SCC121

Fundamentals of Computer Science

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Discrete Maths

WHAT

- discrete objects

Discrete Maths

WHAT

- discrete objects

WHY

- foundation for formal methods:
  - mathematical approaches to software and hardware
  - software engineering and software testing

Overview

- Sets
  - Defining sets
  - Set operations
  - Types of sets

Objectives

- Objectives
  - Understanding the basic ideas about sets
  - Facility with set operations

Sets

- Set – a collection of objects/elements/members
  - in a set there are **no duplicates**
  - a set is **unordered**
- Conventions:
  - Set’s name – upper case (single) letter
  - Set’s elements or members – lower case (single) letters

- Example set:
  - $A = \{1, 2, 3, 4, 5, 6, 7\}$
  - $A = \{7, 6, 5, 4, 3, 2, 1\}$

Membership

- Set membership: “is an element of” or “belongs to”
- Set membership notation: symbol $\in$ (from Greek letter ε)
- We write 1 is an element of set A: $1 \in A$
  - 1 is an element/object/member of the set A
  - 1 belongs to the set A
  - $A = \{1, 2, 3, 4, 5, 6, 7\}$

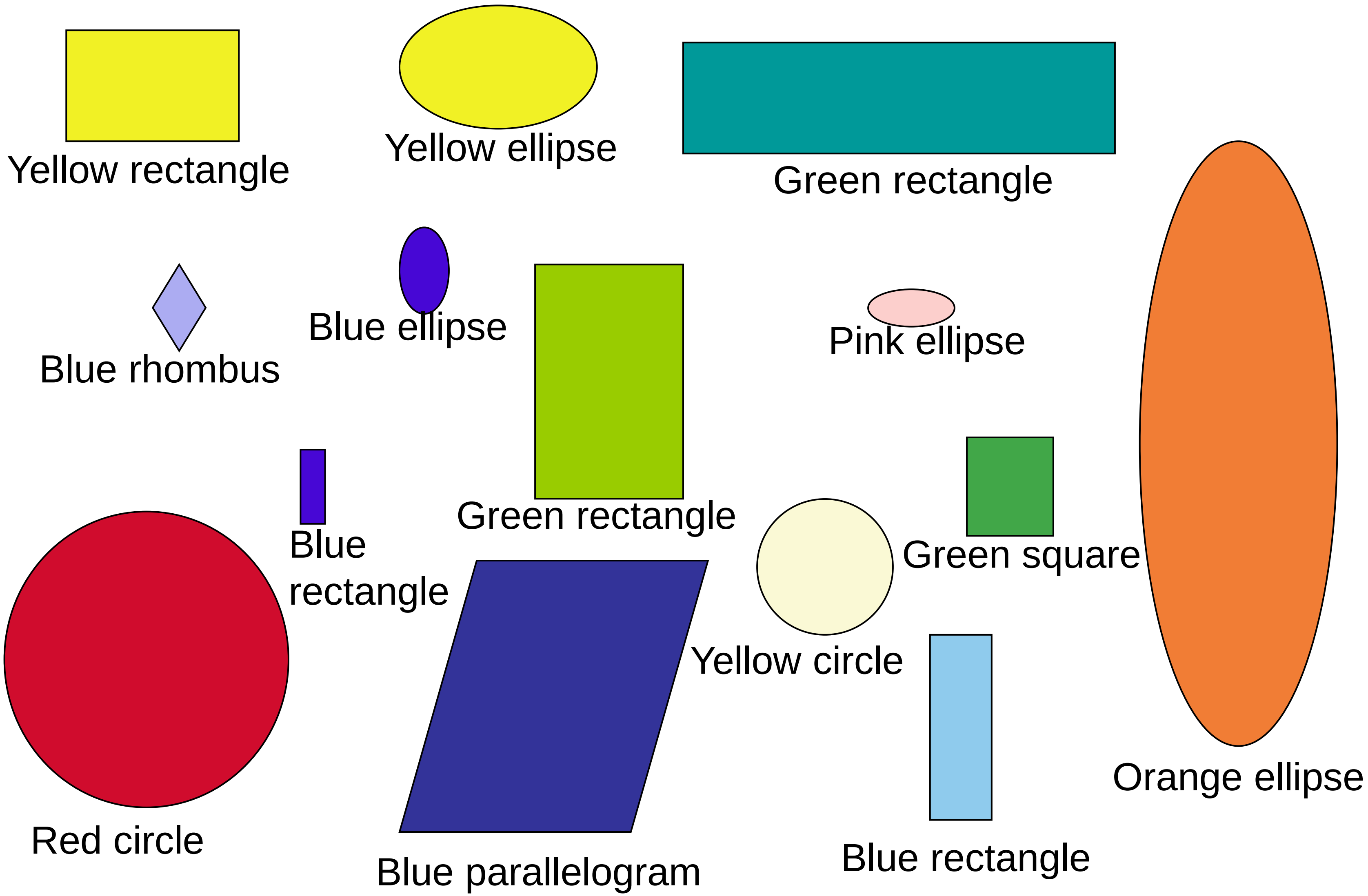
Membership

- Non membership: “is not an element of” or “does not belong to”
- Non membership notation: symbol $\notin$
- We write 1 is not an element of set B: $1 \notin B$
  - 1 does not belong to set B
  - $B = \{2, 3, 4, 5, 6, 7\}$

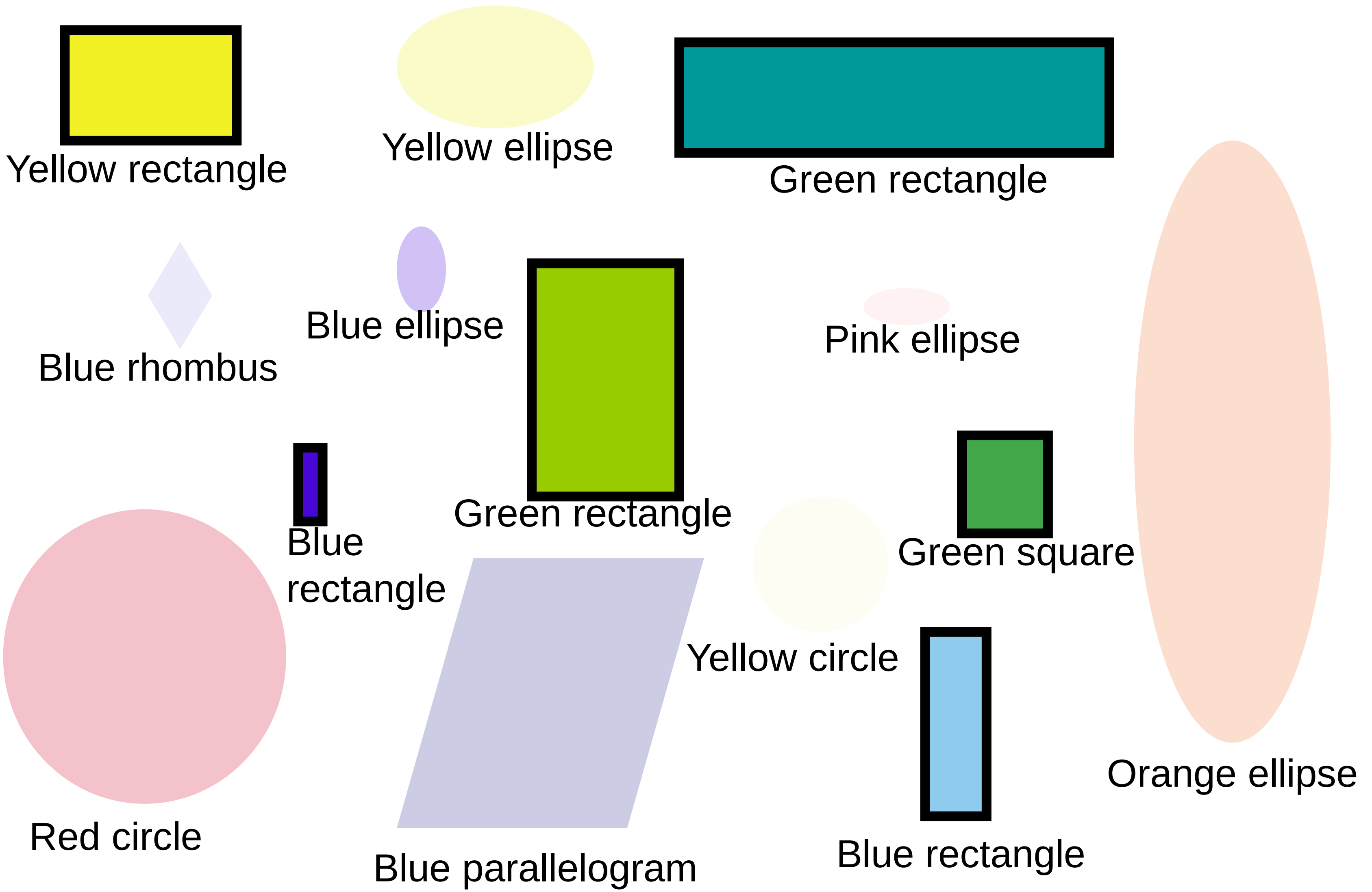
Summary: Set Notations

Symbol	**Symbol name**	**Meaning**
{1, 2}	set	collection of elements
$1 \in A$	is an element of	set membership
$3 \notin A$	is not an element of	non set membership

