Dynamic bycicle model

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1 Simple dynamic model

//TODO Inserire figura

$$\begin{aligned} m_T\ddot{x} &= F_{xF}cos(\psi+\delta) + F_{xR}cos(\psi) - F_{yF}sin(\psi+\delta) - F_{yR}sin(\psi) \\ m_T\ddot{y} &= F_{xF}sin(\psi+\delta) + F_{xR}sin(\psi) + F_{yF}cos(\psi+\delta) + F_{yR}cos(\psi) \\ I_T\ddot{\psi} &= F_{xF}asin(\delta) + F_{yF}acos(\delta) - F_{yR}b \end{aligned} \tag{1}$$

$$\begin{aligned} \alpha_F &= arctan(\frac{\dot{y}+a\dot{\psi}cos(\psi)}{\dot{x}-a\dot{\psi}sin(\psi)}) - (\delta+\psi) \\ \alpha_R &= arctan(\frac{\dot{y}-b\dot{\psi}cos(\psi)}{\dot{x}+b\dot{\psi}sin(\psi)}) - \psi \end{aligned} \tag{2}$$

At first, as state vector this has been used

$$\begin{aligned} z1 &= x \\ z2 &= y \\ z3 &= \psi \\ z4 &= \dot{x} \\ z5 &= \dot{y} \\ z6 &= \dot{\psi} \end{aligned} \tag{3}$$

So that

$$
\begin{aligned}
\dot{z_1} &= z_4 \\
\dot{z_2} &= z_5 \\
\dot{z_3} &= z_6 \\
\dot{z_4} &= \frac{F_{xF}cos(z_3+\delta)+F_{xR}cos(z_3)-F_{yF}sin(z_3+\delta)-F_{yR}sin(z_3)}{m_T} \\
\dot{z_5} &= \frac{F_{xF}sin(z_3+\delta)+F_{xR}sin(z_3)+F_{yF}cos(z_3+\delta)+F_{yR}cos(z_3)}{m_T} \\
\dot{z_6} &= \frac{F_{xF}asin(\delta)+F_{yF}acos(\delta)-F_{yR}b}{I_T}
\end{aligned}
\tag{4}
$$

With slip angles

$$
\begin{aligned}
\alpha_F &= arctan(\frac{z_5+az_6cos(z_3)}{z_4-az_6sin(z_3)})-(\delta+z_3) \\
\alpha_R &= arctan(\frac{z_5-bz_6cos(z_3)}{z_4+bz_6sin(z_3)})-z_3
\end{aligned}
\tag{5}
$$

Now instead of using $\dot{x}$ and $\dot{y}$, v_T and α_T have been used. The trasformations are the following:

$$
\begin{aligned}
\dot{x} &= v_T cos(\psi+\alpha_T) \\
\dot{y} &= v_T sin(\psi+\alpha_T) \\
\\
\ddot{x} &= \dot{v_T}cos(\psi+\alpha_T)-v_T(\dot{\psi}+\dot{\alpha_T})sin(\psi+\alpha_T) \\
\ddot{y} &= \dot{v_T}sin(\psi+\alpha_T)+v_T(\dot{\psi}+\dot{\alpha_T})cos(\psi+\alpha_T)
\end{aligned}
\tag{6}
$$

Substituting and simplyfing with the help of Matlab

$$
\begin{aligned}
\dot{v_T} &= \frac{F_{xF}cos(\alpha_T-\delta)+F_{xR}cos(\alpha_T)+F_{yF}sin(\alpha_T-\delta)+F_{yR}sin(\alpha_T)}{m_T} \\
\dot{\alpha_T} &= \frac{-F_{xF}sin(\alpha_T-\delta)-F_{xR}sin(\alpha_T)+F_{yF}cos(\alpha_T-\delta)+F_{yR}cos(\alpha_T)-m_Tv_T\dot{\psi}}{m_Tv_T} \\
\ddot{\psi} &= \frac{F_{xF}asin(\delta)+F_{yF}acos(\delta)-F_{yR}b}{I_T}
\end{aligned}
\tag{7}
$$

$$\begin{aligned} \alpha_F &= arctan(\frac{v_T sin(\alpha_T) + a\dot{\psi}}{v_T cos(\alpha_T)}) - \delta \\ \alpha_F &= arctan(\frac{v_T sin(\alpha_T) - b\dot{\psi}}{v_T cos(\alpha_T)}) \end{aligned} \tag{8}$$

The new state and the state equations are

$$\begin{aligned} x1 &= x \\ x2 &= y \\ x3 &= \psi \\ x4 &= v_T \\ x5 &= \alpha_T \\ x6 &= \dot{\psi} \end{aligned}$$

$$\begin{aligned} \dot{x_1} &= x_4 cos(x_3 + x_5) \\ \dot{x_2} &= x_5 sin(x_3 + x_5) \\ \dot{x_3} &= x_6 \\ \dot{x_4} &= \frac{F_{xF} cos(x_5 - \delta) + F_{xR} cos(x_5) + F_{yF} sin(x_5 - \delta) + F_{yR} sin(x_5)}{m_T} \\ \dot{x_5} &= \frac{-F_{xF} sin(x_5 - \delta) - F_{xR} sin(x_5) + F_{yF} cos(x_5 - \delta) + F_{yR} cos(x_5) - m_T x_4 x_6}{m_T x_4} \\ \dot{x_6} &= \frac{F_{xF} a sin(\delta) + F_{yF} a cos(\delta) - F_{yR} b}{I_T} \end{aligned} \tag{9}$$