 traing\_acc:0.705 tst\_acc:0.644  
epoch: 122 traing\_acc:0.724 tst\_acc:0.644  
epoch: 123 traing\_acc:0.724 tst\_acc:0.644  
epoch: 124 traing\_acc:0.733 tst\_acc:0.644  
epoch: 125 traing\_acc:0.714 tst\_acc:0.644  
epoch: 126 traing\_acc:0.724 tst\_acc:0.644  
epoch: 127 traing\_acc:0.733 tst\_acc:0.644  
epoch: 128 traing\_acc:0.724 tst\_acc:0.644  
epoch: 129 traing\_acc:0.714 tst\_acc:0.644  
epoch: 130 traing\_acc:0.733 tst\_acc:0.644  
epoch: 131 traing\_acc:0.733 tst\_acc:0.644  
epoch: 132 traing\_acc:0.695 tst\_acc:0.644  
epoch: 133 traing\_acc:0.733 tst\_acc:0.644  
epoch: 134 traing\_acc:0.733 tst\_acc:0.644  
epoch: 135 traing\_acc:0.724 tst\_acc:0.644  
epoch: 136 traing\_acc:0.705 tst\_acc:0.644  
epoch: 137 traing\_acc:0.724 tst\_acc:0.644

epoch: 138 traing\_acc:0.714 tst\_acc:0.667  
epoch: 139 traing\_acc:0.724 tst\_acc:0.667  
epoch: 140 traing\_acc:0.705 tst\_acc:0.667  
epoch: 141 traing\_acc:0.724 tst\_acc:0.667  
epoch: 142 traing\_acc:0.705 tst\_acc:0.667  
epoch: 143 traing\_acc:0.724 tst\_acc:0.689  
epoch: 144 traing\_acc:0.705 tst\_acc:0.689  
epoch: 145 traing\_acc:0.724 tst\_acc:0.689  
epoch: 146 traing\_acc:0.705 tst\_acc:0.689  
epoch: 147 traing\_acc:0.733 tst\_acc:0.667  
epoch: 148 traing\_acc:0.724 tst\_acc:0.689  
epoch: 149 traing\_acc:0.724 tst\_acc:0.689  
epoch: 150 traing\_acc:0.743 tst\_acc:0.667  
epoch: 151 traing\_acc:0.724 tst\_acc:0.689  
epoch: 152 traing\_acc:0.733 tst\_acc:0.667  
epoch: 153 traing\_acc:0.752 tst\_acc:0.689  
epoch: 154 traing\_acc:0.733 tst\_acc:0.689  
epoch: 155 traing\_acc:0.752 tst\_acc:0.689  
epoch: 156 traing\_acc:0.752 tst\_acc:0.689  
epoch: 157 traing\_acc:0.762 tst\_acc:0.689  
epoch: 158 traing\_acc:0.752 tst\_acc:0.689  
epoch: 159 traing\_acc:0.762 tst\_acc:0.689  
epoch: 160 traing\_acc:0.762 tst\_acc:0.667  
epoch: 161 traing\_acc:0.743 tst\_acc:0.689  
epoch: 162 traing\_acc:0.762 tst\_acc:0.689  
epoch: 163 traing\_acc:0.762 tst\_acc:0.689  
epoch: 164 traing\_acc:0.762 tst\_acc:0.689  
epoch: 165 traing\_acc:0.762 tst\_acc:0.711  
epoch: 166 traing\_acc:0.762 tst\_acc:0.689  
epoch: 167 traing\_acc:0.781 tst\_acc:0.689  
epoch: 168 traing\_acc:0.781 tst\_acc:0.689  
epoch: 169 traing\_acc:0.800 tst\_acc:0.689  
epoch: 170 traing\_acc:0.790 tst\_acc:0.711  
epoch: 171 traing\_acc:0.800 tst\_acc:0.689  
epoch: 172 traing\_acc:0.781 tst\_acc:0.711  
epoch: 173 traing\_acc:0.810 tst\_acc:0.689  
epoch: 174 traing\_acc:0.790 tst\_acc:0.711  
epoch: 175 traing\_acc:0.819 tst\_acc:0.711  
epoch: 176 traing\_acc:0.800 tst\_acc:0.733  
epoch: 177 traing\_acc:0.829 tst\_acc:0.711  
epoch: 178 traing\_acc:0.810 tst\_acc:0.733  
epoch: 179 traing\_acc:0.810 tst\_acc:0.733  
epoch: 180 traing\_acc:0.829 tst\_acc:0.778  
epoch: 181 traing\_acc:0.810 tst\_acc:0.778  
epoch: 182 traing\_acc:0.829 tst\_acc:0.778  
epoch: 183 traing\_acc:0.829 tst\_acc:0.778  
epoch: 184 traing\_acc:0.829 tst\_acc:0.778  
epoch: 185 traing\_acc:0.819 tst\_acc:0.778

epoch: 186 traing\_acc:0.829 tst\_acc:0.778  
epoch: 187 traing\_acc:0.838 tst\_acc:0.778  
epoch: 188 traing\_acc:0.829 tst\_acc:0.778  
epoch: 189 traing\_acc:0.838 tst\_acc:0.778  
epoch: 190 traing\_acc:0.857 tst\_acc:0.778  
epoch: 191 traing\_acc:0.838 tst\_acc:0.778  
epoch: 192 traing\_acc:0.867 tst\_acc:0.800  
epoch: 193 traing\_acc:0.857 tst\_acc:0.800  
epoch: 194 traing\_acc:0.867 tst\_acc:0.800  
epoch: 195 traing\_acc:0.886 tst\_acc:0.800  
epoch: 196 traing\_acc:0.886 tst\_acc:0.800  
epoch: 197 traing\_acc:0.886 tst\_acc:0.800  
epoch: 198 traing\_acc:0.895 tst\_acc:0.800  
epoch: 199 traing\_acc:0.886 tst\_acc:0.800  
epoch: 200 traing\_acc:0.895 tst\_acc:0.800  
epoch: 201 traing\_acc:0.895 tst\_acc:0.800  
epoch: 202 traing\_acc:0.886 tst\_acc:0.800  
epoch: 203 traing\_acc:0.886 tst\_acc:0.800  
epoch: 204 traing\_acc:0.886 tst\_acc:0.800  
epoch: 205 traing\_acc:0.886 tst\_acc:0.800  
epoch: 206 traing\_acc:0.895 tst\_acc:0.800  
epoch: 207 traing\_acc:0.895 tst\_acc:0.822  
epoch: 208 traing\_acc:0.886 tst\_acc:0.800  
epoch: 209 traing\_acc:0.895 tst\_acc:0.822  
epoch: 210 traing\_acc:0.895 tst\_acc:0.778  
epoch: 211 traing\_acc:0.895 tst\_acc:0.778  
epoch: 212 traing\_acc:0.886 tst\_acc:0.800  
epoch: 213 traing\_acc:0.895 tst\_acc:0.778  
epoch: 214 traing\_acc:0.895 tst\_acc:0.822  
epoch: 215 traing\_acc:0.905 tst\_acc:0.822  
epoch: 216 traing\_acc:0.895 tst\_acc:0.822  
epoch: 217 traing\_acc:0.895 tst\_acc:0.844  
epoch: 218 traing\_acc:0.924 tst\_acc:0.822  
epoch: 219 traing\_acc:0.905 tst\_acc:0.867  
epoch: 220 traing\_acc:0.924 tst\_acc:0.822  
epoch: 221 traing\_acc:0.905 tst\_acc:0.867  
epoch: 222 traing\_acc:0.914 tst\_acc:0.867  
epoch: 223 traing\_acc:0.924 tst\_acc:0.822  
epoch: 224 traing\_acc:0.905 tst\_acc:0.867  
epoch: 225 traing\_acc:0.924 tst\_acc:0.867  
epoch: 226 traing\_acc:0.914 tst\_acc:0.867  
epoch: 227 traing\_acc:0.924 tst\_acc:0.822  
epoch: 228 traing\_acc:0.905 tst\_acc:0.867  
epoch: 229 traing\_acc:0.924 tst\_acc:0.867  
epoch: 230 traing\_acc:0.924 tst\_acc:0.867  
epoch: 231 traing\_acc:0.924 tst\_acc:0.867  
epoch: 232 traing\_acc:0.914 tst\_acc:0.867  
epoch: 233 traing\_acc:0.914 tst\_acc:0.867

epoch: 234 traing\_acc:0.914 tst\_acc:0.867  
epoch: 235 traing\_acc:0.924 tst\_acc:0.867  
epoch: 236 traing\_acc:0.924 tst\_acc:0.867  
epoch: 237 traing\_acc:0.924 tst\_acc:0.867  
epoch: 238 traing\_acc:0.924 tst\_acc:0.867  
epoch: 239 traing\_acc:0.933 tst\_acc:0.867  
epoch: 240 traing\_acc:0.924 tst\_acc:0.867  
epoch: 241 traing\_acc:0.933 tst\_acc:0.867  
epoch: 242 traing\_acc:0.933 tst\_acc:0.867  
epoch: 243 traing\_acc:0.933 tst\_acc:0.867  
epoch: 244 traing\_acc:0.933 tst\_acc:0.867  
epoch: 245 traing\_acc:0.943 tst\_acc:0.867  
epoch: 246 traing\_acc:0.933 tst\_acc:0.867  
epoch: 247 traing\_acc:0.943 tst\_acc:0.867  
epoch: 248 traing\_acc:0.924 tst\_acc:0.867  
epoch: 249 traing\_acc:0.943 tst\_acc:0.867  
epoch: 250 traing\_acc:0.943 tst\_acc:0.867  
epoch: 251 traing\_acc:0.933 tst\_acc:0.867  
epoch: 252 traing\_acc:0.943 tst\_acc:0.867  
epoch: 253 traing\_acc:0.933 tst\_acc:0.867  
epoch: 254 traing\_acc:0.943 tst\_acc:0.867  
epoch: 255 traing\_acc:0.933 tst\_acc:0.867  
epoch: 256 traing\_acc:0.933 tst\_acc:0.867  
epoch: 257 traing\_acc:0.933 tst\_acc:0.867  
epoch: 258 traing\_acc:0.943 tst\_acc:0.867  
epoch: 259 traing\_acc:0.933 tst\_acc:0.867  
epoch: 260 traing\_acc:0.943 tst\_acc:0.867  
epoch: 261 traing\_acc:0.943 tst\_acc:0.867  
epoch: 262 traing\_acc:0.933 tst\_acc:0.867  
epoch: 263 traing\_acc:0.943 tst\_acc:0.867  
epoch: 264 traing\_acc:0.924 tst\_acc:0.889  
epoch: 265 traing\_acc:0.943 tst\_acc:0.867  
epoch: 266 traing\_acc:0.933 tst\_acc:0.867  
epoch: 267 traing\_acc:0.943 tst\_acc:0.867  
epoch: 268 traing\_acc:0.933 tst\_acc:0.867  
epoch: 269 traing\_acc:0.943 tst\_acc:0.867  
epoch: 270 traing\_acc:0.933 tst\_acc:0.867  
epoch: 271 traing\_acc:0.943 tst\_acc:0.867  
epoch: 272 traing\_acc:0.933 tst\_acc:0.889  
epoch: 273 traing\_acc:0.943 tst\_acc:0.889  
epoch: 274 traing\_acc:0.943 tst\_acc:0.889  
epoch: 275 traing\_acc:0.933 tst\_acc:0.889  
epoch: 276 traing\_acc:0.952 tst\_acc:0.889  
epoch: 277 traing\_acc:0.943 tst\_acc:0.889  
epoch: 278 traing\_acc:0.943 tst\_acc:0.889  
epoch: 279 traing\_acc:0.933 tst\_acc:0.889  
epoch: 280 traing\_acc:0.933 tst\_acc:0.889  
epoch: 281 traing\_acc:0.952 tst\_acc:0.889

epoch: 282 traing\_acc:0.933 tst\_acc:0.889  
epoch: 283 traing\_acc:0.943 tst\_acc:0.889  
epoch: 284 traing\_acc:0.952 tst\_acc:0.889  
epoch: 285 traing\_acc:0.933 tst\_acc:0.889  
epoch: 286 traing\_acc:0.952 tst\_acc:0.889  
epoch: 287 traing\_acc:0.924 tst\_acc:0.889  
epoch: 288 traing\_acc:0.943 tst\_acc:0.889  
epoch: 289 traing\_acc:0.943 tst\_acc:0.889  
epoch: 290 traing\_acc:0.943 tst\_acc:0.889  
epoch: 291 traing\_acc:0.952 tst\_acc:0.889  
epoch: 292 traing\_acc:0.933 tst\_acc:0.889  
epoch: 293 traing\_acc:0.943 tst\_acc:0.889  
epoch: 294 traing\_acc:0.943 tst\_acc:0.889  
epoch: 295 traing\_acc:0.933 tst\_acc:0.889  
epoch: 296 traing\_acc:0.952 tst\_acc:0.889  
epoch: 297 traing\_acc:0.933 tst\_acc:0.889  
epoch: 298 traing\_acc:0.943 tst\_acc:0.889  
epoch: 299 traing\_acc:0.943 tst\_acc:0.889  
epoch: 300 traing\_acc:0.943 tst\_acc:0.889  
epoch: 301 traing\_acc:0.952 tst\_acc:0.889  
epoch: 302 traing\_acc:0.933 tst\_acc:0.889  
epoch: 303 traing\_acc:0.933 tst\_acc:0.889  
epoch: 304 traing\_acc:0.943 tst\_acc:0.889  
epoch: 305 traing\_acc:0.943 tst\_acc:0.889  
epoch: 306 traing\_acc:0.943 tst\_acc:0.889  
epoch: 307 traing\_acc:0.943 tst\_acc:0.889  
epoch: 308 traing\_acc:0.943 tst\_acc:0.889  
epoch: 309 traing\_acc:0.943 tst\_acc:0.889  
epoch: 310 traing\_acc:0.943 tst\_acc:0.889  
epoch: 311 traing\_acc:0.933 tst\_acc:0.889  
epoch: 312 traing\_acc:0.943 tst\_acc:0.889  
epoch: 313 traing\_acc:0.943 tst\_acc:0.889  
epoch: 314 traing\_acc:0.943 tst\_acc:0.889  
epoch: 315 traing\_acc:0.943 tst\_acc:0.889  
epoch: 316 traing\_acc:0.943 tst\_acc:0.889  
epoch: 317 traing\_acc:0.943 tst\_acc:0.889  
epoch: 318 traing\_acc:0.924 tst\_acc:0.889  
epoch: 319 traing\_acc:0.952 tst\_acc:0.889  
epoch: 320 traing\_acc:0.943 tst\_acc:0.889  
epoch: 321 traing\_acc:0.943 tst\_acc:0.889  
epoch: 322 traing\_acc:0.943 tst\_acc:0.889  
epoch: 323 traing\_acc:0.933 tst\_acc:0.889  
epoch: 324 traing\_acc:0.952 tst\_acc:0.889  
epoch: 325 traing\_acc:0.943 tst\_acc:0.889  
epoch: 326 traing\_acc:0.933 tst\_acc:0.889  
epoch: 327 traing\_acc:0.943 tst\_acc:0.889  
epoch: 328 traing\_acc:0.933 tst\_acc:0.889  
epoch: 329 traing\_acc:0.952 tst\_acc:0.889

epoch: 330 traing\_acc:0.943 tst\_acc:0.889  
epoch: 331 traing\_acc:0.943 tst\_acc:0.889  
epoch: 332 traing\_acc:0.943 tst\_acc:0.889  
epoch: 333 traing\_acc:0.933 tst\_acc:0.889  
epoch: 334 traing\_acc:0.943 tst\_acc:0.889  
epoch: 335 traing\_acc:0.933 tst\_acc:0.889  
epoch: 336 traing\_acc:0.952 tst\_acc:0.889  
epoch: 337 traing\_acc:0.943 tst\_acc:0.889  
epoch: 338 traing\_acc:0.933 tst\_acc:0.889  
epoch: 339 traing\_acc:0.943 tst\_acc:0.889  
epoch: 340 traing\_acc:0.943 tst\_acc:0.889  
epoch: 341 traing\_acc:0.933 tst\_acc:0.889  
epoch: 342 traing\_acc:0.943 tst\_acc:0.889  
epoch: 343 traing\_acc:0.943 tst\_acc:0.889  
epoch: 344 traing\_acc:0.943 tst\_acc:0.889  
epoch: 345 traing\_acc:0.962 tst\_acc:0.889  
epoch: 346 traing\_acc:0.943 tst\_acc:0.889  
epoch: 347 traing\_acc:0.952 tst\_acc:0.889  
epoch: 348 traing\_acc:0.933 tst\_acc:0.889  
epoch: 349 traing\_acc:0.933 tst\_acc:0.889  
epoch: 350 traing\_acc:0.943 tst\_acc:0.889  
epoch: 351 traing\_acc:0.933 tst\_acc:0.889  
epoch: 352 traing\_acc:0.952 tst\_acc:0.889  
epoch: 353 traing\_acc:0.933 tst\_acc:0.889  
epoch: 354 traing\_acc:0.962 tst\_acc:0.911  
epoch: 355 traing\_acc:0.943 tst\_acc:0.911  
epoch: 356 traing\_acc:0.943 tst\_acc:0.911  
epoch: 357 traing\_acc:0.943 tst\_acc:0.911  
epoch: 358 traing\_acc:0.943 tst\_acc:0.911  
epoch: 359 traing\_acc:0.933 tst\_acc:0.911  
epoch: 360 traing\_acc:0.952 tst\_acc:0.911  
epoch: 361 traing\_acc:0.933 tst\_acc:0.911  
epoch: 362 traing\_acc:0.943 tst\_acc:0.911  
epoch: 363 traing\_acc:0.943 tst\_acc:0.911  
epoch: 364 traing\_acc:0.943 tst\_acc:0.911  
epoch: 365 traing\_acc:0.943 tst\_acc:0.911  
epoch: 366 traing\_acc:0.943 tst\_acc:0.911  
epoch: 367 traing\_acc:0.952 tst\_acc:0.911  
epoch: 368 traing\_acc:0.943 tst\_acc:0.911  
epoch: 369 traing\_acc:0.943 tst\_acc:0.911  
epoch: 370 traing\_acc:0.943 tst\_acc:0.911  
epoch: 371 traing\_acc:0.924 tst\_acc:0.911  
epoch: 372 traing\_acc:0.943 tst\_acc:0.911  
epoch: 373 traing\_acc:0.952 tst\_acc:0.911  
epoch: 374 traing\_acc:0.933 tst\_acc:0.911  
epoch: 375 traing\_acc:0.952 tst\_acc:0.911  
epoch: 376 traing\_acc:0.952 tst\_acc:0.911  
epoch: 377 traing\_acc:0.943 tst\_acc:0.911

epoch: 378 traing\_acc:0.933 tst\_acc:0.911  
epoch: 379 traing\_acc:0.943 tst\_acc:0.911  
epoch: 380 traing\_acc:0.943 tst\_acc:0.911  
epoch: 381 traing\_acc:0.943 tst\_acc:0.911  
epoch: 382 traing\_acc:0.943 tst\_acc:0.911  
epoch: 383 traing\_acc:0.943 tst\_acc:0.911  
epoch: 384 traing\_acc:0.943 tst\_acc:0.911  
epoch: 385 traing\_acc:0.952 tst\_acc:0.911  
epoch: 386 traing\_acc:0.943 tst\_acc:0.911  
epoch: 387 traing\_acc:0.943 tst\_acc:0.911  
epoch: 388 traing\_acc:0.943 tst\_acc:0.911  
epoch: 389 traing\_acc:0.933 tst\_acc:0.911  
epoch: 390 traing\_acc:0.943 tst\_acc:0.911  
epoch: 391 traing\_acc:0.952 tst\_acc:0.911  
epoch: 392 traing\_acc:0.933 tst\_acc:0.911  
epoch: 393 traing\_acc:0.962 tst\_acc:0.911  
epoch: 394 traing\_acc:0.943 tst\_acc:0.911  
epoch: 395 traing\_acc:0.952 tst\_acc:0.911  
epoch: 396 traing\_acc:0.933 tst\_acc:0.911  
epoch: 397 traing\_acc:0.933 tst\_acc:0.911  
epoch: 398 traing\_acc:0.943 tst\_acc:0.911  
epoch: 399 traing\_acc:0.943 tst\_acc:0.911  
epoch: 400 traing\_acc:0.943 tst\_acc:0.911  
epoch: 401 traing\_acc:0.943 tst\_acc:0.911  
epoch: 402 traing\_acc:0.943 tst\_acc:0.911  
epoch: 403 traing\_acc:0.952 tst\_acc:0.911  
epoch: 404 traing\_acc:0.943 tst\_acc:0.911  
epoch: 405 traing\_acc:0.952 tst\_acc:0.911  
epoch: 406 traing\_acc:0.952 tst\_acc:0.911  
epoch: 407 traing\_acc:0.943 tst\_acc:0.911  
epoch: 408 traing\_acc:0.943 tst\_acc:0.911  
epoch: 409 traing\_acc:0.943 tst\_acc:0.911  
epoch: 410 traing\_acc:0.952 tst\_acc:0.911  
epoch: 411 traing\_acc:0.943 tst\_acc:0.911  
epoch: 412 traing\_acc:0.962 tst\_acc:0.911  
epoch: 413 traing\_acc:0.952 tst\_acc:0.911  
epoch: 414 traing\_acc:0.952 tst\_acc:0.911  
epoch: 415 traing\_acc:0.943 tst\_acc:0.911  
epoch: 416 traing\_acc:0.933 tst\_acc:0.911  
epoch: 417 traing\_acc:0.943 tst\_acc:0.911  
epoch: 418 traing\_acc:0.943 tst\_acc:0.911  
epoch: 419 traing\_acc:0.943 tst\_acc:0.911  
epoch: 420 traing\_acc:0.933 tst\_acc:0.911  
epoch: 421 traing\_acc:0.962 tst\_acc:0.911  
epoch: 422 traing\_acc:0.952 tst\_acc:0.911  
epoch: 423 traing\_acc:0.952 tst\_acc:0.911  
epoch: 424 traing\_acc:0.943 tst\_acc:0.911  
epoch: 425 traing\_acc:0.943 tst\_acc:0.911