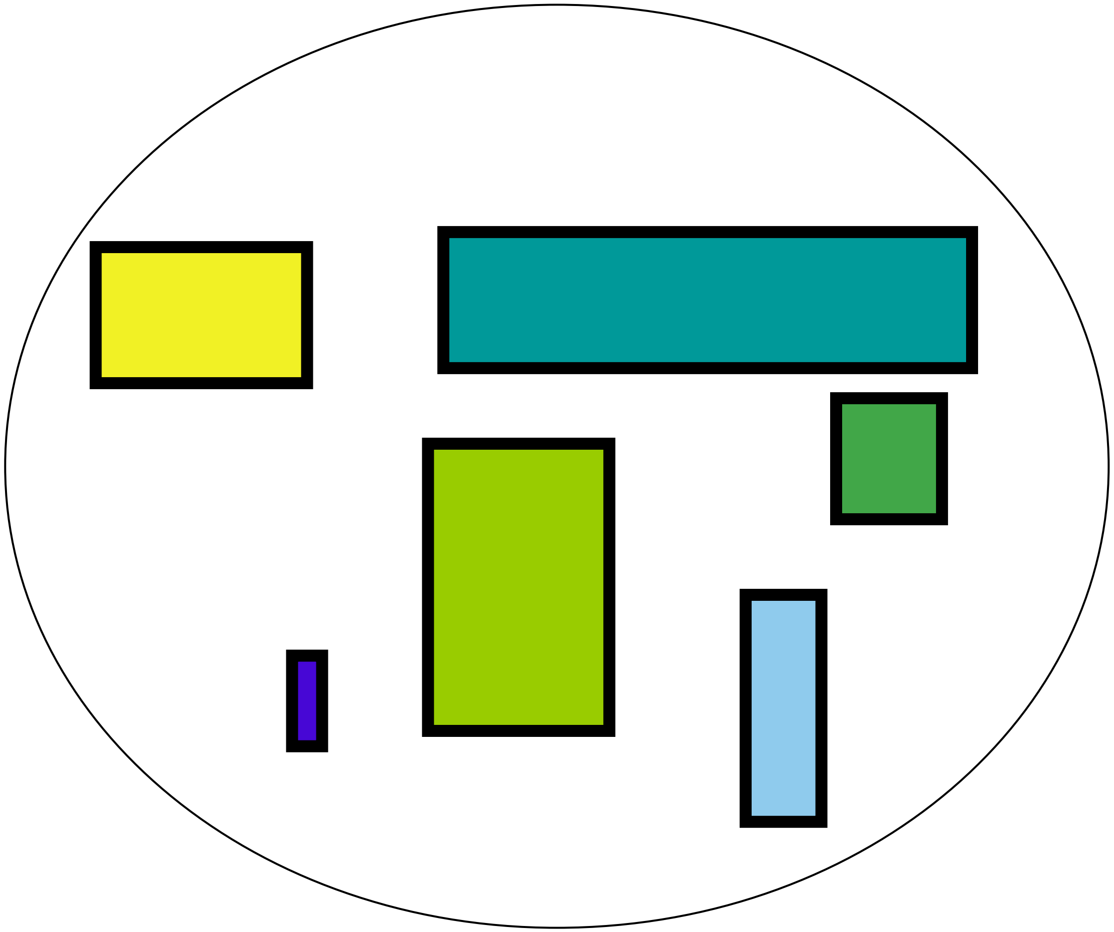
Geometric Figures



Geometric Figures: Rectangles

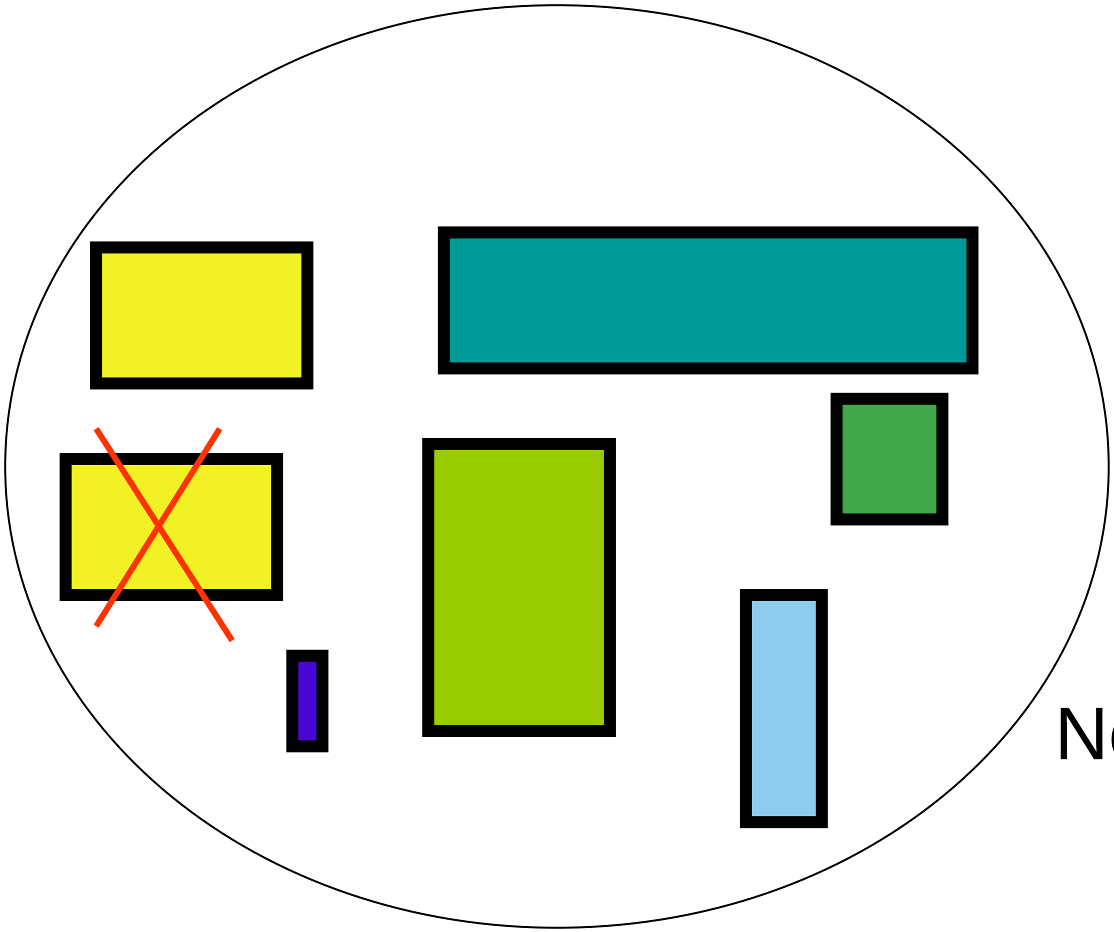


Rectangle Set



Rectangle Set

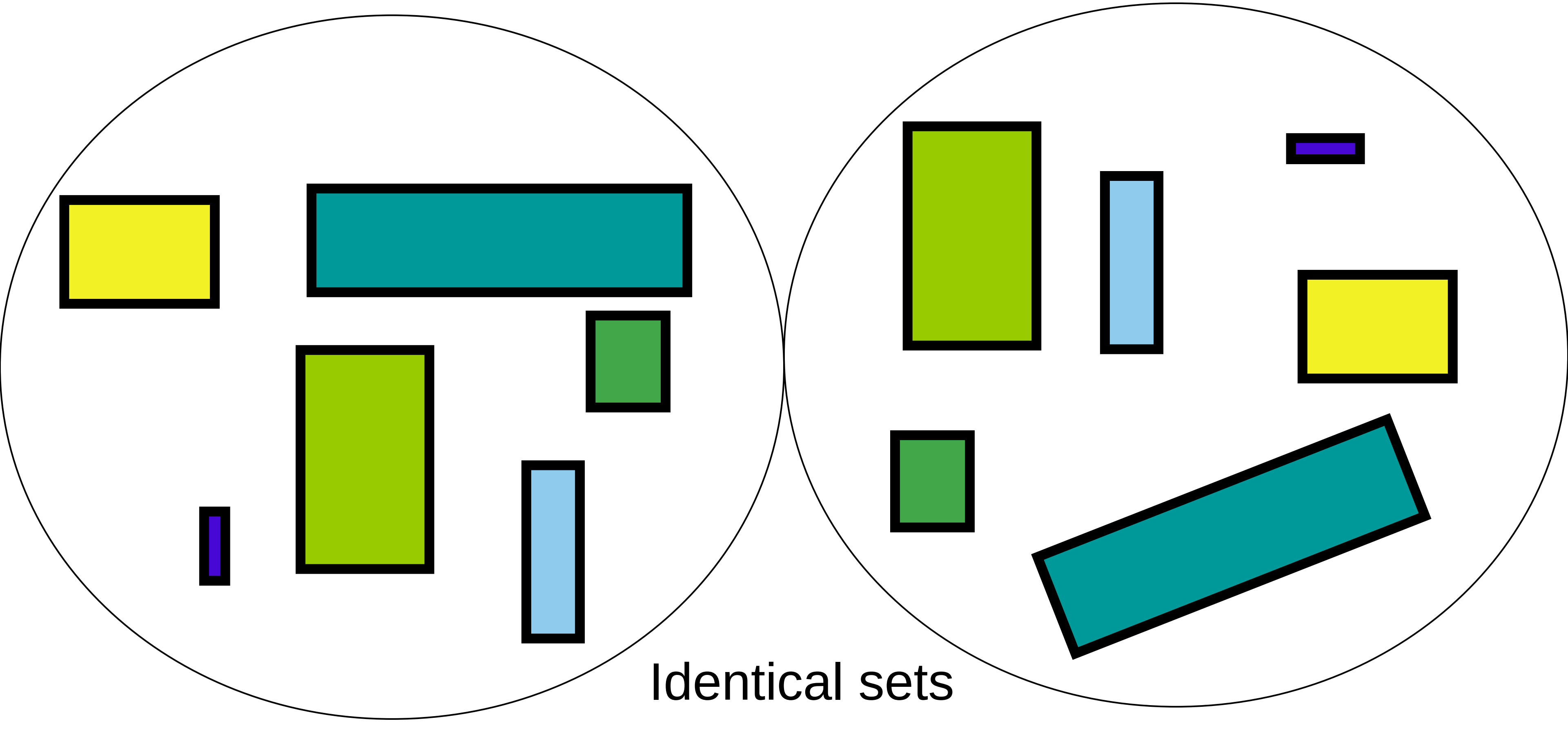
Is this a set?



