SCC121
Fundamentals of Computer Science

Prof Corina Sas

School of Computing and Communications

Overview and Objectives

- Sets
  - Defining sets
  - Set operations
  - Types of sets

Overview and Objectives

- Sets
  - Defining sets
  - Set operations
  - Types of sets
- Objectives
  - Understanding the different types of sets and relationships among them

Recap: Sets and membership

Sets and membership

- A = {*a*, *b*, *c*}

  A = set; a, b and c are its elements;

  "{" and "}" are markers for the beginning and the end of set.
- $a \in A$ (element *a* belongs to/is in set A)
- $m \notin A$ (element *m* does not belong/is not in set A)

Defining a set through a property:

  – the set of natural numbers x, such that x < 10

  – $\{x \mid x \in N \text{ and } x < 10\}$

Recap: Set operations

- Union ($A \cup B$)
  - elements in A, or B, or both
- Intersection ($A \cap B$)
  - elements common to both A and B
- Difference (A - B)
  - elements in A which are not in B
- Cartesian Product (A x B)
  - set of all possible ordered pairs whose first component is a member or element of set A and whose second component is a member of set B

Types of Sets

- Empty set
- Disjoint sets
- Equal sets
- Sets of sets
- Subsets, and proper subsets
- Super sets, and proper supersets
- Universal sets
- Complement sets

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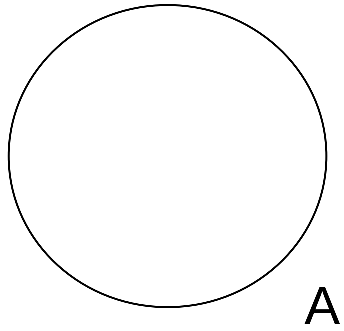
Empty Set

- The **empty set** is the set which contains no objects
- Null set or void set
- Notation for empty set: { } or the symbol $\varnothing$
- Example:
  - A = { } is an empty set
  - A = $\varnothing$



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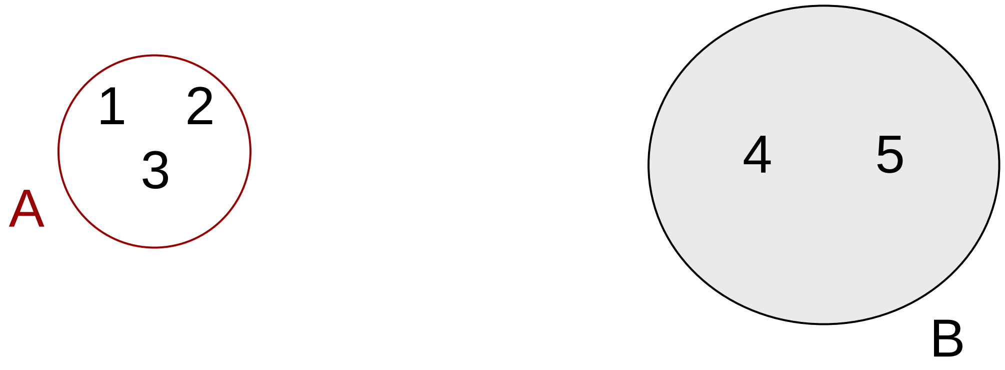
Disjoint Sets

- Two sets are disjoint if the have no elements in common
- Formally, two sets are disjoint if their intersection is the empty set



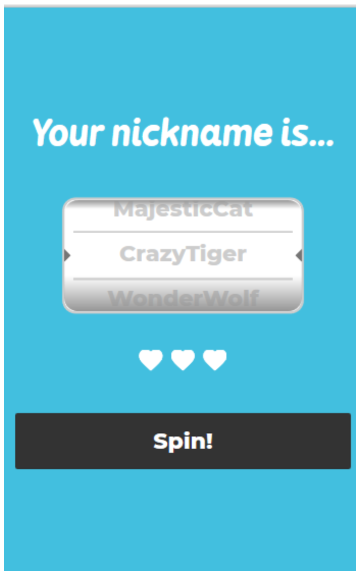
Disjoint Sets

- Two sets are <u>disjoint</u> if their intersection is the empty set
- Examples
  - {New York, Washington} and {3, 4} are disjoint
  - {1, 2} and $\emptyset$ are disjoint
    - Their intersection is the empty set

- Two sets are <u>not disjoint</u> if their intersection is not empty,
- Examples
  - {1, 2, 3} and {3, 4, 5} are not disjoint

Let’s playxercise!