epoch: 426 traing\_acc:0.943 tst\_acc:0.911  
epoch: 427 traing\_acc:0.943 tst\_acc:0.911  
epoch: 428 traing\_acc:0.943 tst\_acc:0.911  
epoch: 429 traing\_acc:0.943 tst\_acc:0.911  
epoch: 430 traing\_acc:0.952 tst\_acc:0.911  
epoch: 431 traing\_acc:0.962 tst\_acc:0.911  
epoch: 432 traing\_acc:0.952 tst\_acc:0.911  
epoch: 433 traing\_acc:0.952 tst\_acc:0.911  
epoch: 434 traing\_acc:0.943 tst\_acc:0.911  
epoch: 435 traing\_acc:0.924 tst\_acc:0.911  
epoch: 436 traing\_acc:0.943 tst\_acc:0.911  
epoch: 437 traing\_acc:0.952 tst\_acc:0.911  
epoch: 438 traing\_acc:0.943 tst\_acc:0.911  
epoch: 439 traing\_acc:0.933 tst\_acc:0.911  
epoch: 440 traing\_acc:0.952 tst\_acc:0.911  
epoch: 441 traing\_acc:0.952 tst\_acc:0.911  
epoch: 442 traing\_acc:0.943 tst\_acc:0.911  
epoch: 443 traing\_acc:0.952 tst\_acc:0.911  
epoch: 444 traing\_acc:0.952 tst\_acc:0.911  
epoch: 445 traing\_acc:0.952 tst\_acc:0.911  
epoch: 446 traing\_acc:0.933 tst\_acc:0.911  
epoch: 447 traing\_acc:0.943 tst\_acc:0.911  
epoch: 448 traing\_acc:0.943 tst\_acc:0.911  
epoch: 449 traing\_acc:0.952 tst\_acc:0.911  
epoch: 450 traing\_acc:0.952 tst\_acc:0.911  
epoch: 451 traing\_acc:0.943 tst\_acc:0.911  
epoch: 452 traing\_acc:0.933 tst\_acc:0.911  
epoch: 453 traing\_acc:0.952 tst\_acc:0.911  
epoch: 454 traing\_acc:0.952 tst\_acc:0.911  
epoch: 455 traing\_acc:0.943 tst\_acc:0.911  
epoch: 456 traing\_acc:0.952 tst\_acc:0.911  
epoch: 457 traing\_acc:0.933 tst\_acc:0.911  
epoch: 458 traing\_acc:0.952 tst\_acc:0.911  
epoch: 459 traing\_acc:0.952 tst\_acc:0.911  
epoch: 460 traing\_acc:0.943 tst\_acc:0.911  
epoch: 461 traing\_acc:0.933 tst\_acc:0.911  
epoch: 462 traing\_acc:0.952 tst\_acc:0.911  
epoch: 463 traing\_acc:0.952 tst\_acc:0.911  
epoch: 464 traing\_acc:0.952 tst\_acc:0.911  
epoch: 465 traing\_acc:0.943 tst\_acc:0.911  
epoch: 466 traing\_acc:0.943 tst\_acc:0.911  
epoch: 467 traing\_acc:0.943 tst\_acc:0.911  
epoch: 468 traing\_acc:0.952 tst\_acc:0.911  
epoch: 469 traing\_acc:0.952 tst\_acc:0.911  
epoch: 470 traing\_acc:0.943 tst\_acc:0.911  
epoch: 471 traing\_acc:0.933 tst\_acc:0.911  
epoch: 472 traing\_acc:0.943 tst\_acc:0.911  
epoch: 473 traing\_acc:0.943 tst\_acc:0.911

epoch: 474 traing\_acc:0.933 tst\_acc:0.911  
epoch: 475 traing\_acc:0.943 tst\_acc:0.911  
epoch: 476 traing\_acc:0.943 tst\_acc:0.911  
epoch: 477 traing\_acc:0.952 tst\_acc:0.911  
epoch: 478 traing\_acc:0.962 tst\_acc:0.911  
epoch: 479 traing\_acc:0.952 tst\_acc:0.911  
epoch: 480 traing\_acc:0.943 tst\_acc:0.911  
epoch: 481 traing\_acc:0.943 tst\_acc:0.911  
epoch: 482 traing\_acc:0.943 tst\_acc:0.911  
epoch: 483 traing\_acc:0.933 tst\_acc:0.911  
epoch: 484 traing\_acc:0.952 tst\_acc:0.911  
epoch: 485 traing\_acc:0.943 tst\_acc:0.911  
epoch: 486 traing\_acc:0.952 tst\_acc:0.911  
epoch: 487 traing\_acc:0.952 tst\_acc:0.911  
epoch: 488 traing\_acc:0.962 tst\_acc:0.911  
epoch: 489 traing\_acc:0.943 tst\_acc:0.911  
epoch: 490 traing\_acc:0.952 tst\_acc:0.911  
epoch: 491 traing\_acc:0.943 tst\_acc:0.911  
epoch: 492 traing\_acc:0.924 tst\_acc:0.911  
epoch: 493 traing\_acc:0.943 tst\_acc:0.911  
epoch: 494 traing\_acc:0.943 tst\_acc:0.911  
epoch: 495 traing\_acc:0.933 tst\_acc:0.911  
epoch: 496 traing\_acc:0.943 tst\_acc:0.911  
epoch: 497 traing\_acc:0.943 tst\_acc:0.911  
epoch: 498 traing\_acc:0.943 tst\_acc:0.911  
epoch: 499 traing\_acc:0.962 tst\_acc:0.911  
epoch: 500 traing\_acc:0.952 tst\_acc:0.911  
epoch: 501 traing\_acc:0.952 tst\_acc:0.911  
epoch: 502 traing\_acc:0.943 tst\_acc:0.911  
epoch: 503 traing\_acc:0.952 tst\_acc:0.911  
epoch: 504 traing\_acc:0.933 tst\_acc:0.911  
epoch: 505 traing\_acc:0.962 tst\_acc:0.911  
epoch: 506 traing\_acc:0.943 tst\_acc:0.911  
epoch: 507 traing\_acc:0.952 tst\_acc:0.911  
epoch: 508 traing\_acc:0.943 tst\_acc:0.911  
epoch: 509 traing\_acc:0.924 tst\_acc:0.911  
epoch: 510 traing\_acc:0.952 tst\_acc:0.911  
epoch: 511 traing\_acc:0.962 tst\_acc:0.911  
epoch: 512 traing\_acc:0.943 tst\_acc:0.911  
epoch: 513 traing\_acc:0.952 tst\_acc:0.911  
epoch: 514 traing\_acc:0.943 tst\_acc:0.911  
epoch: 515 traing\_acc:0.952 tst\_acc:0.911  
epoch: 516 traing\_acc:0.943 tst\_acc:0.911  
epoch: 517 traing\_acc:0.943 tst\_acc:0.911  
epoch: 518 traing\_acc:0.933 tst\_acc:0.911  
epoch: 519 traing\_acc:0.952 tst\_acc:0.911  
epoch: 520 traing\_acc:0.952 tst\_acc:0.911  
epoch: 521 traing\_acc:0.943 tst\_acc:0.911

epoch: 522 traing\_acc:0.952 tst\_acc:0.911  
epoch: 523 traing\_acc:0.933 tst\_acc:0.911  
epoch: 524 traing\_acc:0.962 tst\_acc:0.911  
epoch: 525 traing\_acc:0.943 tst\_acc:0.911  
epoch: 526 traing\_acc:0.952 tst\_acc:0.911  
epoch: 527 traing\_acc:0.933 tst\_acc:0.911  
epoch: 528 traing\_acc:0.933 tst\_acc:0.911  
epoch: 529 traing\_acc:0.952 tst\_acc:0.911  
epoch: 530 traing\_acc:0.952 tst\_acc:0.911  
epoch: 531 traing\_acc:0.943 tst\_acc:0.911  
epoch: 532 traing\_acc:0.952 tst\_acc:0.911  
epoch: 533 traing\_acc:0.952 tst\_acc:0.911  
epoch: 534 traing\_acc:0.952 tst\_acc:0.911  
epoch: 535 traing\_acc:0.943 tst\_acc:0.911  
epoch: 536 traing\_acc:0.933 tst\_acc:0.911  
epoch: 537 traing\_acc:0.943 tst\_acc:0.911  
epoch: 538 traing\_acc:0.943 tst\_acc:0.911  
epoch: 539 traing\_acc:0.943 tst\_acc:0.911  
epoch: 540 traing\_acc:0.933 tst\_acc:0.911  
epoch: 541 traing\_acc:0.952 tst\_acc:0.911  
epoch: 542 traing\_acc:0.943 tst\_acc:0.911  
epoch: 543 traing\_acc:0.962 tst\_acc:0.911  
epoch: 544 traing\_acc:0.952 tst\_acc:0.911  
epoch: 545 traing\_acc:0.952 tst\_acc:0.911  
epoch: 546 traing\_acc:0.943 tst\_acc:0.911  
epoch: 547 traing\_acc:0.943 tst\_acc:0.911  
epoch: 548 traing\_acc:0.943 tst\_acc:0.911  
epoch: 549 traing\_acc:0.933 tst\_acc:0.911  
epoch: 550 traing\_acc:0.943 tst\_acc:0.911  
epoch: 551 traing\_acc:0.933 tst\_acc:0.911  
epoch: 552 traing\_acc:0.962 tst\_acc:0.911  
epoch: 553 traing\_acc:0.952 tst\_acc:0.911  
epoch: 554 traing\_acc:0.952 tst\_acc:0.911  
epoch: 555 traing\_acc:0.943 tst\_acc:0.911  
epoch: 556 traing\_acc:0.943 tst\_acc:0.911  
epoch: 557 traing\_acc:0.952 tst\_acc:0.911  
epoch: 558 traing\_acc:0.933 tst\_acc:0.911  
epoch: 559 traing\_acc:0.943 tst\_acc:0.911  
epoch: 560 traing\_acc:0.952 tst\_acc:0.911  
epoch: 561 traing\_acc:0.943 tst\_acc:0.911  
epoch: 562 traing\_acc:0.962 tst\_acc:0.911  
epoch: 563 traing\_acc:0.952 tst\_acc:0.911  
epoch: 564 traing\_acc:0.952 tst\_acc:0.911  
epoch: 565 traing\_acc:0.943 tst\_acc:0.911  
epoch: 566 traing\_acc:0.933 tst\_acc:0.911  
epoch: 567 traing\_acc:0.943 tst\_acc:0.911  
epoch: 568 traing\_acc:0.943 tst\_acc:0.911  
epoch: 569 traing\_acc:0.943 tst\_acc:0.911

epoch: 570 traing\_acc:0.943 tst\_acc:0.911  
epoch: 571 traing\_acc:0.962 tst\_acc:0.911  
epoch: 572 traing\_acc:0.952 tst\_acc:0.911  
epoch: 573 traing\_acc:0.952 tst\_acc:0.911  
epoch: 574 traing\_acc:0.943 tst\_acc:0.911  
epoch: 575 traing\_acc:0.943 tst\_acc:0.911  
epoch: 576 traing\_acc:0.943 tst\_acc:0.911  
epoch: 577 traing\_acc:0.933 tst\_acc:0.911  
epoch: 578 traing\_acc:0.943 tst\_acc:0.911  
epoch: 579 traing\_acc:0.943 tst\_acc:0.911  
epoch: 580 traing\_acc:0.952 tst\_acc:0.911  
epoch: 581 traing\_acc:0.952 tst\_acc:0.911  
epoch: 582 traing\_acc:0.943 tst\_acc:0.911  
epoch: 583 traing\_acc:0.933 tst\_acc:0.911  
epoch: 584 traing\_acc:0.952 tst\_acc:0.911  
epoch: 585 traing\_acc:0.952 tst\_acc:0.911  
epoch: 586 traing\_acc:0.943 tst\_acc:0.911  
epoch: 587 traing\_acc:0.952 tst\_acc:0.911  
epoch: 588 traing\_acc:0.933 tst\_acc:0.911  
epoch: 589 traing\_acc:0.952 tst\_acc:0.911  
epoch: 590 traing\_acc:0.952 tst\_acc:0.911  
epoch: 591 traing\_acc:0.943 tst\_acc:0.911  
epoch: 592 traing\_acc:0.933 tst\_acc:0.911  
epoch: 593 traing\_acc:0.952 tst\_acc:0.911  
epoch: 594 traing\_acc:0.952 tst\_acc:0.911  
epoch: 595 traing\_acc:0.952 tst\_acc:0.911  
epoch: 596 traing\_acc:0.943 tst\_acc:0.911  
epoch: 597 traing\_acc:0.943 tst\_acc:0.911  
epoch: 598 traing\_acc:0.943 tst\_acc:0.911  
epoch: 599 traing\_acc:0.952 tst\_acc:0.911  
epoch: 600 traing\_acc:0.952 tst\_acc:0.911  
epoch: 601 traing\_acc:0.943 tst\_acc:0.911  
epoch: 602 traing\_acc:0.933 tst\_acc:0.911  
epoch: 603 traing\_acc:0.952 tst\_acc:0.911  
epoch: 604 traing\_acc:0.952 tst\_acc:0.911  
epoch: 605 traing\_acc:0.943 tst\_acc:0.911  
epoch: 606 traing\_acc:0.952 tst\_acc:0.911  
epoch: 607 traing\_acc:0.933 tst\_acc:0.911  
epoch: 608 traing\_acc:0.962 tst\_acc:0.911  
epoch: 609 traing\_acc:0.943 tst\_acc:0.911  
epoch: 610 traing\_acc:0.952 tst\_acc:0.911  
epoch: 611 traing\_acc:0.933 tst\_acc:0.911  
epoch: 612 traing\_acc:0.943 tst\_acc:0.911  
epoch: 613 traing\_acc:0.933 tst\_acc:0.911  
epoch: 614 traing\_acc:0.943 tst\_acc:0.911  
epoch: 615 traing\_acc:0.943 tst\_acc:0.911  
epoch: 616 traing\_acc:0.943 tst\_acc:0.911  
epoch: 617 traing\_acc:0.952 tst\_acc:0.911

epoch: 618 traing\_acc:0.952 tst\_acc:0.911  
epoch: 619 traing\_acc:0.962 tst\_acc:0.911  
epoch: 620 traing\_acc:0.943 tst\_acc:0.911  
epoch: 621 traing\_acc:0.943 tst\_acc:0.911  
epoch: 622 traing\_acc:0.943 tst\_acc:0.911  
epoch: 623 traing\_acc:0.943 tst\_acc:0.911  
epoch: 624 traing\_acc:0.933 tst\_acc:0.911  
epoch: 625 traing\_acc:0.952 tst\_acc:0.911  
epoch: 626 traing\_acc:0.933 tst\_acc:0.911  
epoch: 627 traing\_acc:0.952 tst\_acc:0.911  
epoch: 628 traing\_acc:0.962 tst\_acc:0.911  
epoch: 629 traing\_acc:0.962 tst\_acc:0.911  
epoch: 630 traing\_acc:0.962 tst\_acc:0.911  
epoch: 631 traing\_acc:0.952 tst\_acc:0.911  
epoch: 632 traing\_acc:0.943 tst\_acc:0.911  
epoch: 633 traing\_acc:0.952 tst\_acc:0.911  
epoch: 634 traing\_acc:0.952 tst\_acc:0.911  
epoch: 635 traing\_acc:0.962 tst\_acc:0.911  
epoch: 636 traing\_acc:0.943 tst\_acc:0.911  
epoch: 637 traing\_acc:0.952 tst\_acc:0.911  
epoch: 638 traing\_acc:0.943 tst\_acc:0.911  
epoch: 639 traing\_acc:0.952 tst\_acc:0.911  
epoch: 640 traing\_acc:0.952 tst\_acc:0.911  
epoch: 641 traing\_acc:0.943 tst\_acc:0.911  
epoch: 642 traing\_acc:0.943 tst\_acc:0.911  
epoch: 643 traing\_acc:0.943 tst\_acc:0.911  
epoch: 644 traing\_acc:0.952 tst\_acc:0.911  
epoch: 645 traing\_acc:0.962 tst\_acc:0.911  
epoch: 646 traing\_acc:0.962 tst\_acc:0.911  
epoch: 647 traing\_acc:0.962 tst\_acc:0.911  
epoch: 648 traing\_acc:0.952 tst\_acc:0.911  
epoch: 649 traing\_acc:0.952 tst\_acc:0.911  
epoch: 650 traing\_acc:0.962 tst\_acc:0.911  
epoch: 651 traing\_acc:0.952 tst\_acc:0.911  
epoch: 652 traing\_acc:0.952 tst\_acc:0.911  
epoch: 653 traing\_acc:0.943 tst\_acc:0.911  
epoch: 654 traing\_acc:0.952 tst\_acc:0.911  
epoch: 655 traing\_acc:0.943 tst\_acc:0.911  
epoch: 656 traing\_acc:0.943 tst\_acc:0.911  
epoch: 657 traing\_acc:0.952 tst\_acc:0.911  
epoch: 658 traing\_acc:0.962 tst\_acc:0.911  
epoch: 659 traing\_acc:0.952 tst\_acc:0.911  
epoch: 660 traing\_acc:0.962 tst\_acc:0.911  
epoch: 661 traing\_acc:0.962 tst\_acc:0.911  
epoch: 662 traing\_acc:0.971 tst\_acc:0.911  
epoch: 663 traing\_acc:0.971 tst\_acc:0.911  
epoch: 664 traing\_acc:0.971 tst\_acc:0.911  
epoch: 665 traing\_acc:0.962 tst\_acc:0.911

epoch: 666 traing\_acc:0.962 tst\_acc:0.911  
epoch: 667 traing\_acc:0.952 tst\_acc:0.911  
epoch: 668 traing\_acc:0.952 tst\_acc:0.911  
epoch: 669 traing\_acc:0.952 tst\_acc:0.911  
epoch: 670 traing\_acc:0.943 tst\_acc:0.911  
epoch: 671 traing\_acc:0.943 tst\_acc:0.911  
epoch: 672 traing\_acc:0.952 tst\_acc:0.911  
epoch: 673 traing\_acc:0.952 tst\_acc:0.911  
epoch: 674 traing\_acc:0.962 tst\_acc:0.911  
epoch: 675 traing\_acc:0.952 tst\_acc:0.911  
epoch: 676 traing\_acc:0.962 tst\_acc:0.911  
epoch: 677 traing\_acc:0.962 tst\_acc:0.911  
epoch: 678 traing\_acc:0.952 tst\_acc:0.911  
epoch: 679 traing\_acc:0.962 tst\_acc:0.911  
epoch: 680 traing\_acc:0.952 tst\_acc:0.911  
epoch: 681 traing\_acc:0.971 tst\_acc:0.911  
epoch: 682 traing\_acc:0.971 tst\_acc:0.911  
epoch: 683 traing\_acc:0.971 tst\_acc:0.911  
epoch: 684 traing\_acc:0.962 tst\_acc:0.911  
epoch: 685 traing\_acc:0.962 tst\_acc:0.911  
epoch: 686 traing\_acc:0.952 tst\_acc:0.911  
epoch: 687 traing\_acc:0.962 tst\_acc:0.911  
epoch: 688 traing\_acc:0.952 tst\_acc:0.911  
epoch: 689 traing\_acc:0.952 tst\_acc:0.911  
epoch: 690 traing\_acc:0.962 tst\_acc:0.911  
epoch: 691 traing\_acc:0.962 tst\_acc:0.911  
epoch: 692 traing\_acc:0.971 tst\_acc:0.911  
epoch: 693 traing\_acc:0.971 tst\_acc:0.911  
epoch: 694 traing\_acc:0.962 tst\_acc:0.911  
epoch: 695 traing\_acc:0.971 tst\_acc:0.911  
epoch: 696 traing\_acc:0.962 tst\_acc:0.911  
epoch: 697 traing\_acc:0.962 tst\_acc:0.911  
epoch: 698 traing\_acc:0.962 tst\_acc:0.911  
epoch: 699 traing\_acc:0.952 tst\_acc:0.911  
epoch: 700 traing\_acc:0.962 tst\_acc:0.911  
epoch: 701 traing\_acc:0.971 tst\_acc:0.911  
epoch: 702 traing\_acc:0.952 tst\_acc:0.911  
epoch: 703 traing\_acc:0.971 tst\_acc:0.911  
epoch: 704 traing\_acc:0.971 tst\_acc:0.911  
epoch: 705 traing\_acc:0.962 tst\_acc:0.911  
epoch: 706 traing\_acc:0.971 tst\_acc:0.911  
epoch: 707 traing\_acc:0.971 tst\_acc:0.911  
epoch: 708 traing\_acc:0.962 tst\_acc:0.911  
epoch: 709 traing\_acc:0.971 tst\_acc:0.911  
epoch: 710 traing\_acc:0.971 tst\_acc:0.911  
epoch: 711 traing\_acc:0.952 tst\_acc:0.911  
epoch: 712 traing\_acc:0.962 tst\_acc:0.911  
epoch: 713 traing\_acc:0.971 tst\_acc:0.911

epoch: 714 traing\_acc:0.952 tst\_acc:0.911  
epoch: 715 traing\_acc:0.971 tst\_acc:0.911  
epoch: 716 traing\_acc:0.971 tst\_acc:0.911  
epoch: 717 traing\_acc:0.962 tst\_acc:0.911  
epoch: 718 traing\_acc:0.971 tst\_acc:0.911  
epoch: 719 traing\_acc:0.971 tst\_acc:0.911  
epoch: 720 traing\_acc:0.962 tst\_acc:0.911  
epoch: 721 traing\_acc:0.971 tst\_acc:0.911  
epoch: 722 traing\_acc:0.971 tst\_acc:0.911  
epoch: 723 traing\_acc:0.962 tst\_acc:0.911  
epoch: 724 traing\_acc:0.971 tst\_acc:0.911  
epoch: 725 traing\_acc:0.971 tst\_acc:0.911  
epoch: 726 traing\_acc:0.971 tst\_acc:0.911  
epoch: 727 traing\_acc:0.971 tst\_acc:0.911  
epoch: 728 traing\_acc:0.962 tst\_acc:0.911  
epoch: 729 traing\_acc:0.971 tst\_acc:0.911  
epoch: 730 traing\_acc:0.971 tst\_acc:0.911  
epoch: 731 traing\_acc:0.952 tst\_acc:0.911  
epoch: 732 traing\_acc:0.971 tst\_acc:0.911  
epoch: 733 traing\_acc:0.962 tst\_acc:0.911  
epoch: 734 traing\_acc:0.971 tst\_acc:0.911  
epoch: 735 traing\_acc:0.971 tst\_acc:0.911  
epoch: 736 traing\_acc:0.971 tst\_acc:0.911  
epoch: 737 traing\_acc:0.962 tst\_acc:0.911  
epoch: 738 traing\_acc:0.981 tst\_acc:0.911  
epoch: 739 traing\_acc:0.971 tst\_acc:0.911  
epoch: 740 traing\_acc:0.952 tst\_acc:0.911  
epoch: 741 traing\_acc:0.981 tst\_acc:0.911  
epoch: 742 traing\_acc:0.971 tst\_acc:0.911  
epoch: 743 traing\_acc:0.962 tst\_acc:0.911  
epoch: 744 traing\_acc:0.962 tst\_acc:0.911  
epoch: 745 traing\_acc:0.971 tst\_acc:0.911  
epoch: 746 traing\_acc:0.981 tst\_acc:0.911  
epoch: 747 traing\_acc:0.962 tst\_acc:0.911  
epoch: 748 traing\_acc:0.962 tst\_acc:0.911  
epoch: 749 traing\_acc:0.971 tst\_acc:0.911  
epoch: 750 traing\_acc:0.971 tst\_acc:0.911  
epoch: 751 traing\_acc:0.981 tst\_acc:0.911  
epoch: 752 traing\_acc:0.962 tst\_acc:0.911  
epoch: 753 traing\_acc:0.971 tst\_acc:0.911  
epoch: 754 traing\_acc:0.971 tst\_acc:0.911  
epoch: 755 traing\_acc:0.971 tst\_acc:0.911  
epoch: 756 traing\_acc:0.962 tst\_acc:0.911  
epoch: 757 traing\_acc:0.981 tst\_acc:0.911  
epoch: 758 traing\_acc:0.971 tst\_acc:0.911  
epoch: 759 traing\_acc:0.962 tst\_acc:0.911  
epoch: 760 traing\_acc:0.962 tst\_acc:0.911  
epoch: 761 traing\_acc:0.981 tst\_acc:0.911