No duplicates

Rectangle Set

Unordered



Identical sets

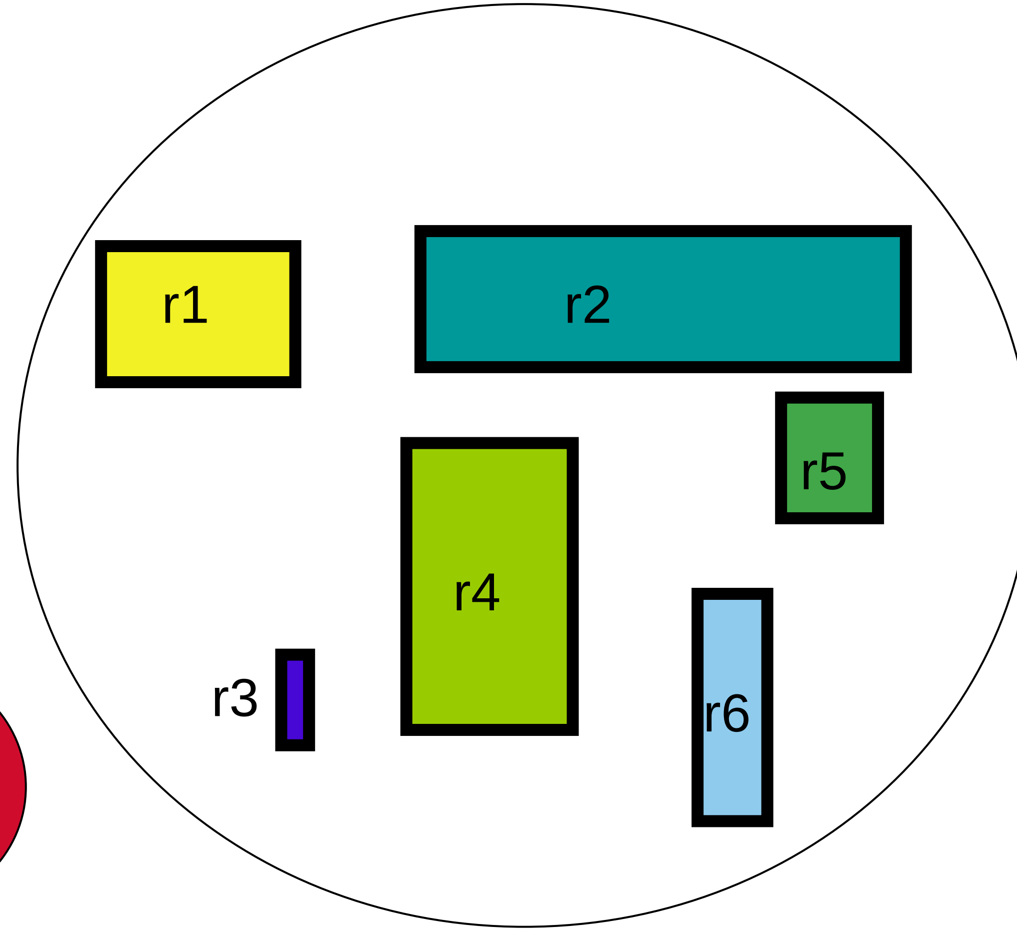
Rectangle Set

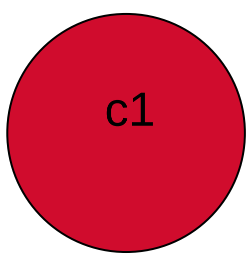
R = {r1, r2, r3, r4, r5, r6}

No duplicates:
R = {r1, ~~r1~~, r2, r3, r4, r5, r6}

Unordered:
R = {r5, r3, r4, r2, r6, r1}

Belongs to:
r1 $\in$ R (rectangle r1 belongs to Set R)
c1 $\notin$ R (circle c1 is not in Set R)





Defining Sets

- Enumerating or listing all its members
  - writing down all the elements
  - finite, small sets: A = {1, 2, 3, 4, 5, 6, 7}
  - infinite sets – impossible to enumerate!

Defining Sets

- Infinite or large sets cannot be enumerated.
- Providing a property that all its members must satisfy

Example:

- Example: A = {1, 2, 3, 4, 5, 6, 7, …. } and so on to infinity
- What property can define this set, or is shared by all its members?
- Every object x **such that** x is an integer and is greater than 0
- A = {x | x is an integer and x > 0}

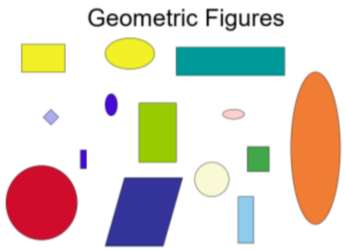
- We read: A is the set of objects x such that x has property P
- **Notation**: {x | P(x)}

Defining Sets

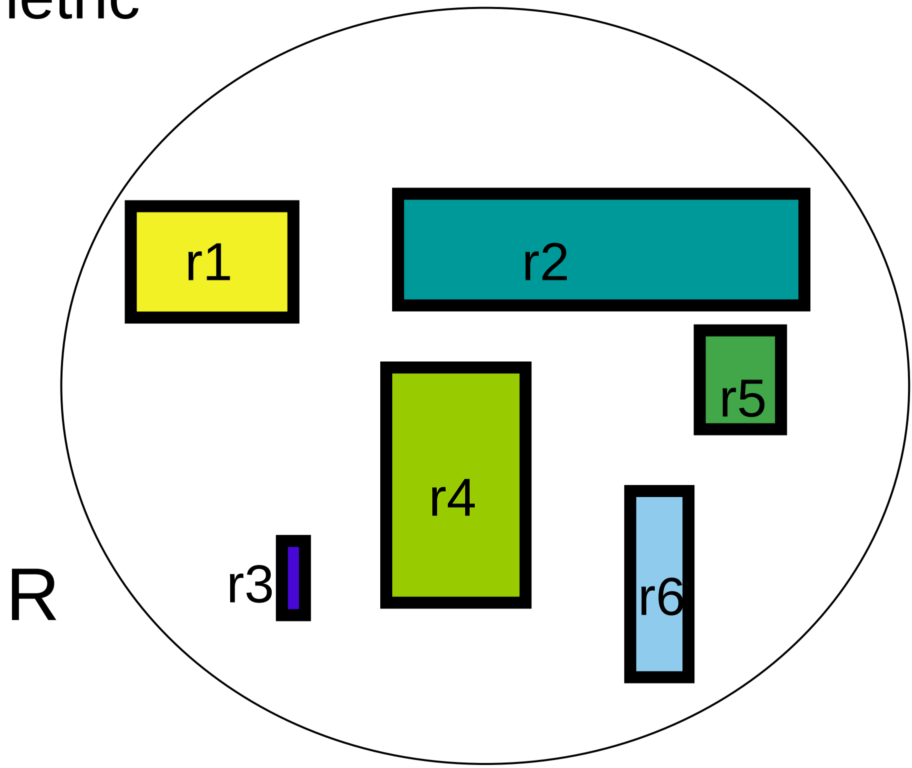
- Properties can also be used for finite sets:
  - How can we define set A = {1, 2, 3, 4, 5, 6, 7} ?
  - every object x such as x is an integer, greater than 0 and smaller than 8
  - {x I x is an integer and $0 < x < 8$}
  - every object x **such that** x is an integer and is greater than 0 and less than 8

Defining Rectangle Set

- Listing all its members (enumeration)
  R = {r1, r2, r3, r4, r5, r6}
- Property
  All the rectangles from the geometric figures drawn before.



