

(Minggu 2 / Sesi 2-3)

Limits & The Derivatives

1. Tentukanlah nilai limit-limit berikut (jika ada):.

$$\lim_{x \to 2} \frac{x - 2}{e^{x - 2}}$$

$$\lim_{x \to 0} \frac{\sin^3 2x}{x^2 \tan 3x}$$

$$\lim_{x \to \infty} \sqrt{\frac{36x^2 - 25x + 16}{9x^2 - 4x + 1}}$$

 $f(x) = \begin{cases} ax + 6, & x < 2 \\ ax^2 - 2x + 3a, & x \ge 2 \end{cases}$ 2. Diberikan fungsi

Tentukan nilai ^a sehingga f(x) kontinu di x = 2.

3. Tentukan derivatif (turunan) pertama dari fungsi-fungsi berikut :

a.
$$y=(x^2+17)(x^3-3x+1)$$

$$y = \frac{x^2 + 1}{x \sin x}$$

c.
$$y = \ln^2[\cos^2(x^2)]$$

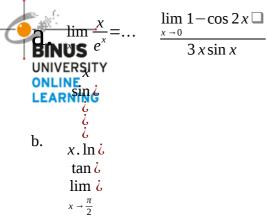
4. Tentukanlah $\frac{dy}{dx}$ jika diketahui persamaan implisit berikut:

a.
$$x^3 - 2x^2y + 3xy^2 - 4y^3 = 5$$

b.
$$e^{5y+2} = y\cos(x^3 - 2)$$

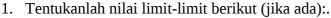
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5. Hitunglah limit bentuk tak tentu berikut :



 $\mathbf{C} \cdot \lim_{x \to 0} (x+1)^{\cot x} = \dots$

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a.
$$\lim_{x\to 2} \frac{x-2}{e^{x-2}} = \frac{(2)-2}{e^{(2)-2}} = \frac{0}{e^0} = \frac{0}{1} = 0$$

b.
$$\lim_{x \to 0} \frac{\sin^3 2x}{x^2 \tan 3x} = \frac{\lim_{x \to 0} \frac{\sin^3 2x}{x^3}}{\frac{x^2 \tan 3x}{x^3}} = \frac{\lim_{x \to 0} \frac{\sin^3 2x}{x^3}}{\frac{\tan 3x}{x}} = \frac{2^3}{3} = \frac{8}{3}$$

C.
$$\lim_{x \to \infty} \sqrt{\frac{36 x^2 - 25 x + 16}{9 x^2 - 4 x + 1}} = \sqrt{\frac{\lim_{x \to \infty} 36 x^2 - 25 x + 16}{9 x^2 - 4 x + 1}}$$

$$i\sqrt{\frac{\lim_{x\to\infty} \frac{36x^2}{x^2} - \frac{25x}{x^2} + \frac{16}{x^2}}{\frac{9x^2}{x^2} - \frac{4x}{x^2} + \frac{1}{x^2}}}$$

$$i\sqrt{\frac{36+0+0}{9+0+0}} = \frac{6}{3} = 2$$

$$f(x) = \begin{cases} ax + 6, & x < 2 \\ ax^2 - 2x + 3a, & x \ge 2 \end{cases}$$
2. Diberikan fungsi

Tentukan nilai a sehingga f(x) kontinu di x = 2. Jawab:

f(x) kontinu jika dan hanya jika
$$\begin{aligned}
x &\to 2 + \mathcal{U}f(x) \\
f(x) &= \mathcal{U}\lim_{\lambda} \mathcal{U} \\
x &\to 2 - \mathcal{U}
\end{aligned}
= L$$

$$\lim_{\lambda} \mathcal{U}$$

$$x \to 2 + \frac{1}{6}ax^{2} - 2x + 3a$$

$$ax + 6 = \frac{1}{6} \lim_{\epsilon} \frac{1}{6}$$

$$x \to 2 - \frac{1}{6} \frac{1}{6}$$

$$\lim_{\epsilon} \frac{1}{6}$$

$$a(2) + 6 = a(2)^{2} - 2(2) + 3a$$

$$2a + 6 = 4a - 4 + 3a$$

$$5a = 10$$

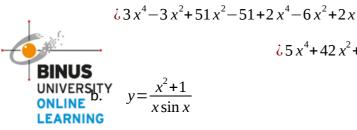
$$a = 2$$

3. Tentukan derivatif (turunan) pertama dari fungsi-fungsi berikut :

a.
$$y=(x^2+17)(x^3-3x+1)$$

Jawab:

