SCC121
Fundamentals of Computer Science

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School of Computing and Communications

Overview

- Functions
  - Definitions
  - Types
  - Operations

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- Functions
  - Definitions
  - Types
  - Operations

Objectives

- Understanding the basic ideas about functions

Function

The concept of *function* has a long history:

- Galileo (1564-1642) – the first statements of dependency of one quantity on another
- Leibniz (1646-1716) - coined the term “function" to mean any quantity varying from point to point of a curve
- Euler (1707-1783) – the first to introduce the notation f(x)

Examples of Functions

- Each UK citizen has a unique National Insurance number, i.e., *QQ123456C*
  - This can be viewed a function mapping UK citizen to their National Insurance number
- Each student at Lancaster University has an eight digit student number

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- When we pull out an attribute of an object
  - The set of rectangles, color as attribute: function that maps figures to colours
  - Cost of mailing is a function of weight

Examples of Functions

- Distance is a function of time

- People’s height is a function of age
- Trees’ growth rings are a function of age

Functions

- A machine which given an input produces a unique output
  - deterministic linkage between two sets of values**:** inputs and outputs

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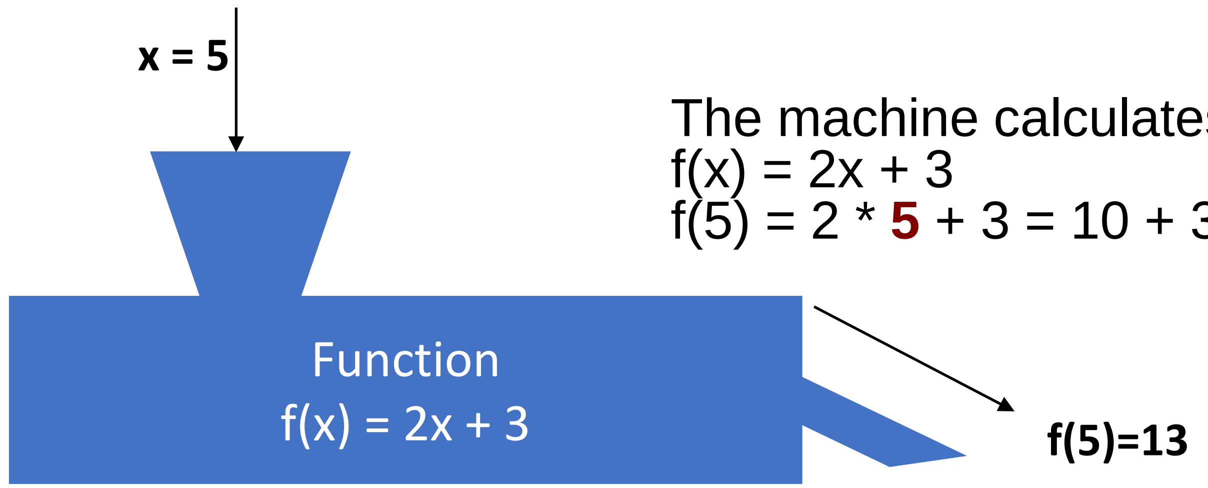
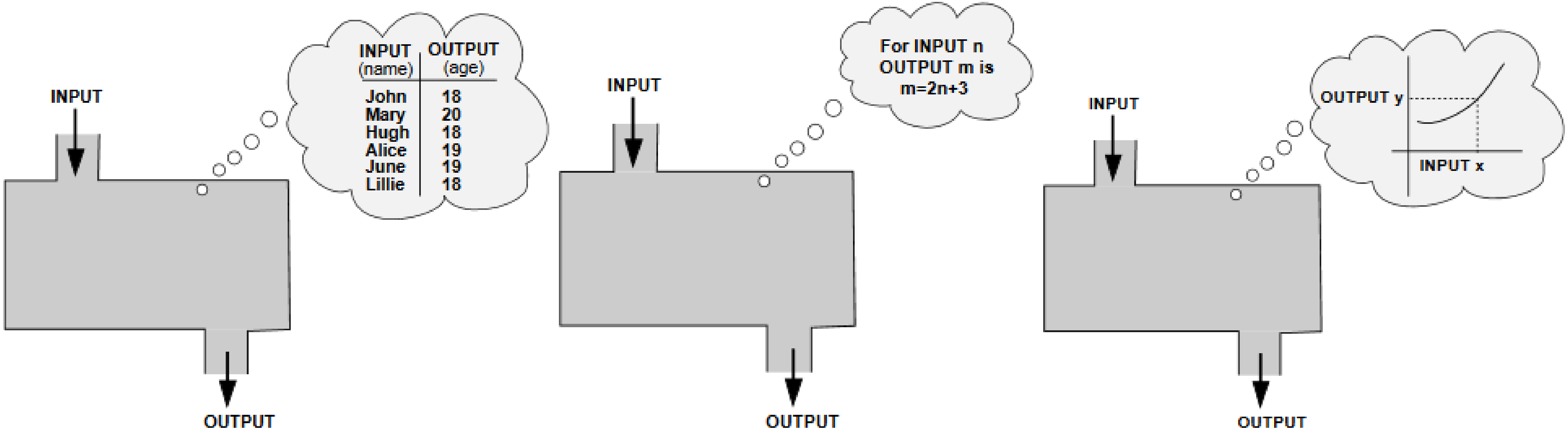


The machine calculates:
$f(x) = 2x + 3$
$f(5) = 2 * \mathbf{5} + 3 = 10 + 3 = 13$

Functions



The function box as a table, a formula and a graph

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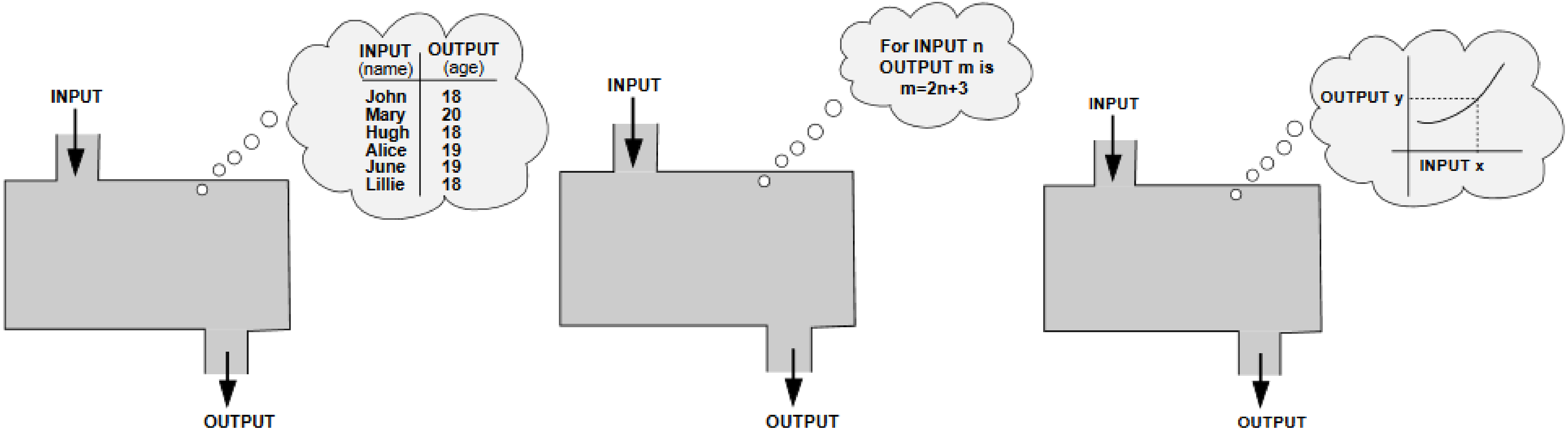
Functions