R = {objects with the property of being geometric figures which are rectangles}



Set Operations

- Union
- Intersection
- Difference
- Cartesian product

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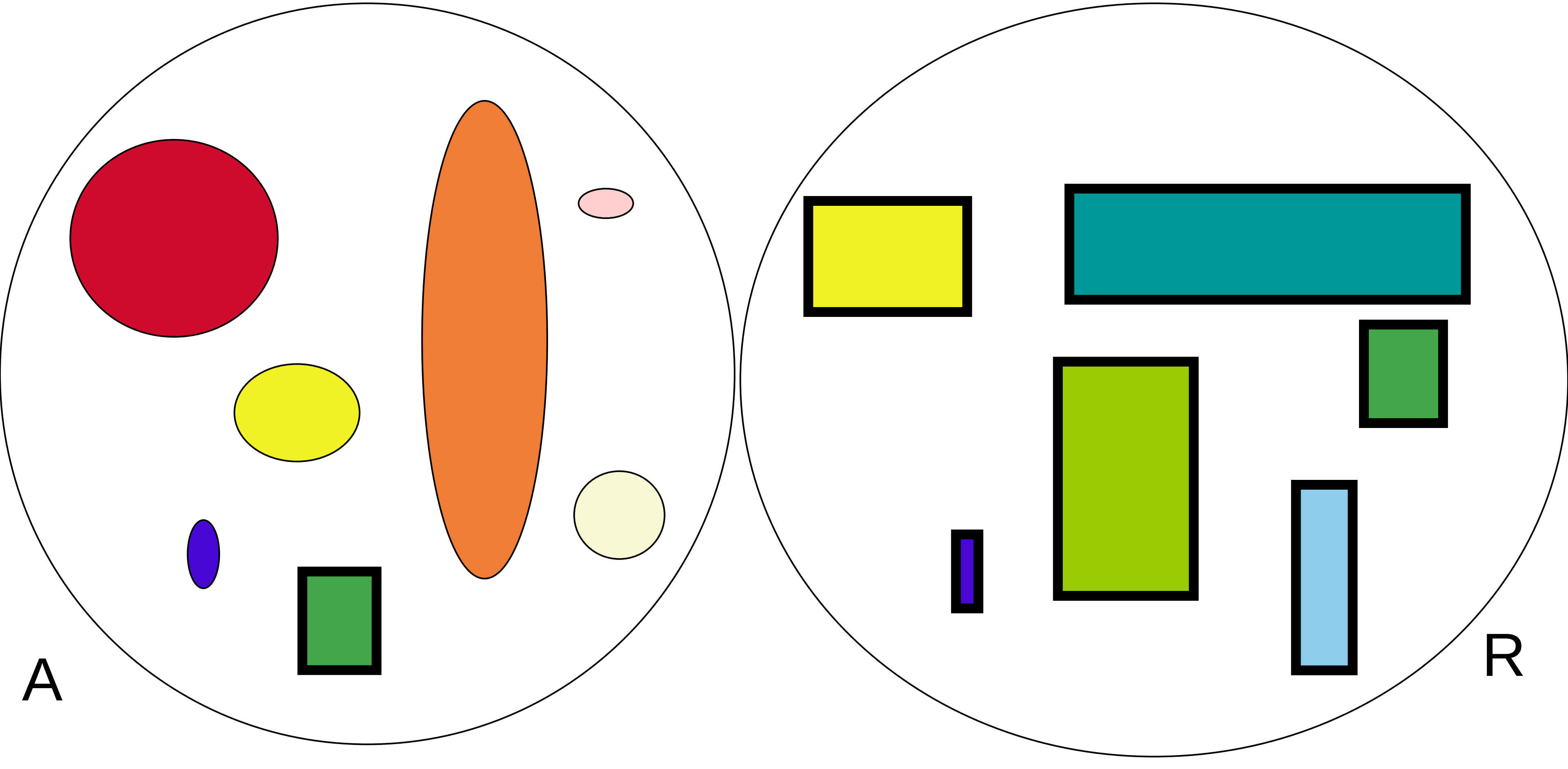
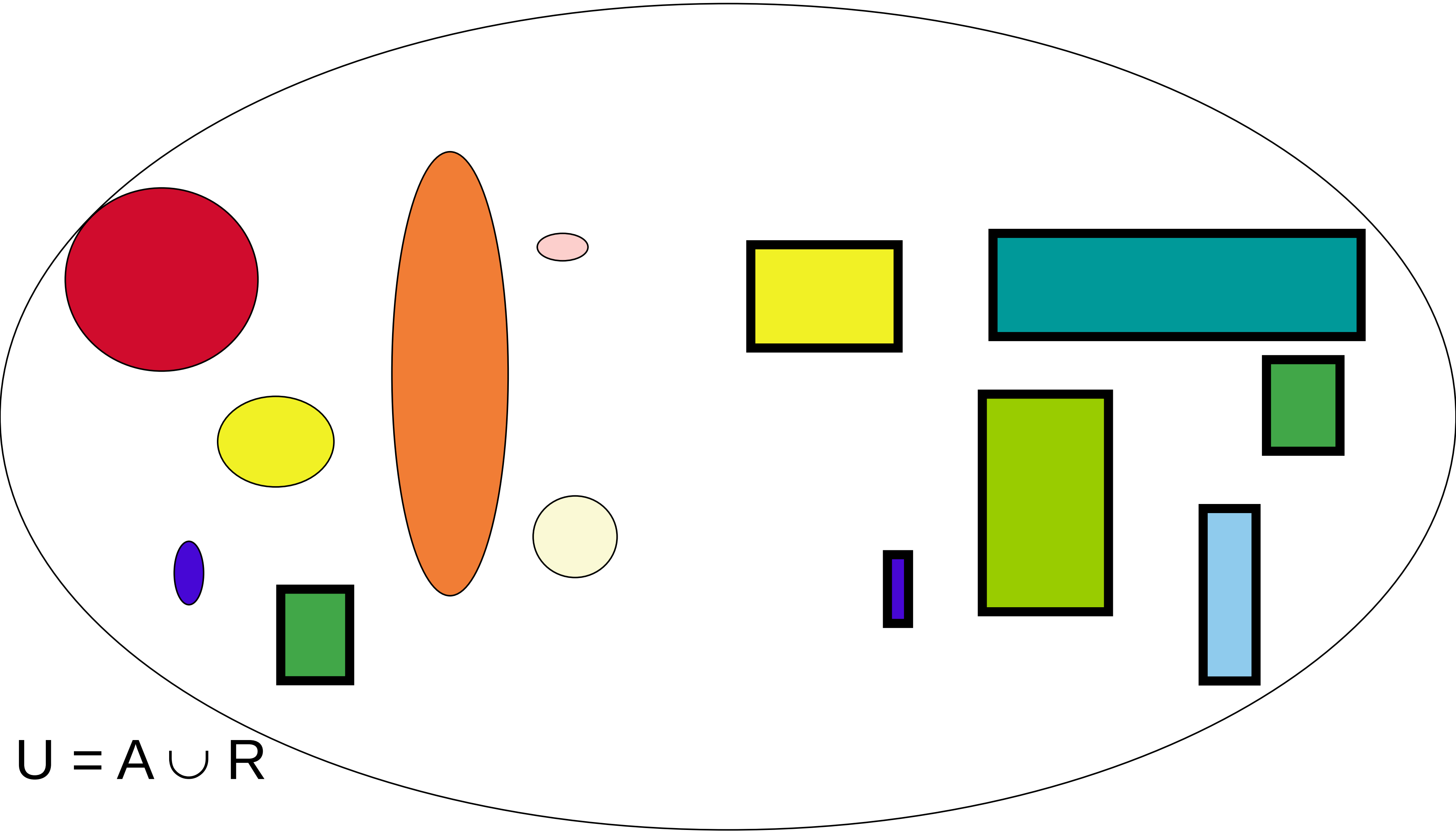
Set Operations: Union (∪)

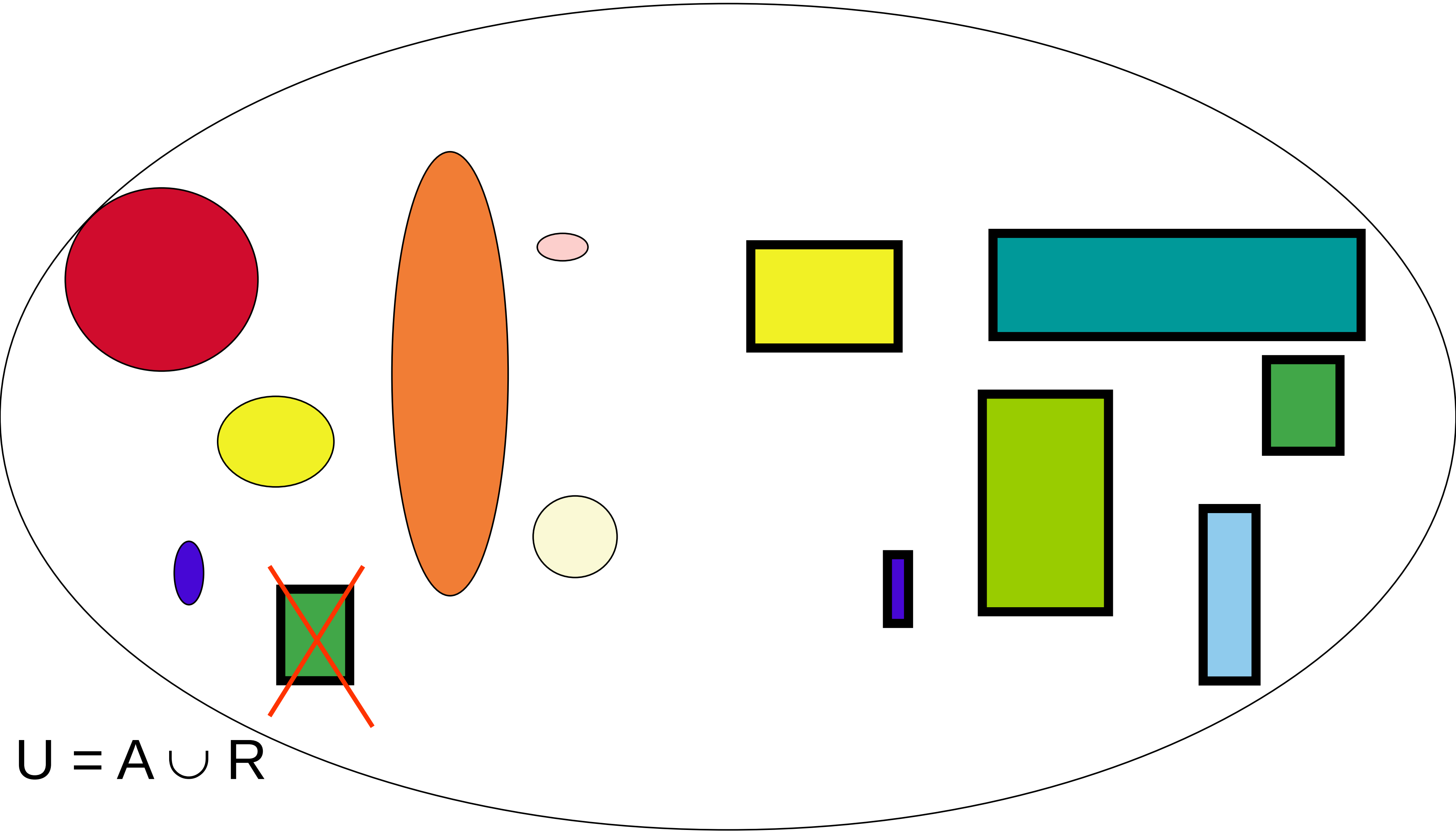


Building a Union (∪)