epoch: 762 traing\_acc:0.971 tst\_acc:0.911  
epoch: 763 traing\_acc:0.952 tst\_acc:0.911  
epoch: 764 traing\_acc:0.981 tst\_acc:0.911  
epoch: 765 traing\_acc:0.971 tst\_acc:0.911  
epoch: 766 traing\_acc:0.962 tst\_acc:0.911  
epoch: 767 traing\_acc:0.962 tst\_acc:0.911  
epoch: 768 traing\_acc:0.981 tst\_acc:0.911  
epoch: 769 traing\_acc:0.971 tst\_acc:0.911  
epoch: 770 traing\_acc:0.962 tst\_acc:0.911  
epoch: 771 traing\_acc:0.962 tst\_acc:0.911  
epoch: 772 traing\_acc:0.971 tst\_acc:0.911  
epoch: 773 traing\_acc:0.981 tst\_acc:0.911  
epoch: 774 traing\_acc:0.971 tst\_acc:0.911  
epoch: 775 traing\_acc:0.962 tst\_acc:0.911  
epoch: 776 traing\_acc:0.971 tst\_acc:0.911  
epoch: 777 traing\_acc:0.971 tst\_acc:0.911  
epoch: 778 traing\_acc:0.962 tst\_acc:0.911  
epoch: 779 traing\_acc:0.981 tst\_acc:0.911  
epoch: 780 traing\_acc:0.971 tst\_acc:0.911  
epoch: 781 traing\_acc:0.971 tst\_acc:0.933  
epoch: 782 traing\_acc:0.962 tst\_acc:0.933  
epoch: 783 traing\_acc:0.962 tst\_acc:0.933  
epoch: 784 traing\_acc:0.981 tst\_acc:0.933  
epoch: 785 traing\_acc:0.981 tst\_acc:0.933  
epoch: 786 traing\_acc:0.952 tst\_acc:0.933  
epoch: 787 traing\_acc:0.981 tst\_acc:0.933  
epoch: 788 traing\_acc:0.981 tst\_acc:0.933  
epoch: 789 traing\_acc:0.971 tst\_acc:0.933  
epoch: 790 traing\_acc:0.962 tst\_acc:0.933  
epoch: 791 traing\_acc:0.981 tst\_acc:0.933  
epoch: 792 traing\_acc:0.981 tst\_acc:0.933  
epoch: 793 traing\_acc:0.962 tst\_acc:0.933  
epoch: 794 traing\_acc:0.971 tst\_acc:0.933  
epoch: 795 traing\_acc:0.971 tst\_acc:0.933  
epoch: 796 traing\_acc:0.981 tst\_acc:0.933  
epoch: 797 traing\_acc:0.971 tst\_acc:0.933  
epoch: 798 traing\_acc:0.971 tst\_acc:0.933  
epoch: 799 traing\_acc:0.971 tst\_acc:0.933  
epoch: 800 traing\_acc:0.981 tst\_acc:0.933  
epoch: 801 traing\_acc:0.981 tst\_acc:0.933  
epoch: 802 traing\_acc:0.981 tst\_acc:0.933  
epoch: 803 traing\_acc:0.962 tst\_acc:0.933  
epoch: 804 traing\_acc:0.981 tst\_acc:0.933  
epoch: 805 traing\_acc:0.971 tst\_acc:0.933  
epoch: 806 traing\_acc:0.971 tst\_acc:0.933  
epoch: 807 traing\_acc:0.981 tst\_acc:0.933  
epoch: 808 traing\_acc:0.971 tst\_acc:0.933  
epoch: 809 traing\_acc:0.962 tst\_acc:0.933



epoch: 810 traing\_acc:0.981 tst\_acc:0.933  
epoch: 811 traing\_acc:0.981 tst\_acc:0.933  
epoch: 812 traing\_acc:0.962 tst\_acc:0.933  
epoch: 813 traing\_acc:0.971 tst\_acc:0.933  
epoch: 814 traing\_acc:0.981 tst\_acc:0.933  
epoch: 815 traing\_acc:0.990 tst\_acc:0.933  
epoch: 816 traing\_acc:0.981 tst\_acc:0.933  
epoch: 817 traing\_acc:0.971 tst\_acc:0.933  
epoch: 818 traing\_acc:0.971 tst\_acc:0.933  
epoch: 819 traing\_acc:0.981 tst\_acc:0.933  
epoch: 820 traing\_acc:0.981 tst\_acc:0.933  
epoch: 821 traing\_acc:0.981 tst\_acc:0.933  
epoch: 822 traing\_acc:0.971 tst\_acc:0.933  
epoch: 823 traing\_acc:0.981 tst\_acc:0.933  
epoch: 824 traing\_acc:0.981 tst\_acc:0.933  
epoch: 825 traing\_acc:0.981 tst\_acc:0.933  
epoch: 826 traing\_acc:0.971 tst\_acc:0.933  
epoch: 827 traing\_acc:0.971 tst\_acc:0.933  
epoch: 828 traing\_acc:0.981 tst\_acc:0.933  
epoch: 829 traing\_acc:0.981 tst\_acc:0.933  
epoch: 830 traing\_acc:0.981 tst\_acc:0.933  
epoch: 831 traing\_acc:0.981 tst\_acc:0.933  
epoch: 832 traing\_acc:0.981 tst\_acc:0.933  
epoch: 833 traing\_acc:0.962 tst\_acc:0.933  
epoch: 834 traing\_acc:0.971 tst\_acc:0.933  
epoch: 835 traing\_acc:0.981 tst\_acc:0.933  
epoch: 836 traing\_acc:0.971 tst\_acc:0.933  
epoch: 837 traing\_acc:0.962 tst\_acc:0.933  
epoch: 838 traing\_acc:0.981 tst\_acc:0.933  
epoch: 839 traing\_acc:0.971 tst\_acc:0.933  
epoch: 840 traing\_acc:0.971 tst\_acc:0.933  
epoch: 841 traing\_acc:0.981 tst\_acc:0.933  
epoch: 842 traing\_acc:0.971 tst\_acc:0.933  
epoch: 843 traing\_acc:0.981 tst\_acc:0.933  
epoch: 844 traing\_acc:0.990 tst\_acc:0.933  
epoch: 845 traing\_acc:0.981 tst\_acc:0.933  
epoch: 846 traing\_acc:0.981 tst\_acc:0.933  
epoch: 847 traing\_acc:0.990 tst\_acc:0.933  
epoch: 848 traing\_acc:0.981 tst\_acc:0.933  
epoch: 849 traing\_acc:0.981 tst\_acc:0.933  
epoch: 850 traing\_acc:0.981 tst\_acc:0.933  
epoch: 851 traing\_acc:0.981 tst\_acc:0.933  
epoch: 852 traing\_acc:0.990 tst\_acc:0.933  
epoch: 853 traing\_acc:0.990 tst\_acc:0.933  
epoch: 854 traing\_acc:0.981 tst\_acc:0.933  
epoch: 855 traing\_acc:0.981 tst\_acc:0.933  
epoch: 856 traing\_acc:0.990 tst\_acc:0.933  
epoch: 857 traing\_acc:0.981 tst\_acc:0.933

epoch: 858 traing\_acc:0.981 tst\_acc:0.933  
epoch: 859 traing\_acc:0.971 tst\_acc:0.933  
epoch: 860 traing\_acc:0.990 tst\_acc:0.933  
epoch: 861 traing\_acc:0.981 tst\_acc:0.933  
epoch: 862 traing\_acc:0.981 tst\_acc:0.933  
epoch: 863 traing\_acc:0.971 tst\_acc:0.933  
epoch: 864 traing\_acc:0.971 tst\_acc:0.933  
epoch: 865 traing\_acc:0.981 tst\_acc:0.933  
epoch: 866 traing\_acc:0.981 tst\_acc:0.933  
epoch: 867 traing\_acc:0.971 tst\_acc:0.933  
epoch: 868 traing\_acc:0.981 tst\_acc:0.933  
epoch: 869 traing\_acc:0.981 tst\_acc:0.933  
epoch: 870 traing\_acc:0.971 tst\_acc:0.933  
epoch: 871 traing\_acc:0.981 tst\_acc:0.933  
epoch: 872 traing\_acc:0.971 tst\_acc:0.933  
epoch: 873 traing\_acc:0.971 tst\_acc:0.933  
epoch: 874 traing\_acc:0.971 tst\_acc:0.933  
epoch: 875 traing\_acc:0.962 tst\_acc:0.933  
epoch: 876 traing\_acc:0.981 tst\_acc:0.933  
epoch: 877 traing\_acc:0.981 tst\_acc:0.933  
epoch: 878 traing\_acc:0.990 tst\_acc:0.933  
epoch: 879 traing\_acc:0.981 tst\_acc:0.933  
epoch: 880 traing\_acc:0.981 tst\_acc:0.933  
epoch: 881 traing\_acc:0.990 tst\_acc:0.933  
epoch: 882 traing\_acc:0.990 tst\_acc:0.933  
epoch: 883 traing\_acc:0.981 tst\_acc:0.933  
epoch: 884 traing\_acc:0.990 tst\_acc:0.933  
epoch: 885 traing\_acc:0.990 tst\_acc:0.933  
epoch: 886 traing\_acc:0.971 tst\_acc:0.933  
epoch: 887 traing\_acc:0.981 tst\_acc:0.933  
epoch: 888 traing\_acc:0.981 tst\_acc:0.933  
epoch: 889 traing\_acc:0.990 tst\_acc:0.933  
epoch: 890 traing\_acc:0.990 tst\_acc:0.933  
epoch: 891 traing\_acc:0.981 tst\_acc:0.933  
epoch: 892 traing\_acc:0.981 tst\_acc:0.933  
epoch: 893 traing\_acc:0.981 tst\_acc:0.933  
epoch: 894 traing\_acc:0.990 tst\_acc:0.933  
epoch: 895 traing\_acc:0.981 tst\_acc:0.933  
epoch: 896 traing\_acc:0.990 tst\_acc:0.933  
epoch: 897 traing\_acc:0.981 tst\_acc:0.933  
epoch: 898 traing\_acc:0.981 tst\_acc:0.933  
epoch: 899 traing\_acc:0.981 tst\_acc:0.933  
epoch: 900 traing\_acc:0.981 tst\_acc:0.933  
epoch: 901 traing\_acc:0.990 tst\_acc:0.933  
epoch: 902 traing\_acc:0.990 tst\_acc:0.933  
epoch: 903 traing\_acc:0.981 tst\_acc:0.933  
epoch: 904 traing\_acc:0.981 tst\_acc:0.933  
epoch: 905 traing\_acc:0.981 tst\_acc:0.933

epoch: 906 traing\_acc:0.971 tst\_acc:0.933  
epoch: 907 traing\_acc:0.981 tst\_acc:0.933  
epoch: 908 traing\_acc:0.971 tst\_acc:0.933  
epoch: 909 traing\_acc:0.981 tst\_acc:0.933  
epoch: 910 traing\_acc:0.981 tst\_acc:0.933  
epoch: 911 traing\_acc:0.981 tst\_acc:0.933  
epoch: 912 traing\_acc:0.962 tst\_acc:0.933  
epoch: 913 traing\_acc:0.981 tst\_acc:0.933  
epoch: 914 traing\_acc:0.971 tst\_acc:0.933  
epoch: 915 traing\_acc:0.971 tst\_acc:0.933  
epoch: 916 traing\_acc:0.990 tst\_acc:0.933  
epoch: 917 traing\_acc:0.981 tst\_acc:0.933  
epoch: 918 traing\_acc:0.971 tst\_acc:0.933  
epoch: 919 traing\_acc:0.981 tst\_acc:0.933  
epoch: 920 traing\_acc:0.990 tst\_acc:0.933  
epoch: 921 traing\_acc:0.990 tst\_acc:0.933  
epoch: 922 traing\_acc:0.990 tst\_acc:0.933  
epoch: 923 traing\_acc:0.981 tst\_acc:0.933  
epoch: 924 traing\_acc:0.981 tst\_acc:0.933  
epoch: 925 traing\_acc:0.981 tst\_acc:0.933  
epoch: 926 traing\_acc:0.981 tst\_acc:0.933  
epoch: 927 traing\_acc:0.990 tst\_acc:0.933  
epoch: 928 traing\_acc:0.990 tst\_acc:0.933  
epoch: 929 traing\_acc:0.981 tst\_acc:0.933  
epoch: 930 traing\_acc:0.981 tst\_acc:0.933  
epoch: 931 traing\_acc:0.981 tst\_acc:0.933  
epoch: 932 traing\_acc:0.981 tst\_acc:0.933  
epoch: 933 traing\_acc:0.990 tst\_acc:0.933  
epoch: 934 traing\_acc:0.990 tst\_acc:0.933  
epoch: 935 traing\_acc:0.981 tst\_acc:0.933  
epoch: 936 traing\_acc:0.981 tst\_acc:0.933  
epoch: 937 traing\_acc:0.981 tst\_acc:0.933  
epoch: 938 traing\_acc:0.990 tst\_acc:0.933  
epoch: 939 traing\_acc:0.990 tst\_acc:0.933  
epoch: 940 traing\_acc:0.971 tst\_acc:0.933  
epoch: 941 traing\_acc:0.981 tst\_acc:0.933  
epoch: 942 traing\_acc:0.981 tst\_acc:0.933  
epoch: 943 traing\_acc:0.981 tst\_acc:0.933  
epoch: 944 traing\_acc:0.990 tst\_acc:0.933  
epoch: 945 traing\_acc:0.990 tst\_acc:0.933  
epoch: 946 traing\_acc:0.971 tst\_acc:0.933  
epoch: 947 traing\_acc:0.981 tst\_acc:0.933  
epoch: 948 traing\_acc:0.971 tst\_acc:0.933  
epoch: 949 traing\_acc:0.981 tst\_acc:0.933  
epoch: 950 traing\_acc:0.981 tst\_acc:0.933  
epoch: 951 traing\_acc:0.981 tst\_acc:0.933  
epoch: 952 traing\_acc:0.981 tst\_acc:0.933  
epoch: 953 traing\_acc:0.971 tst\_acc:0.933

epoch: 954 traing\_acc:0.981 tst\_acc:0.933  
epoch: 955 traing\_acc:0.971 tst\_acc:0.933  
epoch: 956 traing\_acc:0.981 tst\_acc:0.933  
epoch: 957 traing\_acc:0.990 tst\_acc:0.933  
epoch: 958 traing\_acc:0.981 tst\_acc:0.933  
epoch: 959 traing\_acc:0.981 tst\_acc:0.933  
epoch: 960 traing\_acc:0.990 tst\_acc:0.933  
epoch: 961 traing\_acc:0.990 tst\_acc:0.933  
epoch: 962 traing\_acc:0.990 tst\_acc:0.933  
epoch: 963 traing\_acc:0.981 tst\_acc:0.933  
epoch: 964 traing\_acc:0.981 tst\_acc:0.933  
epoch: 965 traing\_acc:0.981 tst\_acc:0.933  
epoch: 966 traing\_acc:0.981 tst\_acc:0.933  
epoch: 967 traing\_acc:0.990 tst\_acc:0.933  
epoch: 968 traing\_acc:0.981 tst\_acc:0.933  
epoch: 969 traing\_acc:0.981 tst\_acc:0.933  
epoch: 970 traing\_acc:0.981 tst\_acc:0.933  
epoch: 971 traing\_acc:0.990 tst\_acc:0.933  
epoch: 972 traing\_acc:0.990 tst\_acc:0.933  
epoch: 973 traing\_acc:0.990 tst\_acc:0.933  
epoch: 974 traing\_acc:0.981 tst\_acc:0.933  
epoch: 975 traing\_acc:0.981 tst\_acc:0.933  
epoch: 976 traing\_acc:0.981 tst\_acc:0.933  
epoch: 977 traing\_acc:0.981 tst\_acc:0.933  
epoch: 978 traing\_acc:0.990 tst\_acc:0.933  
epoch: 979 traing\_acc:0.990 tst\_acc:0.933  
epoch: 980 traing\_acc:0.981 tst\_acc:0.933  
epoch: 981 traing\_acc:0.971 tst\_acc:0.933  
epoch: 982 traing\_acc:0.981 tst\_acc:0.933  
epoch: 983 traing\_acc:0.990 tst\_acc:0.933  
epoch: 984 traing\_acc:0.990 tst\_acc:0.933  
epoch: 985 traing\_acc:0.990 tst\_acc:0.933  
epoch: 986 traing\_acc:0.981 tst\_acc:0.933  
epoch: 987 traing\_acc:0.981 tst\_acc:0.933  
epoch: 988 traing\_acc:0.990 tst\_acc:0.933  
epoch: 989 traing\_acc:0.981 tst\_acc:0.933  
epoch: 990 traing\_acc:0.981 tst\_acc:0.933  
epoch: 991 traing\_acc:0.990 tst\_acc:0.933  
epoch: 992 traing\_acc:0.981 tst\_acc:0.933  
epoch: 993 traing\_acc:0.981 tst\_acc:0.933  
epoch: 994 traing\_acc:0.981 tst\_acc:0.933  
epoch: 995 traing\_acc:0.981 tst\_acc:0.933  
epoch: 996 traing\_acc:0.981 tst\_acc:0.933  
epoch: 997 traing\_acc:0.981 tst\_acc:0.933  
epoch: 998 traing\_acc:0.981 tst\_acc:0.933  
epoch: 999 traing\_acc:0.990 tst\_acc:0.933  
epoch: 1000 traing\_acc:0.981 tst\_acc:0.933  
epoch: 1001 traing\_acc:0.990 tst\_acc:0.933

[illegible]

[illegible]

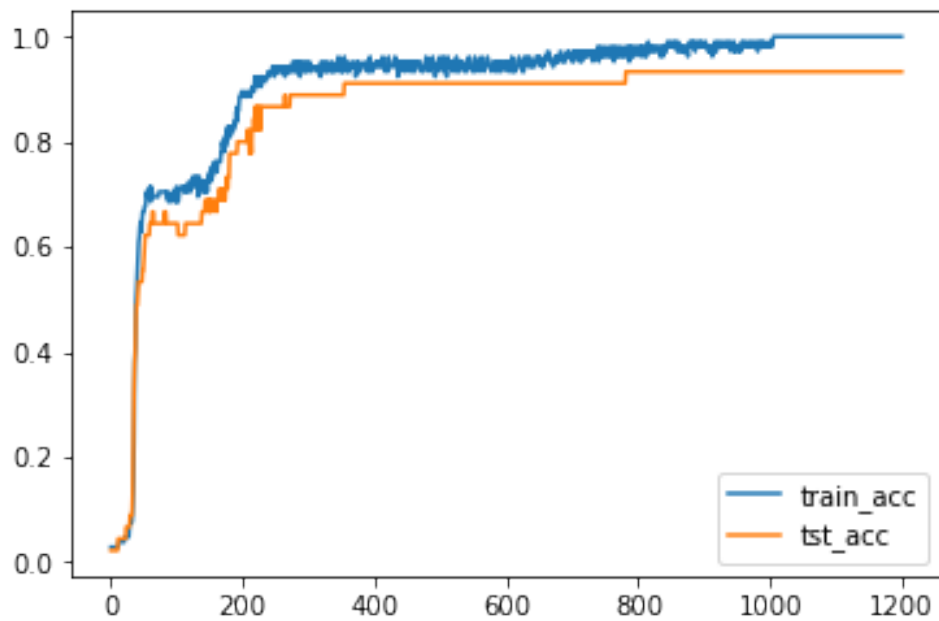
epoch: 1098 traing\_acc:1.000 tst\_acc:0.933  
epoch: 1099 traing\_acc:1.000 tst\_acc:0.933  
epoch: 1100 traing\_acc:1.000 tst\_acc:0.933  
epoch: 1101 traing\_acc:1.000 tst\_acc:0.933  
epoch: 1102 traing\_acc:1.000 tst\_acc:0.933  
epoch: 1103 traing\_acc:1.000 tst\_acc:0.933  
epoch: 1104 traing\_acc:1.000 tst\_acc:0.933  
epoch: 1105 traing\_acc:1.000 tst\_acc:0.933  
epoch: 1106 traing\_acc:1.000 tst\_acc:0.933  
epoch: 1107 traing\_acc:1.000 tst\_acc:0.933  
epoch: 1108 traing\_acc:1.000 tst\_acc:0.933  
epoch: 1109 traing\_acc:1.000 tst\_acc:0.933  
epoch: 1110 traing\_acc:1.000 tst\_acc:0.933  
epoch: 1111 traing\_acc:1.000 tst\_acc:0.933  
epoch: 1112 traing\_acc:1.000 tst\_acc:0.933  
epoch: 1113 traing\_acc:1.000 tst\_acc:0.933  
epoch: 1114 traing\_acc:1.000 tst\_acc:0.933  
epoch: 1115 traing\_acc:1.000 tst\_acc:0.933  
epoch: 1116 traing\_acc:1.000 tst\_acc:0.933  
epoch: 1117 traing\_acc:1.000 tst\_acc:0.933  
epoch: 1118 traing\_acc:1.000 tst\_acc:0.933  
epoch: 1119 traing\_acc:1.000 tst\_acc:0.933  
epoch: 1120 traing\_acc:1.000 tst\_acc:0.933  
epoch: 1121 traing\_acc:1.000 tst\_acc:0.933  
epoch: 1122 traing\_acc:1.000 tst\_acc:0.933  
epoch: 1123 traing\_acc:1.000 tst\_acc:0.933  
epoch: 1124 traing\_acc:1.000 tst\_acc:0.933  
epoch: 1125 traing\_acc:1.000 tst\_acc:0.933  
epoch: 1126 traing\_acc:1.000 tst\_acc:0.933  
epoch: 1127 traing\_acc:1.000 tst\_acc:0.933  
epoch: 1128 traing\_acc:1.000 tst\_acc:0.933  
epoch: 1129 traing\_acc:1.000 tst\_acc:0.933  
epoch: 1130 traing\_acc:1.000 tst\_acc:0.933  
epoch: 1131 traing\_acc:1.000 tst\_acc:0.933  
epoch: 1132 traing\_acc:1.000 tst\_acc:0.933  
epoch: 1133 traing\_acc:1.000 tst\_acc:0.933  
epoch: 1134 traing\_acc:1.000 tst\_acc:0.933  
epoch: 1135 traing\_acc:1.000 tst\_acc:0.933  
epoch: 1136 traing\_acc:1.000 tst\_acc:0.933  
epoch: 1137 traing\_acc:1.000 tst\_acc:0.933  
epoch: 1138 traing\_acc:1.000 tst\_acc:0.933  
epoch: 1139 traing\_acc:1.000 tst\_acc:0.933  
epoch: 1140 traing\_acc:1.000 tst\_acc:0.933  
epoch: 1141 traing\_acc:1.000 tst\_acc:0.933  
epoch: 1142 traing\_acc:1.000 tst\_acc:0.933  
epoch: 1143 traing\_acc:1.000 tst\_acc:0.933  
epoch: 1144 traing\_acc:1.000 tst\_acc:0.933  
epoch: 1145 traing\_acc:1.000 tst\_acc:0.933

[illegible]



```
epoch: 1194 traing_acc:1.000 tst_acc:0.933
epoch: 1195 traing_acc:1.000 tst_acc:0.933
epoch: 1196 traing_acc:1.000 tst_acc:0.933
epoch: 1197 traing_acc:1.000 tst_acc:0.933
epoch: 1198 traing_acc:1.000 tst_acc:0.933
epoch: 1199 traing_acc:1.000 tst_acc:0.933
```

[110]: <matplotlib.legend.Legend at 0x12c6f0dc0>



[110]:

# knn

February 24, 2023

## 1 k-Nearest Neighbor (kNN) exercise

Complete and hand in this completed worksheet (including its outputs and any supporting code outside of the worksheet) with your assignment submission. For more details see the [assignments page](#) on the course website.