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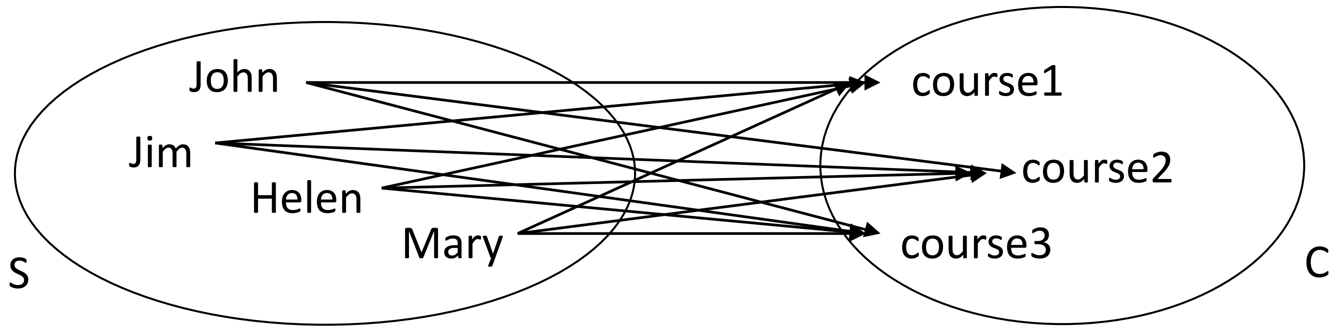
Preliminary

Let S = {John, Jim, Helen, Mary} and C = {course1, course2, course3}



Cartesian product:

S x C = {<John,course1>,<John,course2>,<John,course3>, <Jim,course1>,<Jim,course2>,<Jim,course3>,<Helen,course1>, <Helen,course2>, <Helen,course3>,<Mary,course1>, <Mary,course2>, <Mary,course3>}

Each student can take each course (all possible ordered pairs)

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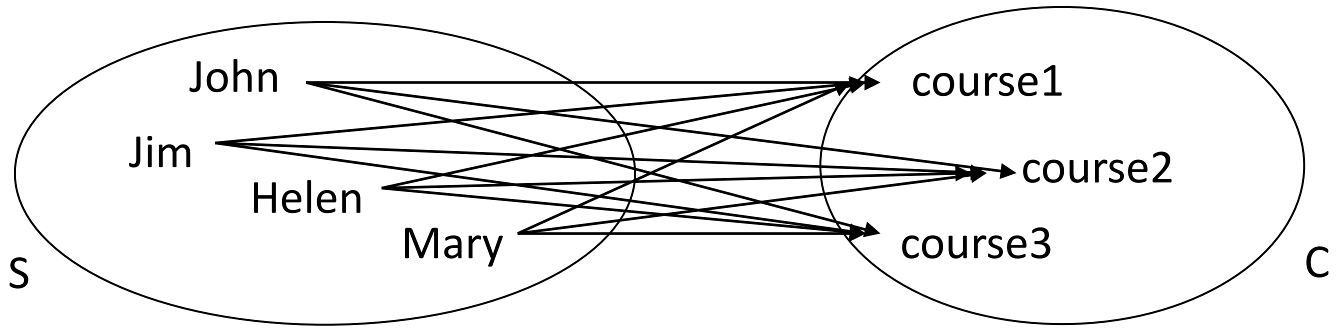

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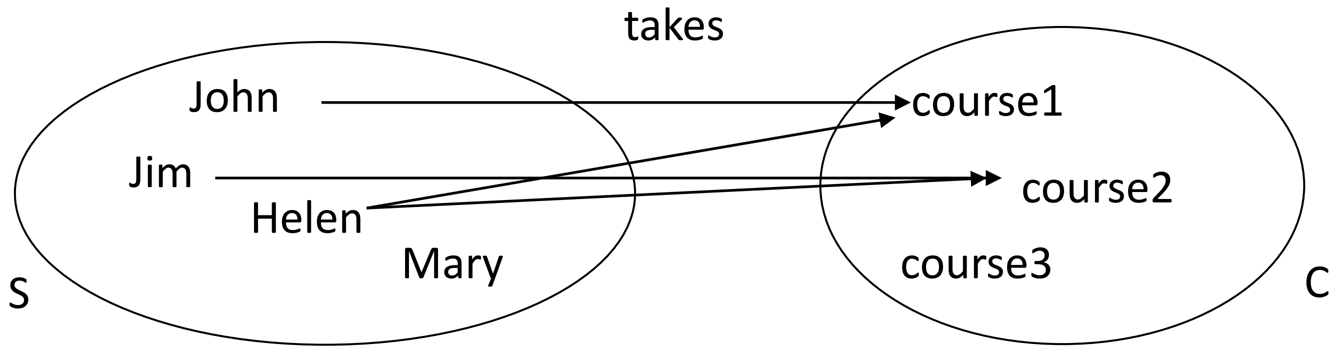

Relation: takes $\subseteq$ S x C

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Some students take some courses (some ordered pairs)

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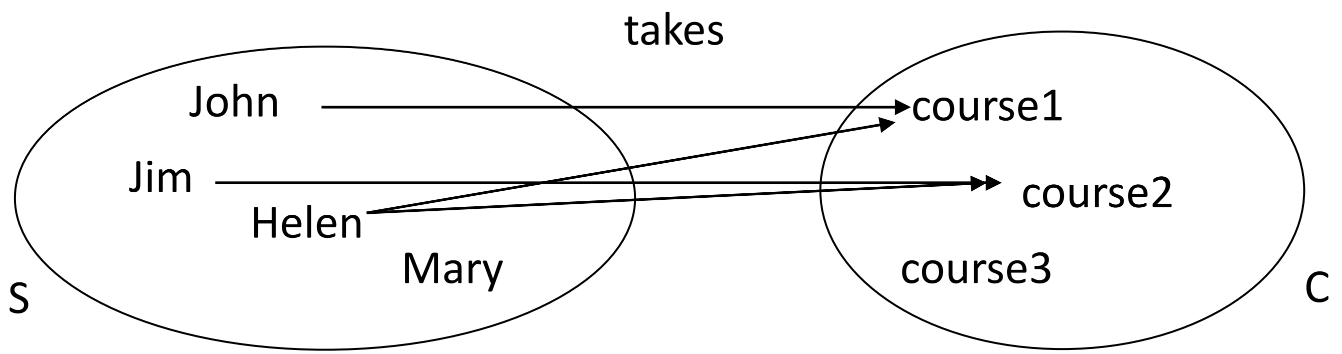

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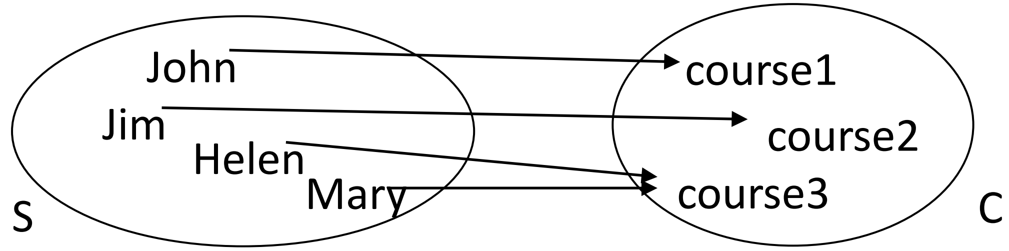
Definition

- A function is a special type of binary relation.
- It associates **each** element of a set with a **unique** element of another set.