$$Dx(x^2+17)(x^3-3x+1)=(x^2+17)(3x^2-3)+(x^3-3x+1)(2x)$$



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$$65x^4 + 42x^2 + 2x - 51$$

$$x$$

$$x^{3}\cos x + x^{2}\sin x + x\cos x + \sin \xi$$

$$\xi$$

$$\xi$$

$$\xi$$

$$\xi$$

$$x$$

$$x\sin \xi$$

$$\xi$$

$$\xi$$

$$x - \xi$$

$$2x^{2}\sin \xi$$

$$\xi \xi$$

$$\begin{array}{c}
x \sin \dot{c} \\
\dot{c} \\
\dot{c} \\
\dot{c}
\end{array}$$

$$\dot{c} \frac{(x^2 - 1)\sin x - (x^3 + x)\cos x}{\dot{c}}$$

c.
$$y = \ln^2 [\cos^2(x^2)]$$

Jawab



Dx

 $\begin{array}{c}
(\dot{c}\dot{c}2) \\
\dot{c} \\
\cos^2 \dot{c} \\
\dot{c} \\
\ln \dot{c} \\
\dot{c}
\end{array}$

x (¿¿2)

 $\frac{\mathcal{L}}{\mathcal{L}}$ $\cos^2 \mathcal{L}$

X

 $\frac{\ln \dot{c}}{\left[Cos^{2}(x^{2})\right] = \dot{c} 2\dot{c}}$ $\ln^{2} \dot{c}$

People Innovation Excellence $x \ (\dot{i}\dot{i}2) \ \dot{i} \ \cos^2{i} \ \dot{i} \ \dot{i} \ (x^2) \ \dot{i} \ \ln{i} \ 2\dot{i} \ \dot{i} \ \dot{i} \ (\dot{i}\dot{i}2) \ 2\cos{i} \ \dot{i} \ x \ (\dot{i}\dot{i}2) \ 2\cos{i} \ \dot{i} \ x \ (\dot{i}\dot{i}2) \ 2\cos{i} \ \dot{i} \ \dot{$

X

(233)

2 cos **¿** (x^2)

$$-\sin \dot{\partial} Dx(x^2)$$

ln ¿

2¿ ¿¿

(235)

 $\cos^2 \mathbf{i}$

i X

(233)

2 cos ¿

 (x^2)

 $-\sin \mathcal{L}(2x)$

ln¿

2 ذ

x (¿¿2)

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cos²¿

 (x^2)

−sin ¿

j (2x)

j ln ¿

2 ¿

 (x^2) cos² ¿

 $\frac{1}{6} - 8x \tan(x^2) \ln \frac{1}{6}$

4. Tentukanlah \overline{dx} jika diketahui persamaan implisit berikut:

a.
$$x^3 - 2x^2y + 3xy^2 - 4y^3 = 5$$
.

Jawab:

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$$\frac{d(x^3)}{dx} - \frac{d(2x^2y)}{dx} + \frac{d(3xy^2)}{dx} - \frac{d(4y^3)}{dx} = \frac{d(5)}{dx}$$
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 $3x^2 - \left((2x^2)\left(\frac{dy}{dx}\right) + (4x)(y)\right) + \left((3x)\left(2y\frac{dy}{dx}\right) + (3)(y^2)\right) - \left(12y^2\frac{dy}{dx}\right) = 0$

$$3x^2 - 4xy + 3y^2 + \frac{dy}{dx} (3x2y - 2x^2 - 12y^2) = 0$$

$$\frac{dy}{dx} = \frac{4xy - 3y^2 - 3x^2}{3x2y - 2x^2 - 12y^2}$$

b.
$$e^{5y+2} = y\cos(x^3 - 2)$$

Jawab :

