

$$\sinh(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$$

$$\Rightarrow \sinh(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$$

$$\lim_{n \rightarrow \infty} \sinh\left(\frac{1}{n}\right) \approx \frac{1}{n}$$

$$-1 < \lim_{n \rightarrow 0} \sinh\left(\frac{1}{n}\right) < 1$$

As n approaches infinity, $\sinh\left(\frac{1}{n}\right)$ is approximately $\frac{1}{n}$ however, as n approaches zero, $\sinh\left(\frac{1}{n}\right)$ continuously moves between -1 and 1 while $\frac{1}{n}$ approaches infinity

$$\text{as } n \rightarrow \infty, \text{ error } \sinh\left(\frac{1}{n}\right) = O(0)$$

$$\text{as } n \rightarrow 0, \text{ error } \sinh\left(\frac{1}{n}\right) = O(\infty)$$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} \dots$$

$$\Rightarrow \cos(x) \approx \frac{(-1)^n x^{2n}}{(2n)!}$$

$$\lim_{n \rightarrow \infty} \cos\left(\frac{1}{n}\right) \approx 1 - \frac{1}{(2 \cdot n^2)}$$

$$\lim_{n \rightarrow 0} \cos\left(\frac{1}{n}\right) \neq 1 - \frac{1}{(2 \cdot n^2)}$$

As n approaches infinity, $\cos\left(\frac{1}{n}\right)$ is approximately $1 - \frac{1}{(2 \cdot n^2)}$ however, as n approaches zero $\cos\left(\frac{1}{n}\right)$ moves between -1 and 1 while $1 - \frac{1}{(2 \cdot n^2)}$ approaches negative infinity.

$$\text{as } n \rightarrow \infty \quad \text{error } \cos\left(\frac{1}{n}\right) = O(0)$$

$$\text{as } n \rightarrow 0 \quad \text{error } \cos\left(\frac{1}{n}\right) = O(\infty)$$