

# Human-Computer Interaction:

## 3A. Movement and Fitts' Law

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22 October 2024

# Human Movement and Fitts' Law

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- Fitts' Law
- Index of Difficulty (ID)
- Building a Fitts' Law Model
- Speed-Accuracy Trade-off
- Throughput in HCI

Learning Objectives: be able to ...

- Identify factors in human performance of input
- Calculate the ID of a pointing task
- Build a Fitts' Law model of the performance with an input device
- Explain how speed-accuracy trade-off and throughput apply to input

# Fitts' Law

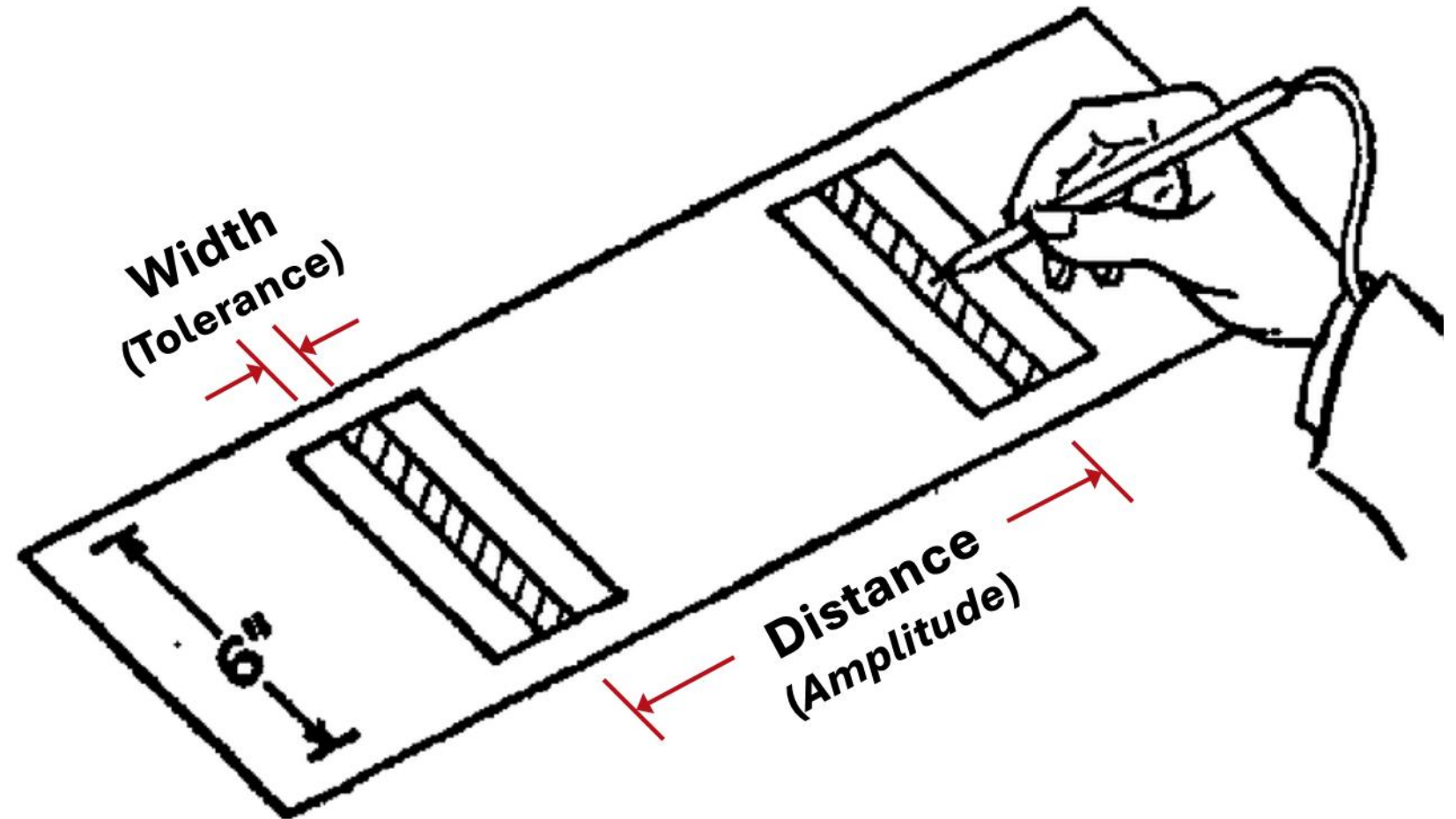
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- Paul Fitts (1912-1965)
  - Psychologist
  - Pioneer in Human Factors
- Fitts' Law is a model of human performance
  - How long does it take to make a movement to a known target?
  - How does this depend on properties of the movement task?
    - Distance and size (width) of the target

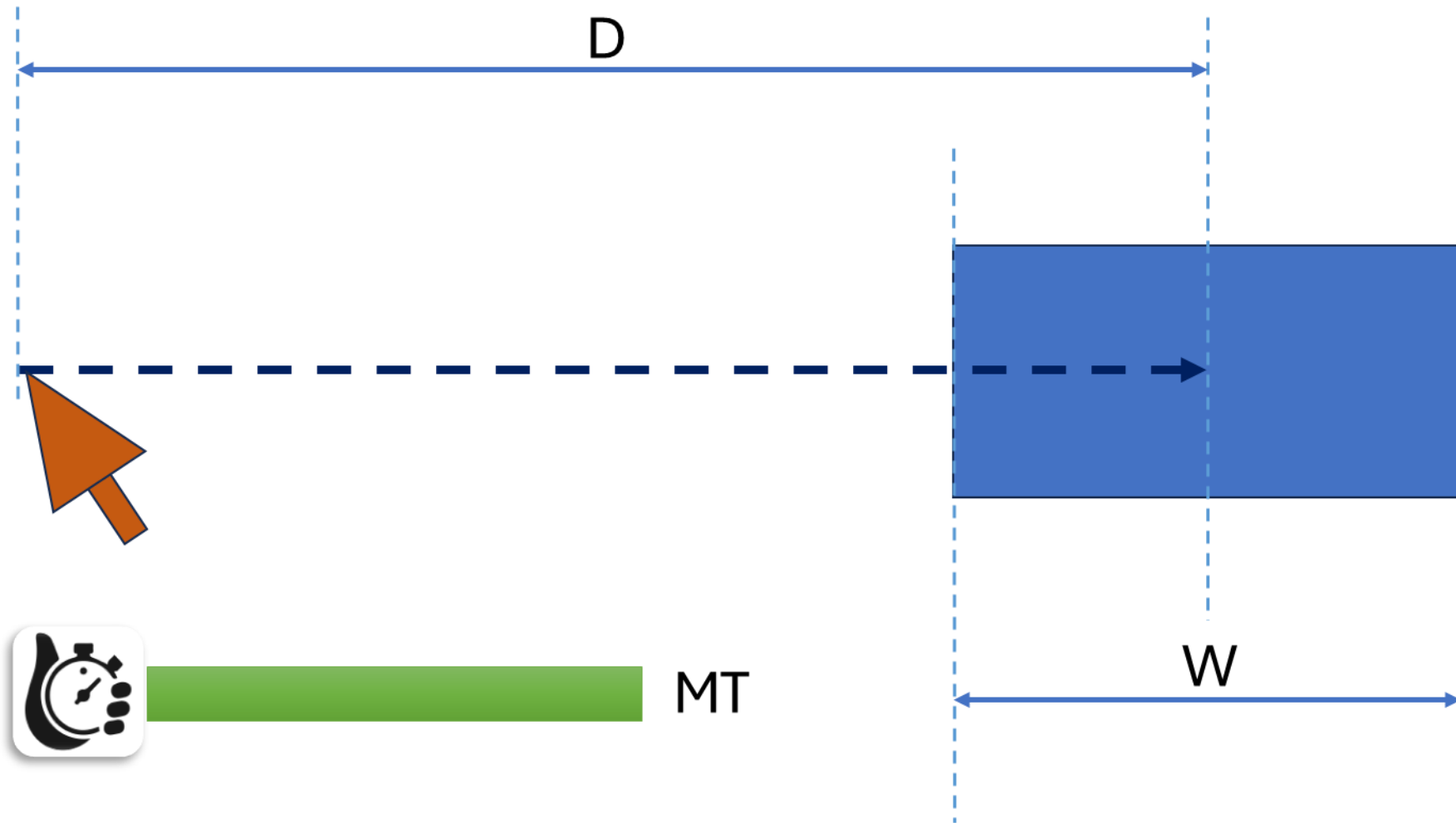


# Fitts' Experiment – Apparatus

- Four distances (D):  
2, 4, 8, 16 inch
- Four widths (W):  
0.25, 0.5, 1, 2 inch
- 16 combinations



