SCC121
Fundamentals of Computer Science

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Week 5 Quiz

- Sets
- Relations
- Functions
- Propositional Logic

Overview

- Propositions
- Truth tables
- Fundamental connectives
- Logical properties of propositions

Objectives

- Understanding basic ideas about propositional logic, propositions and their logical properties
- Facility in the construction of Truth tables and of using fundamental connectives

Overview

- Propositions
- Truth tables
- Fundamental connectives
- Logical properties of propositions

Logic

- Logic: the study of reasoning
  - rational ways of drawing conclusions

Propositions - definition

- Proposition: a claim about how things are
    - a statement that is either true or false, but not both.

- If a proposition is true, then we say it has a truth value of "true”
- if a proposition is false, its truth value is "false“

- We use letters to represent propositions

A = “It is raining”

Notation: proposition A can be

true	T, 1 (one)

or

false	F, 0 (zero)

Propositions - examples

Propositions with the truth value of “true” (T)

- "Grass is green“
- “Snow is white”
- "2+2 = 4“

Propositions with the truth value of “false” (F)

- "2 + 8 = 11“
- "1 = 0“

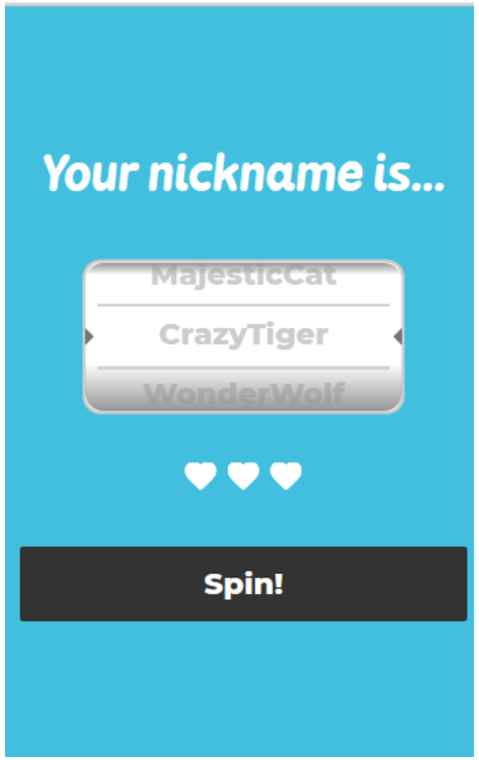
Non propositions - examples

- Non propositions – are not claims about how things are
    - Cannot be said to be true or false

- Is the water warm?
- Where are we?
- “Go for it!”
- Put the phone down!
- Ouch!
- Aha

Let’s playxercise!

- https://kahoot.it/



Overview

- Propositions
- Truth tables
- Fundamental connectives

Atomic and compound propositions

- Atomic proposition – proposition whose true or false value does not depend on that of any other proposition.

- Compound proposition - propositions constructed from atomic propositions by combining them with fundamental connectives.

Truth tables

- Truth table tabulates the value of a compound proposition for all possible values of its atomic propositions and their combination, i.e., one column for each atomic proposition

- Truth table for 2 atomic propositions:

	P	Q	Compound
1	F	F	
2	F	T	
3	T	F	
4	T	T	

Overview

- Propositions
- Truth tables
- Fundamental connectives
- Logical properties of propositions

Fundamental connectives

- AND $\wedge$
- OR $\vee$
- XOR $\oplus$, or $\veebar$
- NOT ~, or $\neg$
- Conditional $\rightarrow$, or $\Rightarrow$
- Biconditional $\leftrightarrow$, $\Leftrightarrow$, or $\equiv$

Fundamental connectives

- AND $\wedge$
- OR $\vee$
- XOR $\oplus$, or $\underline{\vee}$
- NOT $\sim$, or $\neg$
- Conditional $\rightarrow$, or $\Rightarrow$
- Biconditional $\leftrightarrow$, $\Leftrightarrow$, or $\equiv$

AND ($\wedge$)

