Endogenous entrepreneurship and financial frictions

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1 Motivation/Question

2 The Model

- There is a continuum of individuals who differ in their wealth a and entrepreneurial ability z.
- Each period, every individual chooses to be a Worker or an Entrepreneur.
- Entrepreneurs are subject to a collateral constraint.
- Entrepreneurial ability z is drawn from a Pareto distribution with pdf $\mu(z) = \eta z^{-\eta-1}, z \geqslant 1$. Entrepreneurial ability is persistent: each period a new z is drawn with a probability γ.
- Financial intermediaries collect deposits and rent out capital to entrepreneurs at rate R, by no arbitrage then $R = r + \delta$ with r the deposit rate.
- There is a representative public firm.
- The government taxes entrepreneurial profits and revenues at rates τ^{π} and τ^{y} a and rebates all the receipts with lump-sum payments T_t.

2.1 Individuals' problem

Individuals have CRRA preferences over consumption

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma} - 1}{1-\sigma},$$

with inverse elasticity of intertemporal substitution σ.

The budget constraint is

$$c_t + a_{t+1} \leqslant \max\{w_t, \pi(z_t, a_t)\} + (1 + r_t) a_t + T_t,$$

where the max operator encompasses the binary occupation choice.

Profits from operating the production technology are given by

$$\begin{aligned} \pi(z_t, a_t) = \max_{l_t, k_t} & \left\{ (1 - \tau_t^\pi) \left[(1 - \tau_t^y) z_t Z_t \left(k_t^\alpha l_t^{1-\alpha} \right)^{1-\nu} - w_t l_t - (\delta + r_t) k_t \right] \right\}, \\ \text{s.t.} \quad & k_t \leqslant \lambda a_t. \end{aligned}$$

Hence individuals at productivity z_t choose to become entrepreneurs if their wealth exceeds the threshold value $\bar{a}(z_t)$ that solves

$$w_t = \pi(z_t, \bar{a}(z_t)).$$

The technology operated by individual entrepreneurs features decreasing returns to scale

$$y_t = f(z, k, l) = zZ \left(k^\alpha l^{1-\alpha} \right)^{1-\nu},$$

with Z aggregate TFP shocks that varies with time.

2.2 Public firm

A representative public firm operates the CRS technology

$$F(K_{ct}, L_{ct}) = Z_t Z_{ct} K_{ct}^\alpha L_{ct}^{1-\alpha}.$$

From FOCs:

$$r_t = F_K(K_{ct}, L_{ct}) - \delta = \alpha Z_t Z_{ct} \left(\frac{K_{ct}}{L_{ct}}\right)^{\alpha-1} - \delta,$$
$$w_t = F_L(Z_{ct}, K_{ct}, L_{ct}) = (1-\alpha) Z_t Z_{ct} \left(\frac{K_{ct}}{L_{ct}}\right)^{\alpha},$$

and so capital-labor ratio

$$\frac{K_{ct}}{L_{ct}} = \left(\frac{\alpha Z_t Z_{ct}}{r_t + \delta}\right)^{\frac{1}{1-\alpha}}.$$

2.3 Equilibrium

Given an initial distribution $D_0(z, a)$, a competitive equilibrium is allocations

$$\{c_t(z_t, a_t), a_{t+1}(z_t, a_t), l_t(z_t, a_t), k_t(z_t, a_t)\}$$

, distributions $D_t(z_t, a_t)$, and prices $\{w_t, r_t\}$ such that

1. Given prices, the policy functions solve the individuals' problem.

2. Labor market clears:

$$L_{ct} + \int_E l_t(z, a)\, dD_t(a, z) - \int_W dD_t(a, z) = 0.$$

3. Asset market clears:

$$K_{ct} + \int_E k_t(z, a)\, dD_t(a, z) - \int a D_t(a, z) = 0.$$

4. Goods market clears (by Walras's Law).

5. The government's budget is balanced

$$T_t = \int_E \left[\left(\tau_t^{\pi} + \tau_t - \tau_t^{\pi}\tau_t\right) y_t(z, k, l) - \tau_t^{\pi}\left(w_t l_t(z, a) + (\delta + r_t) k_t(z, a)\right)\right] dD_t(a, z).$$

6. The joint distribution evolves according to

$$D^E_{t+1}\left(a_{t+1}, z_{t+1}\right) = \gamma D^E_t\left(a^{-1}_{t+1}\left(z_t, a_t\right), z_t\right) + (1-\gamma)\int_z D^E_t\left(a^{-1}_{t+1}\left(z_t, a_t\right), z_t\right),$$
$$= \left[\gamma I + (1-\gamma)\Pi\right] D^E_t\left(a^{-1}_{t+1}\left(z_t, a_t\right), z_t\right),$$

where $\Pi = \begin{bmatrix} \pi_z \\ \pi_z \\ \cdots \end{bmatrix}$.