- https://kahoot.it/



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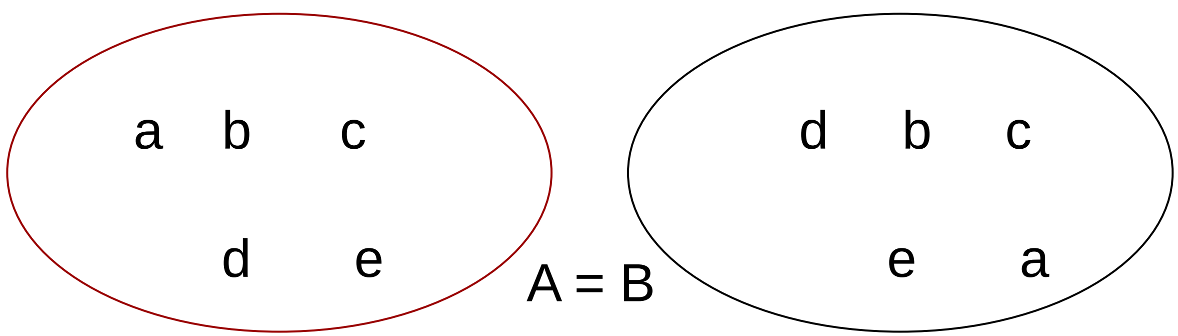
Equal Sets

- Two sets are equal if they have the same elements
  - Written: A = B

- Every element of set A is an element of set B, and every element of set B is an element of set A.

- Example: A = {a, b, c, d, e}; B = {d, b, c, e, a}

  A = B



Not Equal Sets

- Two sets **are not equal** if they do not have identical elements
- there is at least one element in one of the sets which is not an element of the other set
- Written: $A \neq B$

Examples:

- $A = \{a, b, c, d, e\}$; $B = \{d, b, c, e\}$ are not equal sets
  - $a \in A$, but $a \notin B$

Not Equal Sets

- Two sets **are not equal** if they do not have identical elements, or if there is at least one element in one of the sets which is not an element of the other set

Examples:

- C= {1, 2} and D= {2, 1, 3} are not equal sets.
  - $3 \in D$, but $3 \notin C$
- C ={1, 2} and E = {1, 3} are not equal sets.
  - Although both sets have the same number of elements, $3 \in E$, but $3 \notin C$

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Sets of Sets

- Sets can contain atomic elements, i.e., letters, numbers, or pairs of elements.
- Sets can also contain other sets
- Example:
  - $A = \{a,\{b,c\}\}$ is a set containing element a and another set consisting of elements b and c.
  - $B = \{\{a\}\}$
  - $C = \{\emptyset\}$ and note that $\{\emptyset\} \neq \emptyset$

Cardinality of Sets

- Cardinality of a set is the number of its elements
  - Written as $|A|$
- Examples
  - Let $A = \{1, 2, 3, 4, 5\}$. Then $|A| = 5$
  - $|\varnothing| = 0$
  - Let $B = \{\varnothing, \{a\}, \{b\}, \{a, b\}\}$. Then $|B| = 4$

- Singleton set is a set with one element
- Examples:
  - $A = \{a\}$ or $B = \{\{a\}\}$

Cardinality Exercise

https://www.cambridgemaths.org/blogs/geometry-research-russian-dolls/

Answer: |D| = 8

https://en.wikipedia.org/wiki/Matryoshka_doll

Answer: |D| = 5

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Subsets

Subset definition:

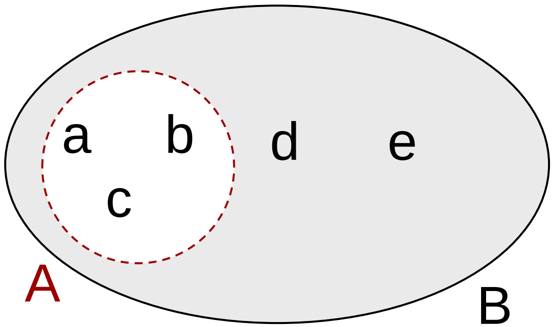
- Set A is a **subset** of set B if every element of set A is also an element of B

- Written: A $\subseteq$ B
- A is subset of B, or A is included in B
- The two sets may also be equal: A = B

Subsets

- A is a subset of B or $A \subseteq B$ if **for every *x*,**
  - **if $x \in A$, then $x \in B$**