# Fitts' Experiment – W and D in 1D Task

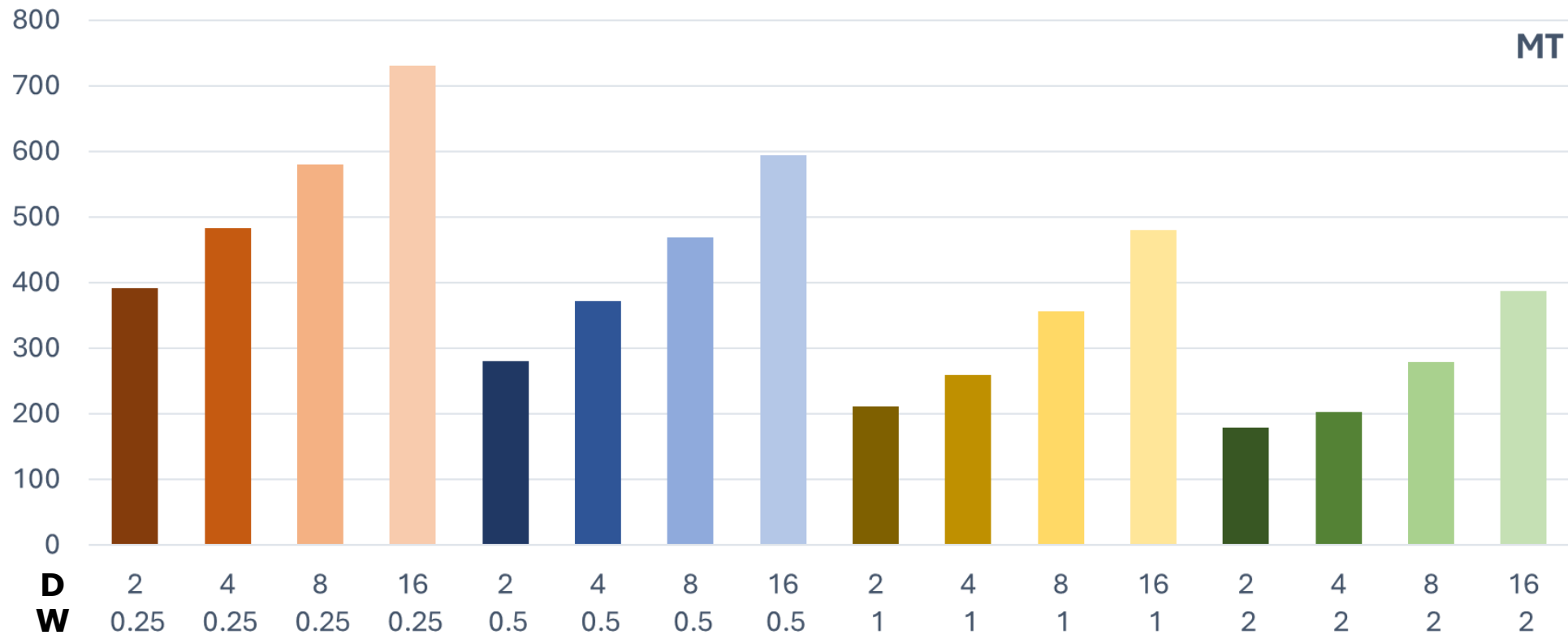


# Fitts' Experiment – Dataset

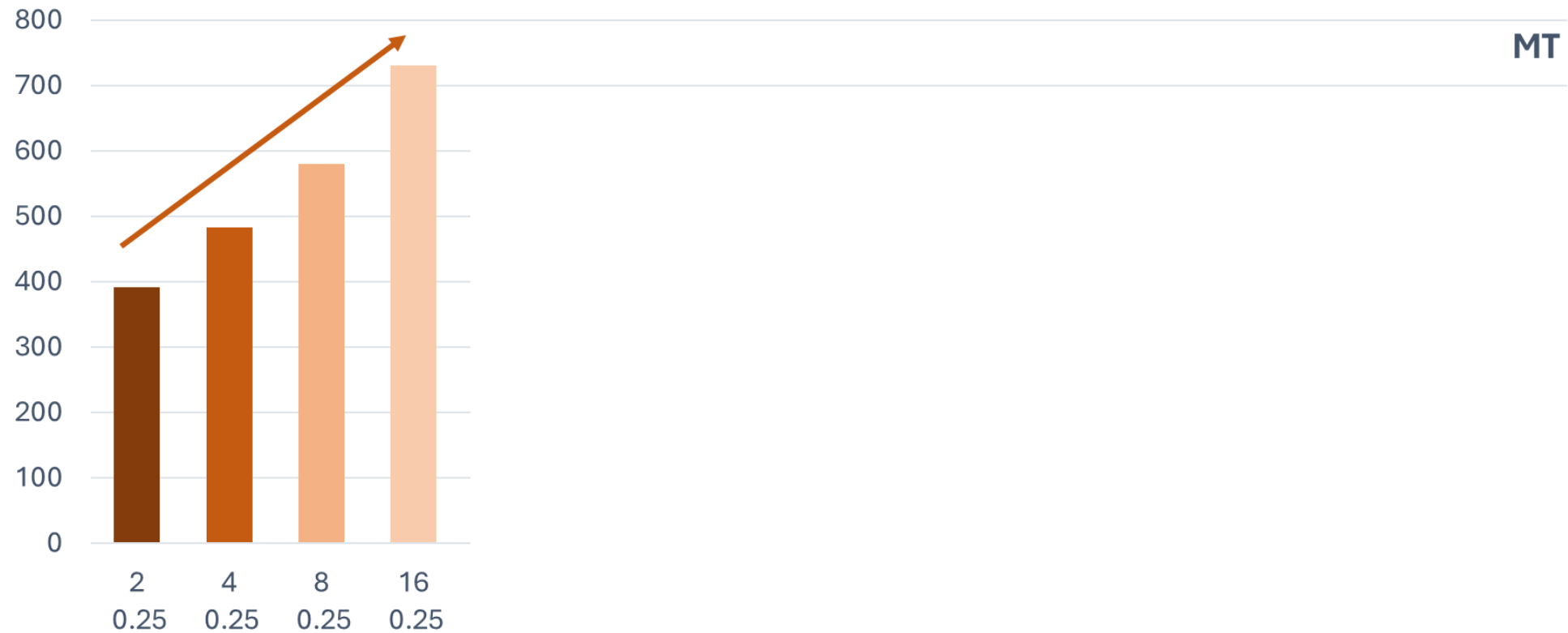
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Amplitude	Width	MT
2	0.25	392
2	0.5	281
2	1	212
2	2	180
4	0.25	484
4	0.5	372
4	1	260
4	2	203
8	0.25	580
8	0.5	469
8	1	357
8	2	279
16	0.25	731
16	0.5	595
16	1	481
16	2	388
Mean		391.5
SD		157.3