The kNN classifier consists of two stages:

- During training, the classifier takes the training data and simply remembers it
- During testing, kNN classifies every test image by comparing to all training images and transferring the labels of the k most similar training examples
- The value of k is cross-validated

In this exercise you will implement these steps and understand the basic Image Classification pipeline, cross-validation, and gain proficiency in writing efficient, vectorized code.

```
[30]: # Run some setup code for this notebook.

import random
import numpy as np
from cs231n.data_utils import load_CIFAR10
import matplotlib.pyplot as plt

# This is a bit of magic to make matplotlib figures appear inline in the
↪ notebook
# rather than in a new window.
%matplotlib inline
plt.rcParams['figure.figsize'] = (10.0, 8.0) # set default size of plots
plt.rcParams['image.interpolation'] = 'nearest'
plt.rcParams['image.cmap'] = 'gray'

# Some more magic so that the notebook will reload external python modules;
# see http://stackoverflow.com/questions/1907993/
↪ autoreload-of-modules-in-ipython
%load_ext autoreload
%autoreload 2
```

The autoreload extension is already loaded. To reload it, use:

```
%reload_ext autoreload
```

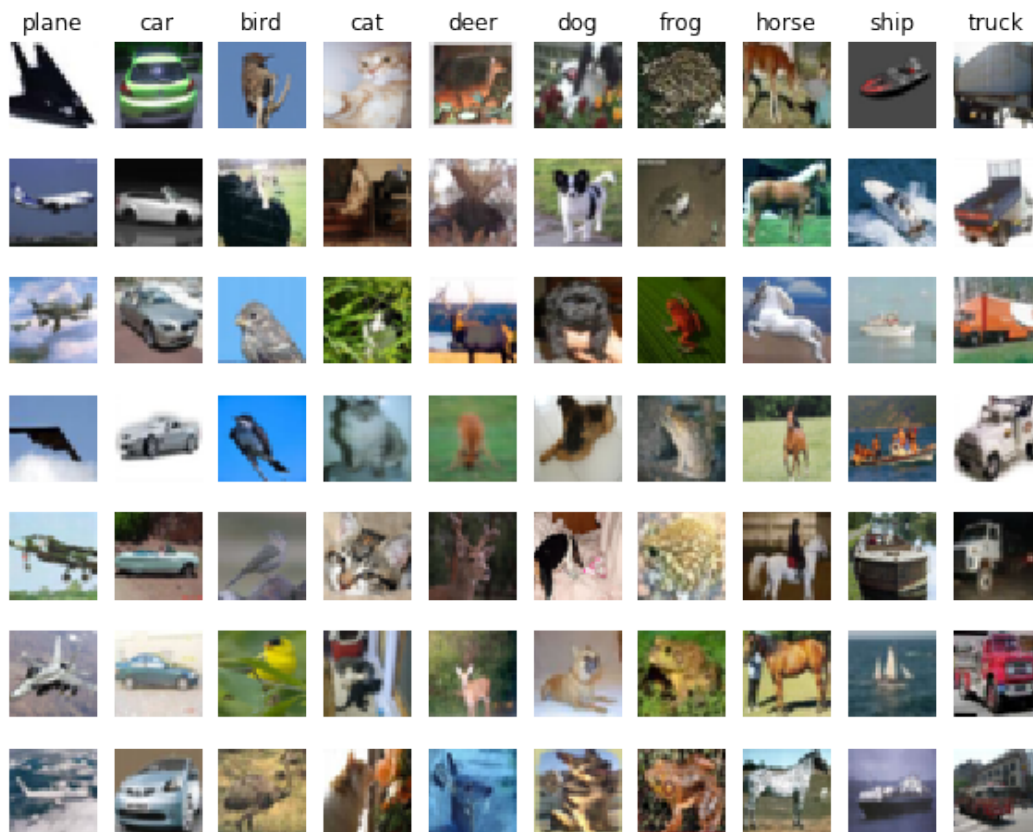
```
[31]: # Load the raw CIFAR-10 data.
cifar10_dir = 'cs231n/datasets/cifar-10-batches-py'
# Cleaning up variables to prevent loading data multiple times (which may cause
↳memory issue)
try:
    del X_train, y_train
    del X_test, y_test
    print('Clear previously loaded data.')
except:
    pass

X_train, y_train, X_test, y_test = load_CIFAR10(cifar10_dir)

# As a sanity check, we print out the size of the training and test data.
print('Training data shape: ', X_train.shape)
print('Training labels shape: ', y_train.shape)
print('Test data shape: ', X_test.shape)
print('Test labels shape: ', y_test.shape)
```

```
Clear previously loaded data.
Training data shape: (50000, 32, 32, 3)
Training labels shape: (50000,)
Test data shape: (10000, 32, 32, 3)
Test labels shape: (10000,)
```

```
[32]: # Visualize some examples from the dataset.
# We show a few examples of training images from each class.
classes = ['plane', 'car', 'bird', 'cat', 'deer', 'dog', 'frog', 'horse',
↳'ship', 'truck']
num_classes = len(classes)
samples_per_class = 7
for y, cls in enumerate(classes):
    idxs = np.flatnonzero(y_train == y)
    idxs = np.random.choice(idxs, samples_per_class, replace=False)
    for i, idx in enumerate(idxs):
        plt_idx = i * num_classes + y + 1
        plt.subplot(samples_per_class, num_classes, plt_idx)
        plt.imshow(X_train[idx].astype('uint8'))
        plt.axis('off')
        if i == 0:
            plt.title(cls)
plt.show()
```



[33]: *# Subsample the data for more efficient code execution in this exercise*

```
num_training = 5000
mask = list(range(num_training))
X_train = X_train[mask]
y_train = y_train[mask]

num_test = 500
mask = list(range(num_test))
X_test = X_test[mask]
y_test = y_test[mask]

# Reshape the image data into rows
X_train = np.reshape(X_train, (X_train.shape[0], -1))
X_test = np.reshape(X_test, (X_test.shape[0], -1))
print(X_train.shape, X_test.shape)
```

(5000, 3072) (500, 3072)

```
[34]: from cs231n.classifiers import KNearestNeighbor

# Create a kNN classifier instance.
# Remember that training a kNN classifier is a noop:
# the Classifier simply remembers the data and does no further processing
classifier = KNearestNeighbor()
classifier.train(X_train, y_train)
```

We would now like to classify the test data with the kNN classifier. Recall that we can break down this process into two steps:

1. First we must compute the distances between all test examples and all train examples.
2. Given these distances, for each test example we find the k nearest examples and have them vote for the label

Lets begin with computing the distance matrix between all training and test examples. For example, if there are  $N_{tr}$  training examples and  $N_{te}$  test examples, this stage should result in a  $N_{te} \times N_{tr}$  matrix where each element  $(i,j)$  is the distance between the  $i$ -th test and  $j$ -th train example.

**Note:** For the three distance computations that we require you to implement in this notebook, you may not use the `np.linalg.norm()` function that numpy provides.

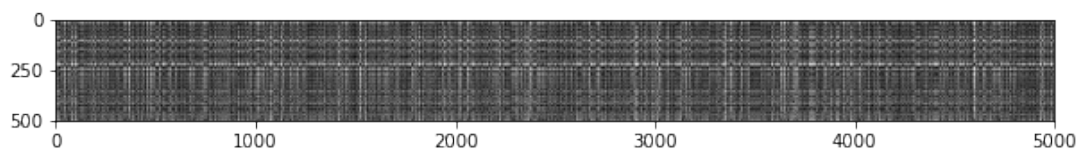
First, open `cs231n/classifiers/k_nearest_neighbor.py` and implement the function `compute_distances_two_loops` that uses a (very inefficient) double loop over all pairs of (test, train) examples and computes the distance matrix one element at a time.

```
[35]: # Open cs231n/classifiers/k_nearest_neighbor.py and implement
# compute_distances_two_loops.

# Test your implementation:
dists = classifier.compute_distances_two_loops(X_test)
print(dists.shape)
```

(500, 5000)

```
[36]: # We can visualize the distance matrix: each row is a single test example and
# its distances to training examples
plt.imshow(dists, interpolation='none')
plt.show()
```



### Inline Question 1

Notice the structured patterns in the distance matrix, where some rows or columns are visible brighter. (Note that with the default color scheme black indicates low distances while white indicates high distances.)

- What in the data is the cause behind the distinctly bright rows?
- What causes the columns?

*Your Answer :*

If a row (say the corresponding test data  $r$ ) is distinctly brighter than others, then it means the corresponding distances from  $r$  to all training points are all distinctly larger than other test points.

If a column (say the corresponding training data  $c$ ) is distinctly brighter than others, then it means the corresponding distances from all test data to  $c$  are distinctly larger than other training points.

```
[37]: # Now implement the function predict_labels and run the code below:
# We use k = 1 (which is Nearest Neighbor).
y_test_pred = classifier.predict_labels(dists, k=1)

# Compute and print the fraction of correctly predicted examples
num_correct = np.sum(y_test_pred == y_test)
accuracy = float(num_correct) / num_test
print('Got %d / %d correct => accuracy: %f' % (num_correct, num_test, accuracy))
```

Got 137 / 500 correct => accuracy: 0.274000

You should expect to see approximately 27% accuracy. Now let's try out a larger  $k$ , say  $k = 5$ :

```
[38]: y_test_pred = classifier.predict_labels(dists, k=5)
num_correct = np.sum(y_test_pred == y_test)
accuracy = float(num_correct) / num_test
print('Got %d / %d correct => accuracy: %f' % (num_correct, num_test, accuracy))
```

Got 139 / 500 correct => accuracy: 0.278000

You should expect to see a slightly better performance than with  $k = 1$ .

### Inline Question 2

We can also use other distance metrics such as L1 distance. For pixel values  $p_{ij}^{(k)}$  at location  $(i, j)$  of some image  $I_k$ ,

the mean  $\mu$  across all pixels over all images is

$$\mu = \frac{1}{nhw} \sum_{k=1}^n \sum_{i=1}^h \sum_{j=1}^w p_{ij}^{(k)}$$

And the pixel-wise mean  $\mu_{ij}$  across all images is

$$\mu_{ij} = \frac{1}{n} \sum_{k=1}^n p_{ij}^{(k)}.$$

The general standard deviation  $\sigma$  and pixel-wise standard deviation  $\sigma_{ij}$  is defined similarly.

Which of the following preprocessing steps will not change the performance of a Nearest Neighbor classifier that uses L1 distance? Select all that apply. 1. Subtracting the mean  $\mu$  ( $\tilde{p}_{ij}^{(k)} = p_{ij}^{(k)} - \mu$ .) 2. Subtracting the per pixel mean  $\mu_{ij}$  ( $\tilde{p}_{ij}^{(k)} = p_{ij}^{(k)} - \mu_{ij}$ .) 3. Subtracting the mean  $\mu$  and dividing by the standard deviation  $\sigma$ . 4. Subtracting the pixel-wise mean  $\mu_{ij}$  and dividing by the pixel-wise standard deviation  $\sigma_{ij}$ . 5. Rotating the coordinate axes of the data.

*Your Answer :*

I choose 1, 2, 3, 5

*Your Explanation :*

The L1 distance is defined as follows:

$$d(I_p, I_q) = \sum_{i=1}^h \sum_{j=1}^w \|p_{i,j}^p - p_{i,j}^q\|_1$$

Then define  $d_i(\cdot, \cdot)$  as the new distance metric for the  $i$ -th preprocessing step.

For  $d_1(\cdot, \cdot)$ , we have

$$d_1(I_p, I_q) = \sum_{i=1}^h \sum_{j=1}^w \|p_{i,j}^p - \mu - p_{i,j}^q + \mu\|_1 = d(I_p, I_q)$$

For  $d_2(\cdot, \cdot)$ , we have

$$d_2(I_p, I_q) = \sum_{i=1}^h \sum_{j=1}^w \|p_{i,j}^p - \mu_{i,j} - p_{i,j}^q + \mu_{i,j}\|_1 = d(I_p, I_q)$$

For  $d_3(\cdot, \cdot)$ , we have

$$d_3(I_p, I_q) = \sum_{i=1}^h \sum_{j=1}^w \left\| \frac{p_{i,j}^p - \mu}{\sigma} - \frac{p_{i,j}^q - \mu}{\sigma} \right\|_1 = \frac{d(I_p, I_q)}{\sigma} \propto d(I_p, I_q)$$

For  $d_4(\cdot, \cdot)$ , we have

$$d_4(I_p, I_q) = \sum_{i=1}^h \sum_{j=1}^w \left\| \frac{p_{i,j}^p - \mu_{i,j}}{\sigma_{i,j}} - \frac{p_{i,j}^q - \mu_{i,j}}{\sigma_{i,j}} \right\|_1 = \sum_{i=1}^h \sum_{j=1}^w \left\| \frac{p_{i,j}^p - p_{i,j}^q}{\sigma_{i,j}} \right\|_1$$

For  $d_5(\cdot, \cdot)$ , we have

$$d_5(I_p, I_q) = \sum_{i=1}^w \sum_{j=1}^h \|p_{i,j}^p - p_{i,j}^q\|_1 = d(I_p, I_q)$$

[39]: *# Now lets speed up distance matrix computation by using partial vectorization  
# with one loop. Implement the function compute\_distances\_one\_loop and run the  
# code below:*  
`dists_one = classifier.compute_distances_one_loop(X_test)`

```

# To ensure that our vectorized implementation is correct, we make sure that it
# agrees with the naive implementation. There are many ways to decide whether
# two matrices are similar; one of the simplest is the Frobenius norm. In case
# you haven't seen it before, the Frobenius norm of two matrices is the square
# root of the squared sum of differences of all elements; in other words,
↳ reshape
# the matrices into vectors and compute the Euclidean distance between them.
difference = np.linalg.norm(dists - dists_one, ord='fro')
print('One loop difference was: %f' % (difference, ))
if difference < 0.001:
    print('Good! The distance matrices are the same')
else:
    print('Uh-oh! The distance matrices are different')

```

One loop difference was: 0.000000  
 Good! The distance matrices are the same

```

[40]: # Now implement the fully vectorized version inside compute_distances_no_loops
# and run the code
dists_two = classifier.compute_distances_no_loops(X_test)

# check that the distance matrix agrees with the one we computed before:
difference = np.linalg.norm(dists - dists_two, ord='fro')
print('No loop difference was: %f' % (difference, ))
if difference < 0.001:
    print('Good! The distance matrices are the same')
else:
    print('Uh-oh! The distance matrices are different')

```

No loop difference was: 0.000000  
 Good! The distance matrices are the same

```

[41]: # Let's compare how fast the implementations are
def time_function(f, *args):
    """
    Call a function f with args and return the time (in seconds) that it took
    ↳ to execute.
    """
    import time
    tic = time.time()
    f(*args)
    toc = time.time()
    return toc - tic

two_loop_time = time_function(classifier.compute_distances_two_loops, X_test)
print('Two loop version took %f seconds' % two_loop_time)

```



```

one_loop_time = time_function(classifier.compute_distances_one_loop, X_test)
print('One loop version took %f seconds' % one_loop_time)

no_loop_time = time_function(classifier.compute_distances_no_loops, X_test)
print('No loop version took %f seconds' % no_loop_time)

# You should see significantly faster performance with the fully vectorized_
↪implementation!

# NOTE: depending on what machine you're using,
# you might not see a speedup when you go from two loops to one loop,
# and might even see a slow-down.

```

Two loop version took 12.713599 seconds  
One loop version took 14.369608 seconds  
No loop version took 0.144269 seconds

### 1.0.1 Cross-validation

We have implemented the k-Nearest Neighbor classifier but we set the value  $k = 5$  arbitrarily. We will now determine the best value of this hyperparameter with cross-validation.

```

[42]: num_folds = 5
      k_choices = [1, 3, 5, 8, 10, 12, 15, 20, 50, 100]

      X_train_folds = []
      y_train_folds = []

      #####
      # TODO:                                     #
      # Split up the training data into folds. After splitting, X_train_folds and   #
      # y_train_folds should each be lists of length num_folds, where               #
      # y_train_folds[i] is the label vector for the points in X_train_folds[i].    #
      # Hint: Look up the numpy array_split function.                             #
      #####
      # *****START OF YOUR CODE (DO NOT DELETE/MODIFY THIS LINE)*****

      X_train_folds = np.array_split(X_train, num_folds)
      y_train_folds = np.array_split(y_train, num_folds)

      # *****END OF YOUR CODE (DO NOT DELETE/MODIFY THIS LINE)*****

      # A dictionary holding the accuracies for different values of k that we find
      # when running cross-validation. After running cross-validation,
      # k_to_accuracies[k] should be a list of length num_folds giving the different
      # accuracy values that we found when using that value of k.
      k_to_accuracies = {}

```

```
#####
# TODO:
# Perform k-fold cross validation to find the best value of k. For each
# possible value of k, run the k-nearest-neighbor algorithm num_folds times,
# where in each case you use all but one of the folds as training data and the
# last fold as a validation set. Store the accuracies for all fold and all
# values of k in the k_to_accuracies dictionary.
#####
# *****START OF YOUR CODE (DO NOT DELETE/MODIFY THIS LINE)*****

for k in k_choices:
    k_to_accuracies[k] = []
    for i in range(num_folds):
        X_train_fold = np.concatenate(X_train_folds[:i] + X_train_folds[i+1:])
        y_train_fold = np.concatenate(y_train_folds[:i] + y_train_folds[i+1:])
        X_val_fold = X_train_folds[i]
        y_val_fold = y_train_folds[i]
        classifier.train(X_train_fold, y_train_fold)
        y_pred = classifier.predict(X_val_fold, k=k)
        k_to_accuracies[k].append(np.sum(y_pred == y_val_fold) / y_val_fold.
↪size)

# *****END OF YOUR CODE (DO NOT DELETE/MODIFY THIS LINE)*****

# Print out the computed accuracies
for k in sorted(k_to_accuracies):
    for accuracy in k_to_accuracies[k]:
        print('k = %d, accuracy = %f' % (k, accuracy))
```

```
k = 1, accuracy = 0.263000
k = 1, accuracy = 0.257000
k = 1, accuracy = 0.264000
k = 1, accuracy = 0.278000
k = 1, accuracy = 0.266000
k = 3, accuracy = 0.239000
k = 3, accuracy = 0.249000
k = 3, accuracy = 0.240000
k = 3, accuracy = 0.266000
k = 3, accuracy = 0.254000
k = 5, accuracy = 0.248000
k = 5, accuracy = 0.266000
k = 5, accuracy = 0.280000
k = 5, accuracy = 0.292000
k = 5, accuracy = 0.280000
k = 8, accuracy = 0.262000
k = 8, accuracy = 0.282000
```

```

k = 8, accuracy = 0.273000
k = 8, accuracy = 0.290000
k = 8, accuracy = 0.273000
k = 10, accuracy = 0.265000
k = 10, accuracy = 0.296000
k = 10, accuracy = 0.276000
k = 10, accuracy = 0.284000
k = 10, accuracy = 0.280000
k = 12, accuracy = 0.260000
k = 12, accuracy = 0.295000
k = 12, accuracy = 0.279000
k = 12, accuracy = 0.283000
k = 12, accuracy = 0.280000
k = 15, accuracy = 0.252000
k = 15, accuracy = 0.289000
k = 15, accuracy = 0.278000
k = 15, accuracy = 0.282000
k = 15, accuracy = 0.274000
k = 20, accuracy = 0.270000
k = 20, accuracy = 0.279000
k = 20, accuracy = 0.279000
k = 20, accuracy = 0.282000
k = 20, accuracy = 0.285000
k = 50, accuracy = 0.271000
k = 50, accuracy = 0.288000
k = 50, accuracy = 0.278000
k = 50, accuracy = 0.269000
k = 50, accuracy = 0.266000
k = 100, accuracy = 0.256000
k = 100, accuracy = 0.270000
k = 100, accuracy = 0.263000
k = 100, accuracy = 0.256000
k = 100, accuracy = 0.263000

```

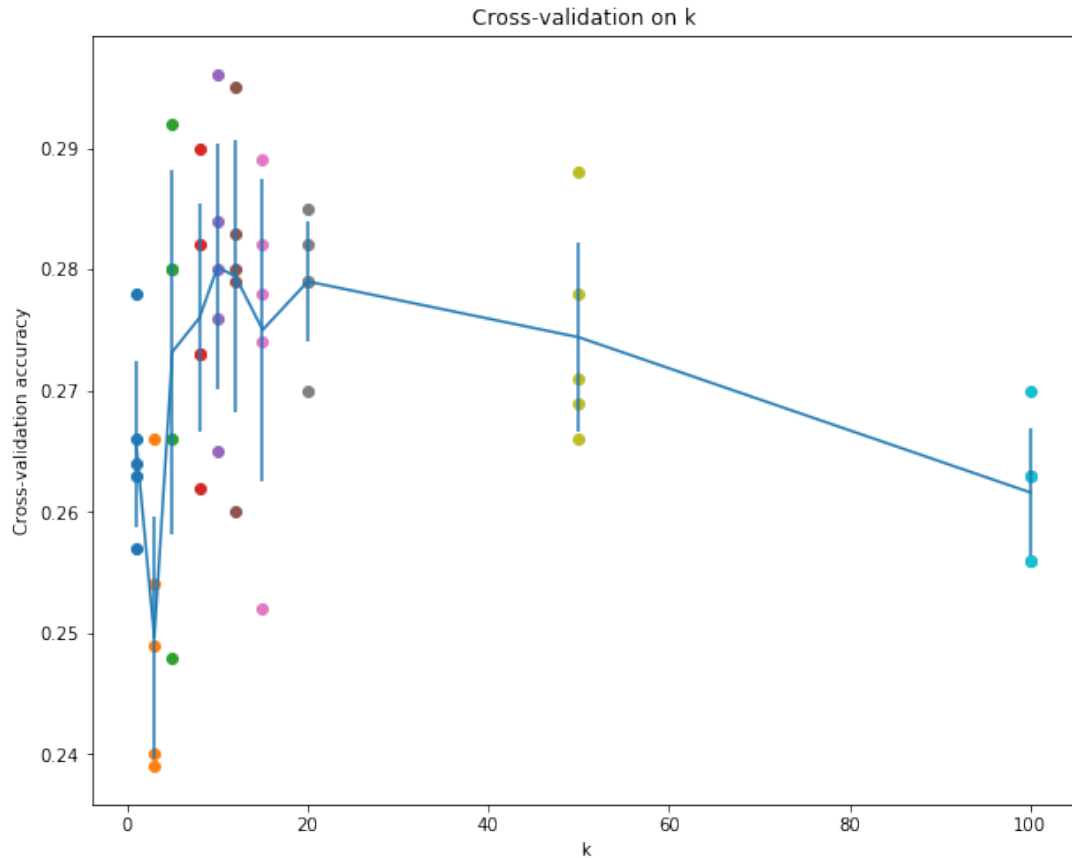
```

[43]: # plot the raw observations
for k in k_choices:
    accuracies = k_to_accuracies[k]
    plt.scatter([k] * len(accuracies), accuracies)

# plot the trend line with error bars that correspond to standard deviation
accuracies_mean = np.array([np.mean(v) for k,v in sorted(k_to_accuracies.
    ↪items())])
accuracies_std = np.array([np.std(v) for k,v in sorted(k_to_accuracies.
    ↪items())])
plt.errorbar(k_choices, accuracies_mean, yerr=accuracies_std)
plt.title('Cross-validation on k')
plt.xlabel('k')

```

```
plt.ylabel('Cross-validation accuracy')
plt.show()
```



```
[44]: # Based on the cross-validation results above, choose the best value for k,
# retrain the classifier using all the training data, and test it on the test
# data. You should be able to get above 28% accuracy on the test data.
best_k = 10

classifier = KNearestNeighbor()
classifier.train(X_train, y_train)
y_test_pred = classifier.predict(X_test, k=best_k)

# Compute and display the accuracy
num_correct = np.sum(y_test_pred == y_test)
accuracy = float(num_correct) / num_test
print('Got %d / %d correct => accuracy: %f' % (num_correct, num_test, accuracy))
```

Got 141 / 500 correct => accuracy: 0.282000

### Inline Question 3

Which of the following statements about  $k$ -Nearest Neighbor ( $k$ -NN) are true in a classification setting, and for all  $k$ ? Select all that apply. 1. The decision boundary of the  $k$ -NN classifier is linear. 2. The training error of a 1-NN will always be lower than that of 5-NN. 3. The test error of a 1-NN will always be lower than that of a 5-NN. 4. The time needed to classify a test example with the  $k$ -NN classifier grows with the size of the training set. 5. None of the above.