- Example: if **A = {a, b, c}** and **B= {a, b, c, d, e}**
  - $a \in A$ and also $a \in B$
  - $b \in A$ and also $b \in B$
  - $c \in A$ and also $c \in B$
  - so all elements in A are also in B
  - so $\mathbf{A \subseteq B}$



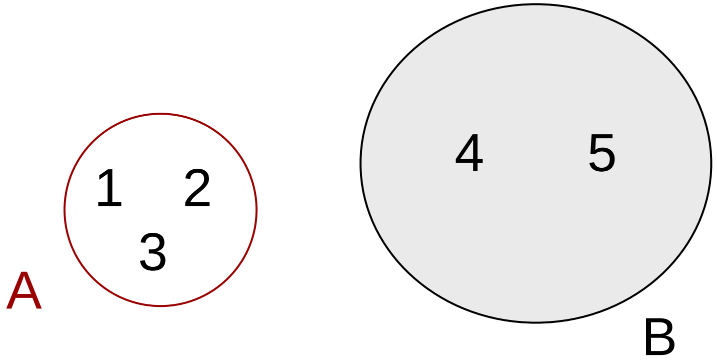
Subsets

- A is **not a subset** of B if there is at least one element in A which is not in B
- Written: A $\not\subseteq$ B

Example: if ***A* = {*1, 2, 3*}** and ***B* = {*4, 5*}**

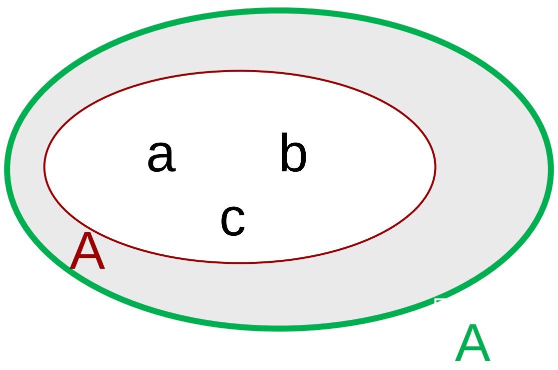
– at least one element in A, i.e.,1 $\notin$ B

– so A $\not\subseteq$ B



Subsets

- Let A= {a, b, c}
- Is every element of A, also an element of A?
- Yes:
  - a $\in$ A, and also a $\in$ A
  - b $\in$ A, and also b $\in$ A
  - c $\in$ A, and also c $\in$ A
- Therefore A is a subset of A; A $\subseteq$ A
- This does not seem very *proper*, as we want our subsets to be *proper.*



Proper Subsets

What is {a, b, c} with respect to {a, b, c, d, e}?

- {a, b, c} is a subset: {a, b, c} $\subseteq$ {a, b, c, d, e}
- But {a, b, c} is a also a proper subset because d, and e elements are only in the second set:

  Write: {a, b, c} $\subset$ {a, b, c, **d, e**}

What is {a, b, c, d, e} with respect to {b, c, d, e, a}?

- {a, b, c, d, e} $\subseteq$ {b, c, d, e, a}

  {a, b, c, d, e} is a subset but not a proper subset
- {a, b, c, d, e} = {b, c, d, e, a}

