SCC121
Fundamentals of Computer Science

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Overview

- Why predicate logic:
  - Rationale and distinction from propositional logic
- Predicate logic’s syntax:
  - Logical concepts: predicates, terms, formulae
  - Operators: connectives, quantifiers
- Predicate logic’s semantics:
  - Interpretation, satisfiable formulae

Objectives

- Understanding basic ideas about predicate logic
- Facility to use predicate logic notations
- Facility to operate with quantifiers
- Understanding the semantics of predicate logic

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2. links between parts of propositions
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Recall modus ponens:

$(P \rightarrow Q)$	If today is Tuesday, then I will go to work
P	Today is Tuesday
$\therefore Q$	Therefore, I will go to work

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Example - consider the propositions:

- All ravens fly.
- Peter is a raven.
- So, Peter flies.

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- Therefore: Aristotle is mortal

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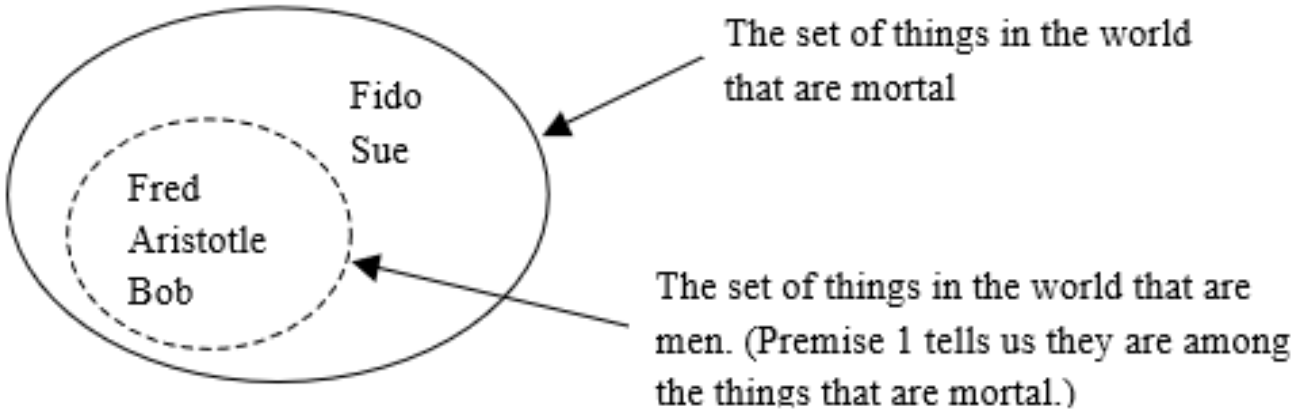

Why Predicate Logic?

Propositional logic does not describe:

1. parts of propositions
2. links between parts of propositions
3. statements about “for all” objects, or about “for some” objects

Limitations of Propositional Logic

Propositional logic focuses on proposition as a whole, and not on the links between its parts.

Example:

- Peter is a raven
- Peter is the father of Pan

Why Predicate Logic?

Propositional logic does not describe:

1. parts of propositions
2. links between parts of propositions
3. statements about “for all” objects, or about “for some” objects

Limitations of Propositional Logic

Propositional logic does not provide information on the quantity of the set of objects which have properties.

Example:

- Jon is a SCC120 student who passed the exam
- Ann is a SCC120 student who passed the exam
- Ken is a SCC120 student who passed the exam

  …

We can express quantities through statements such as:

- **All** SCC120 students passed the exam
- **Some** of the SCC120 students passed the exam

Logicians who Developed Predicate Logic

Charles Peirce

Gottlob Frege

Beatty, R., 1969. Peirce's development of quantifiers and of predicate logic. *Notre Dame Journal of Formal Logic*, *10*(1), pp.64-76.

Why Predicate Logic?

