Problem Set 12.6

2.) $w = x^2 y - y^2 x$

$x = \cos t$

$y = \sin t$

$$\frac{dw}{dt} = (2xy - y^2)(-\sin t) + (x^2 - 2xy)(\cos t)$$

$$= (\sin t + \cos t)(1 - 3\sin t \cos t)$$

3.) $w = e^x \sin y + e^y \sin x$

$x = 3t$

$y = 2t$

$$\frac{dw}{dt} = (e^x \sin y + e^y \cos x)(3) + (e^x \cos y + e^y \sin x)$$

$$= 3e^{3t} \sin 2t + 3e^{2t} \cos 3t + 2e^{3t} \cos 2t + 2e^{2t} \sin 3t$$

4.) $w = \ln\left(\frac{x}{y}\right)$

$x = \tan t$

$y = \sec^2 t$

$$\frac{dw}{dt} = \left(\frac{1}{x}\right) \sec^2 t + \left(-\frac{1}{y}\right)(2 \sec^2 t \tan t)$$

$$= \frac{\sec^2 t}{\tan t} - 2 \tan t$$

$$= \frac{\sec^2 t - 2 \tan^2 t}{\tan t}$$

$$= \frac{1 - \tan^2 t}{\tan t}$$

Problem Set 12.8

2.) $f(x,y) = x^2 + 4y^2 - 2x + 8y - 1$

$\nabla f(x,y) = x^2 - 2x \quad 4y^2 + 8y$

$= (2x - 2), (8y + 8) = (0,0) \longrightarrow (1, -1) \rightarrow \text{stationary}$

$D = f_{xx} f_{yy} - f^2_{xy}$

$= (2)(8) - (0)^2$

$= 16 > 0 \quad \longrightarrow f_{xx} = 2 > 0 \rightarrow \text{local minimum} \ (1, -1)$