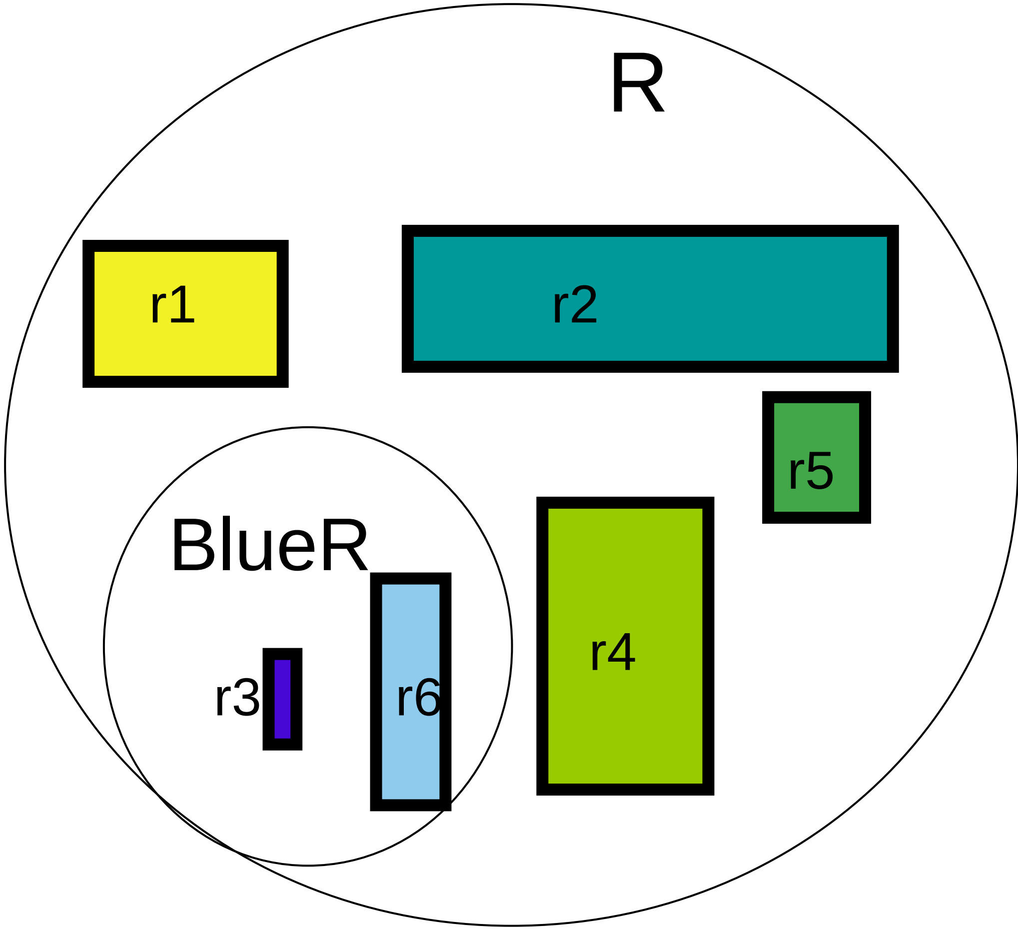
  {a, b, c, d, e} is equal set to {b, c, d, e, a}

Proper Subsets Example

- R – rectangle set
- BlueR – blue rectangle set

BlueR is a subset of
BlueR is contained in R
Write: BlueR $\subseteq$ R

R contains elements that are
not contained in BlueR:
BlueR is a proper subset of R
Write: BlueR $\subset$ R



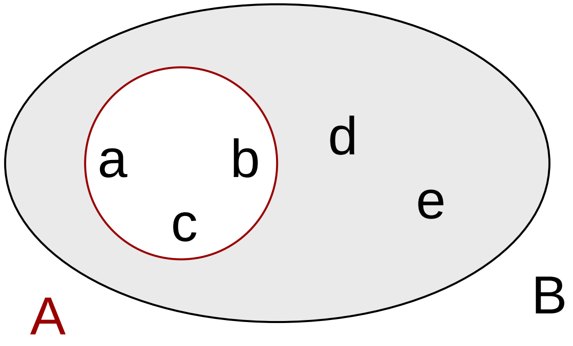
Proper Subsets

- A is a **proper subset** of B if
  - every element in A is also in B, and
  - there is at least one element in B that is not in A
- Written: $A \subset B$
- Note: for proper subset $A \neq B$

Example: A = {a, b, c} and B = {a, b, c, d, e}

- $a \in A$, and also $a \in B$
- $b \in A$, and also $b \in B$
- $c \in A$, and also $c \in B$
- there is also $d, e \in B$ but $\notin A$

- so: $A \subset B$



Proper Subsets

- C = {1, 2} and D = {1, 2, 3}
- The set C is a proper subset of D because
  - C is a subset of D
  - D contains at least one element which is not contained in C
- We write: $C \subset D$

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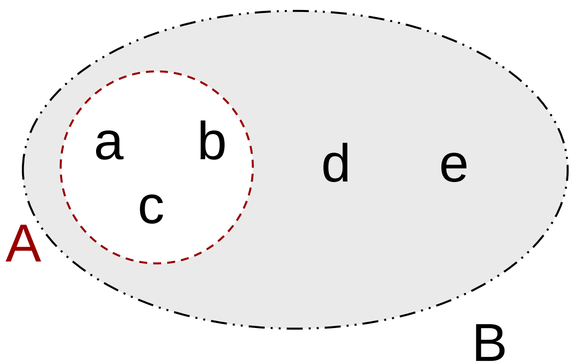
Supersets