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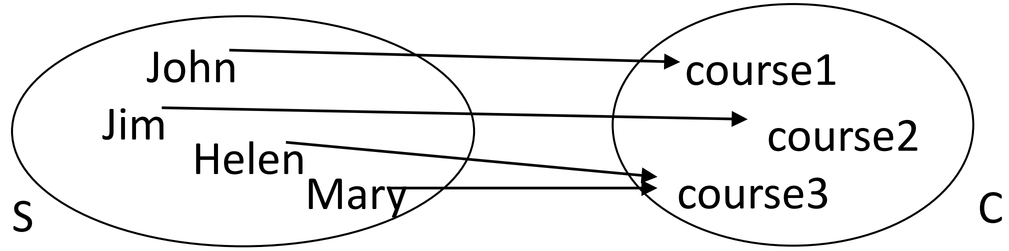

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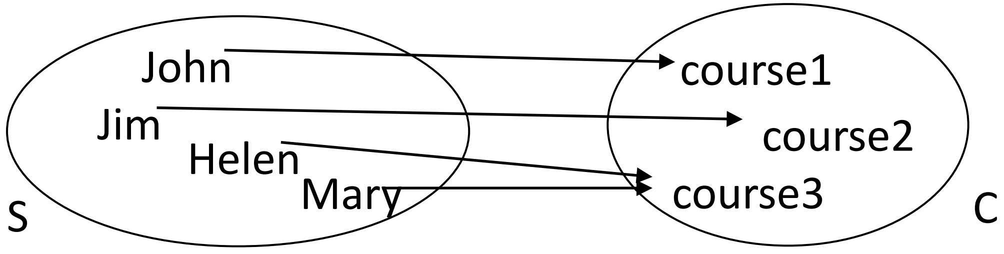

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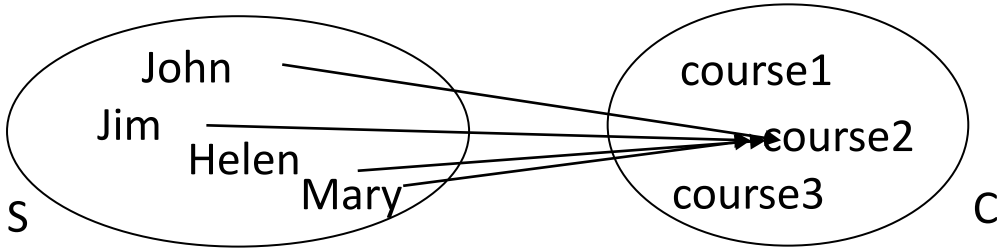

Function2?

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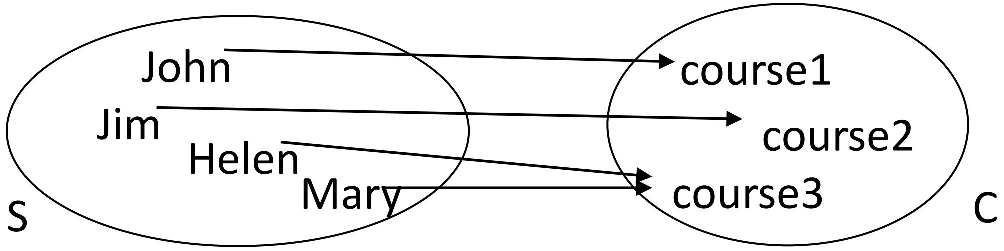

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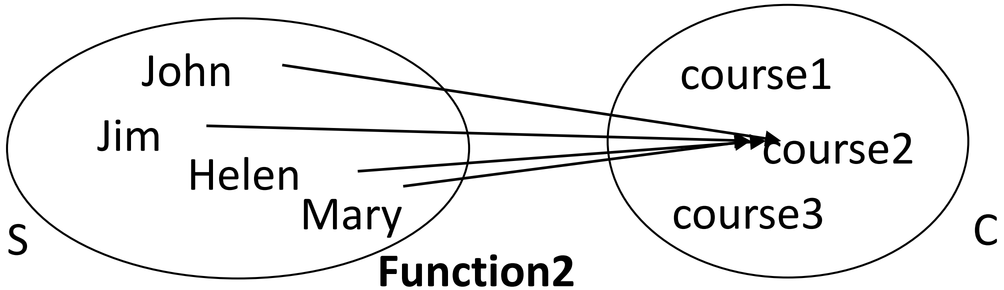

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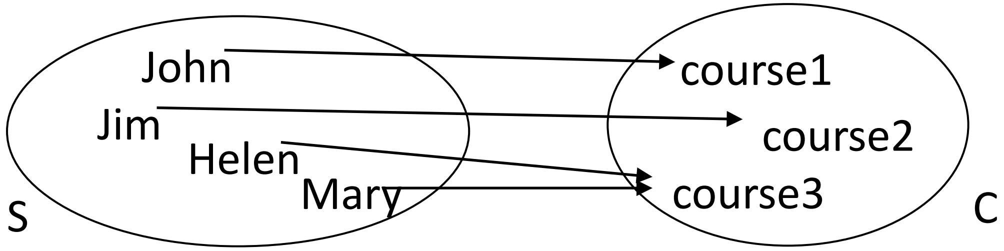

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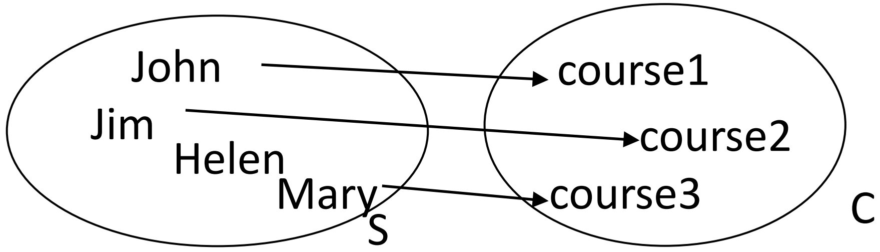
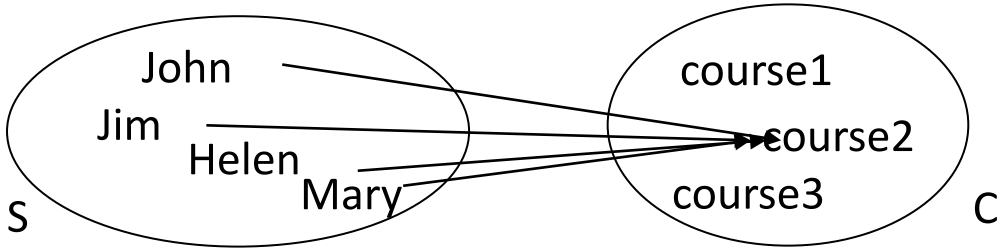


Function3?



Function2

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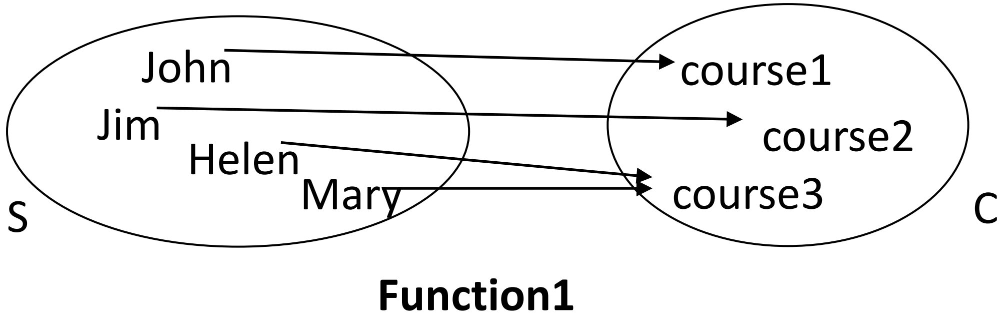