# Fitts' Experiment – Initial Analysis

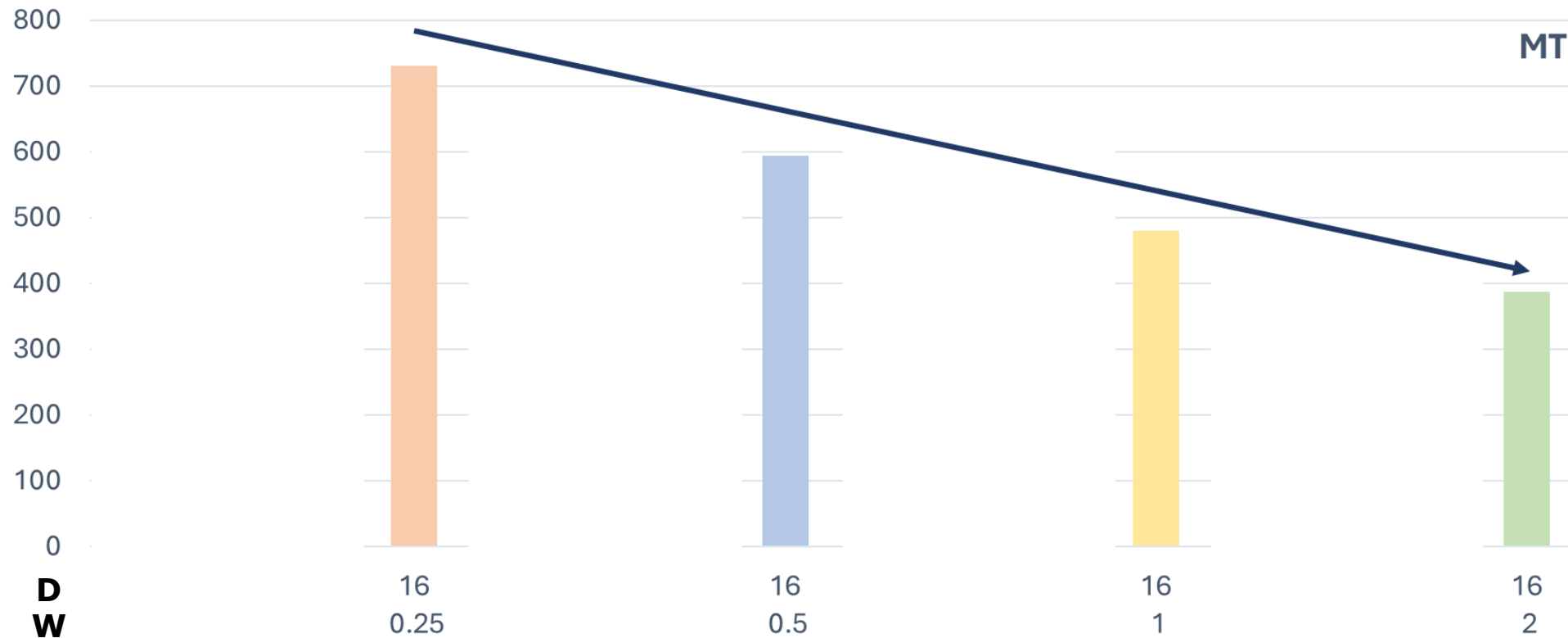


# Fitts' Experiment – Factor of Distance

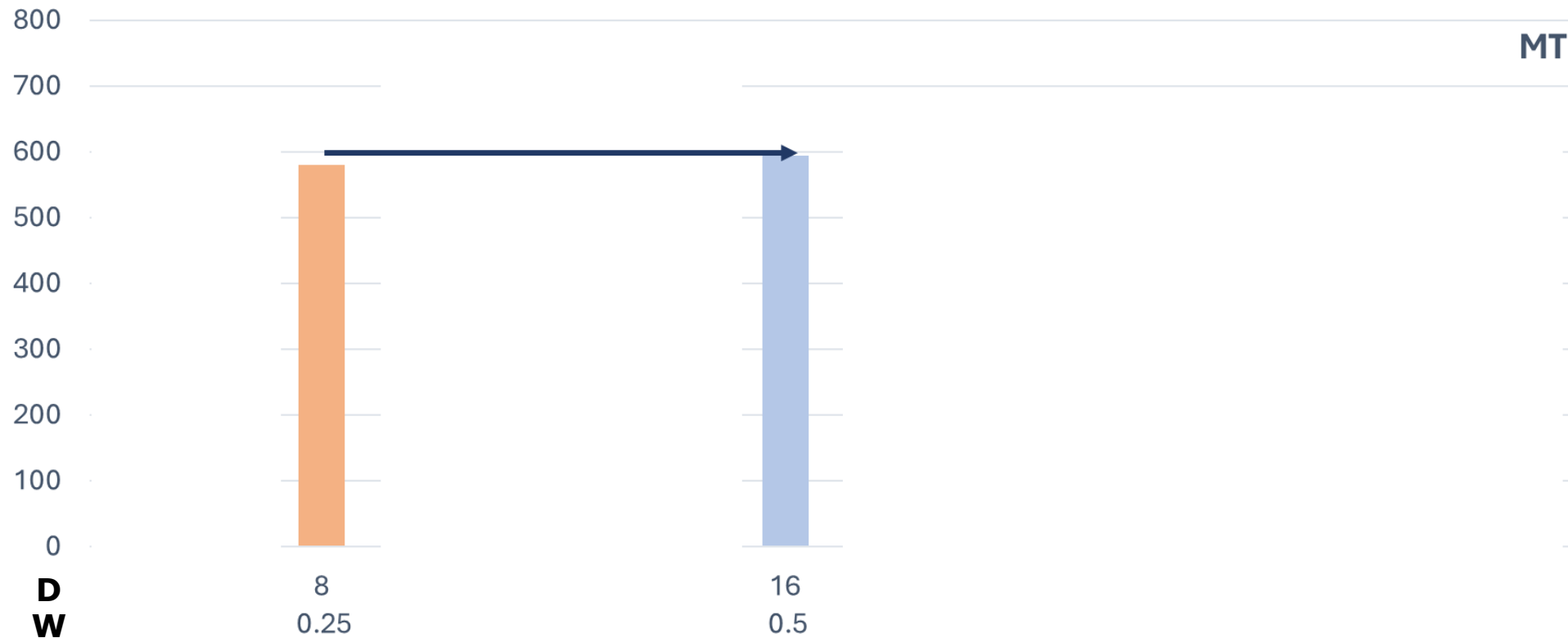




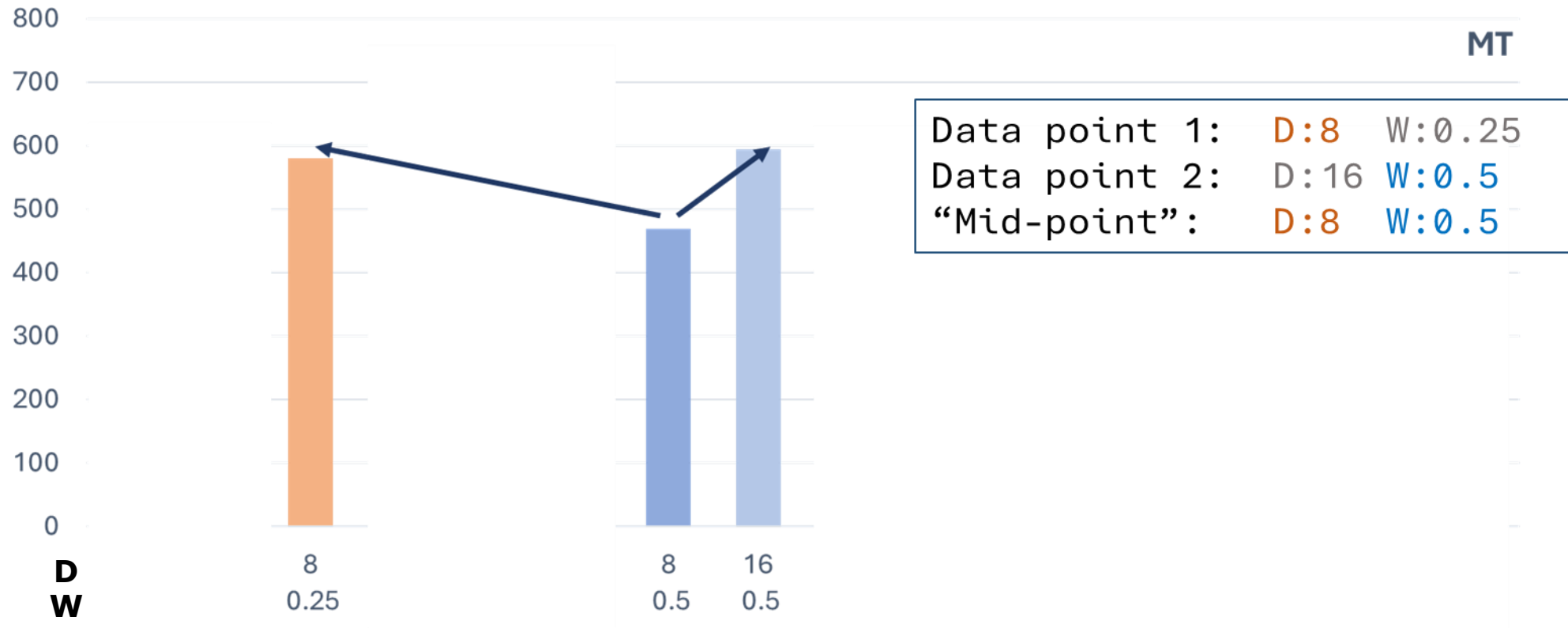
# Fitts' Experiment – Factor of Size



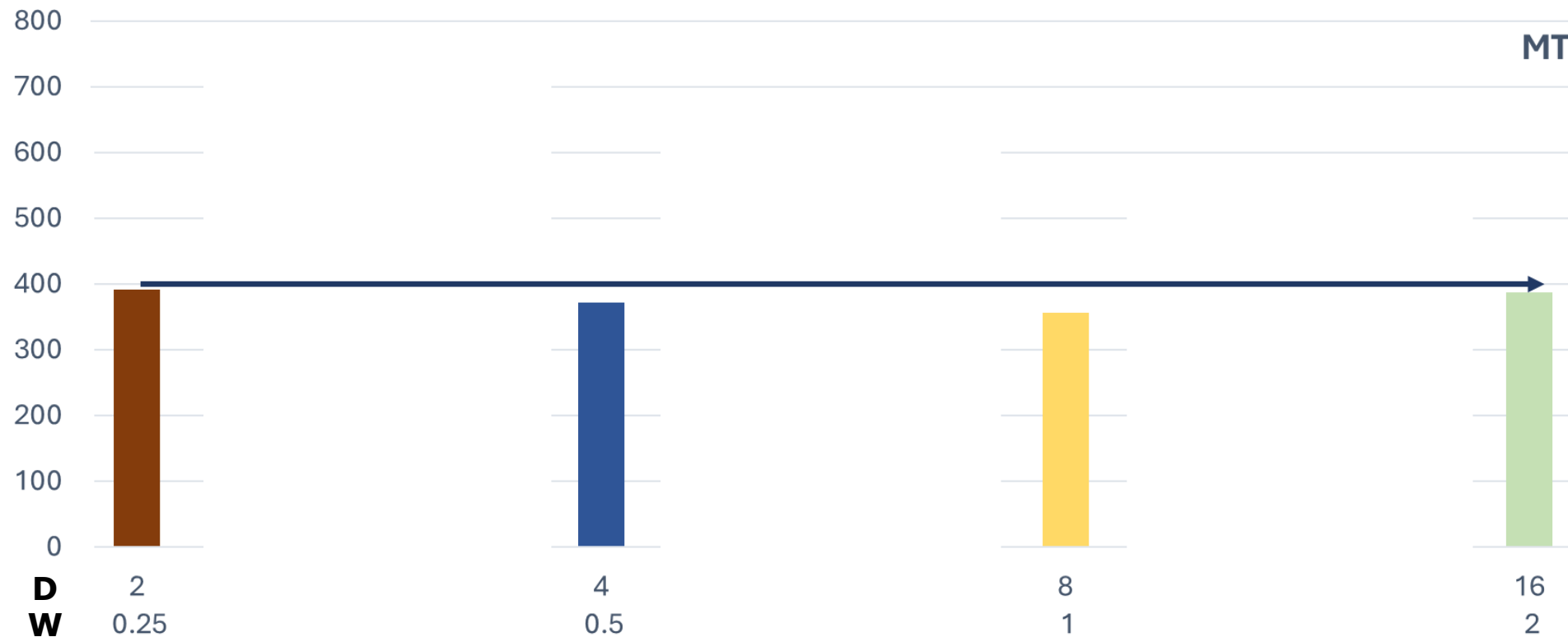
# Fitts' Experiment – Interactions (Size x Distance)