- Set B is a **superset** of set A if every element of A is also an element of B.
- Written: B $\supseteq$ A
- If B is a **superset** of A, or B $\supseteq$ A

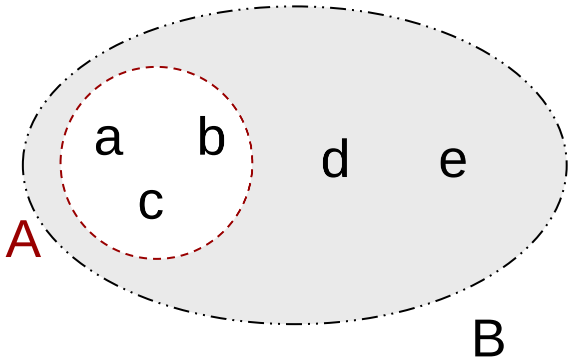
  then A is a subset of B, or A $\subseteq$ B



Proper Supersets

- Set B is a **proper superset** of set A if there is a at least one element in set B which is not in set A
- Written B ⊃ A
- If B is a **proper superset** of A

  then A is a proper subset of B, or A ⊂ B



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Universal Sets

- Universal set - a non-empty set of all of the possible elements (including those of all subsets), relevant to the solution of a specific problem.
- Usually denoted by U.

Examples:

- Set of natural numbers N if we are counting objects
- Set of alphabet letters if we are spelling words
- U = {red, orange, yellow, green, blue, indigo, violet}

Universal Sets

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Complement Sets

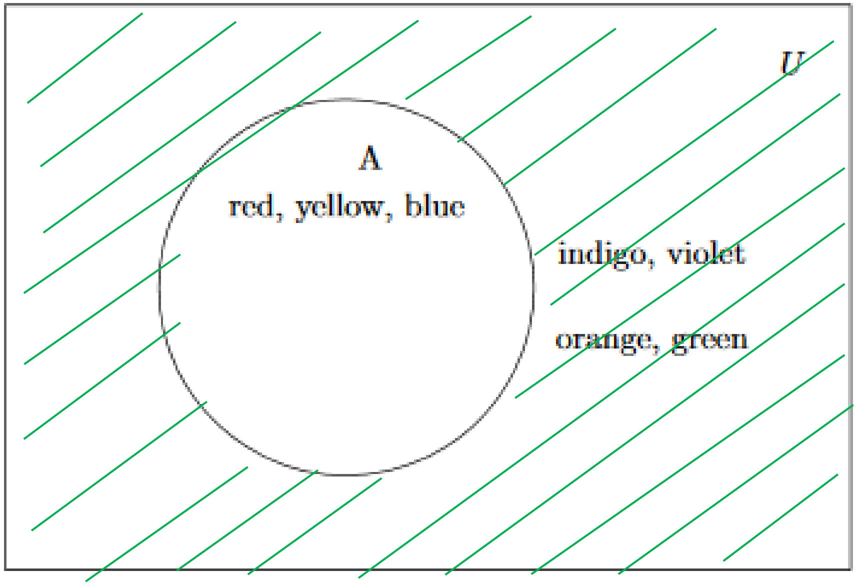
The **complement set** is the difference between the universe and a given set