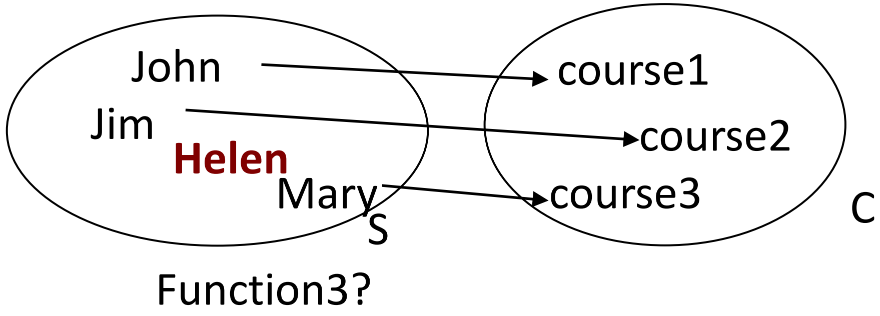

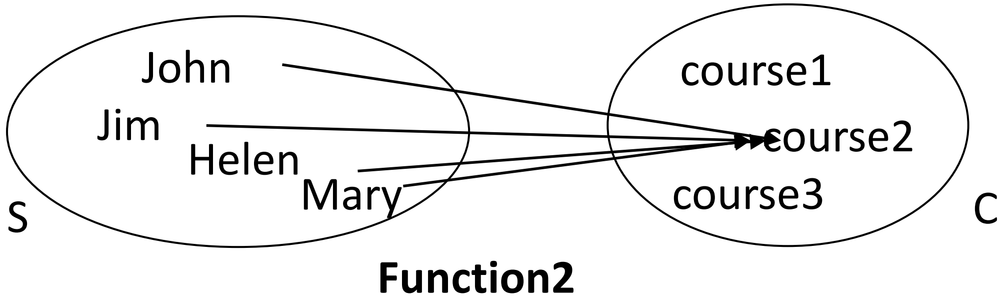

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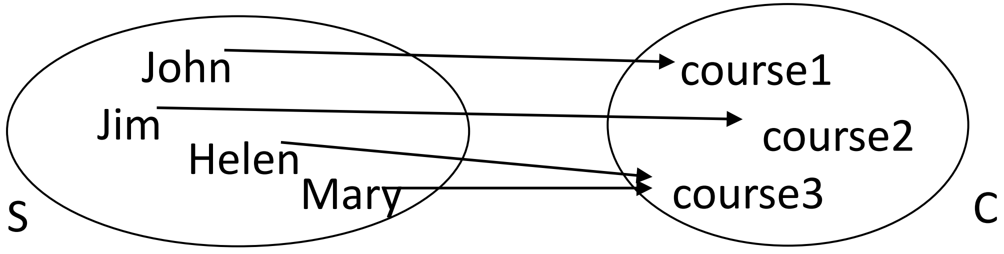

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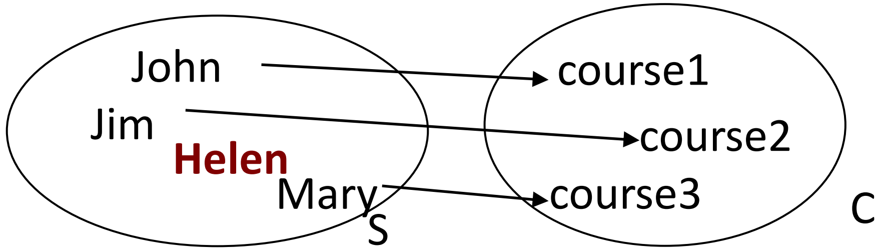

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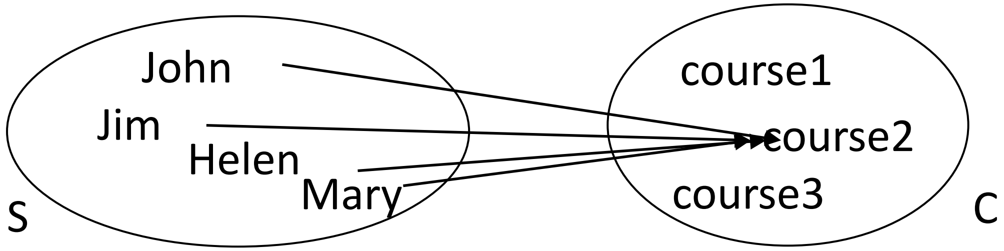

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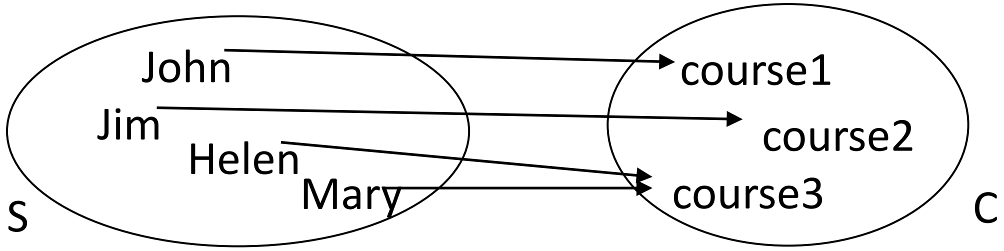

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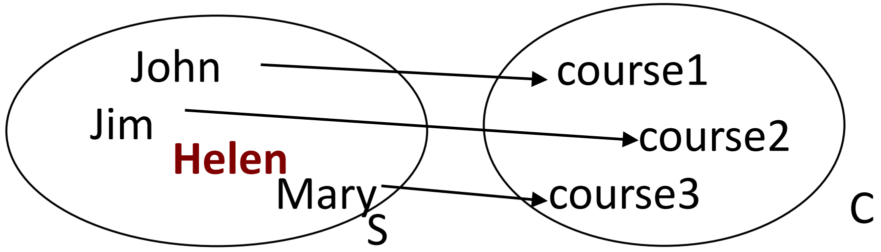

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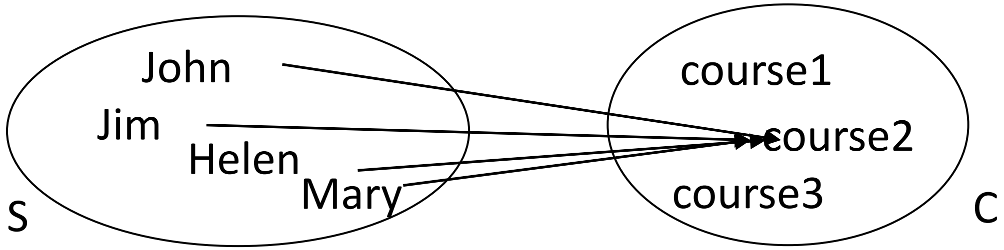

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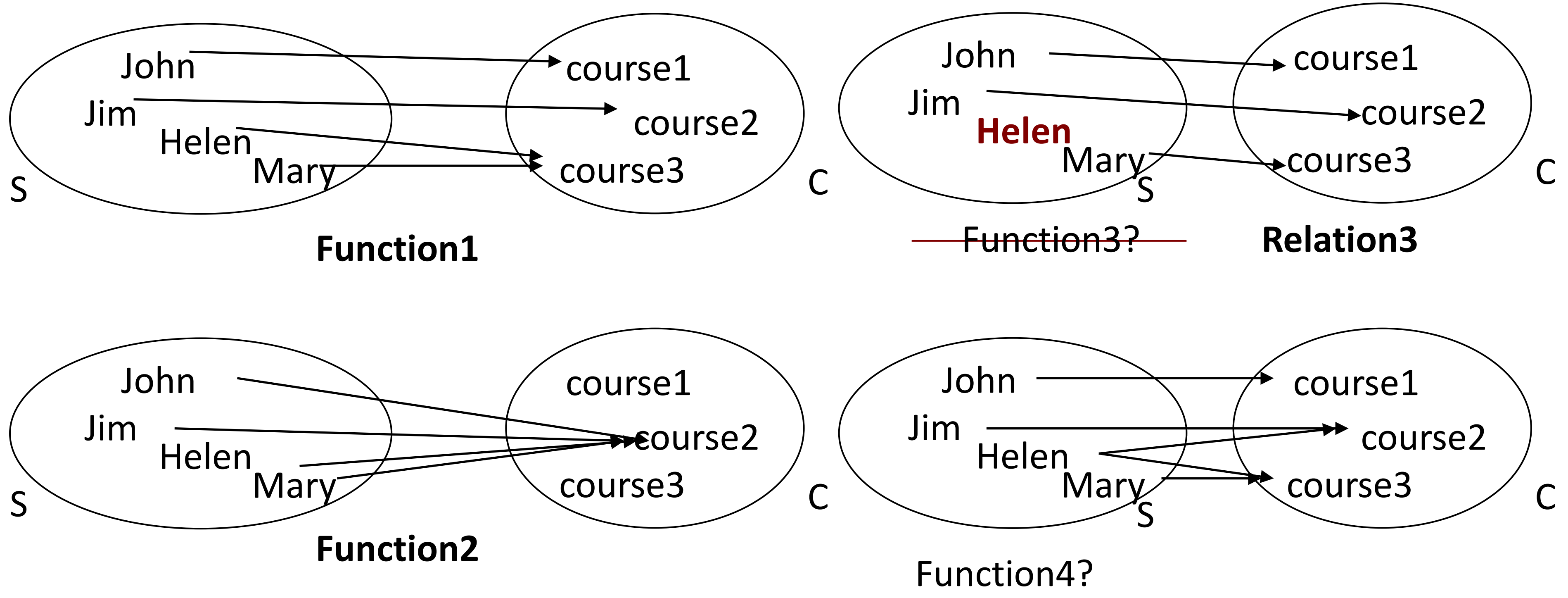