# Fitts' Experiment – Interactions (Effects - Inverse)



# Fitts' Experiment – Interactions (Effects - Direct)



# Fitt's Experiment – Conclusions

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- Movement time depends on the task
- A pointing task has two properties that affect performance
  - Target distance (= Amplitude of the movement)
  - Target width (= Tolerance for landing on the target)
- When a target is nearer, we can reach it faster
- When a target is smaller, we have to slow down to land on it

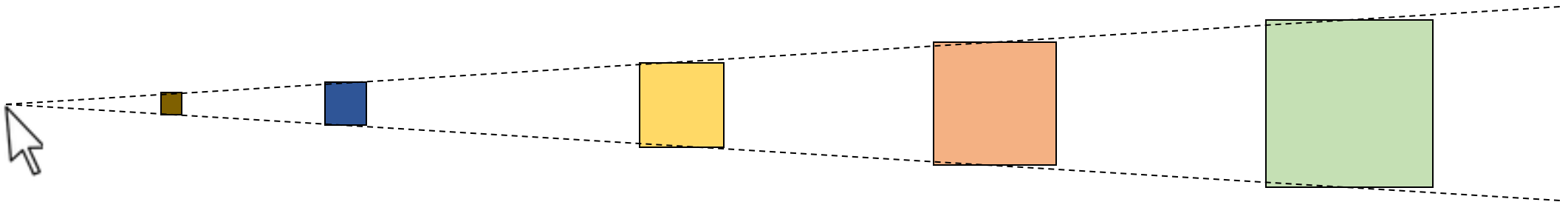
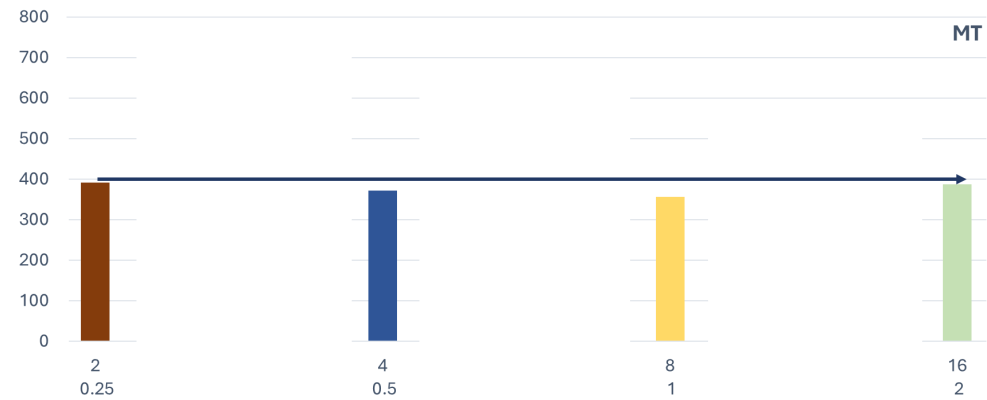
# Human Movement and Fitts' Law

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- Fitts' Law
- **Index of Difficulty (ID)**
- **Building a Fitts' Law Model**
- Speed-Accuracy Trade-off
- Throughput in HCI

# Task Difficulty

- Some tasks take the same time to complete
- They have the same *difficulty*
- We can model task difficulty based on distance and width



# Index of Difficulty (ID)

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- Paul Fitts proposed to combine distance and width into a single *index of difficulty*, measured in bits
- The formulation has become refined through later research, and in HCI we now use:

$$ID = \log_2 \left( \frac{D}{W} + 1 \right)$$

$$ID = \log_2 \left( \frac{2D}{W} \right)$$

Original formulation  
proposed by Paul Fitts

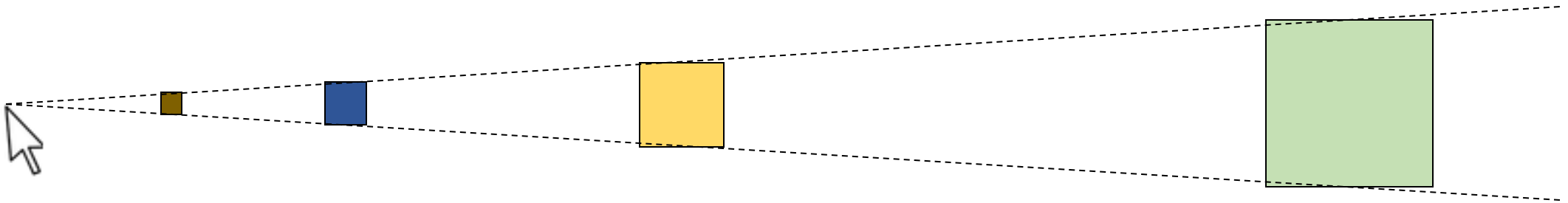
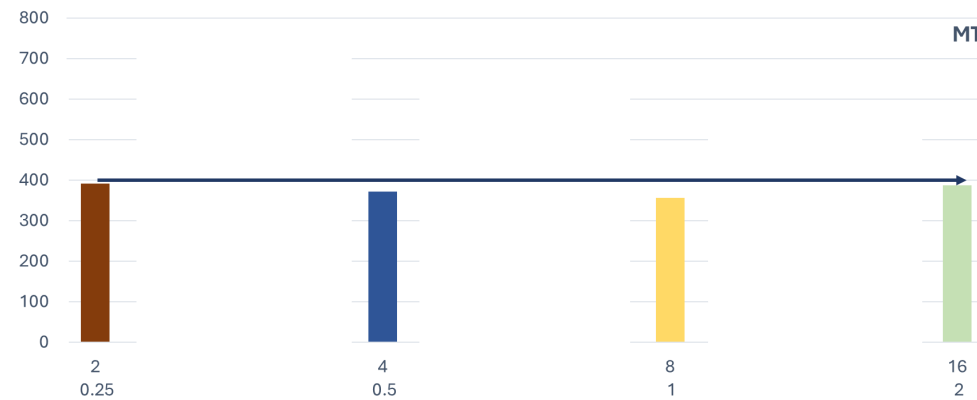


# Index of Difficulty (ID) – Example 1

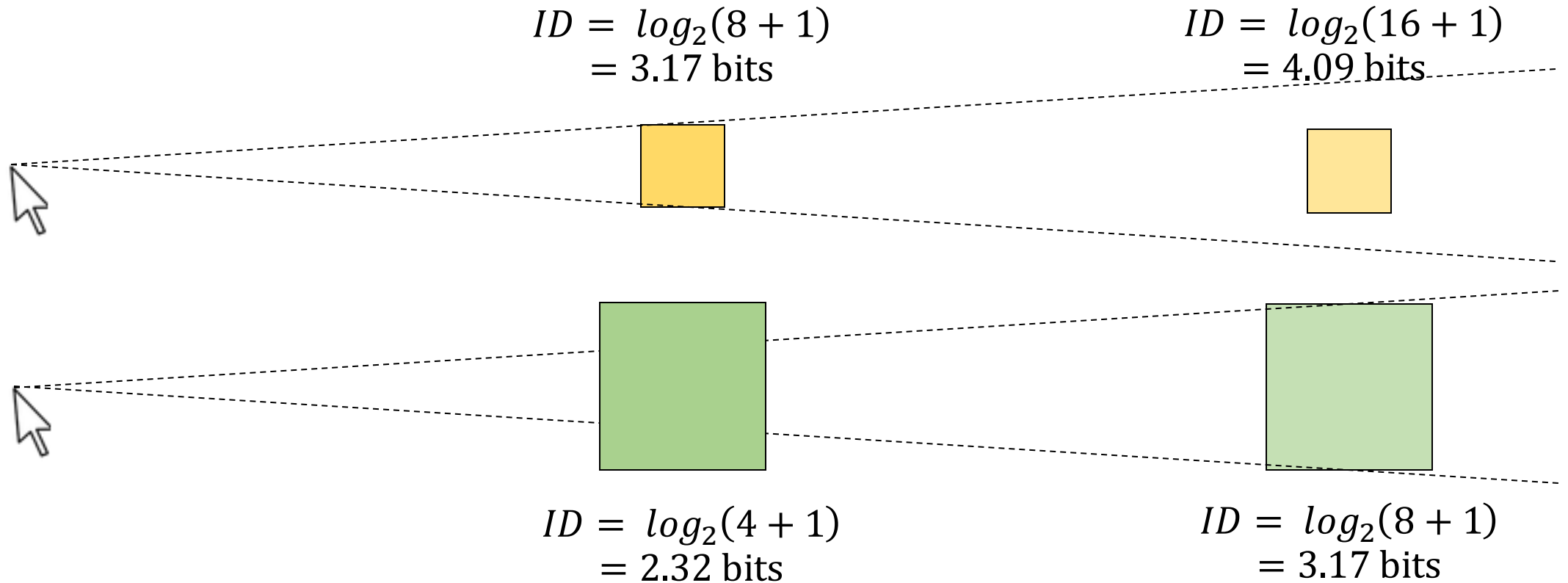
- These four tasks have the same D/W ratio