Predicate logic builds on propositional logic and extends it in new ways, accounting for:

- parts of propositions
- links between parts of propositions
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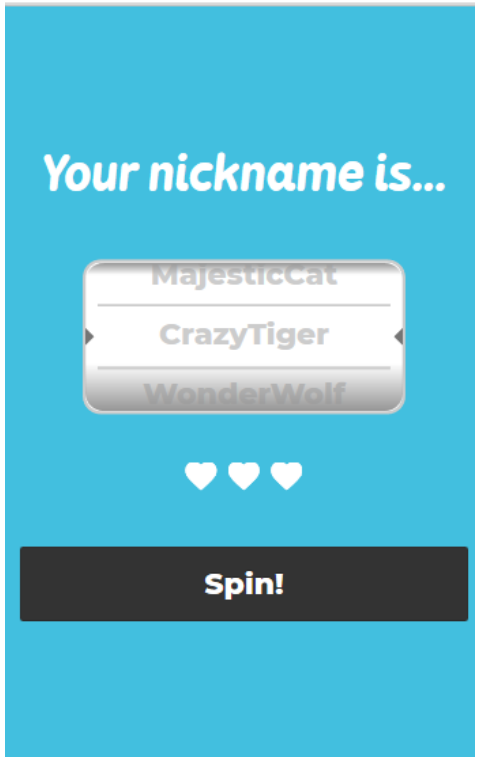
Predicate logic is a generalisation of propositional logic and is more powerful.

Let’s playxercise!

- https://kahoot.it/



Predicate Logic

Predicate logic has two key parts:

- Syntax - notations for concepts and operators used to create formulae
- Semantics - meaning behind these formulae.

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Predicate Logic - Syntax

Concepts

- **terms** - similar to subjects (or objects), expressed through nouns or pronouns
- **predicates** – properties or relations among terms, expressed through verbs

Operators

- connectives
- **quantifiers**

Predicate Logic - Syntax

English language

Sentence

Subject(s)
- noun(s)/ pronoun(s)

Predicate
- verb

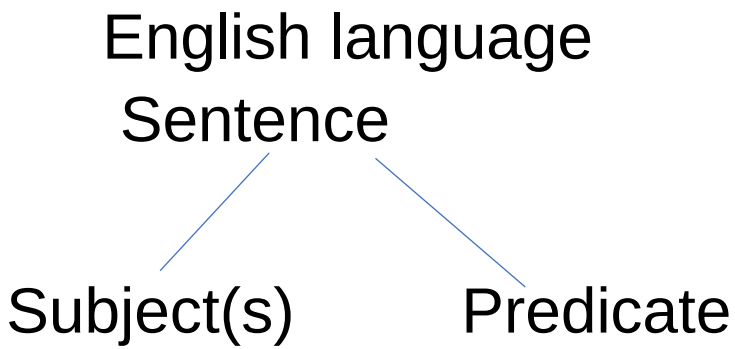
Predicate logic

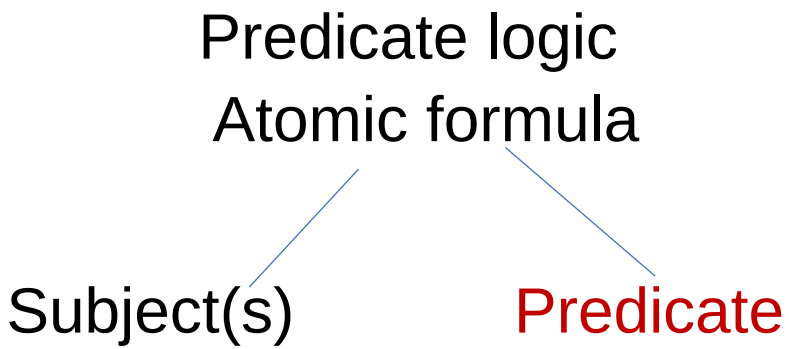
Atomic formula

Subject(s)
- noun(s)/ pronoun(s)

Predicate
- verb

Predicate Logic - Syntax





Predicate Logic - Syntax

- In predicate logic, every atomic formula consists of one predicate and one or more terms: subject(s) (possibly object(s)).

- Convention for writing predicates: upper case letter(s) of the verb.