3 Solving the model

3.1 Solving for entrepreneurial profits

The indirect profit function is

$$\pi\left(z_t, a_t\right) = \max_{l_t, k_t}\left\{\left(1-\tau^\pi_t\right)\left[\left(1-\tau^y_t\right) z_t Z_t\left(k^\alpha_t l^{1-\alpha}_t\right)^{1-\nu} - w_t l_t - \left(\delta + r_t\right) k_t\right]\right\},$$
$$\text{s.t.} \quad k_t \leqslant \lambda a_t.$$

If the collateral constraint does not bind, profit maximization implies

$$k^u\left(z_t\right) = \left[\left(1-\tau^y_t\right) z_t Z_t\right]^{\frac{1}{\nu}}\left(\frac{\alpha(1-\nu)}{r_t+\delta}\right)^{\frac{1-(1-\alpha)(1-\nu)}{\nu}}\left(\frac{(1-\alpha)(1-\nu)}{w_t}\right)^{\frac{(1-\alpha)(1-\nu)}{\nu}}.$$

If the collateral constraint binds (with θ the Lagrange multiplier)

$$k^c\left(a_t\right) = \lambda a_t.$$

Hence the capital policy function is

$$k\left(z_t, a_t\right) = \min\{k^c\left(a_t\right), k^u\left(z_t\right)\}.$$

and the labor policy function is

$$l\left(z_t, a_t\right) = \left(\frac{(1-\alpha)(1-\nu)\left(1-\tau^y_t\right) z_t Z_t}{w_t}\right)^{\frac{1}{1-(1-\alpha)(1-\nu)}} k\left(z_t, a_t\right)^{\frac{\alpha(1-\nu)}{1-(1-\alpha)(1-\nu)}}.$$

and the indirect profit function is given by

$$\pi\left(z_t, a_t\right) = \left(1-\tau_t^{\pi}\right)\left[\left(1-\tau_t^{y}\right) z_t Z_t \left(k\left(z_t, a_t\right)^{\alpha} l\left(z_t, a_t\right)^{1-\alpha}\right)^{1-\nu} - w_t l\left(z_t, a_t\right) - \left(\delta + r_t\right) k\left(z_t, a_t\right)\right].$$

Note that the tax on profits τ_t^{π} does not affect the optimal policies, while the tax on revenue τ_t^{y} does.

3.2 Solving the individuals' problem

We can write the Bellman as

$$\begin{aligned} v_t\left(a_t, z_t\right) &= \max_{c_t, a_{t+1}} \left\{u\left(c_t\right) + \beta\left[\gamma v_{t+1}\left(a_{t+1}, z_t\right) + (1-\gamma)\int v_{t+1}\left(a_{t+1}, z_{t+1}\right)\mu\left(z_{t+1}\right) dz_{t+1}\right]\right\}, \\ &\text{s.t.} \quad c_t + a_{t+1} = M\left(a_{t+1}, z_t\right) + \left(1+r_t\right) a_t + T_t, \end{aligned}$$

where $M\left(a_{t+1}, z_t\right) = \max\{w_t, \pi\left(z_t, a_t\right)\}$ and the Euler equation is

$$\begin{aligned} u'\left(c_t\left(z_t, a_t\right)\right) = &\beta\left[\gamma\left(1 + r_{t+1}^{eff}\left(z_t, a_{t+1}, d_{t+1}\right)\right) u'\left(c_{t+1}\left(z_t, a_{t+1}\right)\right),\right. \\ &\left. + (1-\gamma)\int\left(1 + r_{t+1}^{eff}\left(z_{t+1}, a_{t+1}, d_{t+1}\right)\right) u'\left(c_{t+1}\left(z_{t+1}, a_{t+1}\right)\right)\mu\left(z_{t+1}\right) dz_{t+1}\right], \end{aligned}$$

where the net effective return is defined as

$$r_t^{eff}\left(z_t, a_t\right) = \begin{cases} r_t & \text{if Worker,} \\ r_t + \frac{\partial \pi\left(z_t, a_t\right)}{\partial a_t} & \text{if Entrepreneur,} \end{cases}$$

where

$$\frac{\partial \pi\left(z_t, a_t\right)}{\partial a_t} = \begin{cases} \frac{\partial \pi^{const}\left(z_t, a_t\right)}{\partial a_t} & \text{if constrained Entrepreneur,} \\ 0 & \text{if unconstrained Entrepreneur.} \end{cases}$$

3.2.1 Obtaining policies

Inputs: prices $\{w_t, r_t\}$, government policy $\{\tau_t^{\pi}, T_t\}$ and consumption on next period's asset grid $c_{t+1}\left(z_{t+1}, a_{t+1}\right)$.