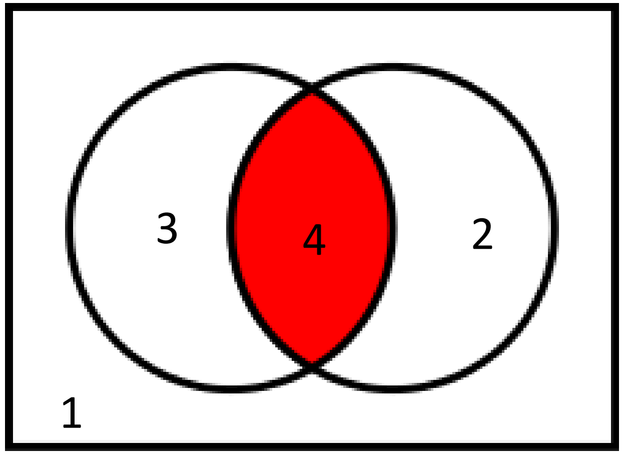
- AND connective takes two proposition P and Q to form a third proposition called conjunction.
- The conjunction is true when both P and Q propositions are true
- Written: $P \wedge Q$

P	Q	$P \wedge Q$
F	F	F
F	T	F
T	F	F
T	T	T

AND (∧)

- AND connective takes two proposition P and Q to form a third proposition called conjunction.
- The conjunction is true when both P and Q propositions are true
- Written: $P \wedge Q$

P	Q	$P \wedge Q$
F	F	F
F	T	F
T	F	F
T	T	T



AND ($\wedge$): examples

- X = “Java is a programming language”
- Y = “ C is a programming language”

- What is $X \wedge Y$?
  - $X \wedge Y$ = “Java and C are programming languages”

Multiple Propositions for AND ($\wedge$)

- We can connect as many propositions as we like
  $R = A \wedge B \wedge C \wedge D \ldots.$
- The rule is :
  - all propositions must be TRUE for the result to be TRUE
  - so, if even only one proposition is FALSE, then the result is FALSE
- So the result of
  $R = T \wedge T \wedge T \wedge T \ldots. \wedge T \wedge F$ is FALSE.

Fundamental connectives

- AND $\wedge$
- OR $\vee$
- XOR $\oplus$, or $\veebar$
- NOT $\sim$, or $\neg$
- Conditional $\rightarrow$, or $\Rightarrow$
- Biconditional $\leftrightarrow$, $\Leftrightarrow$, or $\equiv$

OR (∨)

- OR connective takes two proposition P and Q to form a third proposition called disjunction.
- The disjunction is true when P is true, Q is true, or both P and Q propositions are true
- Written: P ∨ Q

P	Q	P ∨ Q
F	F	F
F	T	T
T	F	T
T	T	T

OR (∨)

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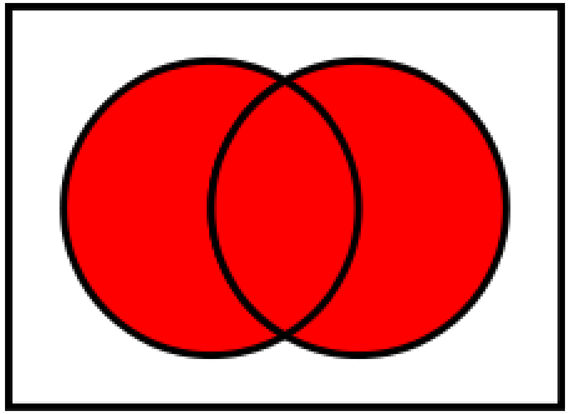
P	Q	P ∨ Q
F	F	F
F	T	T
T	F	T
T	T	T



OR ($\vee$): examples

- C = “I am going to the Lake District”
- D = “I am going to London”

- What is $C \vee D$?
    - $C \vee D$ = “I am going to the Lake District or I am going to London”

Multiple Propositions for OR ($\vee$)

- We can connect as many propositions as we like
  $R = A \vee B \vee C \vee D$ ….
- The rule is :
  - all propositions must be FALSE for the result to be FALSE
  - so, if even only one proposition is TRUE, then the result is TRUE
- So the result of
  $R = F \vee F \vee F \vee F$ …. $\vee\, F \vee T$ is TRUE.