- Two types of predicates that represent:
  - properties
  - relationships

Predicate – as Property

Predicate – a verb describing a property: feature or attribute of the subject

The three propositions:

- “Tom’s car is blue“
- "The sky is blue“
- "The cover of this book is blue“

have the same predicate**:** “is blue”, describing subjects’ property of being blue.

These predicate can be written, following the upper letter(s) convention: as B.

Predicate – as Relationship

Predicate – a verb describing a relationship between 2 terms: 1 subject & 1 object

The three propositions:

- “Jay is the father of Kay”.
- “Jay is Kay’s best friend”.
- “Jay and Kay are neighbors”.

show 3 relationships between Jay and Kay:

- ”is father of”, “is best friend of”, and “is neighbor of”.

These predicate can be written, following the upper letter(s) convention:

- “Jay is the father of Kay” as F
- “Jay is the best friend of Kay” as BF
- “Jay is a neighbor of Kay” as N

Predicate – as Relationship

Example of predicate as relationship among 3 terms: 1 subject, 2 objects

- Chris is sitting between Jay and Kay

  "… is sitting between … and …"

Note the distinction: "and" as part of predicate as relationship vs "AND" as conjunction:

- "Jay and Kay are neighbors".
- "Jay AND Kay are SCC120 students".

These can be written as:

- "Jay is a neighbor of Kay".
- "Jay is a SCC120 student" AND "Kay is a SCC120 student".

Predicate – as Relationship - Exercise

Let’s have three propositions:

- "John gives the book to Mary“
- "Jim gives a loaf of bread to Tom“
- "Jane gives a lecture to Mary"

Which are their predicates?

Predicate – as Relationship - Exercise

Let’s have three propositions:

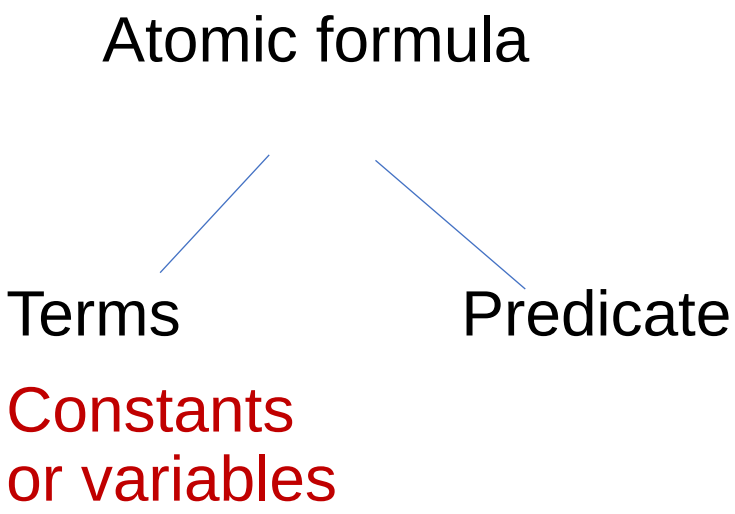
- "John gives the book to Mary“
- "Jim gives a loaf of bread to Tom“
- "Jane gives a lecture to Mary"

Which are their predicates?

Which are the 2 objects?

Answer: They all have the same predicate: GT - “… gives… to…”

Predicate Logic - Syntax



Terms: Constants

Constants - terms describing specific individual entities, i.e., people or objects.

Which are the predicates and constants in following formulae?

- Aristotle is a man
- Socrates is a man
- Bob is a man

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These have similar syntactic form:

- 1 term as subject, i.e., Aristotle, Socrates, Bob, and 1 predicate: “is a man”.

Convention for writing constants - lower case letter of the term:

a for Aristotle, s for Socrates, b for Bob.

Predicate Logic - Notations

Predicates are symbolized by upper case letter(s) of the verb

Terms are symbolized by lower case letter(s)

Atomic formula combines terms and predicate letters:

- predicate letter followed by the term letter in brackets, i.e., P(a)
- terms are also called predicate's arguments

Example:

- Aristotle is a man
- Socrates is a man
- Bob is a man

We write them as: M(a), M(s), M(b), where M stands for the predicate "is a man"

Terms: Constants

Which are the predicates and constants in following formulae?