- Denoted: comp(A) = U - A

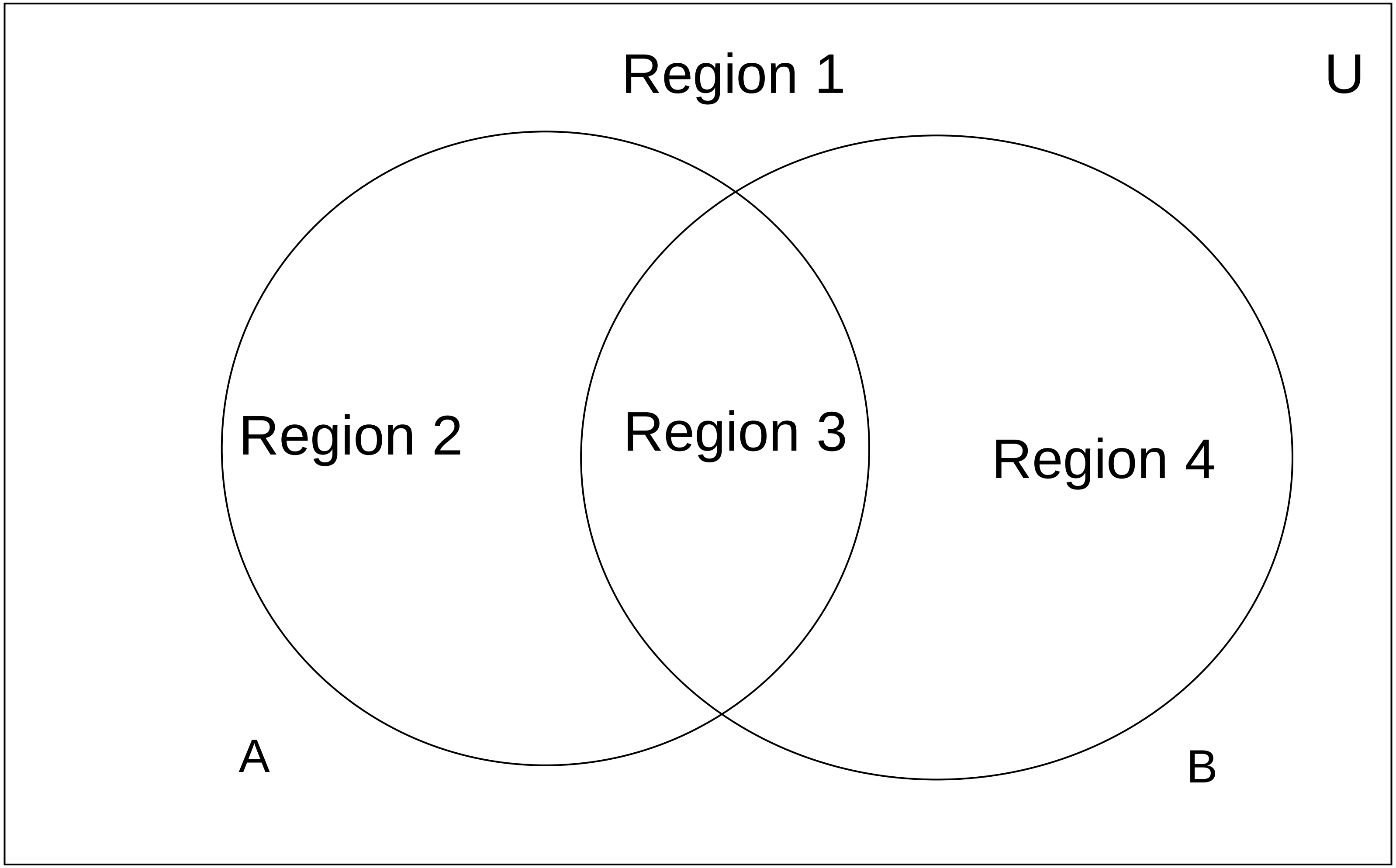
Example: U = {red, orange, yellow, green, blue, indigo, violet}

A = {red, yellow, blue}, and

comp(A) = {orange, green, indigo, violet}



Sets Exercise



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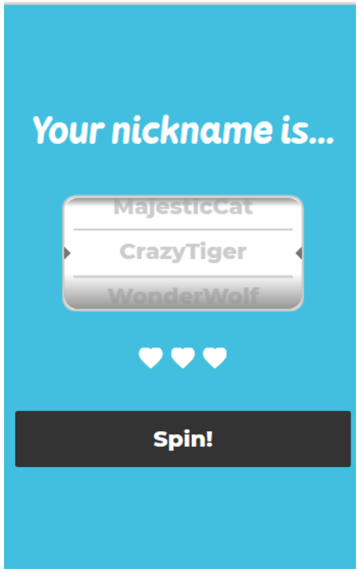

Summary: Types of Sets

Symbol	Symbol name	Meaning
∅	empty set	set with no elements
	disjoint sets	sets whose intersection is the empty set
$A = B$	equal sets	sets with the same elements
$A \neq B$	not equal sets	sets which do not have the same elements
$\lvert A \rvert$	set cardinality	number of elements in a set A

Summary: Types of Sets

Symbol	Symbol name	Meaning
$A \subseteq B$	subset	elements of set A are also in set B
$A \subset B$	proper subset	A is subset and there is at least one element in set B that is not in set A
$B \supseteq A$	superset	elements of set A are also in set B
$B \supset A$	proper superset	B is superset and there is at least one element in set B that is not in set A
U	universal set	set of all of the possible elements relevant to a specific problem
comp(A)	complement set	the difference between the universe and a given set A