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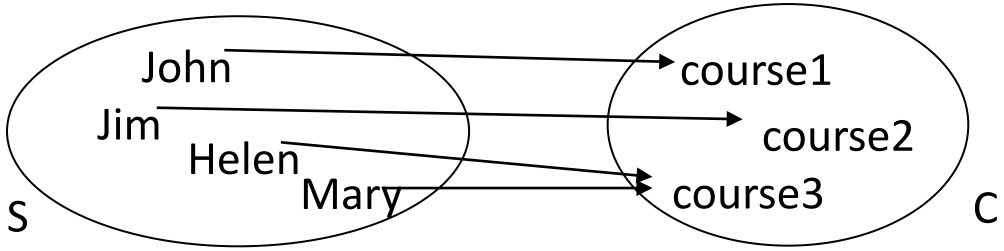

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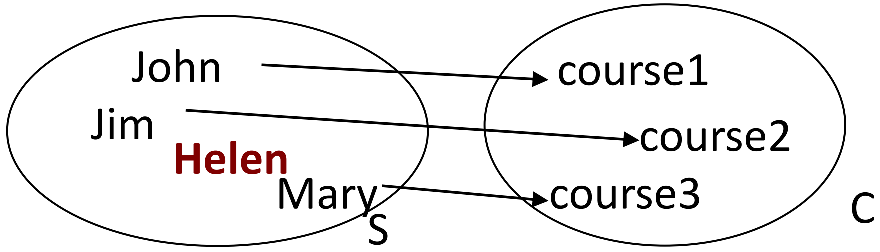

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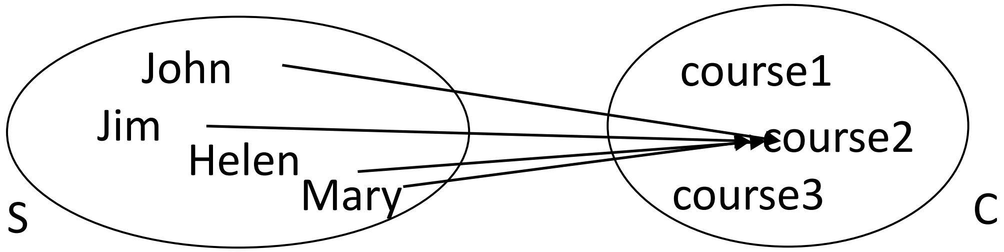

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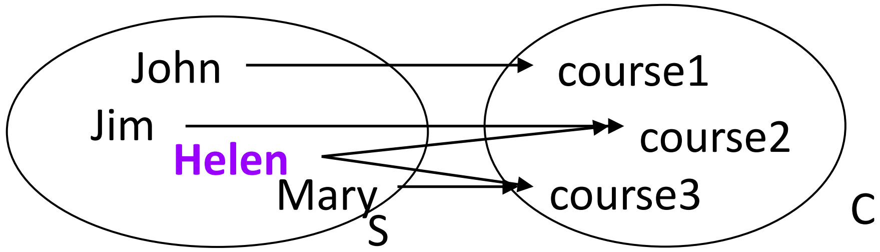

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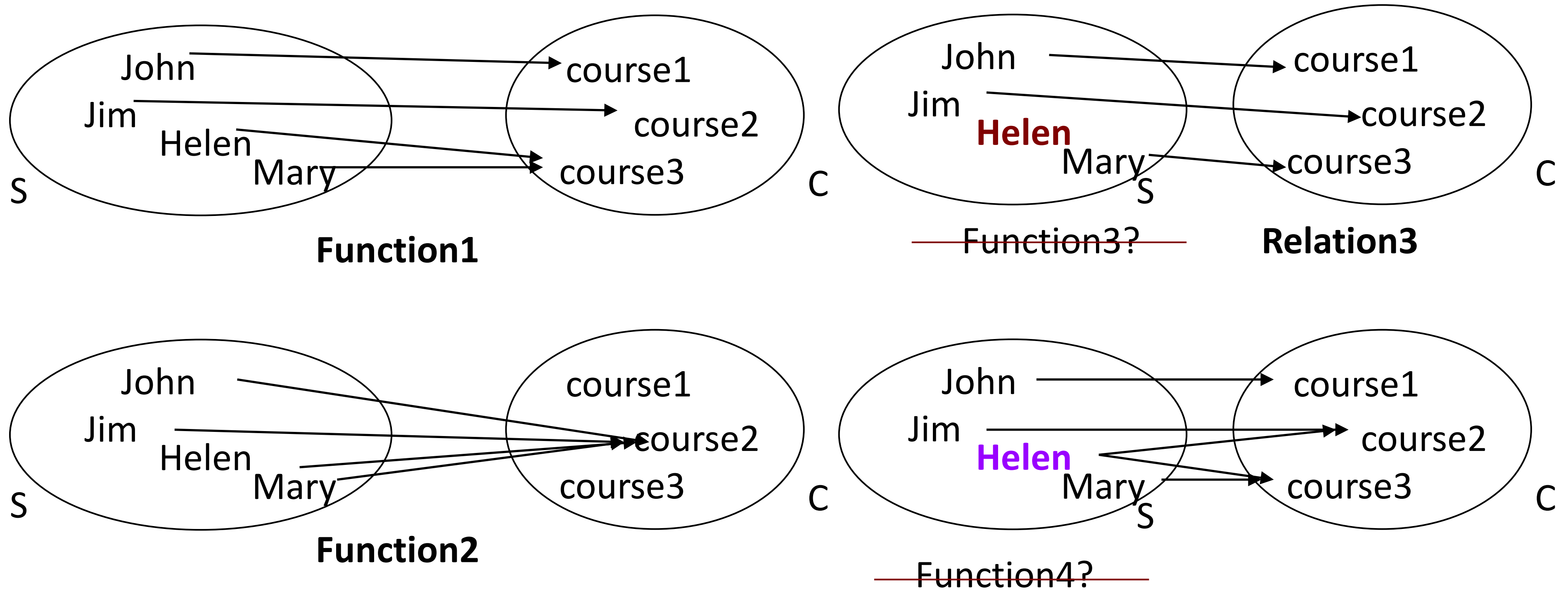

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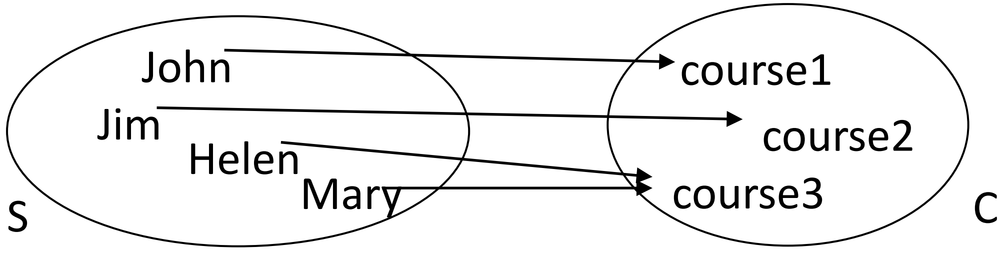

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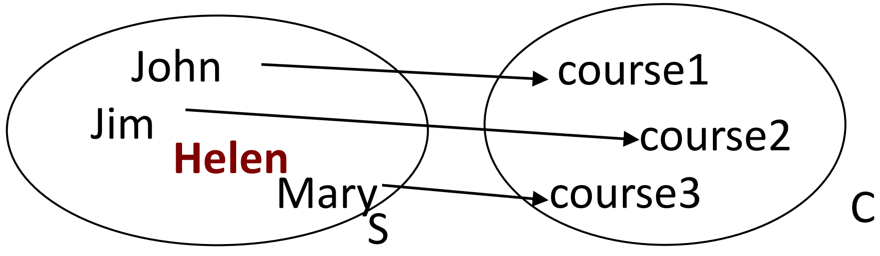