Fundamental connectives

- AND $\wedge$
- OR $\vee$
- XOR $\oplus$, or $\veebar$
- NOT $\sim$, or $\neg$
- Conditional $\rightarrow$, or $\Rightarrow$
- Biconditional $\leftrightarrow$, $\Leftrightarrow$, or $\equiv$

Exclusive OR (XOR) ($\oplus$ or $\veebar$)

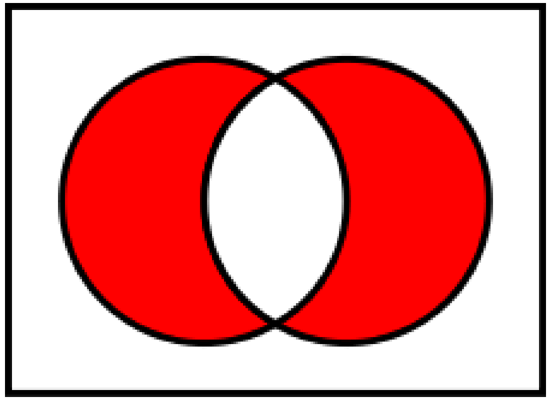
- OR is also called inclusive disjunction
- XOR connective takes two proposition P and Q to form a third proposition called exclusive disjunction.
- The exclusive disjunction is true when P is true, Q is true, but not both P and Q propositions are true.
- Written: $P \oplus Q$

P	Q	$P \oplus Q$
F	F	F
F	T	T
T	F	T
T	T	F

Exclusive OR (XOR) ($\oplus$ or $\veebar$)

- OR is also called inclusive disjunction
- XOR connective takes two proposition P and Q to form a third proposition called exclusive disjunction.
- The exclusive disjunction is true when P is true, Q is true, but not both P and Q propositions are true.
- Written: $P \oplus Q$

P	Q	$P \oplus Q$
F	F	F
F	T	T
T	F	T
T	T	F



Exclusive OR ($\oplus$): examples

- A = “I will have the soup for starters”
- B = “I will have the salad for starters”

- What is A $\oplus$ B?
  - A $\oplus$ B = “I will have either the soup or salad for starters, but not both”

Multiple Propositions for XOR ($\oplus$ or $\underline{\vee}$)

- We can connect as many propositions as we like
  $R = A \oplus B \oplus C \oplus D$ ….
- The rule is :
  - an odd number of propositions must be TRUE for the result to be TRUE
  - an even number of propositions must be TRUE for the result to be FALSE

Fundamental connectives

- AND $\wedge$
- OR $\vee$
- XOR $\oplus$, or $\veebar$
- NOT $\sim$, or $\neg$
- Conditional $\rightarrow$, or $\Rightarrow$
- Biconditional $\leftrightarrow$, $\Leftrightarrow$, or $\equiv$

Negation (~)

- Negation connective takes one proposition P to form a second proposition called negation.
- The negation is True when P is False
- Written: ~P

P	~P
T	F
F	T

Negation (~)

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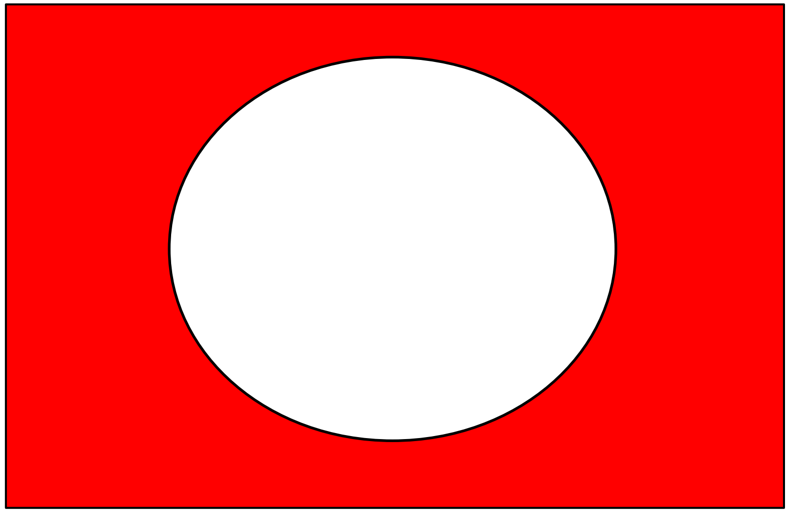
- The negation is True when P is False
- Written: ~P