$$\frac{D}{W} = 8$$

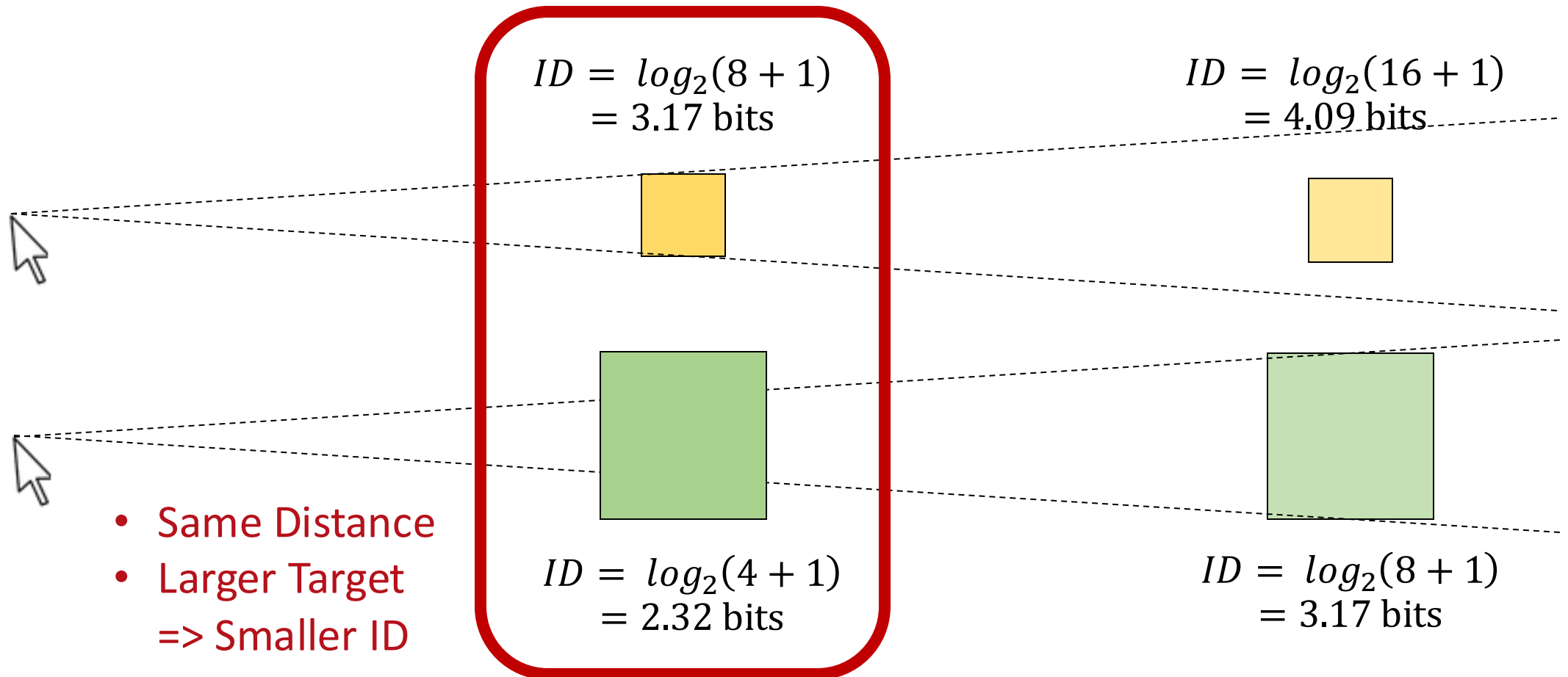
- $ID = \log_2(8 + 1) = 3.17$  bits



# Index of Difficulty (ID) – Example 2



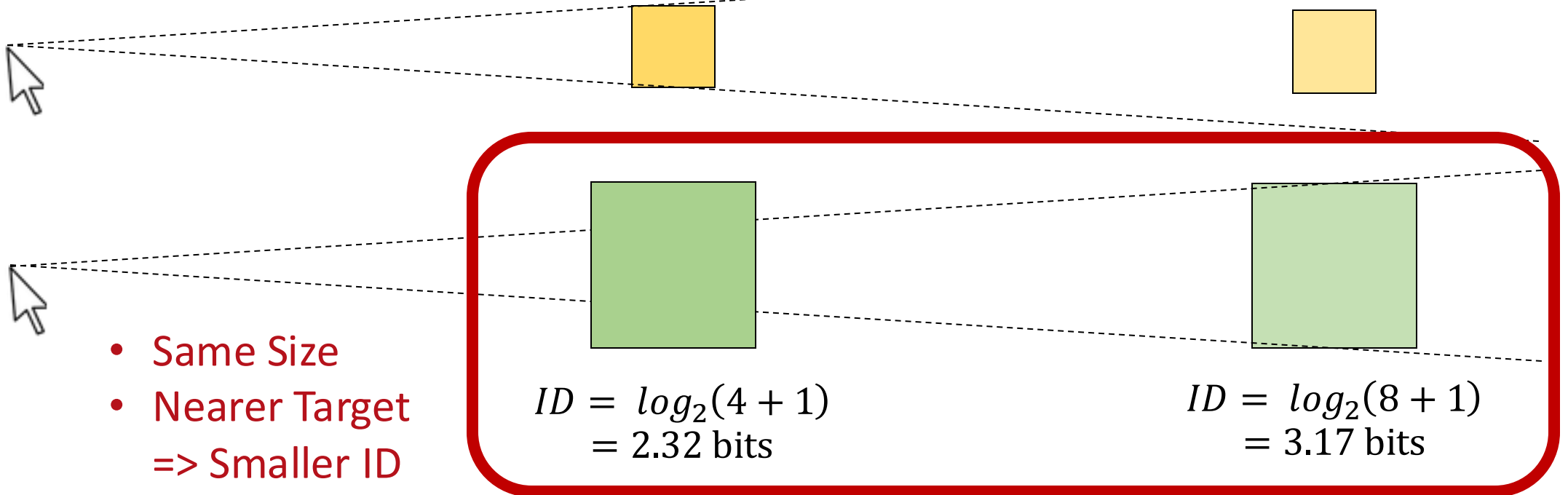
# Index of Difficulty (ID) – Example 2



# Index of Difficulty (ID) – Example 2

$$ID = \log_2(8 + 1) \\ = 3.17 \text{ bits}$$

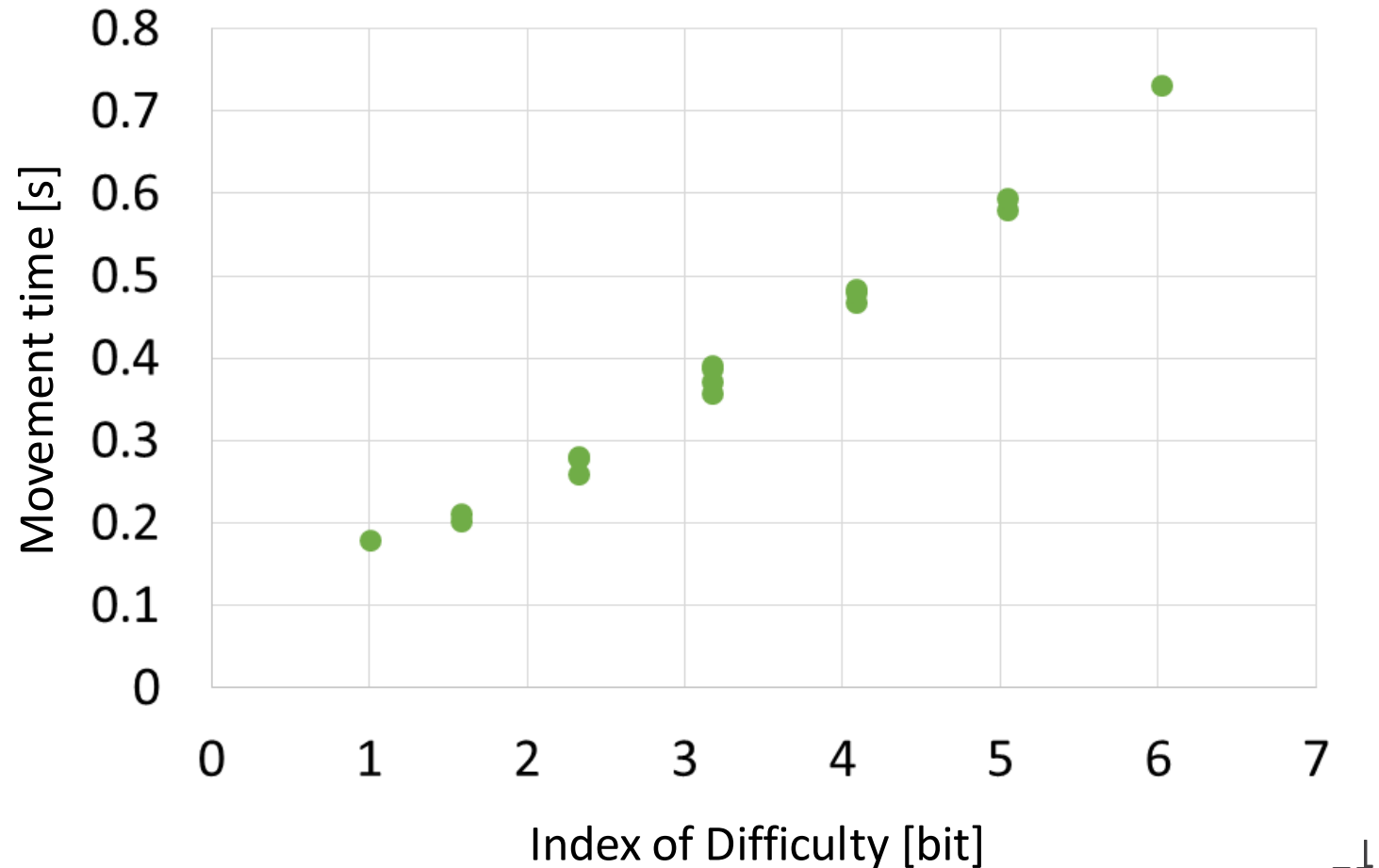
$$ID = \log_2(16 + 1) \\ = 4.09 \text{ bits}$$



# Building a Fitts' Law Model

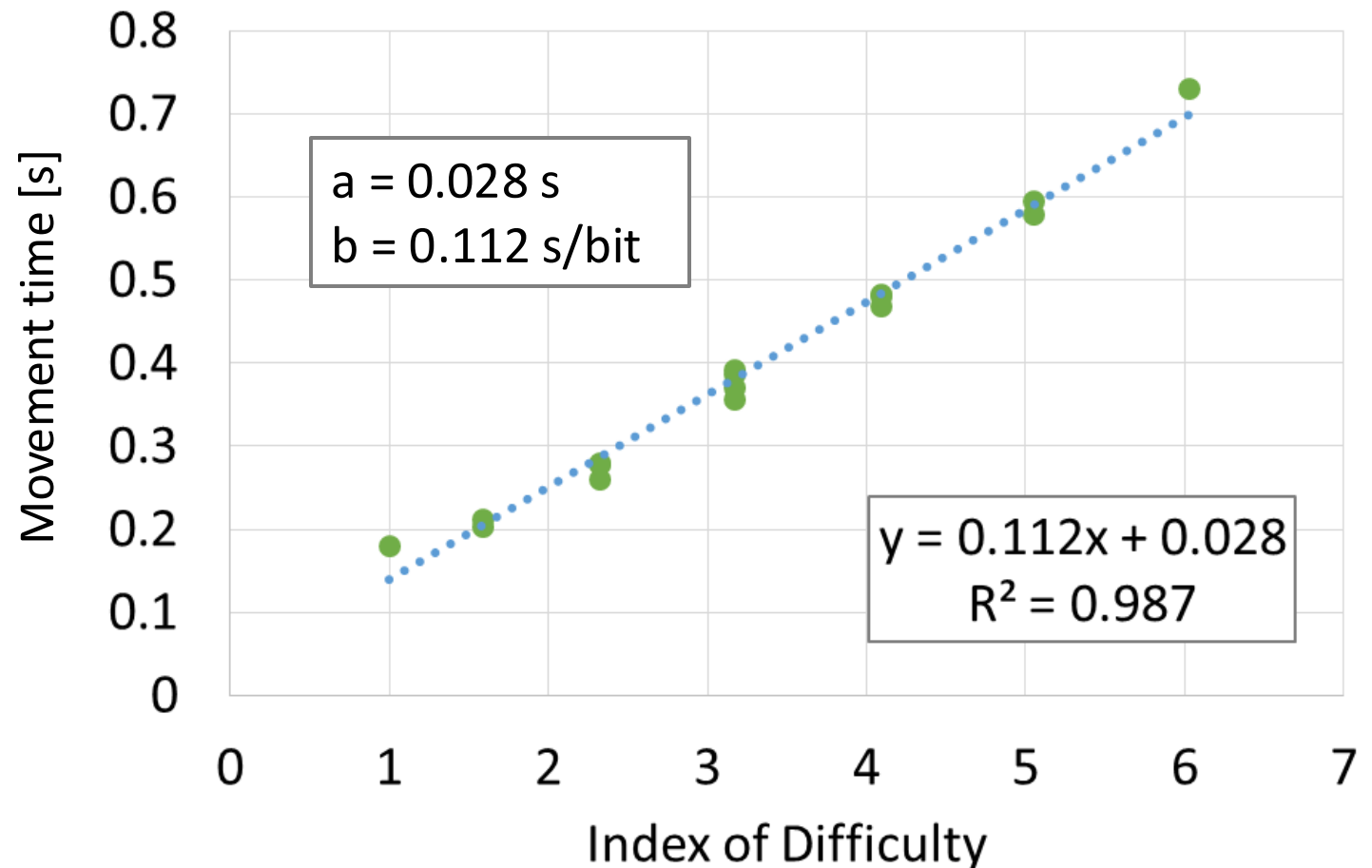
Amplitude	Width	ID	MT
2	0.25	3.17	392
2	0.5	2.32	281
2	1	1.58	212
2	2	1	180
4	0.25	4.09	484
4	0.5	3.17	372
4	1	2.32	260
4	2	1.58	203
8	0.25	5.04	580
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16	1	4.09	481
16	2	3.17	388

**Mean** 391.5  
**SD** 157.3



# Building a Fitts' Law Model - Formulation

- Movement time depends on task difficulty
- The relationship is linear
- Fitts' Law:  
 $MT = a + b * ID$
- The values for  $a$  and  $b$  are specific to the apparatus / device



# Fitts' Law – Generalisation

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$$MT = a + b * ID = a + b * \log_2 \left( \frac{D}{W} + 1 \right)$$