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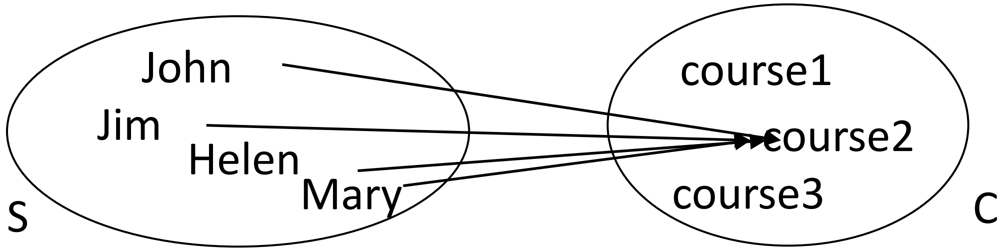
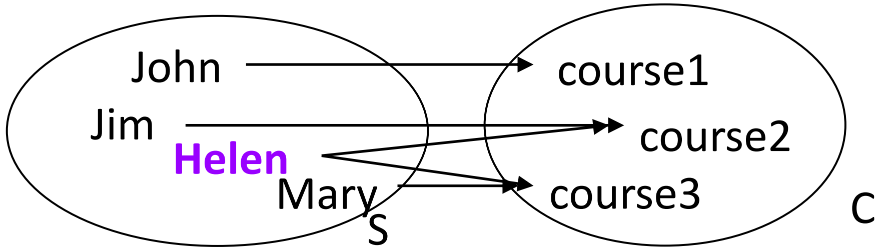


Function2



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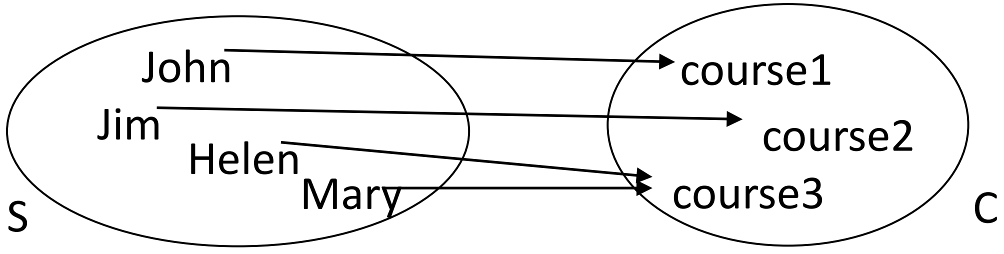

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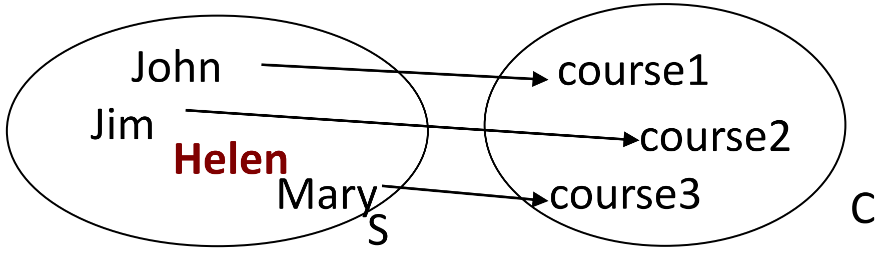

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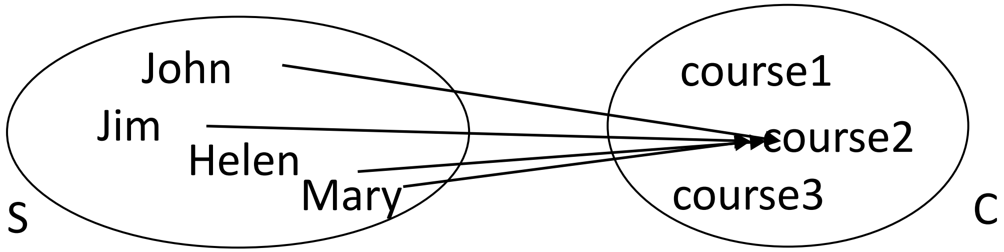

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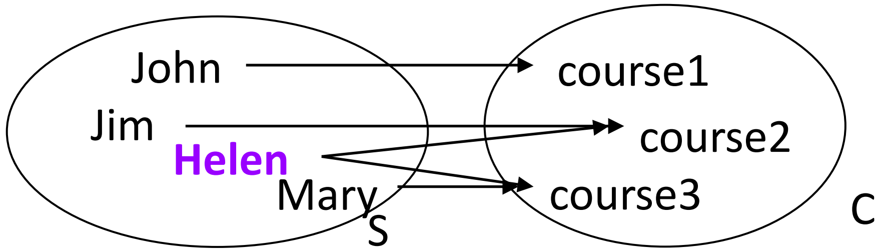

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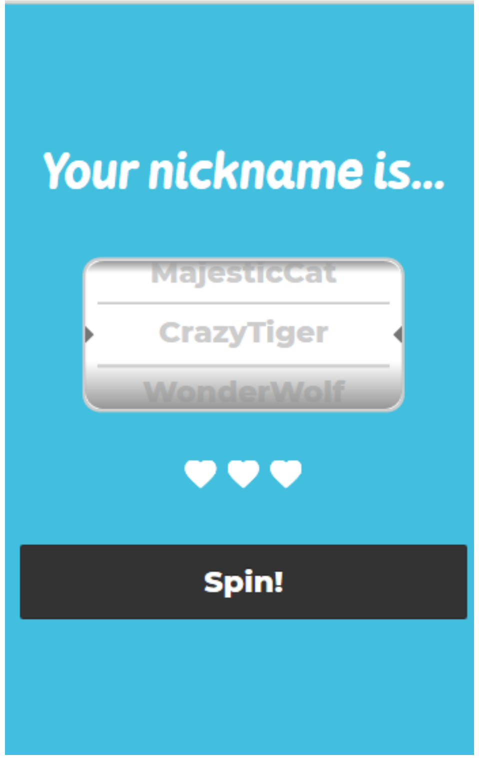
Function1 = {<John, course1>, <Jim, course2>, <Helen, course3>, <Mary, coure3>} is a function from S to C.

Let’s playxercise!

- https://kahoot.it/



Functions and Related Concepts

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- Domain
- Codomain
- Range
- Image
- Preimage

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Functions

Function from set A to set B

- for every $a \in A$, there exists a unique $b \in B$ such that **<a, b>** $\in f$

Notation: $\boldsymbol{f} : A \rightarrow B$

A function from a set ***A*** to a set ***B*** is a relation from **A** to **B** that satisfies:

- for **each** element **a** in **A**, there is an element **b** in **B** such that **<a, b>** is in the relation, and
- that element is **unique**: if **<a, b>** and **<a, c>** are in the relation, then **b = c.**

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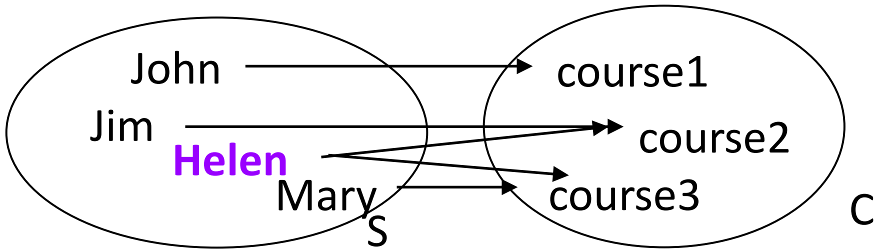
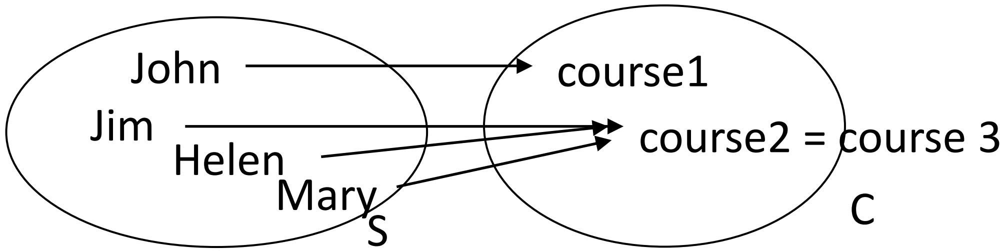