Calculate income $M\left(a_{t+1}, z_t\right)$ on the grids for a and z from the indirect profit function.

Calculate next period's cash-on-hand on the grids for a and z

$$coh\left(z_{t+1}, a_{t+1}\right) = M\left(z_{t+1}, a_{t+1}\right) + \left(1+r_t\right) a_{t+1} + T_t.$$

Compute the RHS of the Euler equation

$$
\begin{aligned}
\mathrm{RHS}\left(a_{t+1}, z_t\right) = \beta \Bigg[\gamma \Bigg\{ & \left(1 + r^{eff}_{t+1}\left(z_t, a_{t+1}\right)\right) u'\left(c_{t+1}\left(z_t, a_{t+1}\right)\right) \\
& + (1-\gamma) \int \Bigg\{ \left(1 + r^{eff}_{t+1}\left(z_{t+1}, a_{t+1}\right)\right) u'\left(c_{t+1}\left(z_{t+1}, a_{t+1}\right)\right) \Bigg\} \mu\left(z_{t+1}\right) dz_{t+1} \Bigg],
\end{aligned}
$$

and invert to get current consumption

$$
c\left(a_{t+1}, z_t\right) = u^{-1}\left(\mathrm{RHS}\left(a_{t+1}, z_t\right)\right).
$$

Calculate the asset policy functions with the mapping from assets today

$$
a_t = \frac{c_t\left(a_{t+1}, z_t\right) + a_{t+1} - M\left(a_{t+1}, z_t\right) - T_t}{1 + r_t},
$$

to assets tomorrow a_{t+1} that we can invert by interpolation to get the policy function $a_{t+1}\left(a_t, z_t\right)$.

Enforce the borrowing constraint, ie enforce $a_{t+1}\left(a_t, z_t\right) \geqslant 0$.

Compute the consumption policy functions as

$$
c_t\left(a_t, z_t\right) = \mathrm{coh}\left(z_t, a_t\right) - a_{t+1}\left(a_t, z_t\right),
$$

and if $c_t\left(a_t, z_t\right) < 0$, set $c_t\left(a_t, z_t\right) = 0$ and $a_{t+1}\left(a_t, z_t\right) = \mathrm{coh}\left(z_t, a_t\right)$.

3.2.2 Obtaining the distribution

Given a distribution at t, the distribution at $t+1$ is obtained as

$$
D_{t+1}\left(a_{t+1}, z_{t+1}\right) = \gamma D_t\left(a^{-1}_{t+1}\left(z_t, a_t; w, r\right), z_t\right) + (1-\gamma) \sum_{\hat{z} \in \mathcal{Z}} \mu\left(\hat{z}\right) D_t\left(a^{-1}_{t+1}\left(\hat{z}, a_t; w, r\right), \hat{z}\right),
$$

and the stationary distribution is obtained by iterating until convergence.

In matrix form, we stack the vector $\mu(z)$ in a matrix Π and we can do

$$
D_{t+1}\left(a_{t+1}, z_{t+1}\right) = \left(\gamma I + (1-\gamma)\Pi\right) D_t\left(a^{-1}_{t+1}\left(z_t, a_t; w, r\right), z_{t+1}\right).
$$

4 Algorithm to solve for the general equilibrium

1. Fix grid for states a and z.

2. Set a guess for the interest rate r.

3. Given r, use the public sector to find w.

4. Given prices (r, w), and a guess for T, find the individuls' policy functions.

5. Given policies, find the stationary distribution.

6. Compute the implied transfers T and if not close, use the updated T and go back to step 4.

7. Aggregate policies and distributions to find assets and savings of the private sector and back out K_c from asset market clearing condition.

8. Compute error in labor market and update r until convergence.

Algorithm 1 Solve General Equilibrium

```
Fix grid for states a and z
Initialize guess for interest rate r
while not converged on r do
    Use public sector to compute w given r
    Initialize guess for T
    while not converged on T do
        Compute individuals' policy functions given prices (r, w) and T
        Compute stationary distribution given policies
        Compute implied transfers T_new
        if |T_new − T| > tolerance then
            Set T = T_new                                   ▷ or some update rule
        else
            Converged on T
        end if
    end while
    Aggregate policies and distributions to compute private sector assets and savings
    Back out K_c from asset market clearing condition
    Compute labor market error using aggregates
    if error > tolerance then
        Update r based on error
    else
        Converged on r
    end if
end while
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