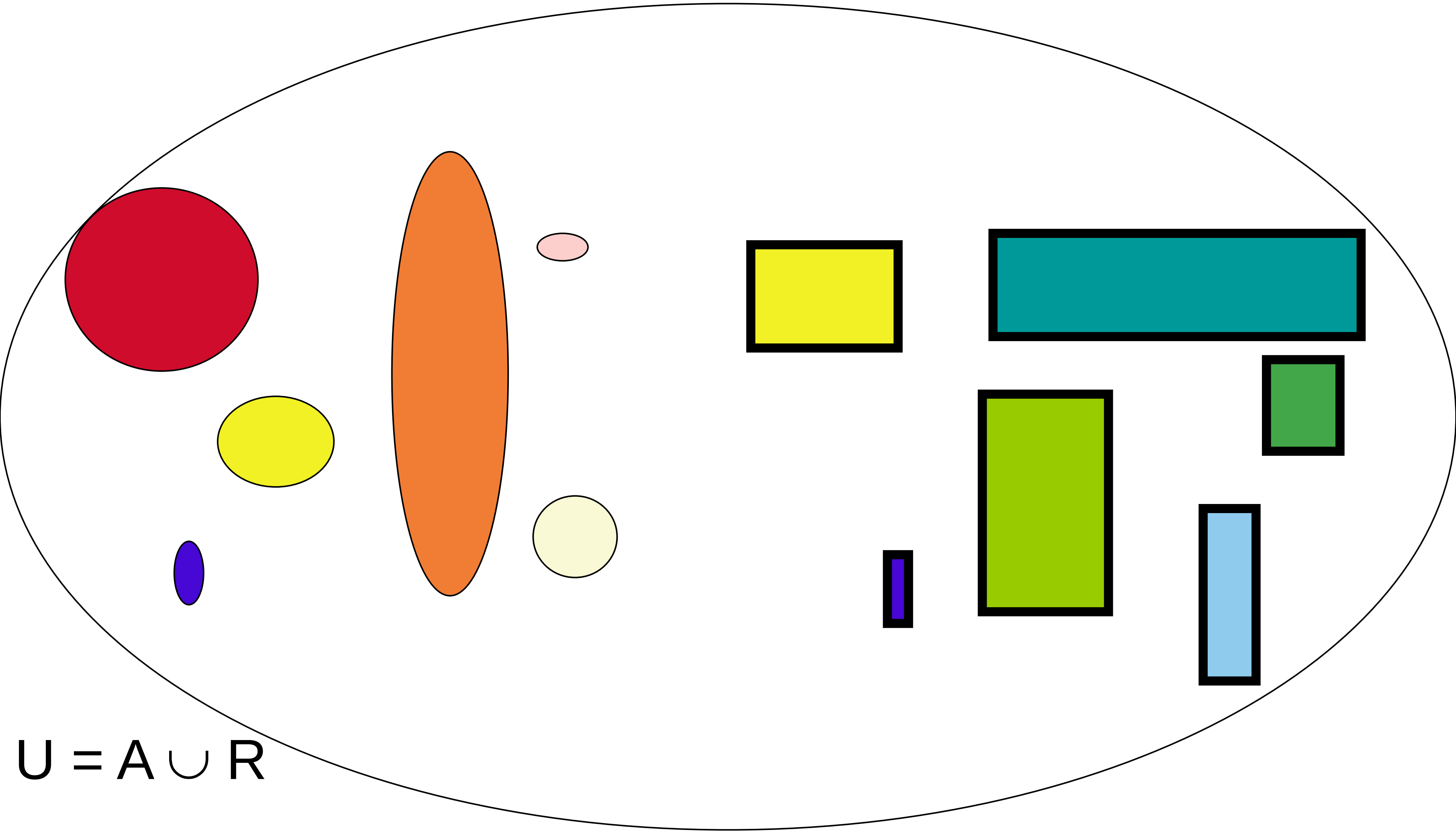


$U = A \cup R$

No duplicates!



Union (∪) Result



Set Operations

UNION (written $\cup$)

- forms a new set from two sets consisting of all elements that are in EITHER of the original sets (or both)
- $A \cup B = \{ x \mid x \in A \text{ or } x \in B \}$

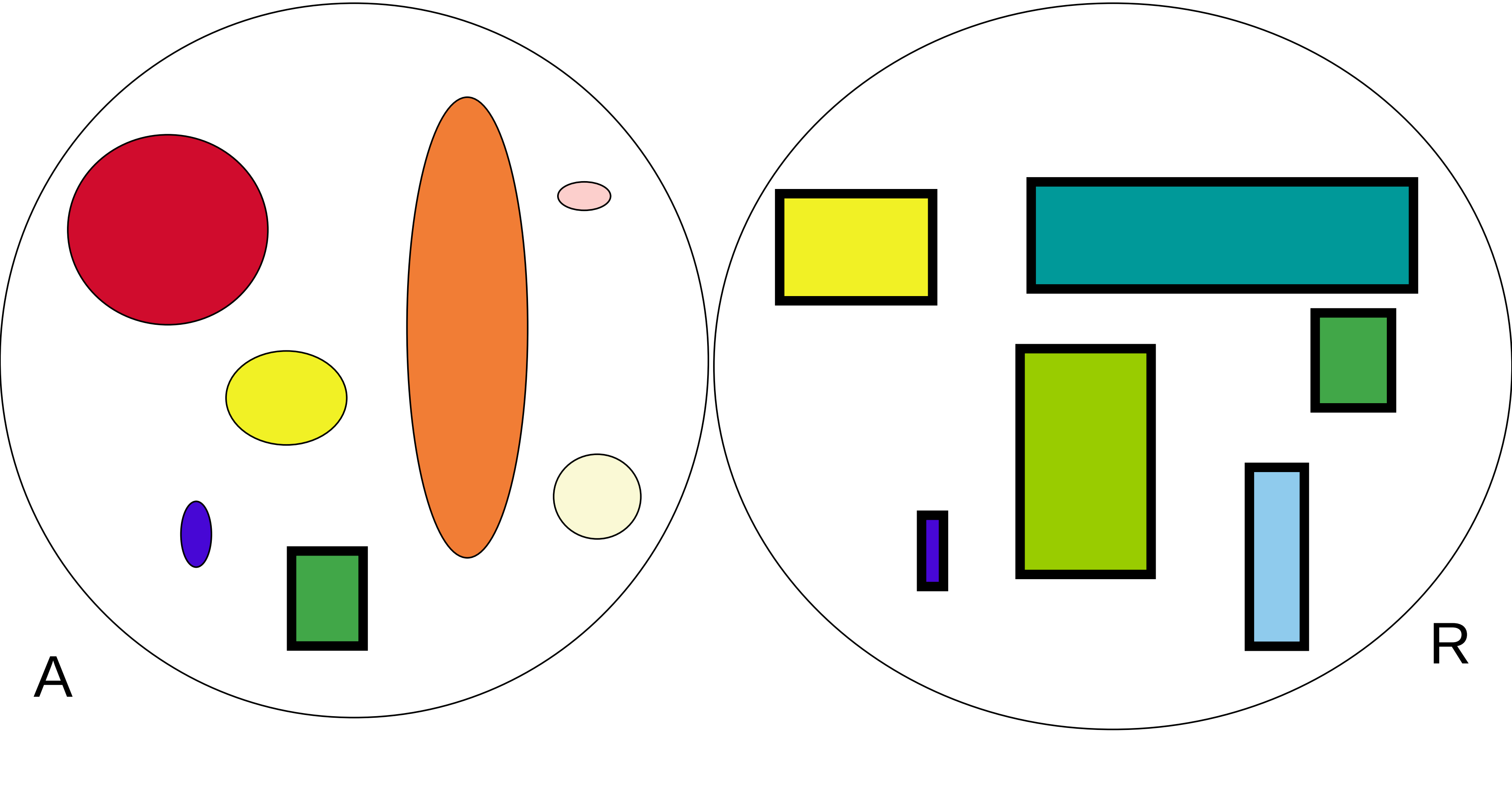
- Examples
  - If $A = \{1, 2, 3\}$ and $B = \{4, 5\}$
  - $A \cup B = \{1, 2, 3, 4, 5\}$

  - If $A = \{1, 2, 3\}$ and $B = \{1, 2, 4, 5\}$
  - $A \cup B = \{1, 2, 3, 4, 5\}$