P	~P
T	F
F	T



Negation (~): examples

- A = “It is raining”
- ~A = “It is not raining”

- X = “The program runs OK”
- ~X = “The program does not run OK”

Conditional or implication (if… then) (→ or ⇒)

- If the train is late, then we will miss our flight.
  - The statement after the "if“ is the antecedent
  - The claim after the "then" is the consequent.
- IF antecedent THEN consequent
- IF the train is late, THEN we will miss our flight.

- If the train is late, but we do not miss our flight, then the above conditional must be false.

Conditional or implication (if… then) ($\rightarrow$ or $\Rightarrow$)

- *If… then* connective combines two propositions P and Q into a third proposition called conditional or implication.
- The conditional or implication is false when the antecedent P is true and its consequent Q is false.

P	Q	$P \rightarrow Q$
F	F	T
F	T	T
T	F	F
T	T	T
antecedent	consequent	conditional

Conditional or implication (if… then) (→ or ⇒)

- *If… then* connective combines two propositions P and Q into a third proposition called conditional or implication.
- The conditional or implication is false when the antecedent P is true and its consequent Q is false; otherwise, a conditional is true.

P	Q	$P \rightarrow Q$
F	F	T
F	T	T
T	F	F
T	T	T

antecedent consequent conditional

Conditional or implication (if… then) ($\rightarrow$ or $\Rightarrow$)

- *If… then* connective combines two propositions P and Q into a third proposition called conditional or implication.
- The conditional or implication is false when the antecedent P is true and its consequent Q is false; otherwise, a conditional is true.

P	Q	$P \rightarrow Q$
F	F	T
F	T	T
T	F	F
T	T	T
antecedent	consequent	conditional

Conditional or implication (if… then) (→ or ⇒)

- *If… then* connective combines two propositions P and Q into a third proposition called conditional or implication.
- The conditional or implication is false when the antecedent P is true and its consequent Q is false; otherwise, a conditional is true.

- Written: $P \to Q$
    - If P is true, then Q is true
        - P is antecedent
        - Q is consequent

P	Q	$P \to Q$
F	F	T
F	T	T
T	F	F
T	T	T

antecedent consequent conditional

Conditional or implication (if… then) ($\rightarrow$ or $\Rightarrow$)

- *If… then* connective combines two propositions P and Q into a third proposition called conditional or implication.
- The conditional or implication is false when the antecedent P is true and its consequent Q is false; otherwise, a conditional is true.
- A conditional also states that false antecedent could imply either false or true consequent. Why?

P	Q	$P \rightarrow Q$
F	F	T
F	T	T
T	F	F
T	T	T
antecedent	consequent	conditional

Conditional or implication - example

"If I am elected, then I will lower taxes" – social contract

- P = "I am elected"
- Q = "I will lower taxes"
- $P \rightarrow Q$ implication

P = elected	Q= lower taxes	P	Q	$P \rightarrow Q$
not elected	not lower taxes	F	F	T
not elected	lower taxes	F	T	T
elected	not lower taxes	T	F	F
elected	lower taxes	T	T	T

Conditional or implication - example

a) seems fair: If I am not elected, then I am not obligated to lower the taxes.

	P = elected	Q= lower taxes	P	Q	$P \rightarrow Q$
a	not elected	not lower taxes	F	F	T
b	not elected	lower taxes	F	T	T
c	elected	not lower taxes	T	F	F
d	elected	lower taxes	T	T	T

Conditional or implication - example

a) seems fair: If I am not elected, then I am not obligated to lower the taxes.

b) seems fair: If I am not elected, I may still in other ways work hard to lower the taxes, although I do not have to for the contract to remain valid.