- Distance (D): Distance of the target
- Width (W): width of the target in the direction of the movement
- ID: index of difficulty of the task, in *bits*
- b: rate at which time increases with task difficulty, in *seconds/bit*
- a is a time constant, in *seconds*
- D, W and ID are properties of the movement task
  - independent of the device used for the movement
- a and b are device-dependent, on the device and body part used to perform the movement

# Human Movement and Fitts' Law

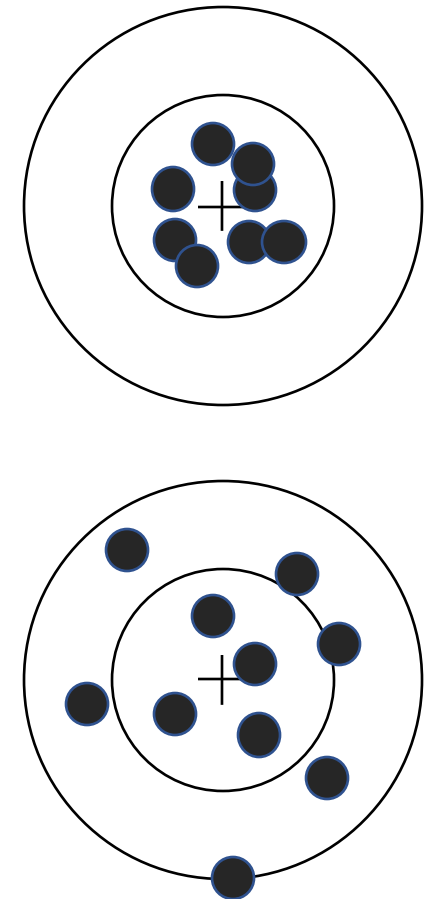
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- Fitts' Law
- Index of Difficulty (ID)
- Building a Fitts' Law Model
- **Speed-Accuracy Trade-off**
- **Throughput in HCI**



# Speed-Accuracy Trade-off

- Fitts' Law captures the *speed-accuracy trade-off* in movement
  - We move faster, when we don't have to be accurate
  - We can be more accurate, when we move more slowly
- The speed-accuracy trade-off is a fundamental property of input in user interfaces
  - When we move faster, we make more errors
  - Pointing with less precision
  - Typing errors