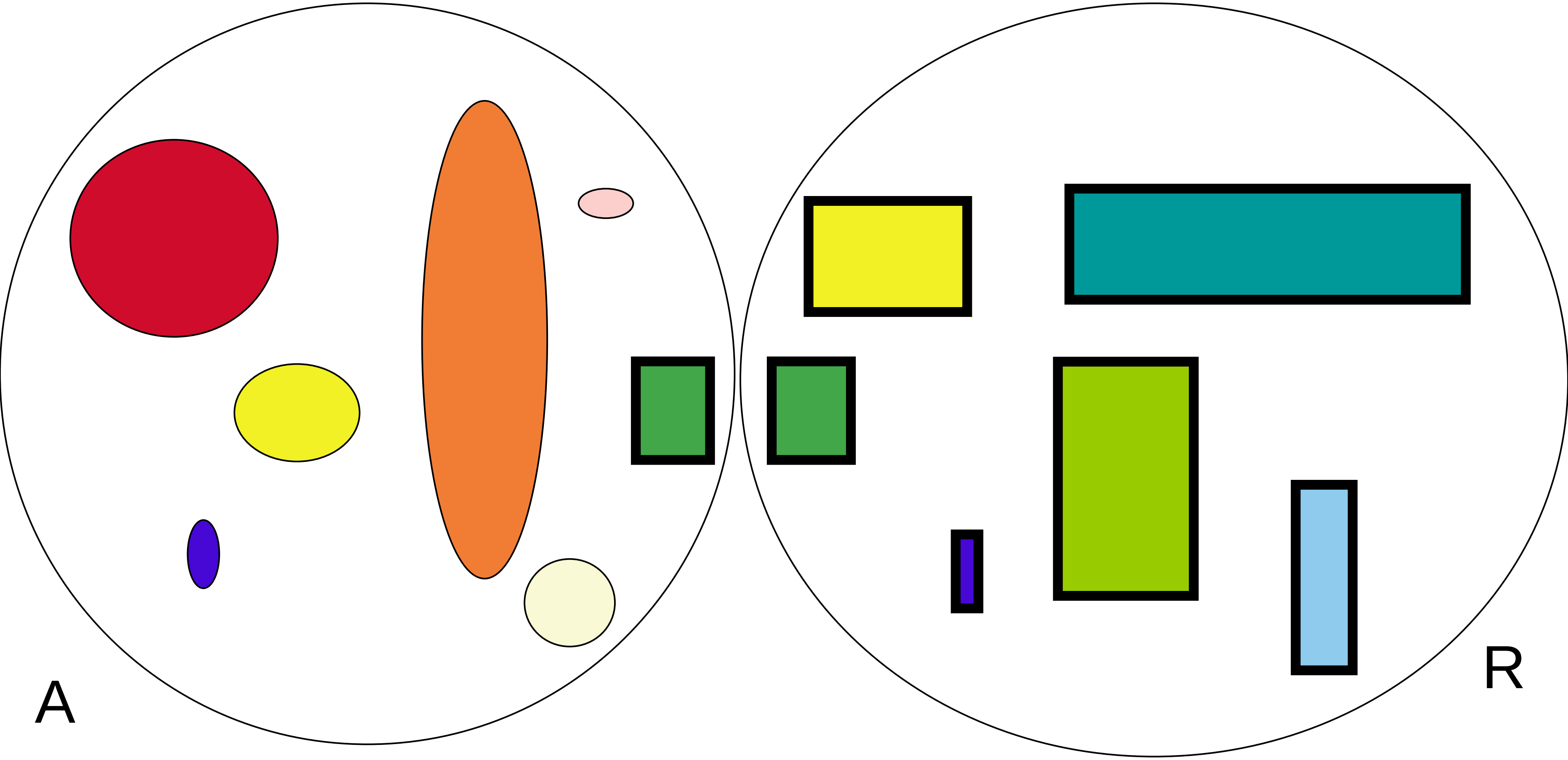
Set Operations

- Union
- Intersection
- Difference
- Cartesian product

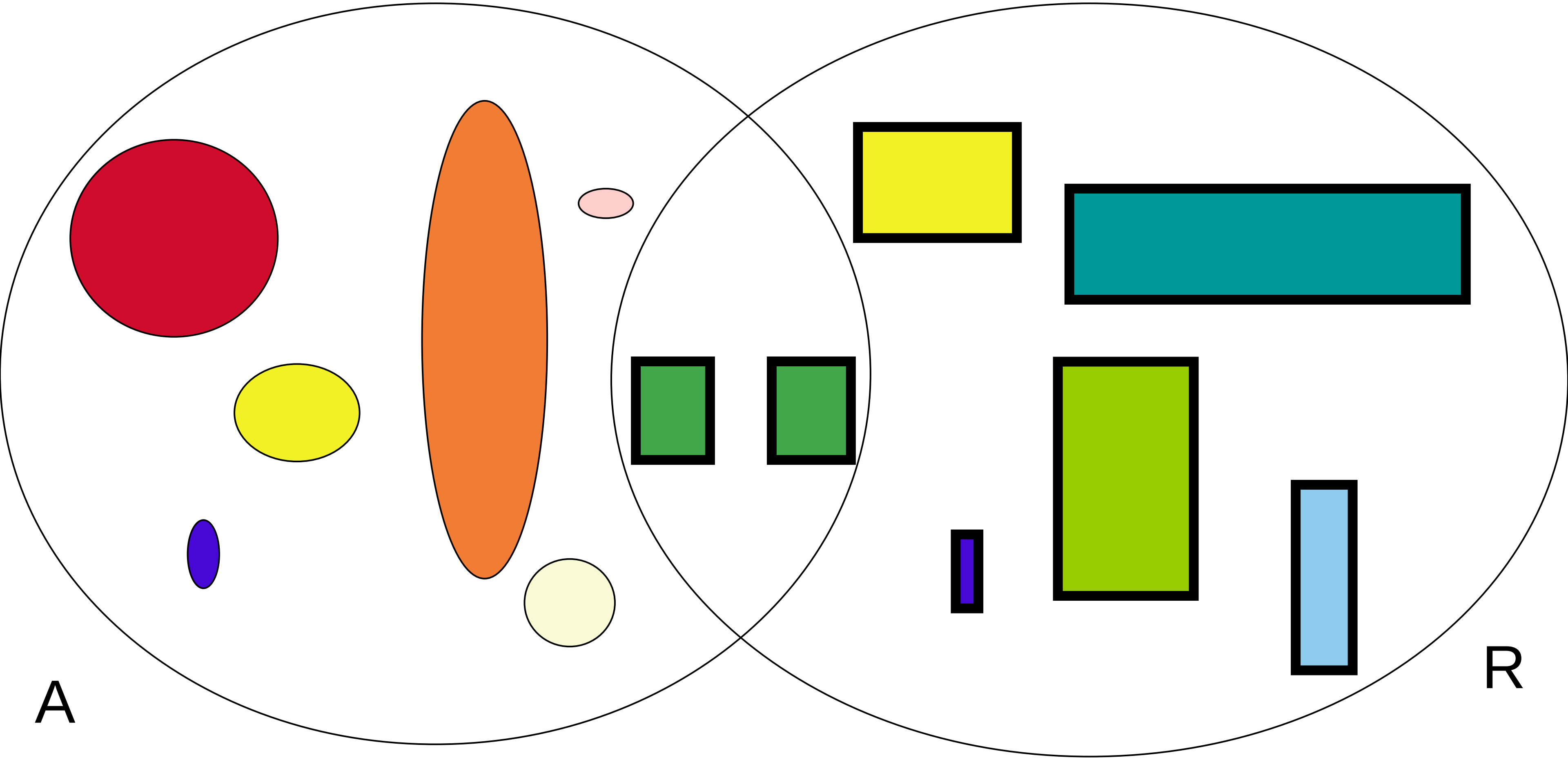
Set Operations: Intersection (∩)



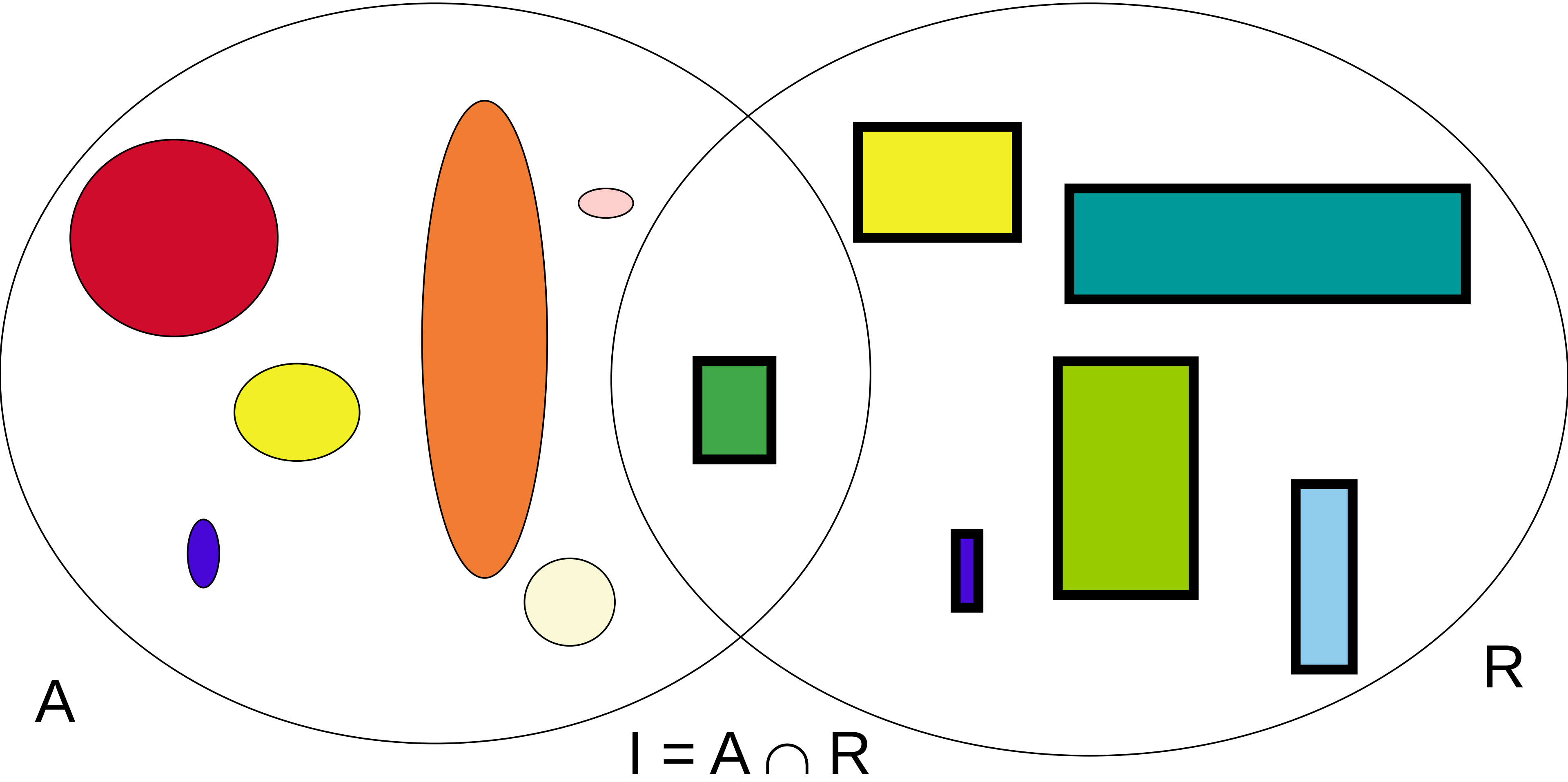
Building an Intersection (∩)