	P = elected	Q= lower taxes	P	Q	$P \rightarrow Q$
a	not elected	not lower taxes	F	F	T
b	not elected	lower taxes	F	T	T
c	elected	not lower taxes	T	F	F
d	elected	lower taxes	T	T	T

Conditional or implication - example

c) seems fair: I was elected, but I didn't keep my part of the bargain and I didn't reduce taxes) I lied when I made my promise. It was a false proposition.

	P = elected	Q= lower taxes	P	Q	$P \to Q$
a	not elected	not lower taxes	F	F	T
b	not elected	lower taxes	F	T	T
c	elected	not lower taxes	T	F	F
d	elected	lower taxes	T	T	T

Conditional or implication - example

c) seems fair: I was elected, but I didn’t keep my part of the bargain and I didn’t reduce taxes) I lied when I made my promise. It was a false proposition.

d) seems fair, I kept my promise. (I was elected, I reduced taxes)

	P = elected	Q= lower taxes	P	Q	$P \to Q$
a	not elected	not lower taxes	F	F	T
b	not elected	lower taxes	F	T	T
c	elected	not lower taxes	T	F	F
d	elected	lower taxes	T	T	T

Fundamental connectives

- AND $\wedge$
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- Biconditional $\leftrightarrow$, $\Leftrightarrow$, or $\equiv$

Biconditional (if and only if) ($\leftrightarrow$, $\Leftrightarrow$, or $\equiv$)

- *If and only if* connective combines two propositions P and Q into a third proposition called biconditional.
- The biconditional is true when both P and Q have the same truth value, and it is false if P and Q have different truth values
- Written: $P \leftrightarrow Q$
  - $P \rightarrow Q$
  - $Q \rightarrow P$
- Equivalent to $(P \rightarrow Q) \land (Q \rightarrow P)$

P	Q	$P \leftrightarrow Q$
F	F	T
F	T	F
T	F	F
T	T	T

Biconditional (if and only if): example

You will be paid on Monday *if and only if* you submit your timesheets today.

2 conditionals:

You will be paid on Monday *if* you submit your timesheets today.

You will be paid on Monday *only if* you submit your timesheets today.

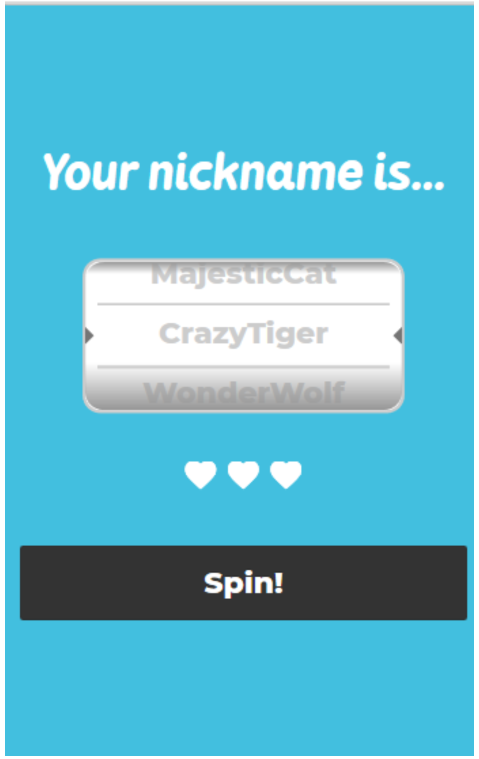
Or:

If you submit your timesheets today then you will be paid on Monday.

If you are paid on Monday then you must have submitted your timesheets today.

Let’s playxercise!

- https://kahoot.it/



Fundamental connectives: precedence

How do we parse this statement?

$\neg X \rightarrow Y \vee Z \rightarrow X \vee Y \wedge Z$

Operator precedence for propositional logic:

$\neg$

$\wedge$

$\vee$

$\rightarrow$

$\leftrightarrow$

$\neg X \rightarrow Y \vee Z \rightarrow X \vee Y \wedge Z$ is short for: $\neg X \rightarrow Y \vee X \rightarrow X \vee Y \wedge Z$

Fundamental connectives: precedence

How do we parse this statement?