# Factor of Device

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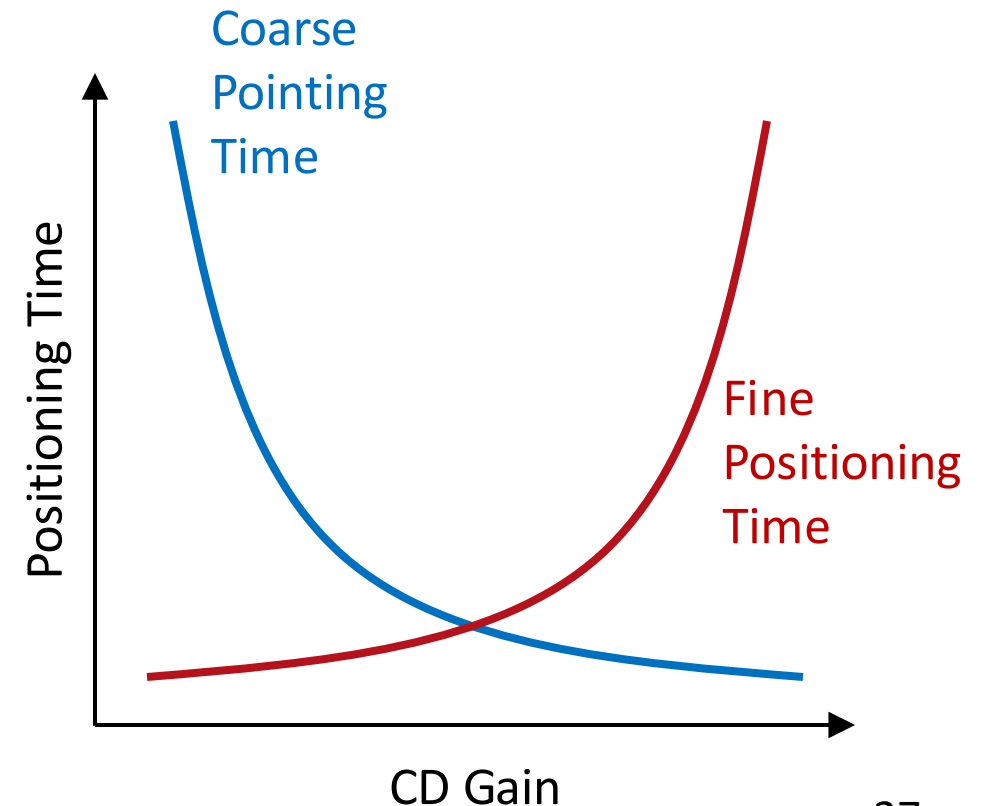
- Speed and accuracy of input depend on the input method
- There are many factors that can influence speed and accuracy
  - The movement of the user that is sensed in *motor space* (eyes, head, hands)
  - Input devices and trackers used (mouse, trackpad, joystick, ...)
  - The mapping of input from motor space to display space (control-display gain, transfer function)
- Collectively we refer to these as *Factor of Device*

# Example: Control-Display Gain (CD Gain)

- CD gain is a scale factor in mapping input from a device to a cursor on the display:

$$CD_{gain} = \frac{V_{display}}{V_{control}}$$

- CD gain > 1: less movement of the input device needed, for faster cursor movement on the display (coarse pointing)
- CD gain < 1: cursor moves more slowly than the input device, for precise input (fine positioning)



# Throughput

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- Throughput is the amount of data that can pass through a system in a given amount of time
  - In communication systems, throughput depends on bandwidth (speed) and signal-to-noise ratio (accuracy)
  - It provides a single metric of a system's efficiency, that combines speed and accuracy, measured in bit/s
- One of the key ideas underlying Fitts' Law is that we can adopt throughput as a single measure of human performance with an input device, for the transfer of information to a computer

# Throughput in Fitts' Law

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- Fitts' Law defines throughput  $TP$  as a measure of input efficiency

$$TP = \frac{ID}{MT} \quad [\text{in bit/s}]$$

- Throughput ( $ID/MT$ ) increases ...
  - When we can complete more difficult tasks (higher  $ID$ ) within a given time (fixed  $MT$ )
  - When we need less time (lower  $MT$ ) for a task of given difficulty (fixed  $ID$ )
- Throughput combines speed and accuracy into a single metric of performance, of a user with an input device

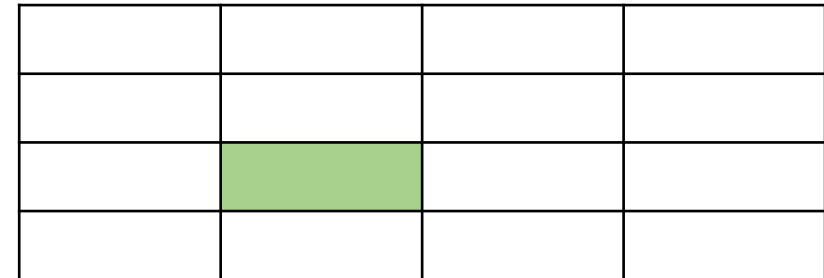
# Throughput – Thought Experiment

- Think of information transfer as selection of 1 from N options
- If we divide the screen into two large buttons (A) then we can select 1 of 2
  - Transferring 1 bit/selection
- If we divide the screen into more buttons, then every selection transfers more information
  - B: 1 of 16  $\Rightarrow$  4 bit/selection
  - C: 1 of 128  $\Rightarrow$  7 bit/selection

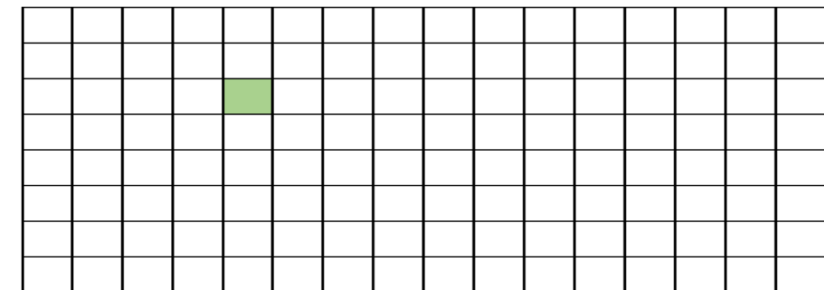
**A**  
1 bit



**B**  
4 bit



**C**  
7 bit



# Throughput – Thought Experiment

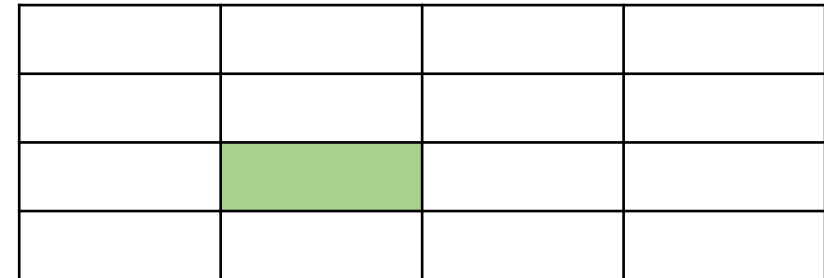
## Scenario 1: No errors

- Scenario 1: we are very careful to avoid any input error
- The time for a selection depends on the button size, say
  - A: MT = 250ms  $\rightarrow$  4 selections/s
  - B: MT = 666ms  $\rightarrow$  1.5 selections/s
  - C: MT = 1000ms  $\rightarrow$  1 selection/s
- Which interface has the highest throughput?

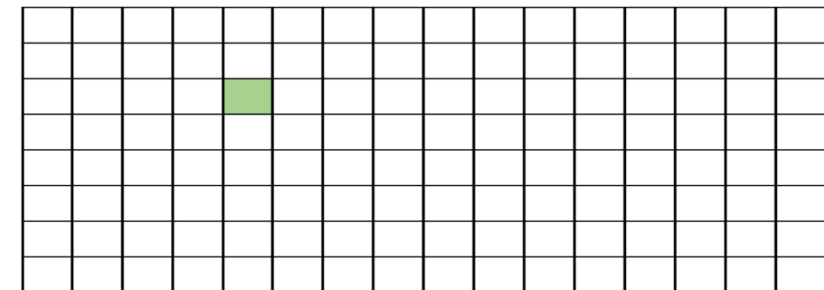
**A**  
1 bit



**B**  
4 bit



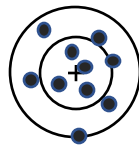
**C**  
7 bit



# Throughput – Thought Experiment

## Scenario 2: High-speed input

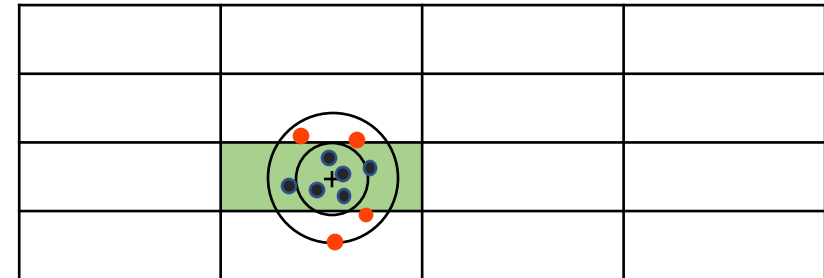
- Scenario 2: we risk error and make all selections at high speed
  - MT = 250ms -> 4 selections/s
- Fast movement means that we will not be precise. Our input will be noisy:
  - A: no errors
  - B: 40% error rate
  - C: 80% error rate
- Which interface has the highest throughput?