Relation



Function

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Domain and Codomain

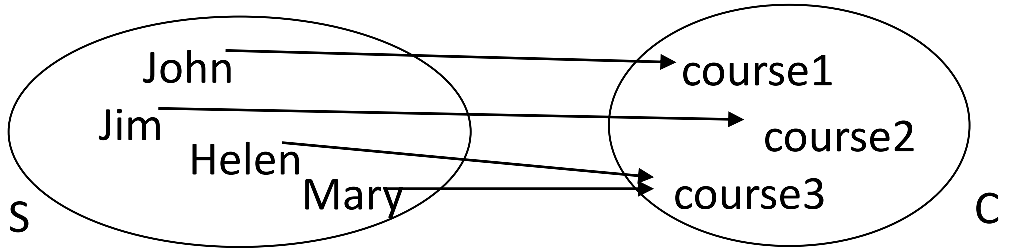
We write a function f from A to B: $\boldsymbol{f} : A \rightarrow B$

The set ***A*** is called the **domain** of **function *f* – all input elements**

The set ***B*** is **codomain** of **function *f* – all possible output elements**

Examples

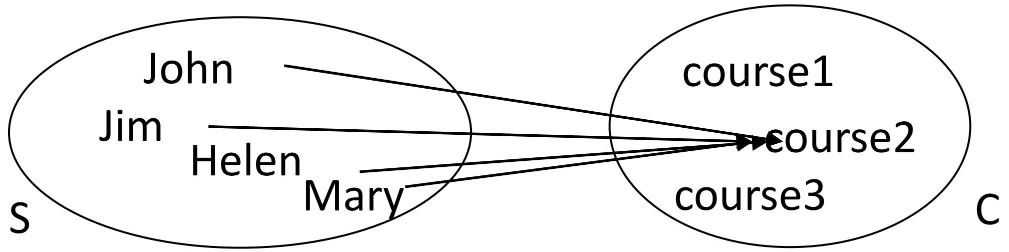
- S = {John, Jim, Helen, Mary}
- C ={course1, course2, course3}
- f1 : S $\rightarrow$ C
- f1 = {<John, course1>, <Jim, course2>, <Helen, course3>, <Mary, course 3>}



- Domain: {John, Jim, Helen, Mary} = S
- Codomain = {course1, course2, course2} = C

Examples

- S = {John, Jim, Helen, Mary}
- C ={course1, course2, course3}
- f2 : S $\rightarrow$ C
- f2 = {<John, course2>, <Jim, course2>, <Helen, course2>, <Mary, course2>}



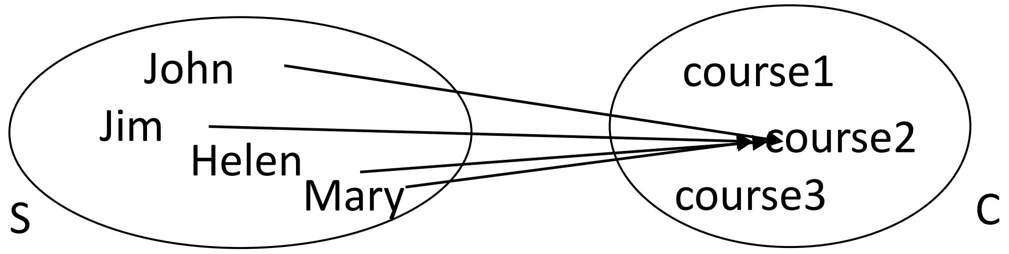
- Domain: {John, Jim, Helen, Mary} = S
- Codomain = {course1, course2, course3}

Range

Range - set of values that actually do come out of a function.

Range is a subset of the Codomain.

- S = {John, Jim, Helen, Mary}
- C ={course1, course2, course3}
- f2 : S $\rightarrow$ C
- Codomain = {course1, course2, course3}
- Range = {course2}



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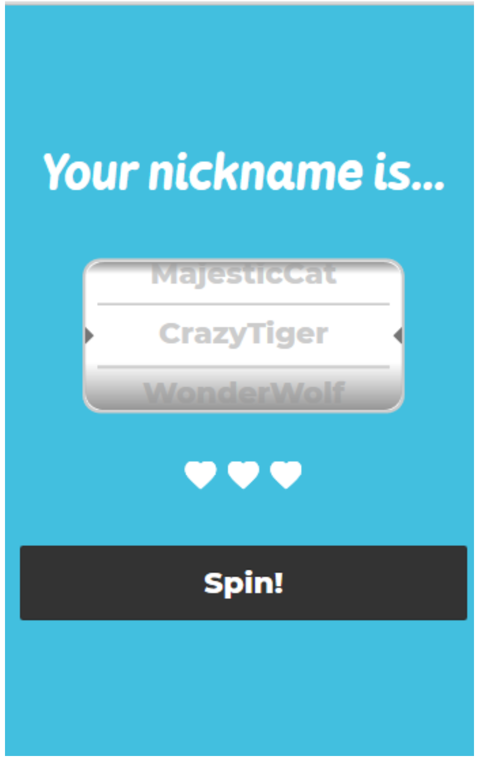

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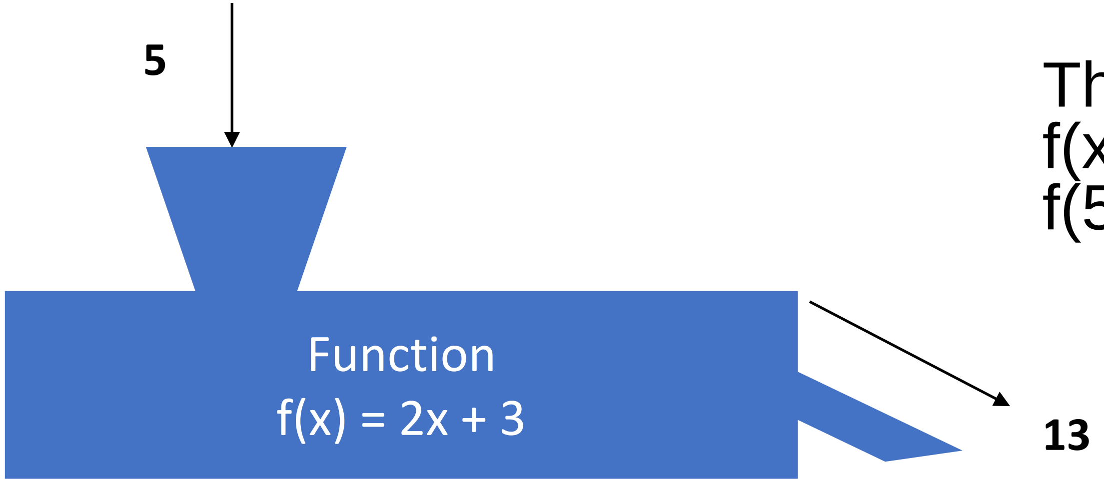
Image and Preimage