*Your Answer :*

I choose 2, 4.

*Your Explanation :*

1. False. The decision boundary of the  $k$ -NN classifier is non-linear since the unit “sphere” under L2 norm is a unit circle in 2D and a unit sphere in 3D which is not linear.
2. True. The training error of a 1-NN is always 0 since the nearest neighbor of a point is itself.
3. False. The test error does not increase with  $k$  just like show in the picture above.
4. True. The process of classifying a test sample is to traverse through all the training data, which is an  $O(n)$  operation.

[44] :

# SVM

February 25, 2023

## 1 Multiclass Support Vector Machine exercise

Complete and hand in this completed worksheet (including its outputs and any supporting code outside of the worksheet) with your assignment submission. For more details see the [assignments page](#) on the course website.

In this exercise you will:

- implement a fully-vectorized **loss function** for the SVM
- implement the fully-vectorized expression for its **analytic gradient**
- **check your implementation** using numerical gradient
- use a validation set to **tune the learning rate and regularization** strength
- **optimize** the loss function with **SGD**
- **visualize** the final learned weights

```
[2]: # Run some setup code for this notebook.
import random
import numpy as np
from cs231n.data_utils import load_CIFAR10
import matplotlib.pyplot as plt

# This is a bit of magic to make matplotlib figures appear inline in the
# notebook rather than in a new window.
%matplotlib inline
plt.rcParams['figure.figsize'] = (10.0, 8.0) # set default size of plots
plt.rcParams['image.interpolation'] = 'nearest'
plt.rcParams['image.cmap'] = 'gray'

# Some more magic so that the notebook will reload external python modules;
# see http://stackoverflow.com/questions/1907993/
↪ autoreload-of-modules-in-ipython
%load_ext autoreload
%autoreload 2
```

## 1.1 CIFAR-10 Data Loading and Preprocessing

```
[3]: # Load the raw CIFAR-10 data.
cifar10_dir = 'cs231n/datasets/cifar-10-batches-py'

# Cleaning up variables to prevent loading data multiple times (which may cause
↳memory issue)
try:
    del X_train, y_train
    del X_test, y_test
    print('Clear previously loaded data.')
except:
    pass

X_train, y_train, X_test, y_test = load_CIFAR10(cifar10_dir)

# As a sanity check, we print out the size of the training and test data.
print('Training data shape: ', X_train.shape)
print('Training labels shape: ', y_train.shape)
print('Test data shape: ', X_test.shape)
print('Test labels shape: ', y_test.shape)
```

Training data shape: (50000, 32, 32, 3)

Training labels shape: (50000,)

Test data shape: (10000, 32, 32, 3)

Test labels shape: (10000,)

```
[4]: # Visualize some examples from the dataset.
# We show a few examples of training images from each class.
classes = ['plane', 'car', 'bird', 'cat', 'deer', 'dog', 'frog', 'horse', '
↳ship', 'truck']
num_classes = len(classes)
samples_per_class = 7
for y, cls in enumerate(classes):
    idxs = np.flatnonzero(y_train == y)
    idxs = np.random.choice(idxs, samples_per_class, replace=False)
    for i, idx in enumerate(idxs):
        plt_idx = i * num_classes + y + 1
        plt.subplot(samples_per_class, num_classes, plt_idx)
        plt.imshow(X_train[idx].astype('uint8'))
        plt.axis('off')
        if i == 0:
            plt.title(cls)
plt.show()
```



```
[5]: # Split the data into train, val, and test sets. In addition we will
# create a small development set as a subset of the training data;
# we can use this for development so our code runs faster.
num_training = 49000
num_validation = 1000
num_test = 1000
num_dev = 500

# Our validation set will be num_validation points from the original
# training set.
mask = range(num_training, num_training + num_validation)
X_val = X_train[mask]
y_val = y_train[mask]

# Our training set will be the first num_train points from the original
# training set.
mask = range(num_training)
X_train = X_train[mask]
y_train = y_train[mask]

# We will also make a development set, which is a small subset of
# the training set.
mask = np.random.choice(num_training, num_dev, replace=False)
X_dev = X_train[mask]
```



```

y_dev = y_train[mask]

# We use the first num_test points of the original test set as our
# test set.
mask = range(num_test)
X_test = X_test[mask]
y_test = y_test[mask]

print('Train data shape: ', X_train.shape)
print('Train labels shape: ', y_train.shape)
print('Validation data shape: ', X_val.shape)
print('Validation labels shape: ', y_val.shape)
print('Test data shape: ', X_test.shape)
print('Test labels shape: ', y_test.shape)

```

```

Train data shape: (49000, 32, 32, 3)
Train labels shape: (49000,)
Validation data shape: (1000, 32, 32, 3)
Validation labels shape: (1000,)
Test data shape: (1000, 32, 32, 3)
Test labels shape: (1000,)

```

```

[6]: # Preprocessing: reshape the image data into rows
X_train = np.reshape(X_train, (X_train.shape[0], -1))
X_val = np.reshape(X_val, (X_val.shape[0], -1))
X_test = np.reshape(X_test, (X_test.shape[0], -1))
X_dev = np.reshape(X_dev, (X_dev.shape[0], -1))

# As a sanity check, print out the shapes of the data
print('Training data shape: ', X_train.shape)
print('Validation data shape: ', X_val.shape)
print('Test data shape: ', X_test.shape)
print('dev data shape: ', X_dev.shape)

```

```

Training data shape: (49000, 3072)
Validation data shape: (1000, 3072)
Test data shape: (1000, 3072)
dev data shape: (500, 3072)

```

```

[7]: # Preprocessing: subtract the mean image
# first: compute the image mean based on the training data
mean_image = np.mean(X_train, axis=0)
print(mean_image[:10]) # print a few of the elements
plt.figure(figsize=(4,4))
plt.imshow(mean_image.reshape((32,32,3)).astype('uint8')) # visualize the mean_
↪ image
plt.show()

```

```

# second: subtract the mean image from train and test data
X_train -= mean_image
X_val -= mean_image
X_test -= mean_image
X_dev -= mean_image

# third: append the bias dimension of ones (i.e. bias trick) so that our SVM
# only has to worry about optimizing a single weight matrix W.
X_train = np.hstack([X_train, np.ones((X_train.shape[0], 1))])
X_val = np.hstack([X_val, np.ones((X_val.shape[0], 1))])
X_test = np.hstack([X_test, np.ones((X_test.shape[0], 1))])
X_dev = np.hstack([X_dev, np.ones((X_dev.shape[0], 1))])

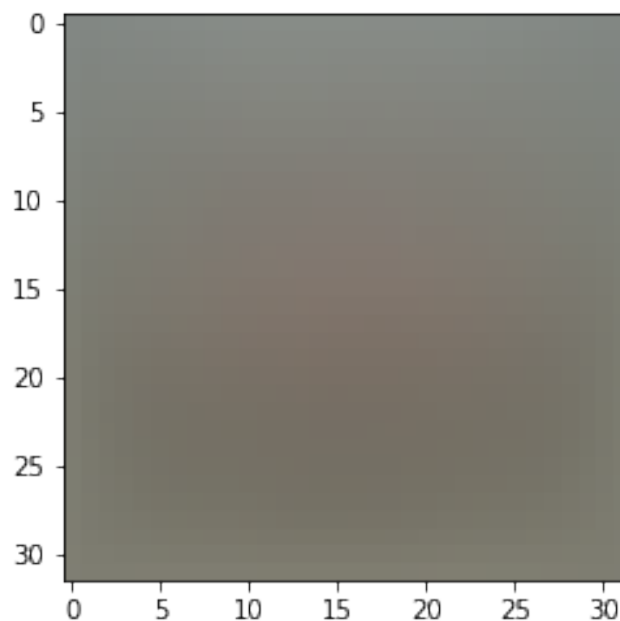
print(X_train.shape, X_val.shape, X_test.shape, X_dev.shape)

```

```

[130.64189796 135.98173469 132.47391837 130.05569388 135.34804082
 131.75402041 130.96055102 136.14328571 132.47636735 131.48467347]

```



```

(49000, 3073) (1000, 3073) (1000, 3073) (500, 3073)

```

## 1.2 SVM Classifier

Your code for this section will all be written inside `cs231n/classifiers/linear_svm.py`.

As you can see, we have prefilled the function `svm_loss_naive` which uses for loops to evaluate the multiclass SVM loss function.

```
[8]: # Evaluate the naive implementation of the loss we provided for you:
from cs231n.classifiers.linear_svm import svm_loss_naive
import time

# generate a random SVM weight matrix of small numbers
W = np.random.randn(3073, 10) * 0.0001

loss, grad = svm_loss_naive(W, X_dev, y_dev, 0.000005)
print('loss: %f' % (loss, ))
```

loss: 9.318401

The `grad` returned from the function above is right now all zero. Derive and implement the gradient for the SVM cost function and implement it inline inside the function `svm_loss_naive`. You will find it helpful to interleave your new code inside the existing function.

To check that you have correctly implemented the gradient correctly, you can numerically estimate the gradient of the loss function and compare the numeric estimate to the gradient that you computed. We have provided code that does this for you:

```
[9]: # Once you've implemented the gradient, recompute it with the code below
# and gradient check it with the function we provided for you

# Compute the loss and its gradient at W.
loss, grad = svm_loss_naive(W, X_dev, y_dev, 0.0)

# Numerically compute the gradient along several randomly chosen dimensions, and
# compare them with your analytically computed gradient. The numbers should
# match
# almost exactly along all dimensions.
from cs231n.gradient_check import grad_check_sparse
f = lambda w: svm_loss_naive(w, X_dev, y_dev, 0.0)[0]
grad_numerical = grad_check_sparse(f, W, grad)

# do the gradient check once again with regularization turned on
# you didn't forget the regularization gradient did you?
loss, grad = svm_loss_naive(W, X_dev, y_dev, 5e1)
f = lambda w: svm_loss_naive(w, X_dev, y_dev, 5e1)[0]
grad_numerical = grad_check_sparse(f, W, grad)
```

```
numerical: 4.066394 analytic: 4.129151, relative error: 7.657412e-03
numerical: -5.600240 analytic: -5.600240, relative error: 7.350527e-11
numerical: 4.651978 analytic: 4.651978, relative error: 8.995426e-13
numerical: -22.334557 analytic: -22.334557, relative error: 1.318483e-11
numerical: -3.009755 analytic: -3.009755, relative error: 1.557965e-10
numerical: -11.344121 analytic: -11.344121, relative error: 4.304613e-13
numerical: 3.158882 analytic: 3.204693, relative error: 7.198895e-03
numerical: -18.502906 analytic: -18.382096, relative error: 3.275296e-03
numerical: -6.049070 analytic: -6.049070, relative error: 7.427929e-12
```

```

numerical: 30.572300 analytic: 30.572300, relative error: 7.611381e-12
numerical: -6.443605 analytic: -6.548512, relative error: 8.074646e-03
numerical: -3.362234 analytic: -3.294436, relative error: 1.018498e-02
numerical: -1.974795 analytic: -1.974795, relative error: 2.956606e-10
numerical: -42.768838 analytic: -42.768838, relative error: 2.457229e-12
numerical: 2.931244 analytic: 2.931244, relative error: 4.977740e-11
numerical: 1.526142 analytic: 1.601676, relative error: 2.414897e-02
numerical: -2.584603 analytic: -2.584603, relative error: 9.321052e-11
numerical: -12.417974 analytic: -12.417974, relative error: 2.307688e-11
numerical: -22.304136 analytic: -22.232664, relative error: 1.604777e-03
numerical: 25.988804 analytic: 25.988804, relative error: 1.067483e-11

```

### Inline Question 1

It is possible that once in a while a dimension in the gradcheck will not match exactly. What could such a discrepancy be caused by? Is it a reason for concern? What is a simple example in one dimension where a gradient check could fail? How would change the margin affect of the frequency of this happening? *Hint: the SVM loss function is not strictly speaking differentiable*

*Your Answer :*

To begin with, the loss function is not totally differentiable at 0. Since the numerical result is get from approximation, the approximation will fail in the position that the loss function is not differentiable.

Given that the error is caused by approximation, it is not a concern.

A simple example in 1-D: consider ReLU function,  $f(x) = \max(0, x)$ , if we approximate the gradient at  $x = -0.02$  and take the interval length  $h = 0.02$ , then the approximated gradient is

$$f'(x) = \frac{f(0.01) - f(-0.03)}{2 \times 0.02} = \frac{1}{4} \neq 0$$

but the true gradient is 0.

The way to reduce the effect is to reduce the interval length  $h$ .

```

[10]: # Next implement the function svm_loss_vectorized; for now only compute the
      ↪ loss;
      # we will implement the gradient in a moment.
      tic = time.time()
      loss_naive, grad_naive = svm_loss_naive(W, X_dev, y_dev, 0.000005)
      toc = time.time()
      print('Naive loss: %e computed in %fs' % (loss_naive, toc - tic))

      from cs231n.classifiers.linear_svm import svm_loss_vectorized
      tic = time.time()
      loss_vectorized, _ = svm_loss_vectorized(W, X_dev, y_dev, 0.000005)
      toc = time.time()
      print('Vectorized loss: %e computed in %fs' % (loss_vectorized, toc - tic))

      # The losses should match but your vectorized implementation should be much
      ↪ faster.

```

```
print('difference: %f' % (loss_naive - loss_vectorized))
```

Naive loss: 9.318401e+00 computed in 0.045376s  
Vectorized loss: 9.318401e+00 computed in 0.008753s  
difference: -0.000000

```
[11]: # Complete the implementation of svm_loss_vectorized, and compute the gradient
      # of the loss function in a vectorized way.

      # The naive implementation and the vectorized implementation should match, but
      # the vectorized version should still be much faster.
      tic = time.time()
      _, grad_naive = svm_loss_naive(W, X_dev, y_dev, 0.000005)
      toc = time.time()
      print('Naive loss and gradient: computed in %fs' % (toc - tic))

      tic = time.time()
      _, grad_vectorized = svm_loss_vectorized(W, X_dev, y_dev, 0.000005)
      toc = time.time()
      print('Vectorized loss and gradient: computed in %fs' % (toc - tic))

      # The loss is a single number, so it is easy to compare the values computed
      # by the two implementations. The gradient on the other hand is a matrix, so
      # we use the Frobenius norm to compare them.
      difference = np.linalg.norm(grad_naive - grad_vectorized, ord='fro')
      print('difference: %f' % difference)
```

Naive loss and gradient: computed in 0.075716s  
Vectorized loss and gradient: computed in 0.009150s  
difference: 0.000000

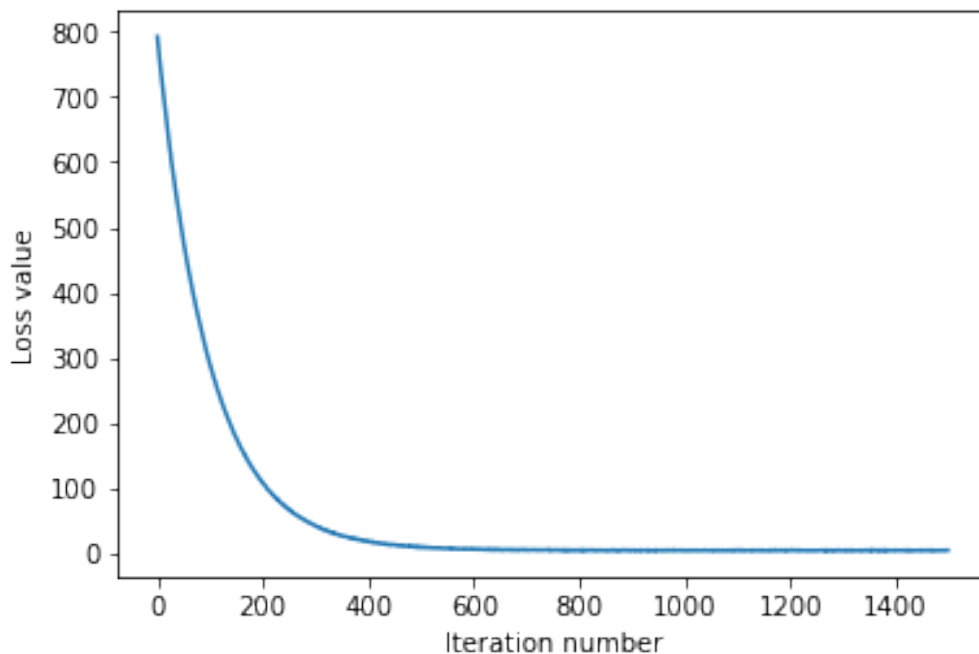
### 1.2.1 Stochastic Gradient Descent

We now have vectorized and efficient expressions for the loss, the gradient and our gradient matches the numerical gradient. We are therefore ready to do SGD to minimize the loss. Your code for this part will be written inside `cs231n/classifiers/linear_classifier.py`.

```
[12]: # In the file linear_classifier.py, implement SGD in the function
      # LinearClassifier.train() and then run it with the code below.
      from cs231n.classifiers import LinearSVM
      svm = LinearSVM()
      tic = time.time()
      loss_hist = svm.train(X_train, y_train, learning_rate=1e-7, reg=2.5e4,
                           num_iters=1500, verbose=True)
      toc = time.time()
      print('That took %fs' % (toc - tic))
```

```
iteration 0 / 1500: loss 791.981194
iteration 100 / 1500: loss 289.379744
iteration 200 / 1500: loss 108.449161
iteration 300 / 1500: loss 42.430152
iteration 400 / 1500: loss 18.899496
iteration 500 / 1500: loss 10.146439
iteration 600 / 1500: loss 7.472539
iteration 700 / 1500: loss 5.959729
iteration 800 / 1500: loss 5.459304
iteration 900 / 1500: loss 5.492587
iteration 1000 / 1500: loss 5.032883
iteration 1100 / 1500: loss 5.570110
iteration 1200 / 1500: loss 5.641019
iteration 1300 / 1500: loss 5.025879
iteration 1400 / 1500: loss 5.251474
That took 7.209703s
```

```
[13]: # A useful debugging strategy is to plot the loss as a function of
      # iteration number:
      plt.plot(loss_hist)
      plt.xlabel('Iteration number')
      plt.ylabel('Loss value')
      plt.show()
```



```
[14]: # Write the LinearSVM.predict function and evaluate the performance on both the
# training and validation set
y_train_pred = svm.predict(X_train)
print('training accuracy: %f' % (np.mean(y_train == y_train_pred), ))
y_val_pred = svm.predict(X_val)
print('validation accuracy: %f' % (np.mean(y_val == y_val_pred), ))
```

training accuracy: 0.369653  
validation accuracy: 0.386000

```
[15]: # Use the validation set to tune hyperparameters (regularization strength and
# learning rate). You should experiment with different ranges for the learning
# rates and regularization strengths; if you are careful you should be able to
# get a classification accuracy of about 0.39 on the validation set.

# Note: you may see runtime/overflow warnings during hyper-parameter search.
# This may be caused by extreme values, and is not a bug.

# results is dictionary mapping tuples of the form
# (learning_rate, regularization_strength) to tuples of the form
# (training_accuracy, validation_accuracy). The accuracy is simply the fraction
# of data points that are correctly classified.
results = {}
best_val = -1 # The highest validation accuracy that we have seen so far.
best_svm = None # The LinearSVM object that achieved the highest validation
    ↪rate.

#####
# TODO: #
# Write code that chooses the best hyperparameters by tuning on the validation #
# set. For each combination of hyperparameters, train a linear SVM on the #
# training set, compute its accuracy on the training and validation sets, and #
# store these numbers in the results dictionary. In addition, store the best #
# validation accuracy in best_val and the LinearSVM object that achieves this #
# accuracy in best_svm. #
# #
# Hint: You should use a small value for num_iters as you develop your #
# validation code so that the SVMs don't take much time to train; once you are #
# confident that your validation code works, you should rerun the validation #
# code with a larger value for num_iters. #
#####

# Provided as a reference. You may or may not want to change these
    ↪hyperparameters
learning_rates = [1e-7, 5e-5]
regularization_strengths = [2.5e4, 5e4]
```

```

# *****START OF YOUR CODE (DO NOT DELETE/MODIFY THIS LINE)*****

for lr in learning_rates:
    for r in regularization_strengths:
        svm = LinearSVM()
        svm.train(X_train, y_train, learning_rate=lr, reg=r, num_iters=1500)
        y_train_pred = svm.predict(X_train)
        y_val_pred = svm.predict(X_val)
        train_accuracy = np.mean(y_train_pred == y_train)
        val_accuracy = np.mean(y_val_pred == y_val)
        results[(lr, r)] = (train_accuracy, val_accuracy)
        if val_accuracy > best_val:
            best_val = val_accuracy
            best_svm = svm

# *****END OF YOUR CODE (DO NOT DELETE/MODIFY THIS LINE)*****

# Print out results.
for lr, reg in sorted(results):
    train_accuracy, val_accuracy = results[(lr, reg)]
    print('lr %e reg %e train accuracy: %f val accuracy: %f' % (
        lr, reg, train_accuracy, val_accuracy))

print('best validation accuracy achieved during cross-validation: %f' %
      best_val)

