

Selection Sort

$$\begin{aligned}
 \sum_{i=0}^{p-1} (C_1 + \sum_{k=i+1}^p (C_2) + C) &= \sum_{i=0}^{p-1} (C_1 + C_2 p - C_2 i) \\
 &= \sum_{i=0}^{p-1} (C_1 + C_2 (p - (i+1) + 1)) = (C_1 + C_2 p)(n - 1 - 0 + 1) - C_2 \sum_{i=0}^{p-1} i \\
 &= \sum_{i=0}^{p-1} (C_1 + C_2 (p - i)) = (C_1 + C_2 p)p - C_2 \left(\frac{(p-1)p}{2} \right) \\
 &= C_2 p^2 + \left(\frac{C_1 + C_2}{2} \right) p + C
 \end{aligned}$$

$$O(p^2)$$

Merge Sort

$$\begin{aligned}
 \text{merge}(n) &= C_1 + \sum_{i=0}^{n/2} C + \sum_{i=n/2}^n C \\
 &= Cn
 \end{aligned}$$

$$\text{mergeSort}(n) = T(n/2) + T(n/2) + Cn$$

$$\Rightarrow 2T(n/2) + Cn$$

$$\Rightarrow 4T(n/4) + 2Cn$$

$$\Rightarrow 8T(n/8) + 3Cn$$

$$\Rightarrow 2^k T(n/2^k) + kCn$$

$$\Rightarrow 2^{\log_2 n} T(1) + (\log_2 n) Cn$$

$$\Rightarrow nC + Cn \log_2 n \Rightarrow O(n \log n)$$

$$\begin{aligned}
 \frac{n}{2^k} &= 1 \Rightarrow 2^k = n \\
 &\Rightarrow k = \log_2 n
 \end{aligned}$$