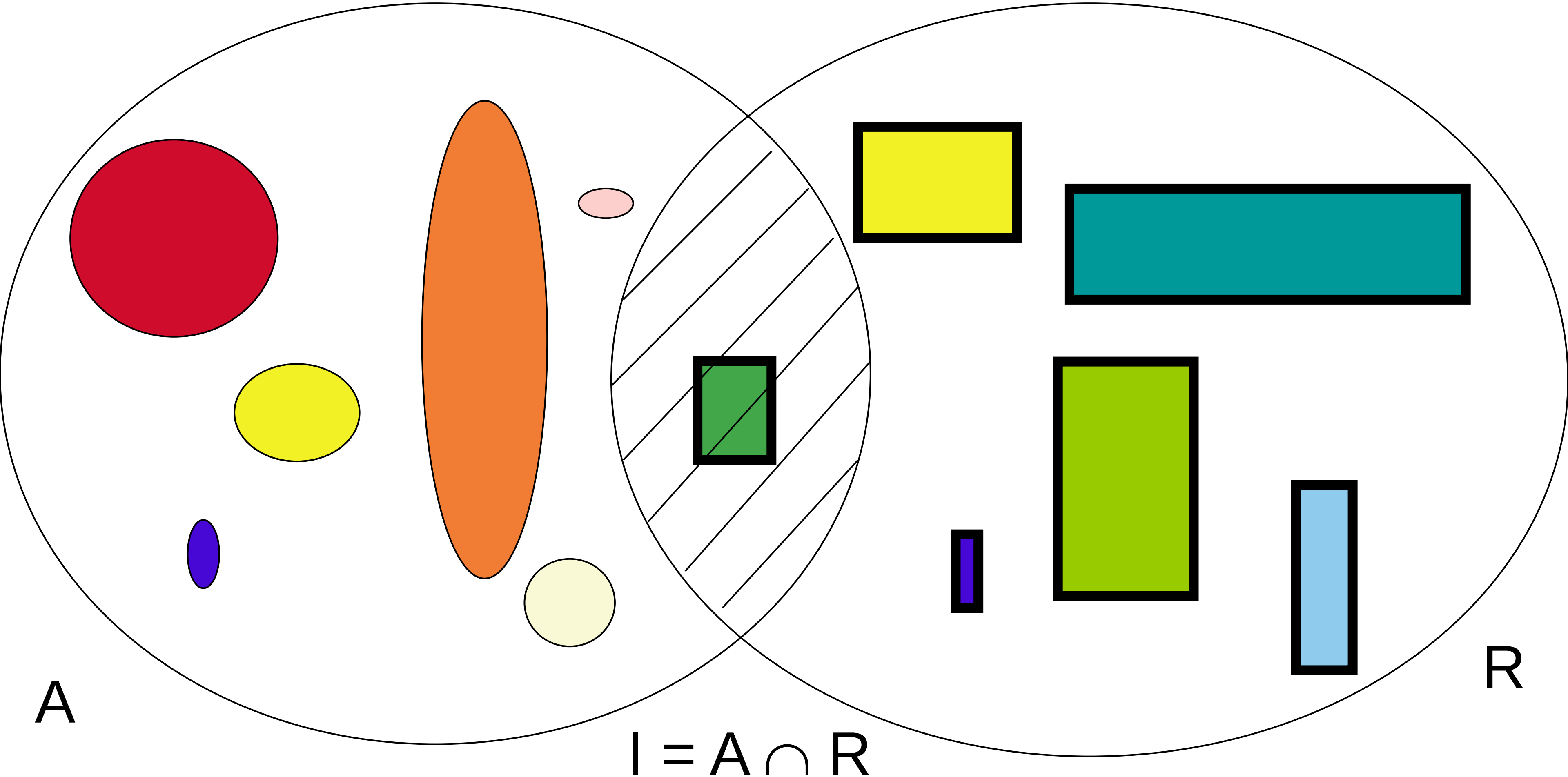
Building an Intersection (∩)



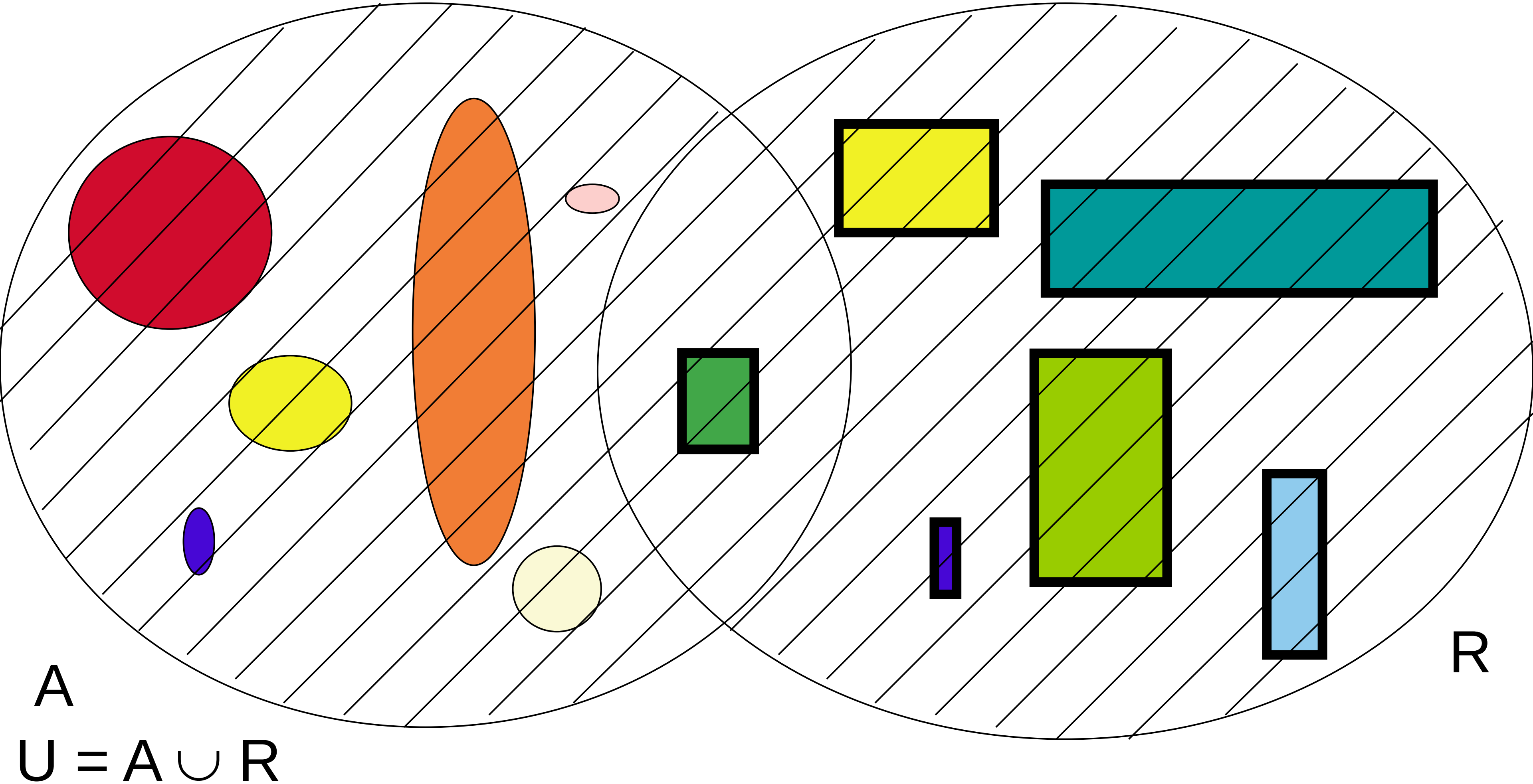
Intersection (∩) Result



Set Operations: Intersection (∩)



How will the Union look like?



Set Operations

INTERSECTION (written $\cap$)

- forms a new set from two sets, consisting of all elements that are in BOTH of the original sets
- $A \cap B = \{\, x \mid x \in A$ **and** $x \in B \,\}$

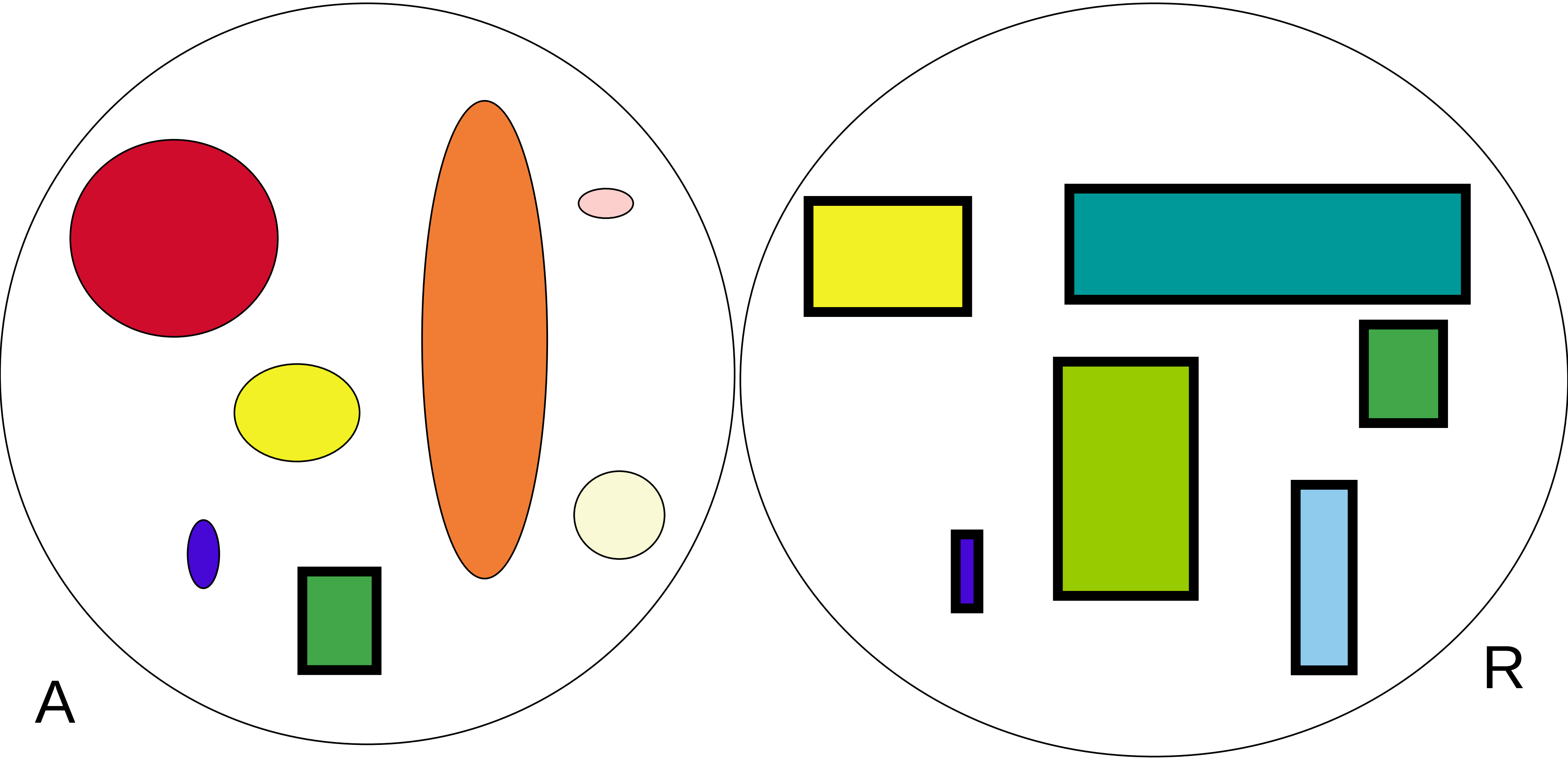
- Examples
  – If $A = \{1, 2, 3\}$ and $B = \{1, 2, 4, 5\}$
  – $A \cap B = \{1, 2\}$