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$$\frac{d(e^{5y+2})}{dx} = \frac{d(y\cos(x^3-2))}{dx}$$

$$\frac{d(e^{5y+2})}{dx} = \frac{d(y\cos(x^3-2))}{dx}$$

$$-\sin(x^3-2)(3x^2)$$

$$e^{5y+2}\frac{d(5y+2)}{dx} = \cos(x^3-2)\left(\frac{dy}{dx}\right) + yi$$

$$5e^{5y+2}\left(\frac{dy}{dx}\right) - \cos(x^3-2)\left(\frac{dy}{dx}\right) = -2x^2y\sin(x^3-2)$$

$$\frac{5e}{dx}$$

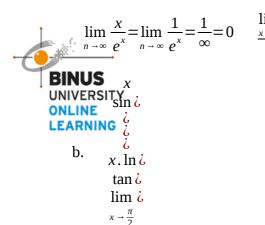
$$\frac{dy}{dx} = \frac{-2x^2y\sin(x^3-2)}{5e^{5y+2}-\cos(x^3-2)}$$

5. Hitunglah limit bentuk tak tentu berikut :

a.
$$\lim_{x \to \infty} \frac{x}{e^x} = \dots$$

Jawab:

Karena bentuk fungsi tersebut jika dilimitkan pada x menuju tak hingga berbentuk tak pasti atau *undefined* maka fungsi tersebut bisa memakai dalil l'hopital.



Jawab:

People Innovation Excellence Karena fungsi tersebut membentuk 0/0 jika limit x mendekati pi/2 maka fugnsi tersebut bisa menggunakan dalil l'hopital

$$\begin{array}{c}
x \\
\sin \lambda \\
\frac{\lambda}{\lambda} \\
\lambda \\
x \\
\sin \lambda \\
\frac{\lambda}{\lambda} \\
\ln \left(\frac{\lambda}{\cot x} \lambda\right) \\
\frac{\lambda}{\cot x} \\
\ln \left(\frac{\lambda}{\cot x} \lambda\right) \\
\ln \left(\frac{\lambda}{\cot x} \lambda\right) \\
\frac{\lambda}{\cot x} \\
\ln \left(\frac{\lambda}{\cot x} \lambda\right) \\
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\ln \left(\frac{\lambda}{\cot x} \lambda\right) \\
\frac{\lambda}{\cot x} \\
\frac{$$

c.
$$\lim_{x \to 0} (x+1)^{\cot x} = \dots$$

$$jawab:$$

$$\lim_{x \to 0} (x+1)^{\cot x} = \lim_{x \to 0} e^{\ln(x+1)^{\cot x}} = \lim_{x \to 0} e^{\cot x \ln(x+1)}$$

Dengan memakai teorema limit komposit

Teorema E Teorema Limit Komposit

Jika $\lim_{x \to c} g(x) = L$ dan jika f kontinu di L, maka

$$\lim_{x \to c} f(g(x)) = f\left(\lim_{x \to c} g(x)\right) = f(L)$$

Khususnya, jika gkontinu dicdan fkontinu di g(c), maka fungsi komposit $f\circ g$ kontinu di c.

$$e^{\lim_{x\to 0}\cot x \ln(x+1)} = e^{\frac{\lim_{x\to 0} (\ln(x+1))}{\cot(x)}}$$

Pada bagian $\frac{\lim_{x\to 0} (\ln (x+1))}{\frac{1}{\cot (x)}}$ menggunakan dalil L'hopital karena jika dilimitkan x menuju 0

akan menghasilkan 0/0, maka

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$$\frac{\lim_{x \to 0} \left(\ln(x+1) \right)}{\frac{1}{\cot(x)}} = \frac{\lim_{x \to 0} \left(\ln(x+1) \right)}{\tan(x)} = \frac{\lim_{x \to 0} \frac{1}{x+1}}{\sec^2 x} = \frac{1}{1} = 1$$

Didapatkan $\frac{\lim_{x\to 0} \left(\ln(x+1)\right)}{\frac{1}{\cot(x)}}$ menghasilkan 1, dan $\frac{\left(\ln(x+1)\right)}{\frac{1}{\cot(x)}}$ kontinu di 0.

Jika dilihat fungsi f(x) pada soal adalah e^x kontinu di 1, maka dapat dicari nilai dari e^x dengan dilimitkan x menuju 1

$$\lim_{x \to 1} e^x = e$$

Nilai dari $\lim_{x\to 0} (x+1)^{\cot x}$ adalah e

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