

Assignment 3

3.2

① function f_1 — quasiconvex.

Sol) • level sets are convex

quasi convex dom f ① convex

→ quasi convex

② sub level sets

• not convex → line with sub-level set is not line segment.

② function f_2 — concave, quasiconcaveSol) f_2 is first not convex, concave

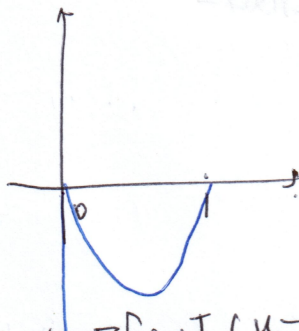
• sub level set is also concave

 $f \rightarrow$ quasi concave.also, all of $\frac{\text{super level} = \text{sub-level}}{\text{convex}} \Rightarrow$ concave

3.13

• negative entropy

• strictly convex.



$$\text{Sol) } f(u) > f(v) + \nabla f(v)^T (u-v)$$

$$\sum_i u_i \log u_i > \sum_i v_i \log v_i + \sum_i (\log v_i + 1) (u_i - v_i)$$

$$> \sum_i v_i \log v_i + u_i \log v_i - v_i \log v_i + u_i - v_i$$

$$> \sum_i (u_i \log v_i + u_i - v_i)$$

$$\rightarrow \sum_i u_i \log u_i - u_i \log v_i - (u_i - v_i) > 0$$

i) $u=v$

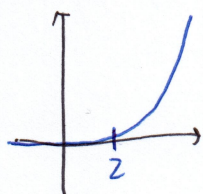
$$\sum_i \frac{u_i \log u_i - u_i \log v_i - (u_i - v_i)}{0} = 0$$

ii) $u \neq v$

$$u_i \log \left(\frac{u_i}{v_i} \right) - (u_i - v_i) > 0. \Rightarrow \text{positive.}$$

3.16

(a)

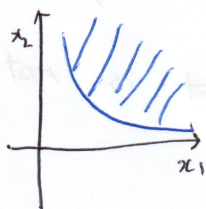


• convex, quasiconvex, quasiconcave

(b)

$$x_1 x_2 \geq \alpha \rightarrow \text{quasiconcave}$$

sol)



(c)

• convex, quasiconvex.

sol)

$$\text{Hessian} = \begin{bmatrix} \frac{1}{(x_1 x_2)^3} & \frac{1}{(x_1 x_2)^2} \\ \frac{1}{(x_1 x_2)^2} & \frac{1}{(x_1 x_2)(x_2^3)} \end{bmatrix} \succeq_0 \Rightarrow \text{positive-semidefinite}$$

(d) quasiconvex / quasiconcave

$$x/x_2$$

3.36

$$(a) f^*(y) = 0$$

$$\nabla f^*(y) = \max_i x_i = 0$$

$$y \geq 0, \sum y_i = 1$$

$$(sol) f^*(y) = \sup (y^T x - f(x))$$

$$= \sup (y^T x - \max x_i)$$

$$y \leq 0 \rightarrow \text{negative sup} \rightarrow -\infty$$

$$y \geq 0 \rightarrow \text{positive sup} \rightarrow \infty$$

$$y \geq 0, \sum y_i = 1 \rightarrow y^T x \leq \max_i x_i = 0$$

2.2 . derive

$$(sol) f(x) = h(g(x)) = h(g_1(x), g_2(x), \dots, g_k(x))$$

$$f''(x) = \underbrace{g'(x)^T}_{\text{convex}} \underbrace{H(g(x))}_{\succeq} \underbrace{g'(x)}_{\succeq} + \underbrace{H(g(x))^T}_{\succeq} \underbrace{g''(x)}_{\succeq}$$

→ positive definite