- Jane is the mother of Mary.
  - Predicate “…is mother of… “ – symbol M. Constants : Jane, Mary.
    Written: M(j, m)
- Mary and Paul are siblings.
  - Predicate: “…is sibling of…” – symbol S. Constants: Mary, Paul.
    Written: S(m, p)
- The sum of 2 and 3 is 5.
  - Predicate: “the sum of.. and.. is…” – symbol S. Constants: 2, 3 and 5.
    Written: S(2, 3, 5)
- Tom is a cat.
  - Predicate: “is a cat” – symbol C. Constant : Tom.
    Written: C(t)

Terms: Variables

How can we write more succinctly multiple atomic formulae with identical predicates and different constants?

- Aristotle is a man; M(a)
- Socrates is a man; M(s)
- Bob is a man; M(b)

We need a different formula for each of these constants.

Terms: Variables

- Variables represent general terms: individuals of the same type, or a class rather than specific individuals.
- Variables abstract away from specific entities, and they do not correspond to anything in the real world; constants do.

Convention for writing variables: lower case letters from the end of alphabet, i.e., x, y

Example:

- Aristotle is a man; M(a)
- Socrates is a man; M(s)
- Bob is a man; M(b)

We can use a single atomic formula written M(x), consisting of:

- a variable x to denote male humans, and predicate M “is a man”

Predicate Logic – Notations Example

"John gives the book to Mary“. Notation: GT (j, b, m):

- GT stands for predicate “… gives … to…”
- j for term John, b for term book, and m for term Mary

“The cover is blue“. Notation: B(c):

- with B standing for predicate “is blue”
- c for term: “the cover”

Predicate Logic – Notations Exercise

How do we express in predicate logic this proposition:

- “Joan is a student”

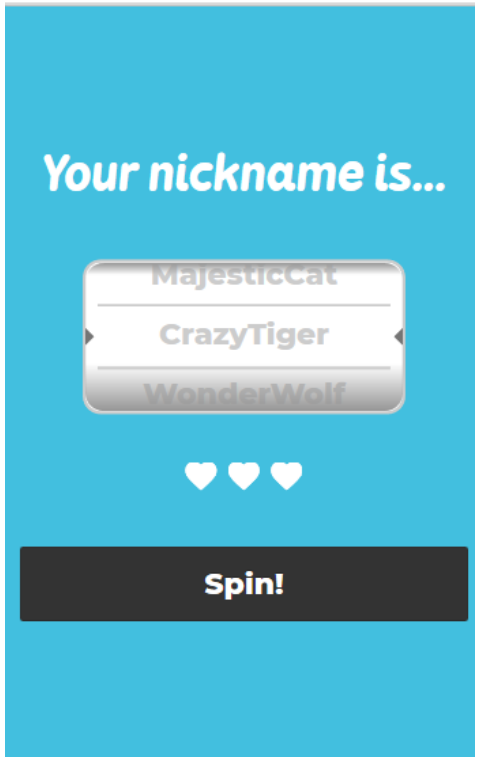
Answer:

S(j), where

S = “is a student” – as predicate, j = Joan – as subject.

Let’s Playxercise!

- https://kahoot.it/



Atomic Formulae – Truth Values

Atomic formulae are functions that take input one or more arguments, and return Boolean values: True or False.

Input elements are called arguments, i.e., constants or variables.

Example: two atomic formulae with one argument: constant or variable.

- M(a) = “Aristotle is a man”, a = Aristotle, M = “is a man”
- T(x) = “Person x is tall”, x = variable, T = “is tall”

Truth values:

- M(a) is a proposition and has truth value, i.e., True if and only if it correctly describes a situation in which Aristotle is a man in the world.
- T(x) is not a proposition; its truth values depend on the truth values of propositions created by replacing variable x with specific constants.

Atomic Formulae – Truth Values

What is the truth value of these formulae?

- Aristotle is a man; M(a) which is True
- Socrates is a man; M(s) which is True
- Bob is a man; M(b) which is True
- Kermit is a man; M(k) which is False (Kermit is a frog)

What is the truth value of the formula with variable term, M(x)?

- M(x) can be True or False, depending on x
  - for x = a, s or b, M(a), M(s), M(b) are True
  - for x = k, M(k) is False

Types of Atomic Formulae

Closed (or ground) formulae:

- predicate's arguments: only constants, no variable terms
- these are propositions and have truth values

Open (or unground) formulae:

- predicate's arguments: at least 1 variable, possible constant terms
- these are not propositions and have no truth value

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Open (or unground) formulae:

- predicate's arguments: at least 1 variable, possible constant terms
- these are not propositions and have no truth value
- but become propositions if their variables are replaced with constants:
    - T(x) "person x is tall"; if x = f, then T(f) = "Freya is tall" is True.

Atomic Formulae – Truth Values Example

Atomic formula: SCG(xs, xc, xg) on student-course-grade relation.

If variables xs, xc, xg, are substituted for three constants:

- SCG(j, course1, 65) is a closed or ground atomic formula, with 3 constants: John, course1, 65
- True if John took course1 and got grade 65
- False otherwise.

- SGC(xs, course1, xg) is an open or unground atomic formula, with 1 constant: course1 and 2 variables. No truth value, but can become:
  - True if variables xs and xg take on any pair of values such that xs is a student who took course1, and got grade xg
  - False otherwise

Atomic Formulae – Truth Values Example

- Let P(x) be an atomic open (or unground) formula: “x > 3”

  What are the truth values of P(4) and P(2)?

  Answer:
- P(4): “4 > 3”, which has the truth vale: True
- P(2): “2 > 3”, which has the truth vale: False

Atomic Formulae – Truth Values Example

- Let Q(x, y) be an atomic open (or unground) formula: “x = y + 3”

  What are the truth values of the propositions
  Q(1, 2) and Q(3, 0)?

  Answer:

- Q(1, 2): “1 = 2 + 3”, which has the truth vale: False
- Q(3, 0): “3 = 0 + 3”, which has the truth vale: True

Terms: Variables – Universe of Discourse

Which values can a variable take?

- Variables take values from the Universe (or domain) of discourse.
- Universe of discourse – all entities that can replace a variable in an atomic formula.

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Example:

- Person x is a man = M(x)
- Universe for x can be the set living men.

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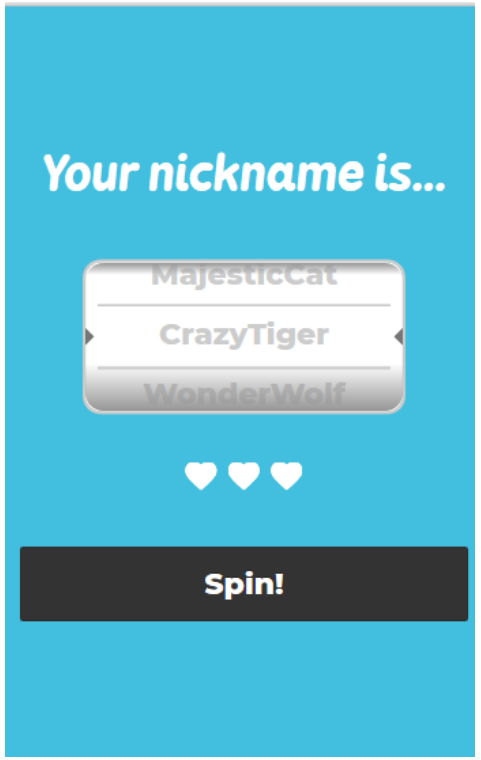

Summary

- Predicate – a verb describing a property of the term subject, or a relationship among subject and object(s).
- Terms – predicate's arguments, similar to subjects and objects in English sentences.
- Constants - specific terms from the Universe of Discourse.
- Variables - generic terms from the Universe of Discourse.
- Atomic formula – expression consisting of one predicate and one or more terms.
- Closed or ground formula – formula which only has constants as arguments of their predicate.
- Open or unground formula - formula which has constants or variables as arguments, but at least one variable.
- Universe of discourse - the set of constants that can replace a variable in an atomic formula.