  – If $A = \{1, 2, 3\}$ and $B = \{4, 5\}$
  – $A \cap B = ?$

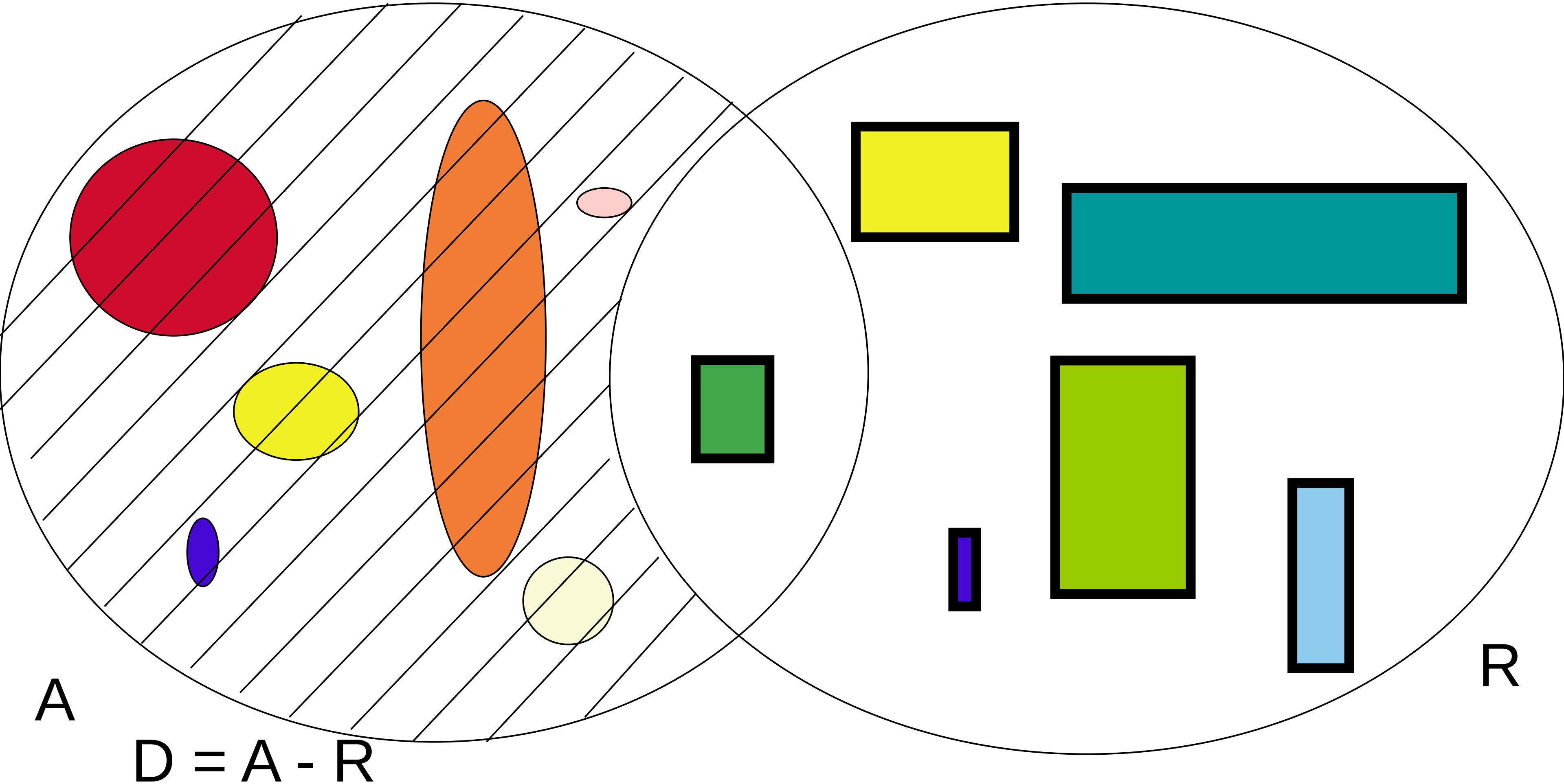
Set Operations

- Union
- Intersection
- Difference
- Cartesian product

Set Operations: Difference (-)



Set Operations: Difference (-)



Set Operations

DIFFERENCE (written –)

- forms a new set from two sets, consisting of all elements from the first set that are not in the second
- $A - B = \{ x \mid x \in A$ **and** $x \notin B \}$

- Examples
  - If $A = \{1, 2, 3\}$ and $B = \{1, 2, 4, 5\}$
  - $A - B = \{3\}$

  - If $A = \{1, 2, 3\}$ and $B = \{4, 5\}$
  - $A - B = \{1, 2, 3\}$

Set Operations

- Union
- Intersection
- Difference
- Cartesian product

Cartesian Product

Ordered pair

- is a pair of objects, with an order associated with them

Convention:

- If objects are represented by *x* and *y*, then we write the ordered pair as <*x*, *y*>

Equality

- Two ordered pairs <*a*, *b*> and <*c*, *d*> are equal if and only if *a* = *c* and *b* = *d*.

Example of ordered pairs

- <*1*, *2*> and <*2*, *1*> are not equal.

Cartesian Product: A x R

- A= {a1, a2, a3, a4, a5, a6, a7}
- R = {r1, r2, r3, r4, r5, r6}

Rene Descartes

A x R = { <**a1**, **r1**>, <a1, **r2**>, <a1, **r3**>, <a1, **r4**>, <a1, **r5**>, <a1, **r6**>,
<**a2**, r1>, <a2, r2>, <a2, r3>, <a2, r4>, <a2, r5>, <a2, r6>,
<**a3**, r1>, <a3, r2>, <a3, r3>, <a3, r4>, <a3, r5>, <a3, r6>,
<**a4**, r1>, <a4, r2>, <a4, r3>, <a4, r4>, <a4, r5>, <a4, r6>,
<**a5**, r1>, <a5, r2>, <a5, r3>, <a5, r4>, <a5, r5>, <a5, r6>,
<**a6**, r1>, <a6, r2>, <a6, r3>, <a6, r4>, <a6, r5>, <a6, r6>,
<**a7**, r1>, <a7, r2>, <a7, r3>, <a7, r4>, <a7, r5>, <a7, r6> }

Cartesian product

Cartesian product of *A* and *B*

- The set of all ordered pairs $<a, b>$
  - where *a* is an element of *A* and *b* is an element of *B*
- written $A \times B$.

Example:

$A = \{1, 2, 3\}$ and $B = \{a, b\}$. Then

- $A \times B = \{<1, a>, <1, b>, <2, a>, <2, b>, <3, a>, <3, b>\}$
- $B \times A = \{<a, 1>, <a, 2>, <a, 3>, <b, 1>, <b, 2>, <b, 3>\}$

Compound Set Operations

Compound operations on sets

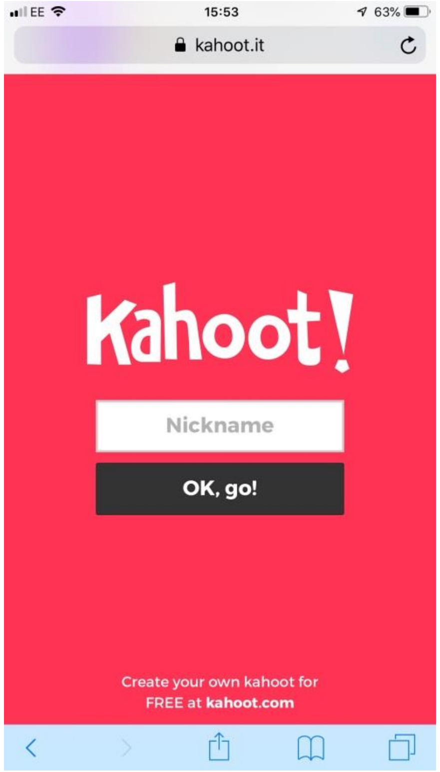
- Union, intersection, difference, Cartesian product are all equal in the order of precedence
- $A \cap B - C$ does not make sense. Which operation is first?
- $(A \cap B) - C$ makes sense
- always do anything in parentheses first

Example: $A = \{a1, a2, x, y\}$, $B = \{b1, x, y\}$, $C = \{y\}$

$(A \cap B) = \{x, y\}$
$(A \cap B) – C = \{x, y\} – \{y\} = \{x\}$

Let’s playxercise!

- https://kahoot.it/
- Game Pin:
- Nickname: Student ID



Summary: Set Operations

Symbol	**Symbol name**	**Meaning**
$A \cup B$	union	objects that belong to set A or set B
$A \cap B$	intersection	objects that belong to set A and set B
$A - B$	difference	objects that belong to set A but not set B
A x B	Cartesian product	all ordered pairs with the first element from set A and the second from set B