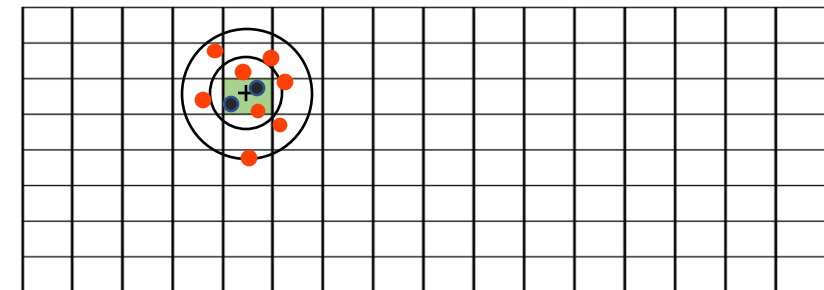
**A**  
1 bit



**B**  
4 bit



**C**  
7 bit



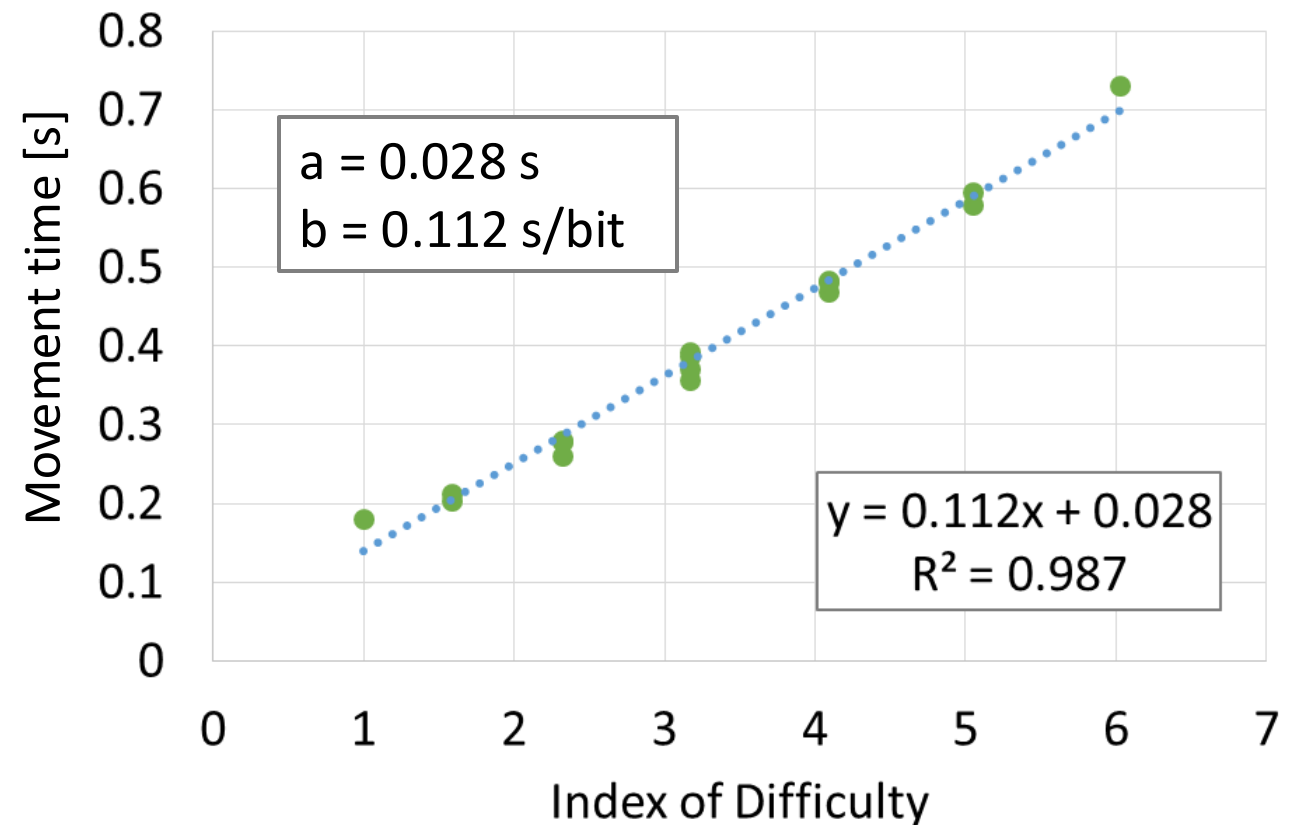


# Throughput – Fitts' Law Visualisation

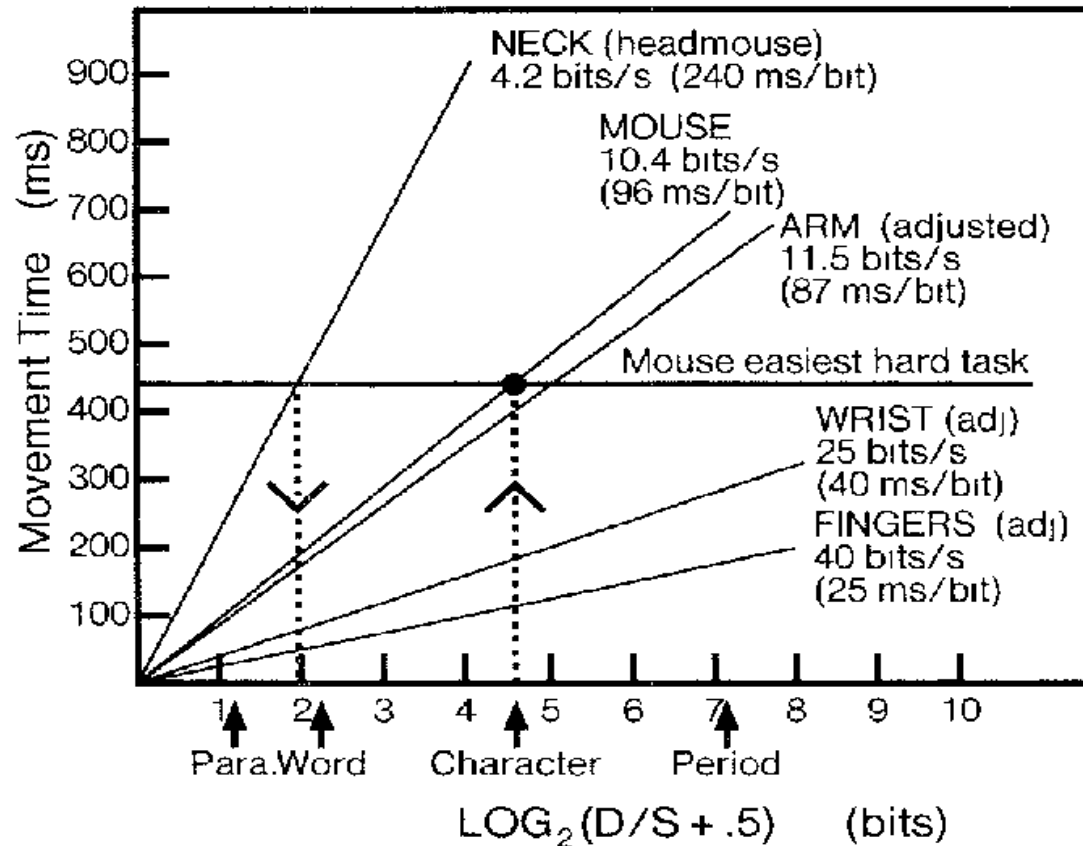
- $MT = a + b * ID$
- $TP = \frac{ID}{MT}$
- We can approximate throughput as the inverse of the slope

$$TP = \frac{1}{b}$$

- The flatter the slope, the higher the throughput



# Throughput of devices / muscle groups



- Mouse tasks, perceived difficulty:
  - Select a word: “hardest easy task”
  - Select a char.: “easiest hard task”
- Mouse throughput 10.4 bit/s
  - 1991 data! Further optimization since
- Fingers have higher throughput, 40 bit/s
  - For adjacent buttons, not in general
- Head pointing is less efficient, 4.2 bit/s
  - Select a word in the time mouse pointing can select a character

# Movement and Fitts' Law – Key Points

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- Movement and input are subject to a speed-accuracy trade-off
- Most input in HCI is based on *aimed* movements
  - Reaching for controls, pointing with a mouse, typing on keyboard, ...
- Aimed movements can be modelled using Fitts' Law
  - Modelling the difficulty of input tasks
  - Modelling the performance with different devices
- Throughput is a measure of input performance that takes both speed and accuracy into account

# Next Lecture

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- Input models
- Fitts' Law Application
- Pointing and Crossing
- Steering Law
- Keystroke-level Model (KLM)