```

```

/Users/caleblee/Desktop/CS280/CS280-Spring23-Assignment1/Homework1_partB/cs231n/
classifiers/linear_svm.py:88: RuntimeWarning: overflow encountered in scalar
multiply
    loss += reg * np.sum(W * W)
/Users/caleblee/Library/Python/3.9/lib/python/site-
packages/numpy/core/fromnumeric.py:86: RuntimeWarning: overflow encountered in
reduce
    return ufunc.reduce(obj, axis, dtype, out, **passkwargs)
/Users/caleblee/Desktop/CS280/CS280-Spring23-Assignment1/Homework1_partB/cs231n/
classifiers/linear_svm.py:88: RuntimeWarning: overflow encountered in multiply
    loss += reg * np.sum(W * W)
/Users/caleblee/Desktop/CS280/CS280-Spring23-Assignment1/Homework1_partB/cs231n/
classifiers/linear_svm.py:106: RuntimeWarning: overflow encountered in multiply
    dW += reg * 2 * W
/Users/caleblee/Desktop/CS280/CS280-Spring23-Assignment1/Homework1_partB/cs231n/
classifiers/linear_svm.py:83: RuntimeWarning: invalid value encountered in
matmul
    loss = X @ W
/Users/caleblee/Desktop/CS280/CS280-Spring23-Assignment1/Homework1_partB/cs231n/
classifiers/linear_classifier.py:77: RuntimeWarning: invalid value encountered
in subtract
    self.W -= learning_rate * grad

```



```

lr 1.000000e-07 reg 2.500000e+04 train accuracy: 0.366571 val accuracy: 0.375000
lr 1.000000e-07 reg 5.000000e+04 train accuracy: 0.358327 val accuracy: 0.371000
lr 5.000000e-05 reg 2.500000e+04 train accuracy: 0.043796 val accuracy: 0.049000
lr 5.000000e-05 reg 5.000000e+04 train accuracy: 0.100265 val accuracy: 0.087000
best validation accuracy achieved during cross-validation: 0.375000

```

```

[16]: # Visualize the cross-validation results
import math
import pdb

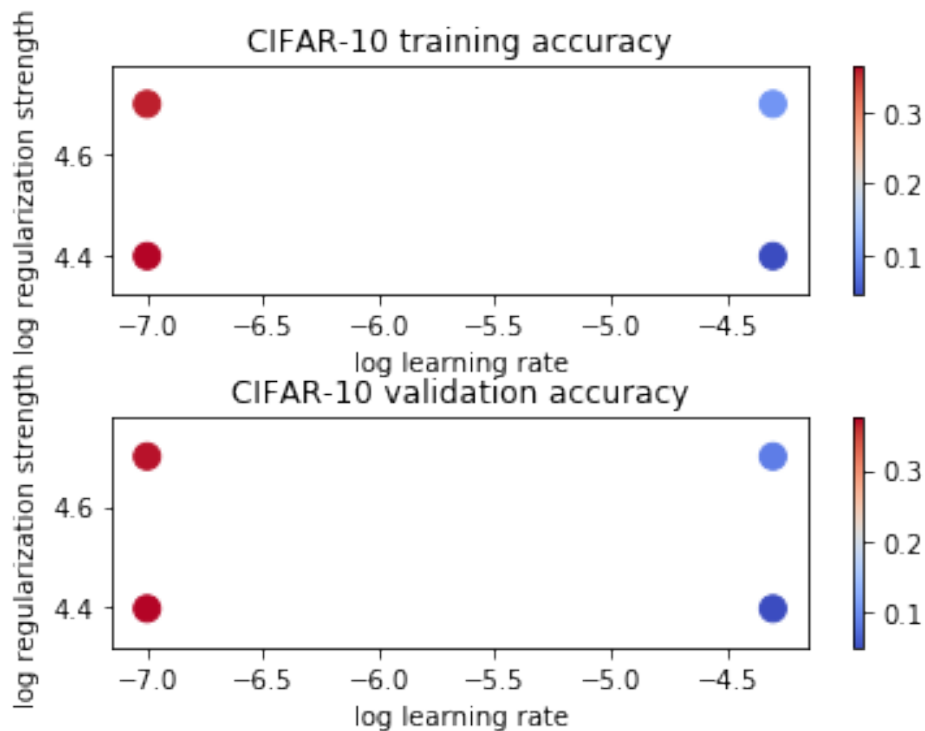
# pdb.set_trace()

x_scatter = [math.log10(x[0]) for x in results]
y_scatter = [math.log10(x[1]) for x in results]

# plot training accuracy
marker_size = 100
colors = [results[x][0] for x in results]
plt.subplot(2, 1, 1)
plt.tight_layout(pad=3)
plt.scatter(x_scatter, y_scatter, marker_size, c=colors, cmap=plt.cm.coolwarm)
plt.colorbar()
plt.xlabel('log learning rate')
plt.ylabel('log regularization strength')
plt.title('CIFAR-10 training accuracy')

# plot validation accuracy
colors = [results[x][1] for x in results] # default size of markers is 20
plt.subplot(2, 1, 2)
plt.scatter(x_scatter, y_scatter, marker_size, c=colors, cmap=plt.cm.coolwarm)
plt.colorbar()
plt.xlabel('log learning rate')
plt.ylabel('log regularization strength')
plt.title('CIFAR-10 validation accuracy')
plt.show()

```



```
[17]: # Evaluate the best svm on test set
y_test_pred = best_svm.predict(X_test)
test_accuracy = np.mean(y_test == y_test_pred)
print('linear SVM on raw pixels final test set accuracy: %f' % test_accuracy)
```

linear SVM on raw pixels final test set accuracy: 0.369000

```
[18]: # Visualize the learned weights for each class.
# Depending on your choice of learning rate and regularization strength, these
# may
# or may not be nice to look at.
w = best_svm.W[:-1,:] # strip out the bias
w = w.reshape(32, 32, 3, 10)
w_min, w_max = np.min(w), np.max(w)
classes = ['plane', 'car', 'bird', 'cat', 'deer', 'dog', 'frog', 'horse', '
    ↪ship', 'truck']
for i in range(10):
    plt.subplot(2, 5, i + 1)

    # Rescale the weights to be between 0 and 255
    wimg = 255.0 * (w[:, :, :, i].squeeze() - w_min) / (w_max - w_min)
    plt.imshow(wimg.astype('uint8'))
```

```
plt.axis('off')
plt.title(classes[i])
```



### Inline question 2

Describe what your visualized SVM weights look like, and offer a brief explanation for why they look they way that they do.

*Your Answer :*

Those pictures look like the “average” pictures of each class. This is because the angle between  $w_k$  and  $x_i$  affects the value of  $w_k^T x_i$ . Thus, the smaller the angle is, the more similar  $w_k$  is to  $x_i$  and the higher the probability that  $x_i$  is classified into class  $k$ .

[18]:

# softmax

February 24, 2023

## 1 Softmax exercise

Complete and hand in this completed worksheet (including its outputs and any supporting code outside of the worksheet) with your assignment submission. For more details see the [assignments page](#) on the course website.

This exercise is analogous to the SVM exercise. You will:

- implement a fully-vectorized **loss function** for the Softmax classifier
- implement the fully-vectorized expression for its **analytic gradient**
- **check your implementation** with numerical gradient
- use a validation set to **tune the learning rate and regularization** strength
- **optimize** the loss function with **SGD**
- **visualize** the final learned weights

```
[1]: import random
import numpy as np
from cs231n.data_utils import load_CIFAR10
import matplotlib.pyplot as plt

%matplotlib inline
plt.rcParams['figure.figsize'] = (10.0, 8.0) # set default size of plots
plt.rcParams['image.interpolation'] = 'nearest'
plt.rcParams['image.cmap'] = 'gray'

# for auto-reloading external modules
# see http://stackoverflow.com/questions/1907993/
↳ autoreload-of-modules-in-ipython
%load_ext autoreload
%autoreload 2

[2]: def get_CIFAR10_data(num_training=49000, num_validation=1000, num_test=1000,
↳ num_dev=500):
    """
    Load the CIFAR-10 dataset from disk and perform preprocessing to prepare
    it for the linear classifier. These are the same steps as we used for the
    SVM, but condensed to a single function.
    """
    # Load the raw CIFAR-10 data
```

```

cifar10_dir = 'cs231n/datasets/cifar-10-batches-py'

# Cleaning up variables to prevent loading data multiple times (which may
↳ cause memory issue)
try:
    del X_train, y_train
    del X_test, y_test
    print('Clear previously loaded data.')
except:
    pass

X_train, y_train, X_test, y_test = load_CIFAR10(cifar10_dir)

# subsample the data
mask = list(range(num_training, num_training + num_validation))
X_val = X_train[mask]
y_val = y_train[mask]
mask = list(range(num_training))
X_train = X_train[mask]
y_train = y_train[mask]
mask = list(range(num_test))
X_test = X_test[mask]
y_test = y_test[mask]
mask = np.random.choice(num_training, num_dev, replace=False)
X_dev = X_train[mask]
y_dev = y_train[mask]

# Preprocessing: reshape the image data into rows
X_train = np.reshape(X_train, (X_train.shape[0], -1))
X_val = np.reshape(X_val, (X_val.shape[0], -1))
X_test = np.reshape(X_test, (X_test.shape[0], -1))
X_dev = np.reshape(X_dev, (X_dev.shape[0], -1))

# Normalize the data: subtract the mean image
mean_image = np.mean(X_train, axis = 0)
X_train -= mean_image
X_val -= mean_image
X_test -= mean_image
X_dev -= mean_image

# add bias dimension and transform into columns
X_train = np.hstack([X_train, np.ones((X_train.shape[0], 1))])
X_val = np.hstack([X_val, np.ones((X_val.shape[0], 1))])
X_test = np.hstack([X_test, np.ones((X_test.shape[0], 1))])
X_dev = np.hstack([X_dev, np.ones((X_dev.shape[0], 1))])

return X_train, y_train, X_val, y_val, X_test, y_test, X_dev, y_dev

```

```
# Invoke the above function to get our data.
X_train, y_train, X_val, y_val, X_test, y_test, X_dev, y_dev = get_CIFAR10_data()
print('Train data shape: ', X_train.shape)
print('Train labels shape: ', y_train.shape)
print('Validation data shape: ', X_val.shape)
print('Validation labels shape: ', y_val.shape)
print('Test data shape: ', X_test.shape)
print('Test labels shape: ', y_test.shape)
print('dev data shape: ', X_dev.shape)
print('dev labels shape: ', y_dev.shape)
```

```
Train data shape: (49000, 3073)
Train labels shape: (49000,)
Validation data shape: (1000, 3073)
Validation labels shape: (1000,)
Test data shape: (1000, 3073)
Test labels shape: (1000,)
dev data shape: (500, 3073)
dev labels shape: (500,)
```

## 1.1 Softmax Classifier

Your code for this section will all be written inside `cs231n/classifiers/softmax.py`.

```
[3]: # First implement the naive softmax loss function with nested loops.
# Open the file cs231n/classifiers/softmax.py and implement the
# softmax_loss_naive function.

from cs231n.classifiers.softmax import softmax_loss_naive
import time

# Generate a random softmax weight matrix and use it to compute the loss.
W = np.random.randn(3073, 10) * 0.0001
loss, grad = softmax_loss_naive(W, X_dev, y_dev, 0.0)

# As a rough sanity check, our loss should be something close to -log(0.1).
print('loss: %f' % loss)
print('sanity check: %f' % (-np.log(0.1)))
```

```
loss: 2.370673
sanity check: 2.302585
```

### Inline Question 1

Why do we expect our loss to be close to  $-\log(0.1)$ ? Explain briefly.\*\*

*Your Answer :*

Since the weight  $W$  is randomly initialized, then the probability  $p_i$  should be close to  $1/\{\text{number of class}\}=0.1$  for each class  $i$ . Thus, the value of loss function can be approximated as follows:

$$L = \frac{1}{n} \sum_{i=1}^n -\log p_i = -\log\left(\prod_{i=1}^n p_i\right)^{\frac{1}{n}} \approx -\log((0.1)^n)^{(1/n)} = -\log 0.1$$

```
[4]: # Complete the implementation of softmax_loss_naive and implement a (naive)
# version of the gradient that uses nested loops.
loss, grad = softmax_loss_naive(W, X_dev, y_dev, 0.0)

# As we did for the SVM, use numeric gradient checking as a debugging tool.
# The numeric gradient should be close to the analytic gradient.
from cs231n.gradient_check import grad_check_sparse
f = lambda w: softmax_loss_naive(w, X_dev, y_dev, 0.0)[0]
grad_numerical = grad_check_sparse(f, W, grad, 10)

# similar to SVM case, do another gradient check with regularization
loss, grad = softmax_loss_naive(W, X_dev, y_dev, 5e1)
f = lambda w: softmax_loss_naive(w, X_dev, y_dev, 5e1)[0]
grad_numerical = grad_check_sparse(f, W, grad, 10)
```

```
numerical: 1.052860 analytic: 1.052860, relative error: 2.201441e-08
numerical: -1.549930 analytic: -1.549930, relative error: 1.647477e-08
numerical: 0.119256 analytic: 0.119256, relative error: 3.456103e-07
numerical: -0.219556 analytic: -0.219556, relative error: 8.991791e-08
numerical: 0.083568 analytic: 0.083568, relative error: 5.460927e-07
numerical: 1.443529 analytic: 1.443529, relative error: 8.874940e-09
numerical: 0.796921 analytic: 0.796921, relative error: 4.137497e-08
numerical: -4.262104 analytic: -4.262104, relative error: 4.176475e-09
numerical: -1.373423 analytic: -1.373423, relative error: 8.834602e-10
numerical: 0.489926 analytic: 0.489926, relative error: 2.588278e-09
numerical: 1.725037 analytic: 1.725037, relative error: 1.961117e-08
numerical: -0.927769 analytic: -0.927769, relative error: 1.667269e-08
numerical: -0.186378 analytic: -0.186378, relative error: 1.390873e-07
numerical: 1.496357 analytic: 1.496357, relative error: 5.722065e-08
numerical: 0.658420 analytic: 0.658420, relative error: 6.588213e-08
numerical: 0.621139 analytic: 0.621139, relative error: 7.778886e-08
numerical: 1.582019 analytic: 1.582019, relative error: 4.703156e-08
numerical: -0.340721 analytic: -0.340721, relative error: 1.002202e-07
numerical: 0.169129 analytic: 0.169129, relative error: 5.742597e-08
numerical: -0.260551 analytic: -0.260551, relative error: 2.454967e-07
```

```
[5]: # Now that we have a naive implementation of the softmax loss function and its
      ↪ gradient,
# implement a vectorized version in softmax_loss_vectorized.
```

```

# The two versions should compute the same results, but the vectorized version
↳ should be
# much faster.
tic = time.time()
loss_naive, grad_naive = softmax_loss_naive(W, X_dev, y_dev, 0.000005)
toc = time.time()
print('naive loss: %e computed in %fs' % (loss_naive, toc - tic))

from cs231n.classifiers.softmax import softmax_loss_vectorized
tic = time.time()
loss_vectorized, grad_vectorized = softmax_loss_vectorized(W, X_dev, y_dev, 0.
↳ 000005)
toc = time.time()
print('vectorized loss: %e computed in %fs' % (loss_vectorized, toc - tic))

# As we did for the SVM, we use the Frobenius norm to compare the two versions
# of the gradient.
grad_difference = np.linalg.norm(grad_naive - grad_vectorized, ord='fro')
print('Loss difference: %f' % np.abs(loss_naive - loss_vectorized))
print('Gradient difference: %f' % grad_difference)

```

```

naive loss: 2.370673e+00 computed in 0.037366s
vectorized loss: 2.370673e+00 computed in 0.005350s
Loss difference: 0.000000
Gradient difference: 0.000000

```

```

[6]: # Use the validation set to tune hyperparameters (regularization strength and
# learning rate). You should experiment with different ranges for the learning
# rates and regularization strengths; if you are careful you should be able to
# get a classification accuracy of over 0.35 on the validation set.

from cs231n.classifiers import Softmax
results = {}
best_val = -1
best_softmax = None

#####
# TODO:
# Use the validation set to set the learning rate and regularization strength. #
# This should be identical to the validation that you did for the SVM; save   #
# the best trained softmax classifier in best_softmax.                         #
#####

# Provided as a reference. You may or may not want to change these
↳ hyperparameters
learning_rates = [1e-7, 5e-7]
regularization_strengths = [2.5e4, 5e4]

```



```

# *****START OF YOUR CODE (DO NOT DELETE/MODIFY THIS LINE)*****

for lr in learning_rates:
    for r in regularization_strengths:
        softmax = Softmax()
        loss_hist = softmax.train(X_train, y_train, learning_rate=lr, reg=r,
                                   num_iters=1500, verbose=False)
        y_train_pred = softmax.predict(X_train)
        y_val_pred = softmax.predict(X_val)
        train_accuracy = np.mean(y_train == y_train_pred)
        val_accuracy = np.mean(y_val == y_val_pred)
        results[(lr, r)] = (train_accuracy, val_accuracy)
        if val_accuracy > best_val:
            best_val = val_accuracy
            best_softmax = softmax

# *****END OF YOUR CODE (DO NOT DELETE/MODIFY THIS LINE)*****

# Print out results.
for lr, reg in sorted(results):
    train_accuracy, val_accuracy = results[(lr, reg)]
    print('lr %e reg %e train accuracy: %f val accuracy: %f' % (
        lr, reg, train_accuracy, val_accuracy))

print('best validation accuracy achieved during cross-validation: %f' %
      best_val)

```

```

lr 1.000000e-07 reg 2.500000e+04 train accuracy: 0.328388 val accuracy: 0.340000
lr 1.000000e-07 reg 5.000000e+04 train accuracy: 0.303184 val accuracy: 0.318000
lr 5.000000e-07 reg 2.500000e+04 train accuracy: 0.326327 val accuracy: 0.331000
lr 5.000000e-07 reg 5.000000e+04 train accuracy: 0.298510 val accuracy: 0.311000
best validation accuracy achieved during cross-validation: 0.340000

```

```

[7]: # evaluate on test set
# Evaluate the best softmax on test set
y_test_pred = best_softmax.predict(X_test)
test_accuracy = np.mean(y_test == y_test_pred)
print('softmax on raw pixels final test set accuracy: %f' % (test_accuracy, ))

```

```
softmax on raw pixels final test set accuracy: 0.344000
```

### Inline Question 2 - True or False

Suppose the overall training loss is defined as the sum of the per-datapoint loss over all training examples. It is possible to add a new datapoint to a training set that would leave the SVM loss unchanged, but this is not the case with the Softmax classifier loss.

*Your Answer :*

Yes, it's possible.

*Your Explanation :*

Given that the loss function of SVM is as follows:

$$L = \frac{1}{n} \sum_{i=1}^n \sum_{j \neq y_i} \max(0, 1 - (w_j - w_{y_i})x_i)$$

if the new added point  $x_i$  has the property that  $(w_j - w_{y_i})x_i < 1$  for all  $j$  then the loss value will not change

However, for softmax, the loss function takes every training points into consideration. Thus, the value will change if we add a new point  $x_i$  into training set.

```
[8]: # Visualize the learned weights for each class
w = best_softmax.W[:-1,:] # strip out the bias
w = w.reshape(32, 32, 3, 10)

w_min, w_max = np.min(w), np.max(w)

classes = ['plane', 'car', 'bird', 'cat', 'deer', 'dog', 'frog', 'horse', 'ship', 'truck']
for i in range(10):
    plt.subplot(2, 5, i + 1)

    # Rescale the weights to be between 0 and 255
    wimg = 255.0 * (w[:, :, :, i].squeeze() - w_min) / (w_max - w_min)
    plt.imshow(wimg.astype('uint8'))
    plt.axis('off')
    plt.title(classes[i])
```



[8]:

## two\_layer\_net

February 24, 2023

### 1 Implementing a Neural Network

In this exercise we will develop a neural network with fully-connected layers to perform classification, and test it out on the CIFAR-10 dataset.

```
[16]: # A bit of setup

import numpy as np
import matplotlib.pyplot as plt

from cs231n.classifiers.neural_net import TwoLayerNet

%matplotlib inline
plt.rcParams['figure.figsize'] = (10.0, 8.0) # set default size of plots
plt.rcParams['image.interpolation'] = 'nearest'
plt.rcParams['image.cmap'] = 'gray'

# for auto-reloading external modules
# see http://stackoverflow.com/questions/1907993/
# ↪ autoreload-of-modules-in-ipython
%load_ext autoreload
%autoreload 2

def rel_error(x, y):
    """ returns relative error """
    return np.max(np.abs(x - y) / (np.maximum(1e-8, np.abs(x) + np.abs(y))))
```

The autoreload extension is already loaded. To reload it, use:

```
%reload_ext autoreload
```

We will use the class `TwoLayerNet` in the file `cs231n/classifiers/neural_net.py` to represent instances of our network. The network parameters are stored in the instance variable `self.params` where keys are string parameter names and values are numpy arrays. Below, we initialize toy data and a toy model that we will use to develop your implementation.

```
[17]: # Create a small net and some toy data to check your implementations.
# Note that we set the random seed for repeatable experiments.

input_size = 4
```

```

hidden_size = 10
num_classes = 3
num_inputs = 5

def init_toy_model():
    np.random.seed(0)
    return TwoLayerNet(input_size, hidden_size, num_classes, std=1e-1)

def init_toy_data():
    np.random.seed(1)
    X = 10 * np.random.randn(num_inputs, input_size)
    y = np.array([0, 1, 2, 2, 1])
    return X, y

net = init_toy_model()
X, y = init_toy_data()

```

## 2 Forward pass: compute scores

Open the file `cs231n/classifiers/neural_net.py` and look at the method `TwoLayerNet.loss`. This function is very similar to the loss functions you have written for the SVM and Softmax exercises: It takes the data and weights and computes the class scores, the loss, and the gradients on the parameters.