For the function f from A to B: ***f*** : A $\rightarrow$ B

if **<a, b>** $\in$ ***f*****,** then b is denoted by *f*(a), or **f(a) = b**



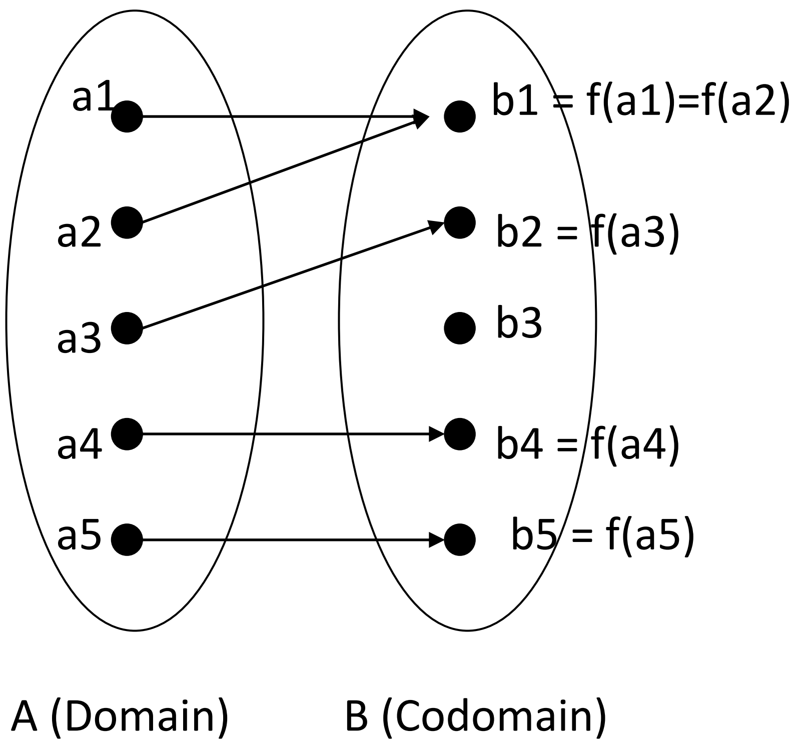
The machine calculates:
f(x) = 2x + 3
f(5) = 2 * 5 + 3 = 10 + 3 = 13

- **a** is the **preimage** of **b** under ***f***
- **b** is the **image** of **a** under ***f***

For f(x) = 2x + 3
5 is preimage of 13
13 is image of 5

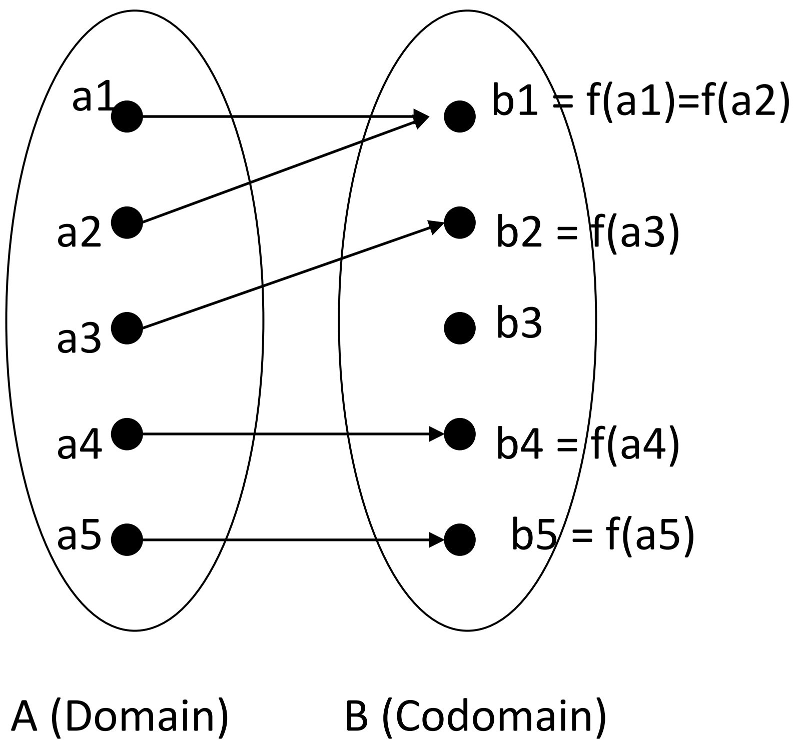
Example

- ***f*** : A $\rightarrow$B
- Domain = A = {a1, a2, a3, a4, a5}
- Codomain = B = {b1, b2, b3, b4, b5}
- Preimages:
  - a1 is the preimage of b1
  - a2 is the preimage of b1
  - a3 is the preimage of b2
  - a4 is the preimage of b4
  - a5 is the preimage of b5



Example

- $f : A \to B$
- Domain = A = {a1, a2, a3, a4, a5}
- Codomain = B = {b1, b2, b3, b4, b5}
- Images:
  - b1 is the image of a1
  - b1 is the image of a2
  - b2 is the image of a3
  - b4 is the image of a4
  - b5 is the image of a5
- Range = {b1, b2, b4, b5}
- Range = set of all images of function f
- Range $\subseteq$ B



Example

- A = {-1, 0, 1, 2, 3}
- B = {0, 1, 4, 9, 16}
- ***f*** : A $\rightarrow$ B
- ***f*** (x) = x^2

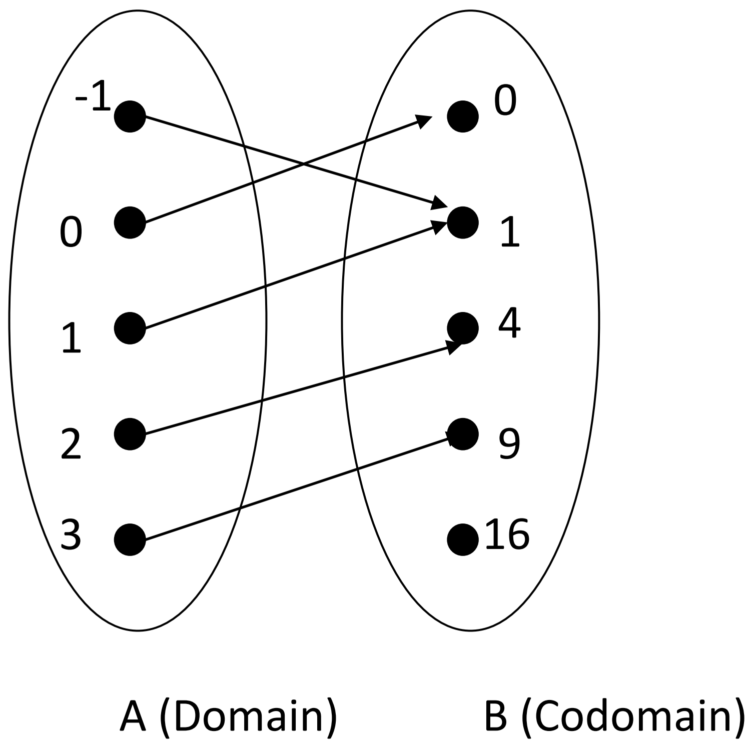
  f (-1) = $(-1)^2 = 1$

  f (0) = $0^2 = 0$

  f (1) = $1^2 = 1$

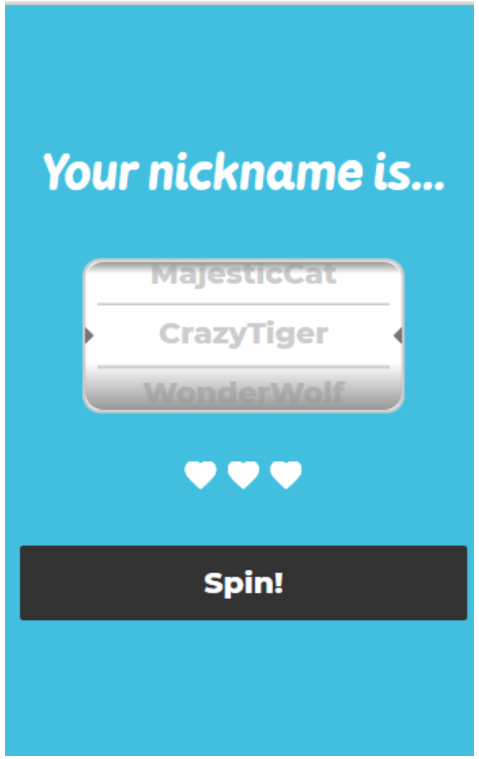
  f (2) = $2^2 = 4$

  f (3) = $3^2 = 9$
- Range = {0, 1, 4, 9}



Let’s playxercise!

- https://kahoot.it/



Summary

- Function $f : A \rightarrow B$ is a special type of binary relation that associates **each** element $a \in A$ with a **unique** element of $b \in B$.
- If <a, b> $\in f$, then $b \in B$ is the image of a under f, and $a \in A$ is the preimage of b under f.
- Set A is the domain of function f.
- Set B is the codomain of function f.
- Range is the set of all images of f, and is a subset of the codomain.