$\neg X \rightarrow Y \vee Z \rightarrow X \vee Y \wedge Z$

Operator precedence for propositional logic:

$\neg$

$\wedge$

$\vee$

$\rightarrow$

$\leftrightarrow$

$\neg X \rightarrow Y \vee Z \rightarrow X \vee Y \wedge Z$ is short for: $(\neg X) \rightarrow ((Y \vee Z) \rightarrow (X \vee (Y \wedge Z)))$

Overview

- Propositions
- Truth tables
- Fundamental connectives
- Logical properties of propositions

Logical properties

- Tautologies
- Contradictions
- Contingencies

Logical properties of two propositions:

- Logical equivalence

Logical properties - tautologies

Tautologies - propositions which are always true, regardless of the truth values of its atomic propositions.

- Example: Q v ~Q
  - Q = “I passed the exam”, ~Q = “I did not pass the exam”
  - Q v ~Q = “I passed the exam OR I did not pass the exam” is always true, regardless of the truth values of Q.

The truth table for a tautology has “T” (true) in every row.

Q	~Q	Q ∨ ~Q
T	F	T
F	T	T

Logical properties - contradictions

Contradictions - propositions which are always false, regardless of the truth values of its atomic propositions.

- Example: $Q \wedge \sim Q$
  - Q = “I passed the exam”, ~Q = “I did not pass the exam”
  - $Q \wedge \sim Q$ = “I passed the exam AND I did not pass the exam” is always false, regardless of the truth values of Q.

The truth table for a contradiction has “F”(false) in every row.

Q	$\sim Q$	$Q \wedge \sim Q$
T	F	F
F	T	F

Logical properties - contingencies

Contingencies – propositions that are neither tautologies nor contradictions

- Example: P, ~P
- P = “I passed the exam”, ~P = “I did not pass the exam”

Contingencies have both “T”s and “F”s in their truth tables.

P	~P
T	F
F	T

Logical properties - equivalence

- Two propositions are logically equivalent if they have exactly the same truth value under all circumstances.
  - P and Q are logically equivalent if they are both true, or both false.
  - Written: $P \equiv Q$
- Whenever we find logically equivalent propositions, we should feel free to replace one with another as we wish.

Summary: propositions

- Proposition - a claim about how things are that is either true or false, but not both.
- Atomic proposition – proposition whose truth or falsity does not depend on the truth or falsity of any other proposition.
- Compound proposition - propositions constructed from atomic propositions by combining them with connectives.
- Truth table – table with the value of a compound proposition for all possible values of its atomic propositions.

Summary: fundamental connectives

- AND - fundamental connective that takes two proposition P and Q to form a third: conjunction proposition which is true when both P and Q are true ($P \wedge Q$)
- OR - fundamental connective that takes two proposition P and Q to form a third: inclusive disjunction proposition which is true when P is true, Q is true, or both P and Q are true ($P \vee Q$)
- XOR - fundamental connective that takes two proposition P and Q to form a third: exclusive disjunction proposition which is true when P is true, Q is true, but not both P and Q are true ($P \oplus V$)
- Negation - fundamental connective that takes one proposition P to form a second proposition called negation which is true when P is false ($\sim P$).
- Conditional – fundamental connective that combines two propositions P and Q into a third proposition called conditional or implication which is false when the antecedent P is true and its consequent Q is false; otherwise, a conditional is true ($P \rightarrow Q$)
- Biconditional – fundamental connective that combines two propositions P and Q into a third proposition called biconditional which is true when both P and Q have the same truth value, and it is false if P and Q have different truth values ($P \leftrightarrow Q$)

Summary: logical properties

- Tautologies - propositions which are always true, regardless of the truth values of its atomic propositions.
- Contradictions - propositions which are always false, regardless of the truth values of its atomic propositions.
- Contingencies – propositions that are neither tautologies nor contradictions.
- Logically equivalent propositions – propositions which have exactly the same truth value under all circumstances.