Implement the first part of the forward pass which uses the weights and biases to compute the scores for all inputs.

```

[18]: scores = net.loss(X)
print('Your scores:')
print(scores)
print()
print('correct scores:')
correct_scores = np.asarray([
    [-0.81233741, -1.27654624, -0.70335995],
    [-0.17129677, -1.18803311, -0.47310444],
    [-0.51590475, -1.01354314, -0.8504215 ],
    [-0.15419291, -0.48629638, -0.52901952],
    [-0.00618733, -0.12435261, -0.15226949]])
print(correct_scores)
print()

# The difference should be very small. We get < 1e-7
print('Difference between your scores and correct scores:')
print(np.sum(np.abs(scores - correct_scores)))

```

Your scores:  
 [[-0.81233741 -1.27654624 -0.70335995]

```

[-0.17129677 -1.18803311 -0.47310444]
[-0.51590475 -1.01354314 -0.8504215 ]
[-0.15419291 -0.48629638 -0.52901952]
[-0.00618733 -0.12435261 -0.15226949]]

```

correct scores:

```

[[-0.81233741 -1.27654624 -0.70335995]
 [-0.17129677 -1.18803311 -0.47310444]
 [-0.51590475 -1.01354314 -0.8504215 ]
 [-0.15419291 -0.48629638 -0.52901952]
 [-0.00618733 -0.12435261 -0.15226949]]

```

Difference between your scores and correct scores:  
3.6802720745909845e-08

### 3 Forward pass: compute loss

In the same function, implement the second part that computes the data and regularization loss.

```

[23]: loss, _ = net.loss(X, y, reg=0.05)
      correct_loss = 1.30378789133

      # should be very small, we get < 1e-12
      print('Difference between your loss and correct loss:')
      print(np.sum(np.abs(loss - correct_loss)))

```

Difference between your loss and correct loss:  
1.7985612998927536e-13

### 4 Backward pass

Implement the rest of the function. This will compute the gradient of the loss with respect to the variables W1, b1, W2, and b2. Now that you (hopefully!) have a correctly implemented forward pass, you can debug your backward pass using a numeric gradient check:

```

[24]: from cs231n.gradient_check import eval_numerical_gradient

      # Use numeric gradient checking to check your implementation of the backward
      ↪pass.
      # If your implementation is correct, the difference between the numeric and
      # analytic gradients should be less than 1e-8 for each of W1, W2, b1, and b2.

      loss, grads = net.loss(X, y, reg=0.05)

      # these should all be less than 1e-8 or so
      for param_name in grads:
          f = lambda W: net.loss(X, y, reg=0.05)[0]

```

```

    param_grad_num = eval_numerical_gradient(f, net.params[param_name],
↪ verbose=False)
    print('%s max relative error: %e' % (param_name, rel_error(param_grad_num,
↪ grads[param_name])))

```

```

W2 max relative error: 3.440708e-09
b2 max relative error: 4.447656e-11
W1 max relative error: 3.561318e-09
b1 max relative error: 2.738421e-09

```

## 5 Train the network

To train the network we will use stochastic gradient descent (SGD), similar to the SVM and Softmax classifiers. Look at the function `TwoLayerNet.train` and fill in the missing sections to implement the training procedure. This should be very similar to the training procedure you used for the SVM and Softmax classifiers. You will also have to implement `TwoLayerNet.predict`, as the training process periodically performs prediction to keep track of accuracy over time while the network trains.

Once you have implemented the method, run the code below to train a two-layer network on toy data. You should achieve a training loss less than 0.02.

```

[25]: net = init_toy_model()
stats = net.train(X, y, X, y,
                  learning_rate=1e-1, reg=5e-6,
                  num_iters=100, verbose=False)

print('Final training loss: ', stats['loss_history'][-1])

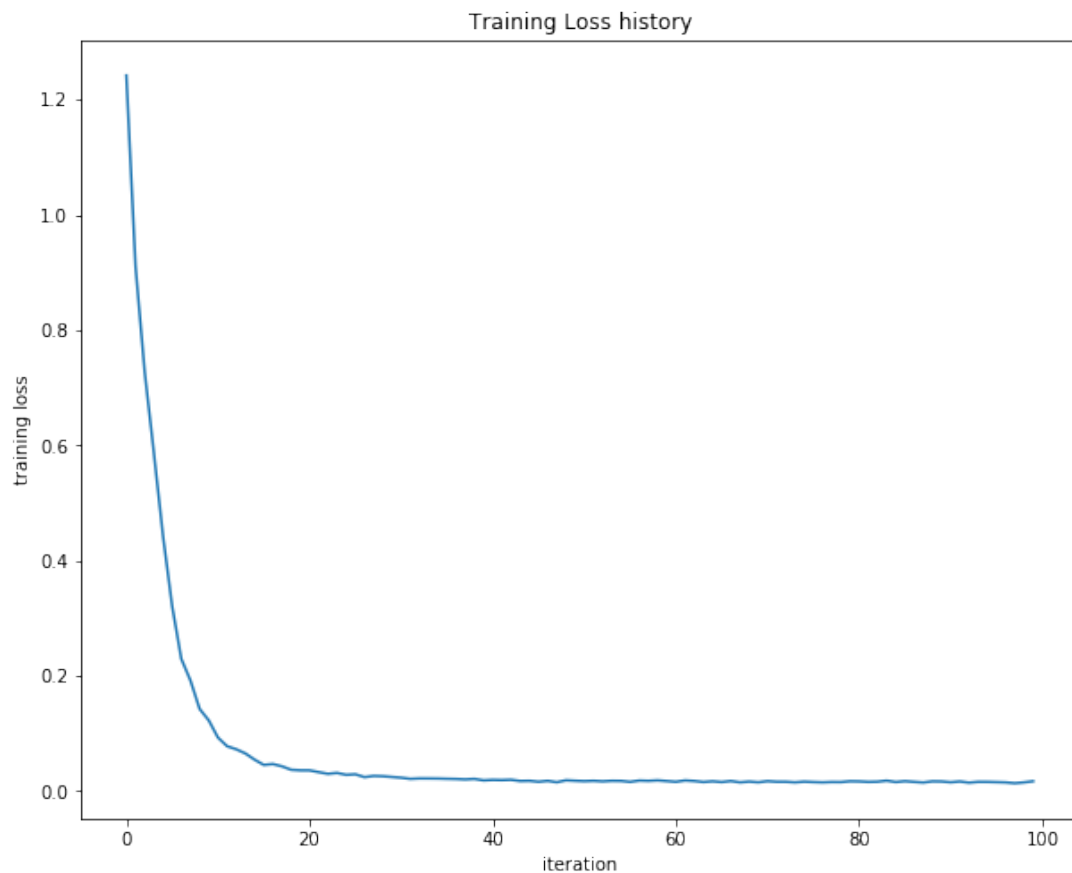
# plot the loss history
plt.plot(stats['loss_history'])
plt.xlabel('iteration')
plt.ylabel('training loss')
plt.title('Training Loss history')
plt.show()

```

```

Final training loss: 0.017149607938732048

```



## 6 Load the data

Now that you have implemented a two-layer network that passes gradient checks and works on toy data, it's time to load up our favorite CIFAR-10 data so we can use it to train a classifier on a real dataset.

```
[26]: from cs231n.data_utils import load_CIFAR10

def get_CIFAR10_data(num_training=49000, num_validation=1000, num_test=1000):
    """
    Load the CIFAR-10 dataset from disk and perform preprocessing to prepare
    it for the two-layer neural net classifier. These are the same steps as
    we used for the SVM, but condensed to a single function.
    """
    # Load the raw CIFAR-10 data
    cifar10_dir = 'cs231n/datasets/cifar-10-batches-py'
```



```

    # Cleaning up variables to prevent loading data multiple times (which may
    ↳ cause memory issue)
    try:
        del X_train, y_train
        del X_test, y_test
        print('Clear previously loaded data.')
    except:
        pass

X_train, y_train, X_test, y_test = load_CIFAR10(cifar10_dir)

# Subsample the data
mask = list(range(num_training, num_training + num_validation))
X_val = X_train[mask]
y_val = y_train[mask]
mask = list(range(num_training))
X_train = X_train[mask]
y_train = y_train[mask]
mask = list(range(num_test))
X_test = X_test[mask]
y_test = y_test[mask]

# Normalize the data: subtract the mean image
mean_image = np.mean(X_train, axis=0)
X_train -= mean_image
X_val -= mean_image
X_test -= mean_image

# Reshape data to rows
X_train = X_train.reshape(num_training, -1)
X_val = X_val.reshape(num_validation, -1)
X_test = X_test.reshape(num_test, -1)

return X_train, y_train, X_val, y_val, X_test, y_test

# Invoke the above function to get our data.
X_train, y_train, X_val, y_val, X_test, y_test = get_CIFAR10_data()
print('Train data shape: ', X_train.shape)
print('Train labels shape: ', y_train.shape)
print('Validation data shape: ', X_val.shape)
print('Validation labels shape: ', y_val.shape)
print('Test data shape: ', X_test.shape)
print('Test labels shape: ', y_test.shape)

```

```

Train data shape: (49000, 3072)
Train labels shape: (49000,)

```

```
Validation data shape: (1000, 3072)
Validation labels shape: (1000,)
Test data shape: (1000, 3072)
Test labels shape: (1000,)
```

## 7 Train a network

To train our network we will use SGD. In addition, we will adjust the learning rate with an exponential learning rate schedule as optimization proceeds; after each epoch, we will reduce the learning rate by multiplying it by a decay rate.

```
[27]: input_size = 32 * 32 * 3
      hidden_size = 50
      num_classes = 10
      net = TwoLayerNet(input_size, hidden_size, num_classes)

      # Train the network
      stats = net.train(X_train, y_train, X_val, y_val,
                        num_iters=1000, batch_size=200,
                        learning_rate=1e-4, learning_rate_decay=0.95,
                        reg=0.25, verbose=True)

      # Predict on the validation set
      val_acc = (net.predict(X_val) == y_val).mean()
      print('Validation accuracy: ', val_acc)
```

```
iteration 0 / 1000: loss 2.302954
iteration 100 / 1000: loss 2.302550
iteration 200 / 1000: loss 2.297648
iteration 300 / 1000: loss 2.259602
iteration 400 / 1000: loss 2.204170
iteration 500 / 1000: loss 2.118565
iteration 600 / 1000: loss 2.051535
iteration 700 / 1000: loss 1.988466
iteration 800 / 1000: loss 2.006591
iteration 900 / 1000: loss 1.951473
Validation accuracy: 0.287
```

## 8 Debug the training

With the default parameters we provided above, you should get a validation accuracy of about 0.29 on the validation set. This isn't very good.

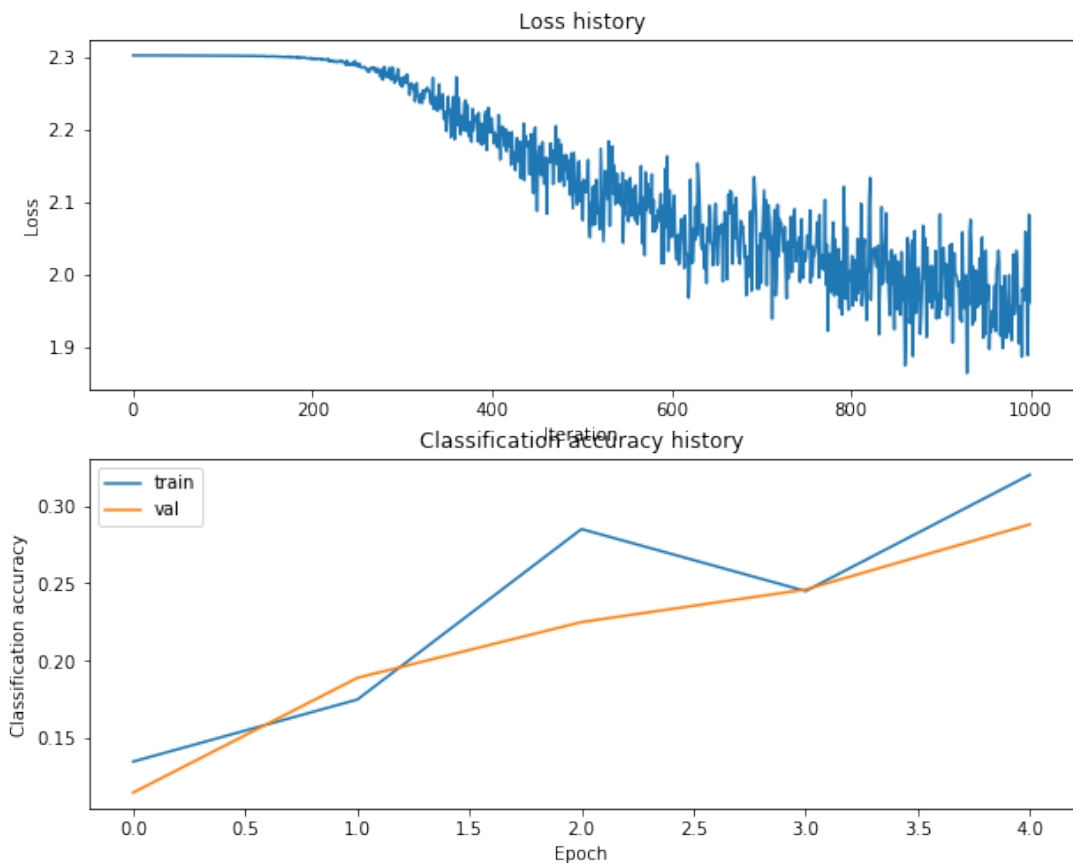
One strategy for getting insight into what's wrong is to plot the loss function and the accuracies on the training and validation sets during optimization.

Another strategy is to visualize the weights that were learned in the first layer of the network. In most neural networks trained on visual data, the first layer weights typically show some visible

structure when visualized.

```
[28]: # Plot the loss function and train / validation accuracies
plt.subplot(2, 1, 1)
plt.plot(stats['loss_history'])
plt.title('Loss history')
plt.xlabel('Iteration')
plt.ylabel('Loss')

plt.subplot(2, 1, 2)
plt.plot(stats['train_acc_history'], label='train')
plt.plot(stats['val_acc_history'], label='val')
plt.title('Classification accuracy history')
plt.xlabel('Epoch')
plt.ylabel('Classification accuracy')
plt.legend()
plt.show()
```

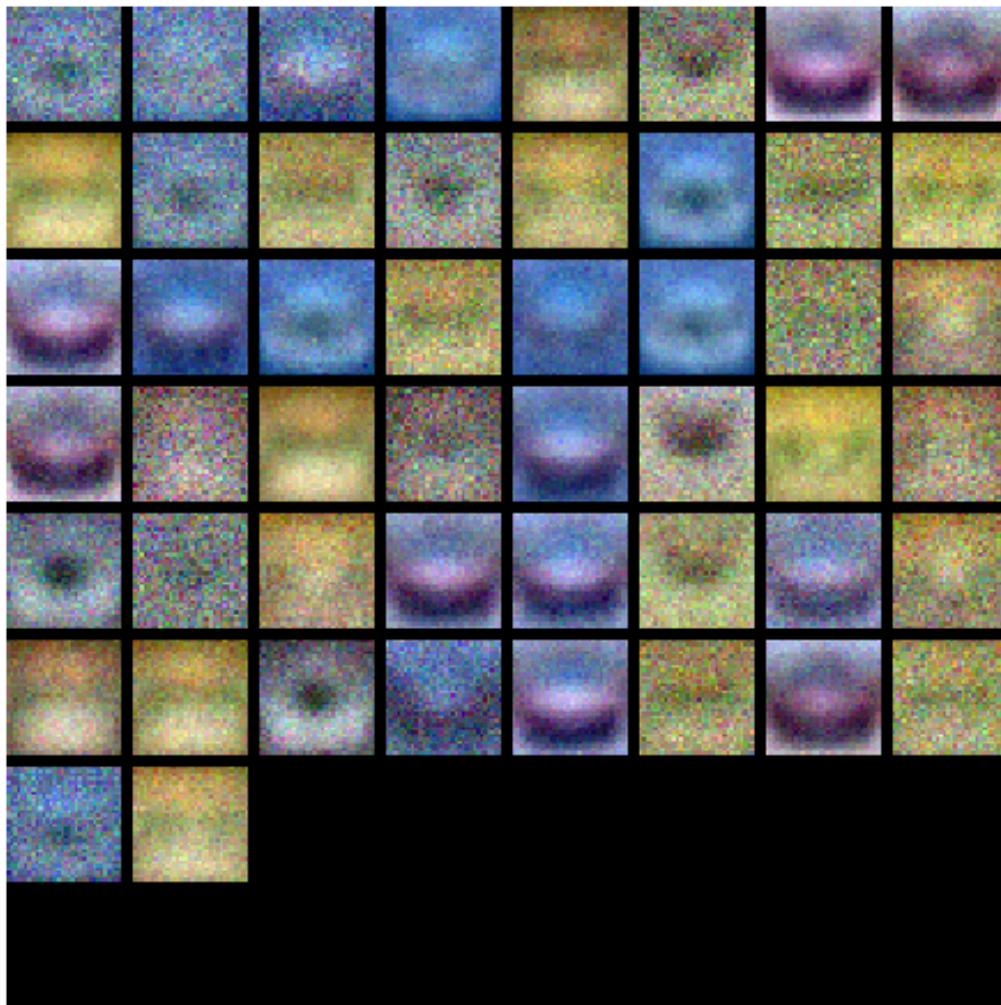


```
[29]: from cs231n.vis_utils import visualize_grid

# Visualize the weights of the network

def show_net_weights(net):
    W1 = net.params['W1']
    W1 = W1.reshape(32, 32, 3, -1).transpose(3, 0, 1, 2)
    plt.imshow(visualize_grid(W1, padding=3).astype('uint8'))
    plt.gca().axis('off')
    plt.show()

show_net_weights(net)
```



## 9 Tune your hyperparameters

**What's wrong?.** Looking at the visualizations above, we see that the loss is decreasing more or less linearly, which seems to suggest that the learning rate may be too low. Moreover, there is no gap between the training and validation accuracy, suggesting that the model we used has low capacity, and that we should increase its size. On the other hand, with a very large model we would expect to see more overfitting, which would manifest itself as a very large gap between the training and validation accuracy.

**Tuning.** Tuning the hyperparameters and developing intuition for how they affect the final performance is a large part of using Neural Networks, so we want you to get a lot of practice. Below, you should experiment with different values of the various hyperparameters, including hidden layer size, learning rate, number of training epochs, and regularization strength. You might also consider tuning the learning rate decay, but you should be able to get good performance using the default value.

**Approximate results.** You should be aim to achieve a classification accuracy of greater than 48% on the validation set. Our best network gets over 52% on the validation set.

**Experiment:** Your goal in this exercise is to get as good of a result on CIFAR-10 as you can (52% could serve as a reference), with a fully-connected Neural Network. Feel free implement your own techniques (e.g. PCA to reduce dimensionality, or adding dropout, or adding features to the solver, etc.).

**Explain your hyperparameter tuning process below.**

*Your Answer :*

I experimented with different hyperparameters combinations, in detail, I choose two different learning\_rates: 5e-4 and 1e-3 and two different batch\_size: 200 and 400.

The reason why I choose to tune learning\_rate is that learning rate is the plot above shows that the loss fluctuated heavily during the training process, then we can increase the learning rate to by increasing 5e-4 each time. As for batch\_size, increase batch size will help the model to get more accurate gradient during training, so we compare the result of original(200) parameter and new (400) parameter.

I traversed through all possible combinations of hyperparameters and save the model with the highest validation accuracy as the best model.

```
[42]: best_net = None # store the best model into this

#####
# TODO: Tune hyperparameters using the validation set. Store your best trained
#
# model in best_net.
#
#
# To help debug your network, it may help to use visualizations similar to the
```

```

# ones we used above; these visualizations will have significant qualitative
↪#
# differences from the ones we saw above for the poorly tuned network.
↪#
#
↪#
# Tweaking hyperparameters by hand can be fun, but you might find it useful to
↪#
# write code to sweep through possible combinations of hyperparameters
↪#
# automatically like we did on the previous exercises.
↪#
#####
# *****START OF YOUR CODE (DO NOT DELETE/MODIFY THIS LINE)*****
learning_rates = [5e-4, 1e-3]
batch_size = [200, 400]
results = {}
best_val = -1
best_stats = None

for lr in learning_rates:
    for b in batch_size:
        net = TwoLayerNet(input_size, 200, num_classes)
        stats = net.train(X_train, y_train, X_val, y_val,
                           num_iters=1000, batch_size=b,
                           learning_rate=lr, learning_rate_decay=0.95,
                           reg=0.25, verbose=False)
        train_acc = (net.predict(X_train) == y_train).mean()
        val_acc = (net.predict(X_val) == y_val).mean()
        results[(lr, b)] = (train_acc, val_acc)
        if val_acc > best_val:
            best_val = val_acc
            best_net = net
            best_stats = stats
        print(f"lr={lr}, batch_size={b}, train_acc={train_acc},
↪val_acc={val_acc}")

plt.subplot(2, 1, 1)
plt.plot(best_stats['loss_history'])
plt.title('Loss history')
plt.xlabel('Iteration')
plt.ylabel('Loss')

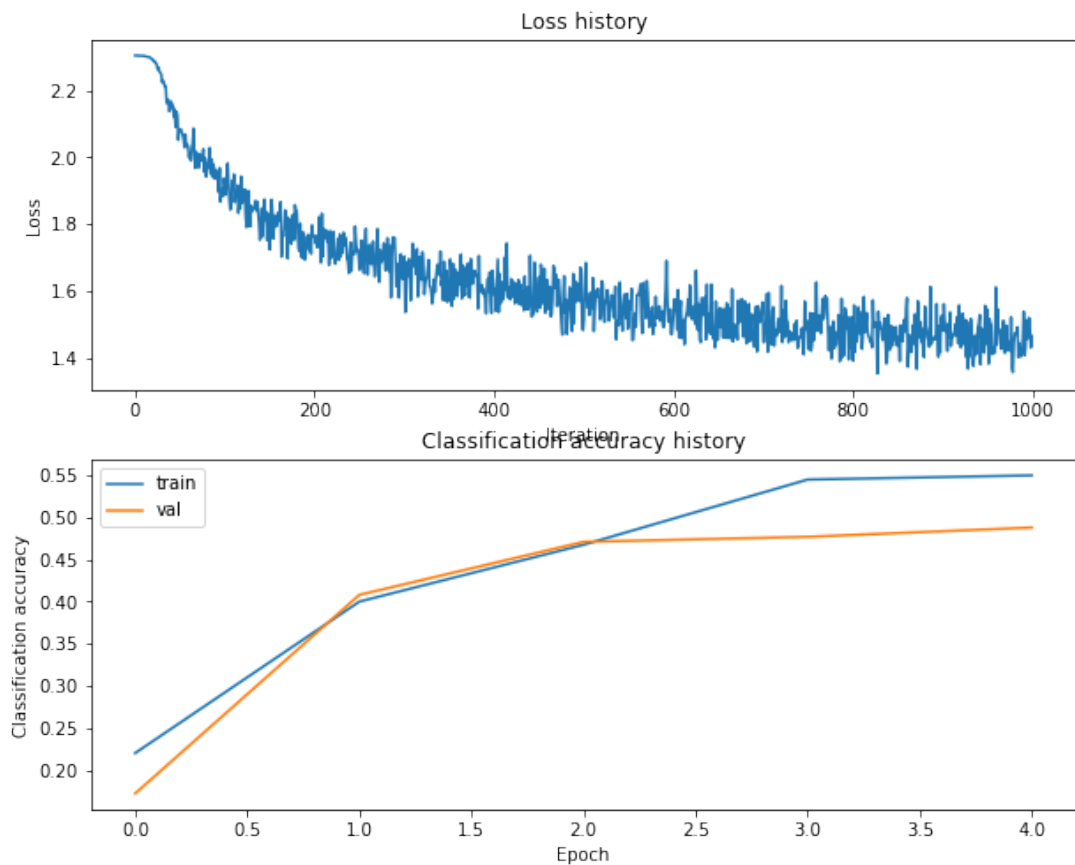
plt.subplot(2, 1, 2)
plt.plot(best_stats['train_acc_history'], label='train')
plt.plot(best_stats['val_acc_history'], label='val')

```

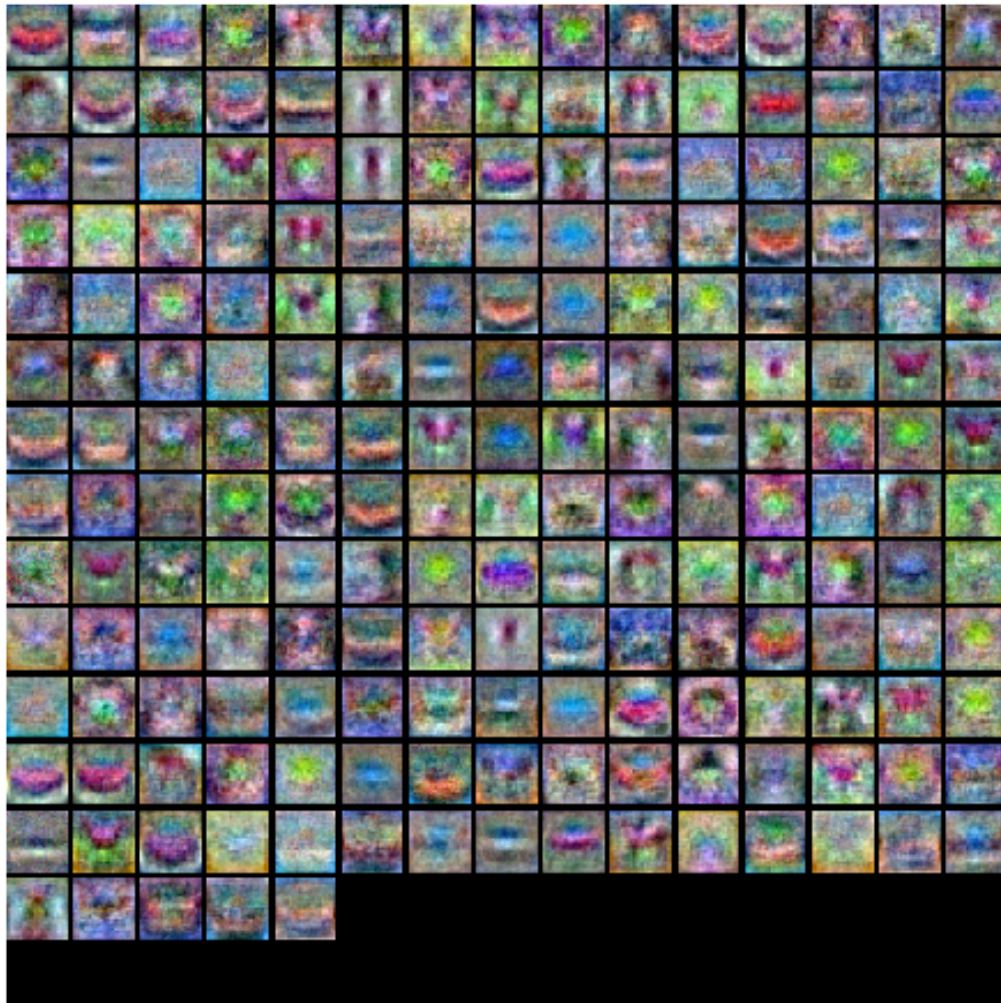
```
plt.title('Classification accuracy history')
plt.xlabel('Epoch')
plt.ylabel('Classification accuracy')
plt.legend()
plt.show()

show_net_weights(best_net)
# *****END OF YOUR CODE (DO NOT DELETE/MODIFY THIS LINE)*****
```

```
lr=0.0005, batch_size=200, train_acc=0.4638775510204082, val_acc=0.449
lr=0.0005, batch_size=400, train_acc=0.4653673469387755, val_acc=0.468
lr=0.001, batch_size=200, train_acc=0.5078571428571429, val_acc=0.481
lr=0.001, batch_size=400, train_acc=0.5173673469387755, val_acc=0.497
```





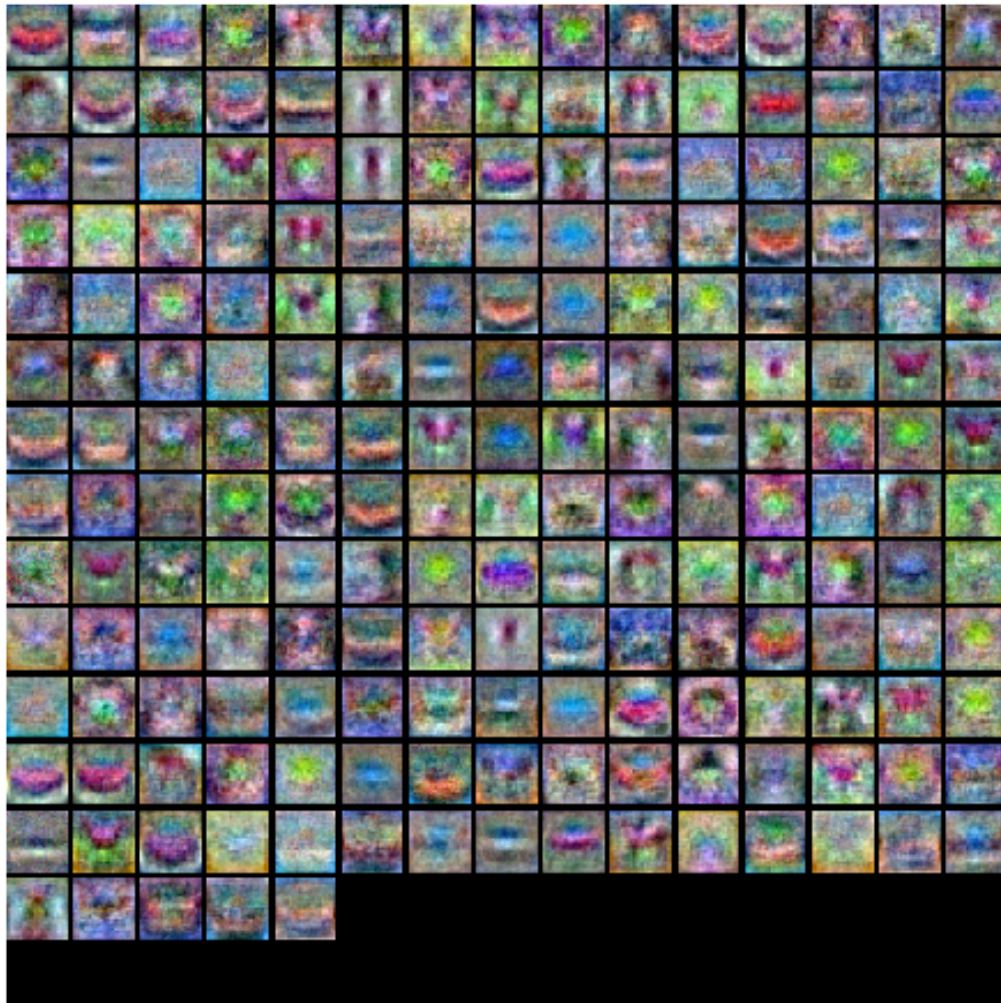


```
[43]: # Print your validation accuracy: this should be above 48%
val_acc = (best_net.predict(X_val) == y_val).mean()
print('Validation accuracy: ', val_acc)
```

Validation accuracy: 0.497

```
[44]: # Visualize the weights of the best network
show_net_weights(best_net)
```





## 10 Run on the test set

When you are done experimenting, you should evaluate your final trained network on the test set; you should get above 48%.

```
[45]: # Print your test accuracy: this should be above 48%
test_acc = (best_net.predict(X_test) == y_test).mean()
print('Test accuracy: ', test_acc)
```

Test accuracy: 0.502

### Inline Question

Now that you have trained a Neural Network classifier, you may find that your testing accuracy

is much lower than the training accuracy. In what ways can we decrease this gap? Select all that apply.

1. Train on a larger dataset.
2. Add more hidden units.
3. Increase the regularization strength.
4. None of the above.

*Your Answer :*

I choose 1, 2, 3.

*Your Explanation :*

Actually this problem is equivalent to how to avoid overfitting because overfitting means the model nearly remembered the whole training set. For 1, the more data the model meet, the less likely that it remember all of them. For 2, the more hidden units the model has, the harder for training and the model is less likely to remember all the dataset. For 3, regularization constrains the L2 norm of weights won't be extremely large which means the model won't approximation the implicit function in an extremely complex way.

[ ]:

# features

February 24, 2023

## 1 Image features exercise

Complete and hand in this completed worksheet (including its outputs and any supporting code outside of the worksheet) with your assignment submission. For more details see the [assignments page](#) on the course website.

We have seen that we can achieve reasonable performance on an image classification task by training a linear classifier on the pixels of the input image. In this exercise we will show that we can improve our classification performance by training linear classifiers not on raw pixels but on features that are computed from the raw pixels.

All of your work for this exercise will be done in this notebook.

```
[1]: import random
import numpy as np
from cs231n.data_utils import load_CIFAR10
import matplotlib.pyplot as plt

%matplotlib inline
plt.rcParams['figure.figsize'] = (10.0, 8.0) # set default size of plots
plt.rcParams['image.interpolation'] = 'nearest'
plt.rcParams['image.cmap'] = 'gray'

# for auto-reloading external modules
# see http://stackoverflow.com/questions/1907993/
# ↪ autoreload-of-modules-in-ipython
%load_ext autoreload
%autoreload 2
```

### 1.1 Load data

Similar to previous exercises, we will load CIFAR-10 data from disk.

```
[2]: from cs231n.features import color_histogram_hsv, hog_feature

def get_CIFAR10_data(num_training=49000, num_validation=1000, num_test=1000):
    # Load the raw CIFAR-10 data
    cifar10_dir = 'cs231n/datasets/cifar-10-batches-py'
```

```

    # Cleaning up variables to prevent loading data multiple times (which may
    ↪ cause memory issue)
    try:
        del X_train, y_train
        del X_test, y_test
        print('Clear previously loaded data.')
    except:
        pass

X_train, y_train, X_test, y_test = load_CIFAR10(cifar10_dir)

# Subsample the data
mask = list(range(num_training, num_training + num_validation))
X_val = X_train[mask]
y_val = y_train[mask]
mask = list(range(num_training))
X_train = X_train[mask]
y_train = y_train[mask]
mask = list(range(num_test))
X_test = X_test[mask]
y_test = y_test[mask]

return X_train, y_train, X_val, y_val, X_test, y_test

X_train, y_train, X_val, y_val, X_test, y_test = get_CIFAR10_data()

```

## 1.2 Extract Features

For each image we will compute a Histogram of Oriented Gradients (HOG) as well as a color histogram using the hue channel in HSV color space. We form our final feature vector for each image by concatenating the HOG and color histogram feature vectors.

Roughly speaking, HOG should capture the texture of the image while ignoring color information, and the color histogram represents the color of the input image while ignoring texture. As a result, we expect that using both together ought to work better than using either alone. Verifying this assumption would be a good thing to try for your own interest.

The `hog_feature` and `color_histogram_hsv` functions both operate on a single image and return a feature vector for that image. The `extract_features` function takes a set of images and a list of feature functions and evaluates each feature function on each image, storing the results in a matrix where each column is the concatenation of all feature vectors for a single image.

```

[3]: from cs231n.features import *

num_color_bins = 10 # Number of bins in the color histogram
feature_fns = [hog_feature, lambda img: color_histogram_hsv(img,
    ↪ nbin=num_color_bins)]

```

```

X_train_feats = extract_features(X_train, feature_fns, verbose=True)
X_val_feats = extract_features(X_val, feature_fns)
X_test_feats = extract_features(X_test, feature_fns)

# Preprocessing: Subtract the mean feature
mean_feat = np.mean(X_train_feats, axis=0, keepdims=True)
X_train_feats -= mean_feat
X_val_feats -= mean_feat
X_test_feats -= mean_feat

# Preprocessing: Divide by standard deviation. This ensures that each feature
# has roughly the same scale.
std_feat = np.std(X_train_feats, axis=0, keepdims=True)
X_train_feats /= std_feat
X_val_feats /= std_feat
X_test_feats /= std_feat

# Preprocessing: Add a bias dimension
X_train_feats = np.hstack([X_train_feats, np.ones((X_train_feats.shape[0], 1))])
X_val_feats = np.hstack([X_val_feats, np.ones((X_val_feats.shape[0], 1))])
X_test_feats = np.hstack([X_test_feats, np.ones((X_test_feats.shape[0], 1))])

```

```

Done extracting features for 1000 / 49000 images
Done extracting features for 2000 / 49000 images
Done extracting features for 3000 / 49000 images
Done extracting features for 4000 / 49000 images
Done extracting features for 5000 / 49000 images
Done extracting features for 6000 / 49000 images
Done extracting features for 7000 / 49000 images
Done extracting features for 8000 / 49000 images
Done extracting features for 9000 / 49000 images
Done extracting features for 10000 / 49000 images
Done extracting features for 11000 / 49000 images
Done extracting features for 12000 / 49000 images
Done extracting features for 13000 / 49000 images
Done extracting features for 14000 / 49000 images
Done extracting features for 15000 / 49000 images
Done extracting features for 16000 / 49000 images
Done extracting features for 17000 / 49000 images
Done extracting features for 18000 / 49000 images
Done extracting features for 19000 / 49000 images
Done extracting features for 20000 / 49000 images
Done extracting features for 21000 / 49000 images
Done extracting features for 22000 / 49000 images
Done extracting features for 23000 / 49000 images
Done extracting features for 24000 / 49000 images
Done extracting features for 25000 / 49000 images

```

```

Done extracting features for 26000 / 49000 images
Done extracting features for 27000 / 49000 images
Done extracting features for 28000 / 49000 images
Done extracting features for 29000 / 49000 images
Done extracting features for 30000 / 49000 images
Done extracting features for 31000 / 49000 images
Done extracting features for 32000 / 49000 images
Done extracting features for 33000 / 49000 images
Done extracting features for 34000 / 49000 images
Done extracting features for 35000 / 49000 images
Done extracting features for 36000 / 49000 images
Done extracting features for 37000 / 49000 images
Done extracting features for 38000 / 49000 images
Done extracting features for 39000 / 49000 images
Done extracting features for 40000 / 49000 images
Done extracting features for 41000 / 49000 images
Done extracting features for 42000 / 49000 images
Done extracting features for 43000 / 49000 images
Done extracting features for 44000 / 49000 images
Done extracting features for 45000 / 49000 images
Done extracting features for 46000 / 49000 images
Done extracting features for 47000 / 49000 images
Done extracting features for 48000 / 49000 images
Done extracting features for 49000 / 49000 images

```

### 1.3 Train SVM on features

Using the multiclass SVM code developed earlier in the assignment, train SVMs on top of the features extracted above; this should achieve better results than training SVMs directly on top of raw pixels.

```

[5]: # Use the validation set to tune the learning rate and regularization strength

from cs231n.classifiers.linear_classifier import LinearSVM

learning_rates = [1e-9, 1e-8, 1e-7]
regularization_strengths = [5e4, 5e5, 5e6]

results = {}
best_val = -1
best_svm = None

#####
# TODO:
# Use the validation set to set the learning rate and regularization strength. #
# This should be identical to the validation that you did for the SVM; save #
# the best trained classifier in best_svm. You might also want to play #
# with different numbers of bins in the color histogram. If you are careful #

```

```

# you should be able to get accuracy of near 0.44 on the validation set.      #
#####
# *****START OF YOUR CODE (DO NOT DELETE/MODIFY THIS LINE)*****

for lr in learning_rates:
    for r in regularization_strengths:
        svm = LinearSVM()
        svm.train(X_train_feats, y_train, learning_rate=1e-7, reg=2.5e4,
num_iters=1500)
        y_train_pred = svm.predict(X_train_feats)
        y_val_pred = svm.predict(X_val_feats)
        train_accuracy = np.mean(y_train_pred == y_train)
        val_accuracy = np.mean(y_val_pred == y_val)
        results[(lr, r)] = (train_accuracy, val_accuracy)
        if val_accuracy > best_val:
            best_val = val_accuracy
            best_svm = svm

# *****END OF YOUR CODE (DO NOT DELETE/MODIFY THIS LINE)*****

# Print out results.
for lr, reg in sorted(results):
    train_accuracy, val_accuracy = results[(lr, reg)]
    print('lr %e reg %e train accuracy: %f val accuracy: %f' % (
        lr, reg, train_accuracy, val_accuracy))

print('best validation accuracy achieved during cross-validation: %f' %
best_val)

```

```

lr 1.000000e-09 reg 5.000000e+04 train accuracy: 0.412286 val accuracy: 0.419000
lr 1.000000e-09 reg 5.000000e+05 train accuracy: 0.411429 val accuracy: 0.416000
lr 1.000000e-09 reg 5.000000e+06 train accuracy: 0.414633 val accuracy: 0.423000
lr 1.000000e-08 reg 5.000000e+04 train accuracy: 0.418000 val accuracy: 0.424000
lr 1.000000e-08 reg 5.000000e+05 train accuracy: 0.415939 val accuracy: 0.420000
lr 1.000000e-08 reg 5.000000e+06 train accuracy: 0.416163 val accuracy: 0.419000
lr 1.000000e-07 reg 5.000000e+04 train accuracy: 0.412673 val accuracy: 0.409000
lr 1.000000e-07 reg 5.000000e+05 train accuracy: 0.411714 val accuracy: 0.413000
lr 1.000000e-07 reg 5.000000e+06 train accuracy: 0.411959 val accuracy: 0.409000
best validation accuracy achieved during cross-validation: 0.424000

```

```

[6]: # Evaluate your trained SVM on the test set: you should be able to get at least
0.40
y_test_pred = best_svm.predict(X_test_feats)
test_accuracy = np.mean(y_test == y_test_pred)
print(test_accuracy)

```

0.423



```
[7]: # An important way to gain intuition about how an algorithm works is to
# visualize the mistakes that it makes. In this visualization, we show examples
# of images that are misclassified by our current system. The first column
# shows images that our system labeled as "plane" but whose true label is
# something other than "plane".

examples_per_class = 8
classes = ['plane', 'car', 'bird', 'cat', 'deer', 'dog', 'frog', 'horse', 'ship',
          'truck']
for cls, cls_name in enumerate(classes):
    idxs = np.where((y_test != cls) & (y_test_pred == cls))[0]
    idxs = np.random.choice(idxs, examples_per_class, replace=False)
    for i, idx in enumerate(idxs):
        plt.subplot(examples_per_class, len(classes), i * len(classes) + cls +
          1)

        plt.imshow(X_test[idx].astype('uint8'))
        plt.axis('off')
        if i == 0:
            plt.title(cls_name)
plt.show()
```



### 1.3.1 Inline question 1:

Describe the misclassification results that you see. Do they make sense?



*Your Answer :*

The misclassified images show features of both their corresponding labels and the labels they are classified into. For example, in the first column, most of the images have the streamline shape which is also an important feature of plane. Moreover, the color of images in “dog” column all have grey style, which is also the color of some typical dogs (e.g. husky and alaskan dog). In conclusion, it’s reasonable for the model to misclassify.

## 1.4 Neural Network on image features

Earlier in this assignment we saw that training a two-layer neural network on raw pixels achieved better classification performance than linear classifiers on raw pixels. In this notebook we have seen that linear classifiers on image features outperform linear classifiers on raw pixels.

For completeness, we should also try training a neural network on image features. This approach should outperform all previous approaches: you should easily be able to achieve over 55% classification accuracy on the test set; our best model achieves about 60% classification accuracy.

```
[8]: # Preprocessing: Remove the bias dimension
# Make sure to run this cell only ONCE
print(X_train_feats.shape)
X_train_feats = X_train_feats[:, :-1]
X_val_feats = X_val_feats[:, :-1]
X_test_feats = X_test_feats[:, :-1]

print(X_train_feats.shape)
```

```
(49000, 155)
```

```
(49000, 154)
```

```
[14]: from cs231n.classifiers.neural_net import TwoLayerNet

input_dim = X_train_feats.shape[1]
hidden_dim = 500
num_classes = 10

best_net = None

#####
# TODO: Train a two-layer neural network on image features. You may want to #
# cross-validate various parameters as in previous sections. Store your best #
# model in the best_net variable.                                           #
#####
# *****START OF YOUR CODE (DO NOT DELETE/MODIFY THIS LINE)*****

learning_rates = [0.5, 1]
batch_size = [200, 400]

best_val = -1
```

```

for lr in learning_rates:
    for b in batch_size:
        net = TwoLayerNet(input_dim, hidden_dim, num_classes)
        net.train(X_train_feats, y_train, X_val_feats, y_val, learning_rate=lr,
↪learning_rate_decay=0.95, reg=2.5e-3, num_iters=1500, batch_size=b)
        y_train_pred = net.predict(X_train_feats)
        y_val_pred = net.predict(X_val_feats)
        train_accuracy = np.mean(y_train_pred == y_train)
        val_accuracy = np.mean(y_val_pred == y_val)
        if val_accuracy > best_val:
            best_val = val_accuracy
            best_net = net
        print(f"lr={lr}, batch_size={b}, train_acc={train_accuracy},
↪val_acc={val_accuracy}")
# *****END OF YOUR CODE (DO NOT DELETE/MODIFY THIS LINE)*****

```

```

lr=0.5, batch_size=200, train_acc=0.6035918367346939, val_acc=0.573
lr=0.5, batch_size=400, train_acc=0.6208775510204082, val_acc=0.588
lr=1, batch_size=200, train_acc=0.5911224489795919, val_acc=0.565
lr=1, batch_size=400, train_acc=0.5933265306122449, val_acc=0.536

```

[15]: *# Run your best neural net classifier on the test set. You should be able  
# to get more than 55% accuracy.*

```

test_acc = (best_net.predict(X_test_feats) == y_test).mean()
print(test_acc)

```

0.567

[ ]: