POLITECNICO DI MILANO

Slip Control (ABS/Traction control)

Automation and Control in Vehicles

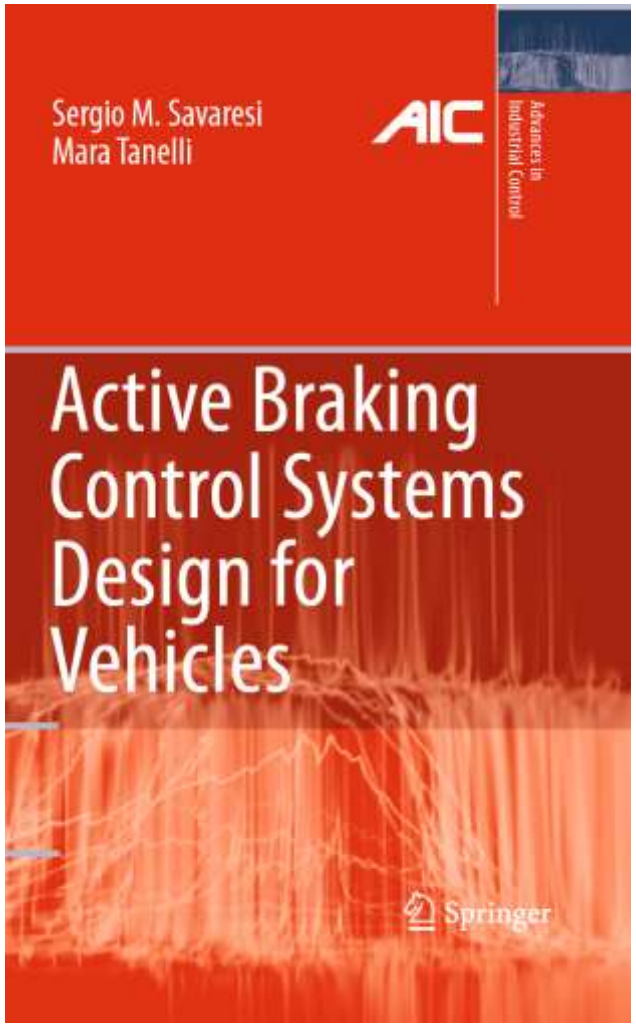
Sergio M. Savaresi

http://move.dei.polimi.it/

Main references:

TA Johansen, I. Petersen, Kalkkuhl J., J. Lüdemann (2003). "Gain-Scheduled Wheel Slip Control in Automotive Brake Systems." IEEE Transactions on Control Systems Technology, Vol 11, No. 6, pp.799-811.

Savaresi s.m., M. Tanelli, C. Cantoni (2007). Mixed slip-deceleration control in automotive braking systems. ASME Transactions: Journal of Dynamic Systems, Measurement and Control, Vol.129, No. 1, pp.20-31 [*2008 ASME Dynamic Systems and Control Rudolf Kalman Best Paper Award*][Patent]



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Dissemination papers

Tecnica | Dossier Traction control
Sistemi di guida

di Sergio Matteo Savaresi
Illustrazioni di Patrizia Renoldi

Nome dell'imputato:

TRACTION CONTROL

Conta più il pilota o l'elettronica? Abbiamo chiesto a un esperto di spiegarci come funziona davvero il controllo di trazione, quali sono le sue potenzialità, i suoi limiti e i suoi sviluppi futuri. Scoprendo che nella battaglia tra uomo ed elettronica non tutto è perduto: a patto di fare le scelte giuste

Dopo lo sfavillante successo della Ducati nel motomondiale 2007, e mentre il *traction control* veniva vietato in F1 a partire dal 2008, le discussioni sui sistemi elettronici hanno acceso il dibattito. Traction control: sì o no?
Soprattutto in Italia, siamo abituati a schierarci su qualunque terna; figuriamoci su questo. Negli ultimi mesi opinionisti e addetti ai lavori si sono scatenati a dire la loro, da tutte le direzioni ed in tutte le direzioni, contribuendo a rendere la discussione molto accesa ed incerta. Intanto, nel primo inverno trascorso da campioni del mondo, i tecnici Ducati hanno annunciato e messo in produzione la 1098R, la prima moto di serie dotata di un controllo di trazione abbastanza sofisticato da lavorare per aumentare le prestazioni, e non solo la sicurezza.

Nel mondo della moto il tema delle prestazioni è centrale, intatti le reazioni non si sono fatte attendere. Piloti dal cuore, e soprattutto dal polso destro particolarmente sensibile, hanno gettato altra benzina su questo improvviso fuoco di opinioni, ed il dibattito ha preso una piega più ruspante e radicale, trasformandosi nella più generale questione "elettronica sì o elettronica no?" e facendo convergere sul traction-control tutte le passioni e le paure che la parola "elettronica" suscita negli appassionati di moto (ma non solo).
Al Dipartimento di Elettronica e Informazione del Politecnico di Milano, ci occupiamo da anni di sistemi elettronici di controllo per veicoli, in collaborazione con le maggiori aziende. Questo articolo ha dunque l'ambizione di spiegare come stanno le cose non con luoghi comuni, ma con qualche fondamento tecnico (e qualche inevitabile semplificazione).

Il controllo di trazione (*Traction Control*, TC) è fondamentalmente un controllo del pneumatico: si occupa cioè di far lavorare il pneumatico al meglio delle prestazioni che può fornire. Per capire come funziona e cosa deve fare, bisogna quindi sapere **non tanto come funziona il motore, quanto come funziona il pneumatico**. Una volta capito questo aspetto, avremo tutti gli elementi per capire cosa succede quando si apre la manetta del gas: ieri, oggi… e domani!

Perdere aderenza... un po'

Partiamo subito con il concetto chiave dello **slittamento**. Lo slittamento del pneumatico (figura 1) è la differenza percentuale fra la sua velocità di rotolamento e la velocità di avanzamento effettiva del veicolo. Per farla semplice: se viaggiamo lentamente a velocità costante possiamo dire che lo slittamento è zero, perché le

Dissemination papers

CONTROLLO DI TRAZIONE

Il limite perfetto

Con il sistema APRC dell'Aprilia RSV4, l'elettronica delle moto è arrivata nel futuro. Abbiamo fatto quattro chiacchiere con chi la conosce bene

di Alberto Dell'Orto, foto Alex Photo, grafici di Raffaele Poblete

64 ++ duemote Febbraio 2011

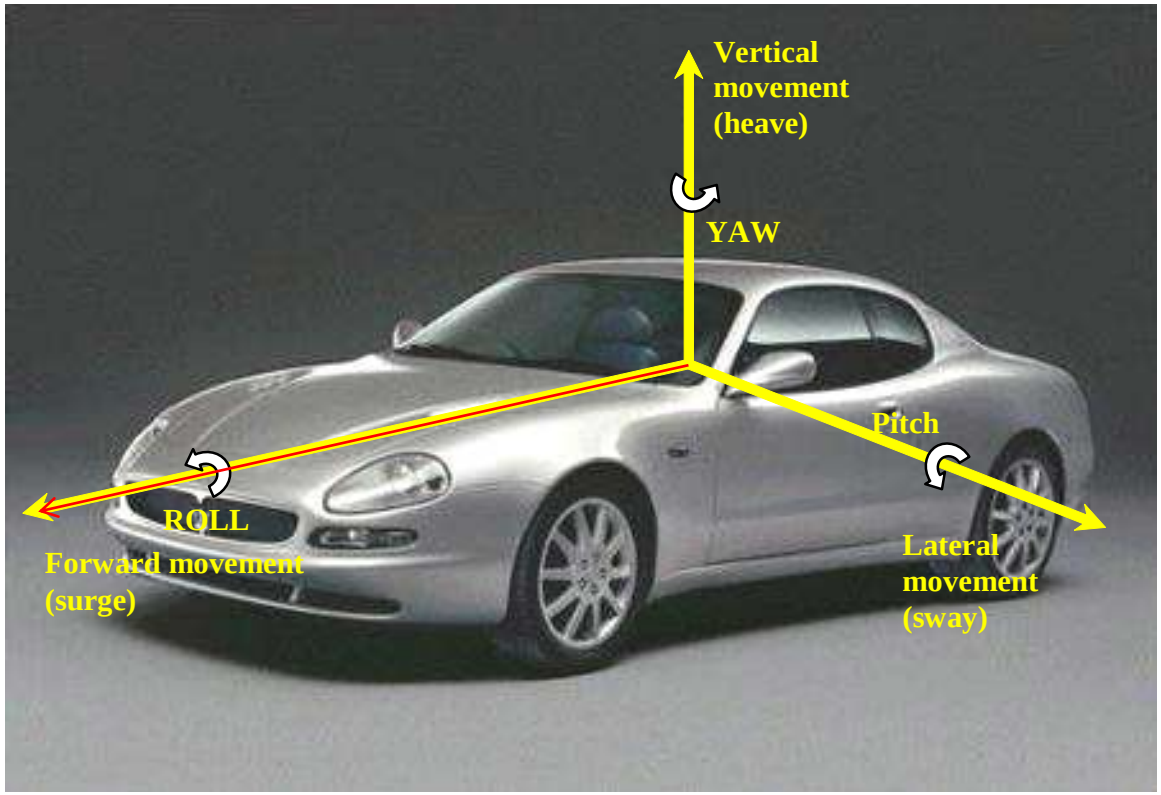
Dissemination papers

IL "TRACTION" FA GUADAGNARE POCHI DECIMI SUL GIRO SECCO, MA SUL PASSO GARA DÀ VANTAGGI MOLTO MAGGIORI PERCHÉ RIDUCE L'USURA GOMME

Ducati) e dell'appeal che può avere un sistema così completo. Il risultato è che questa moto offre ben 216 combinazioni se nel "pacchetto elettronico" comprendiamo anche le mappature (Sport, Track, Road) e il Launch Control (ALC) regolabile su 3 posizioni: una PlayStation a due ruote? Qualcosa di simile, ma presa dimesti

L'UNIVERSITÀ DELLA DERAPATA (CONTROLLATA)
Il controllo di trazione Aprilia può essere regolato su 8 livelli (tramite comandi al manubrio sinistro) anche durante la marcia, contrariamente alle altre sportive; inoltre, si adatta autonomamente a differenti pneumatici. La centralina "gestisce" l'erogazione del motore dopo aver elaborato i dati che riceve dal RBW, dalle ruote foniche (i piccoli dischi fissati ai mozzi ruota, foto sopra) e dalla piattaforma inerziale. La forcella è una Öhlins Racing upside-down da 43 mm con steli trattati superficialmente (Tin) e piedini forgiati in alluminio. La sua escursione è di 120 mm. Il cruscotto è stato rivisto per mostrare meglio al pilota, in modalità *race*, le informazioni dedicate all'uso in pista. La spia gialla tra la folle e la riserva lampeggia quando il controllo di trazione entra in azione. Il sistema aPRC di Aprilia è stato sviluppato anche attraverso una intensa collaborazione con il gruppo di ricerca "MOVE" del Dipartimento di Elettronica e Informazione del Politecnico di Milano, specializzato nello sviluppo di sistemi di controllo elettronici per veicoli, diretto dal Prof. Sergio Savaresi (http://move.dei.polimi.it/). I sistemi sviluppati sono stati brevettati (tre brevetti indipendenti, ciascuno riguardante uno specifico modulo del sistema), e fanno uso di tecniche molto avanzate di controllo automatico e di stima parametrica. Questa collaborazione rappresenta uno dei rari casi in cui una intensa collaborazione fra una azienda motociclistica ed una Università riesce a far nascere un prodotto d'avanguardia, contribuendo a creare un vantaggio competitivo per l'industria italiana basato su innovazione e tecnologia"

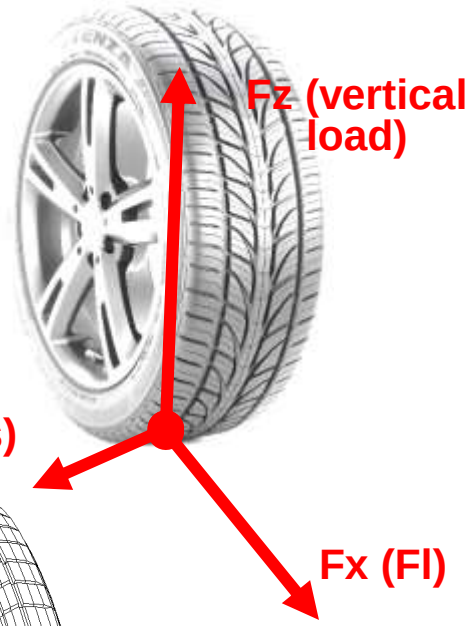
6dof...



Braking&traction-control
= longitudinal (forward) movement

Tire-road contact forces: introduction

Separable (along the road surface) into two orthogonal components: Fz (vertical load) and
longitudinal (*Fl* or *Fx*) and lateral/side (*Fs* or *Fy*)

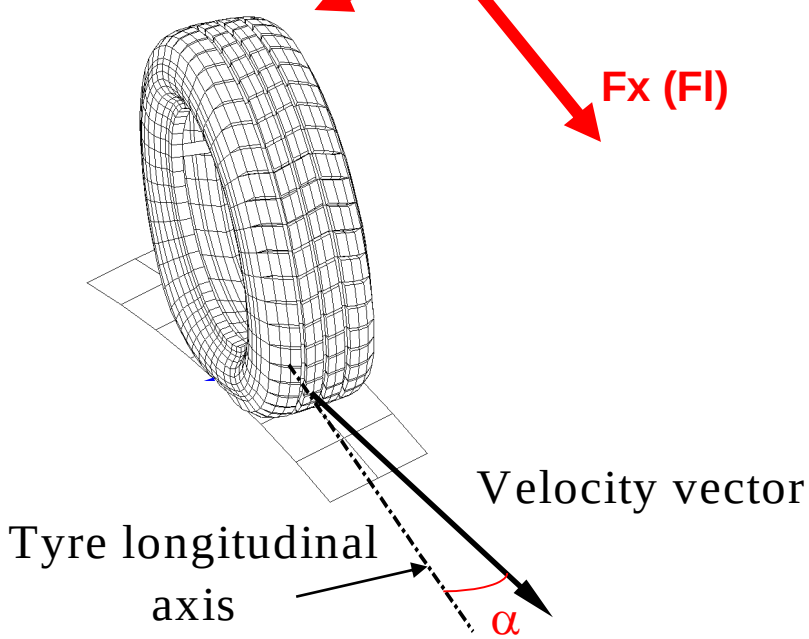


$$F_l(\lambda, \alpha, F_z, \gamma) \qquad \mu_l := \frac{F_l}{F_z}$$

$$F_s(\lambda, \alpha, F_z, \gamma) \qquad \mu_s := \frac{F_s}{F_z}$$

μ : friction coefficient

Property of road&tire



λ= longitudinal slip
α= Slip angle (side-slip)
[γ= Camber angle]
Fz= Vertical load

Road-tire contact forces: the definition of (longitudinal) slip

$$\lambda = \frac{v - \omega r}{\max\{ v, \omega r \}}$$

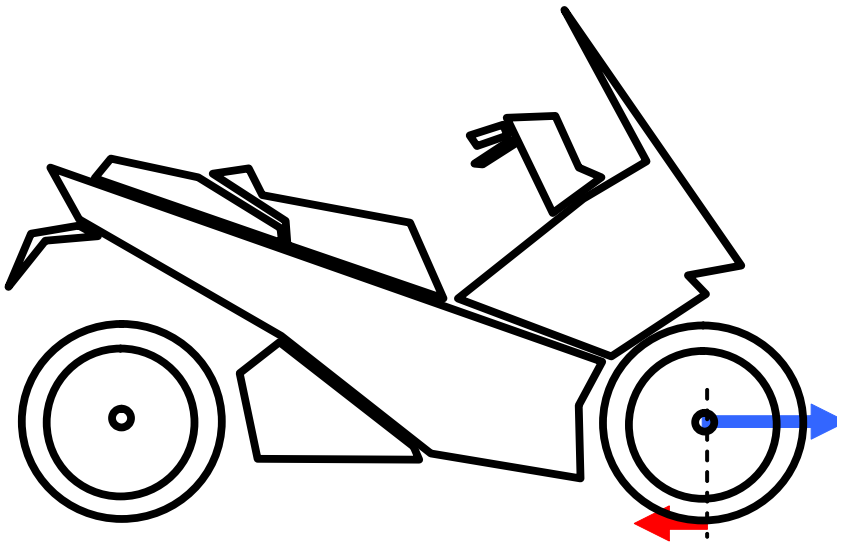
If brake:

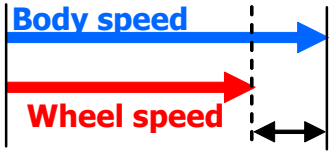
$$\lambda := \frac{v - \omega r}{v}$$

If traction:

$$\lambda = \frac{-(v - \omega r)}{\omega r}$$

v: body speed
r: rolling radius
ω: angular wheel speed





"SLIP" $\lambda = \dfrac{\text{Body speed} - \text{Wheel speed}}{\text{Body speed}}$

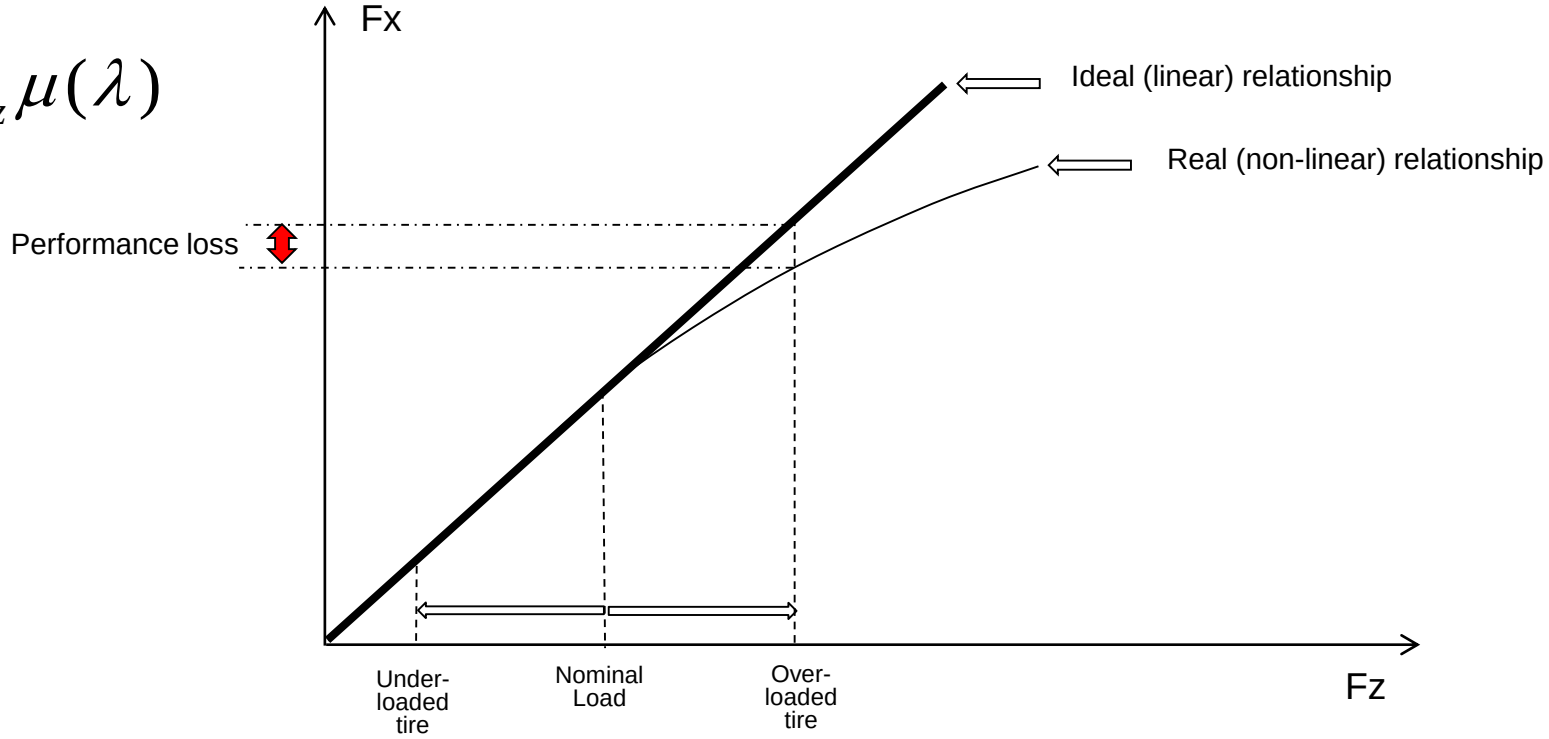
Linearity with respect to Fz - REMARK

Consider the simplified relationship for Fx (alpha=0; gamma=0)

$$F_x(F_z, \lambda)$$

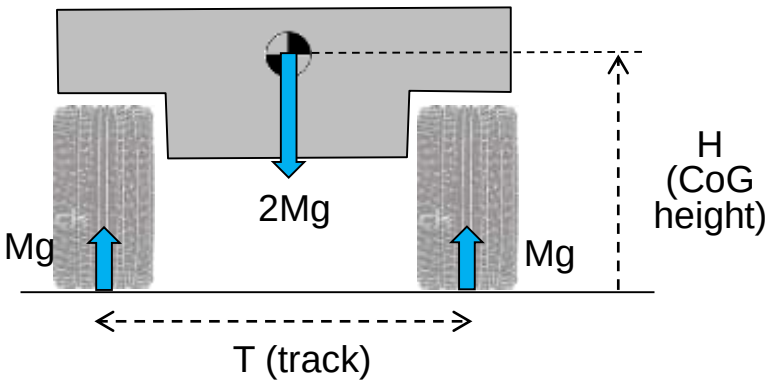
Usually it is approximated by assuming a LINEAR relationship:

$$F_x = F_z \mu(\lambda)$$



Similar behaviour for Fy

Linearity with respect to Fz
example: lateral load transfer

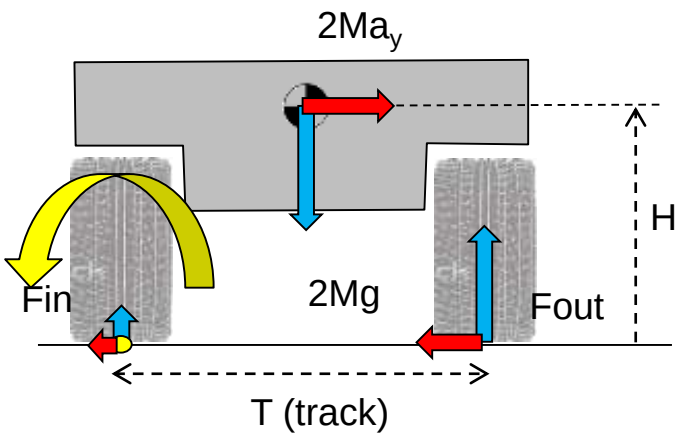


$$TF_{out} = \frac{T}{2} 2Mg + H\, 2Ma_y$$

$$F_{out} = Mg + 2\frac{H}{T} Ma_y$$

Rollover limit (Fin=0; Fout=2Mg):

$$a_y = g\left(0.5\frac{T}{H}\right)$$



Similar equations for front-rear load-transfer

Tire-road contact forces: dynamics

$$F_l(\lambda, \alpha, F_z, \gamma)$$
$$F_s(\lambda, \alpha, F_z, \gamma)$$

are algebraic relations

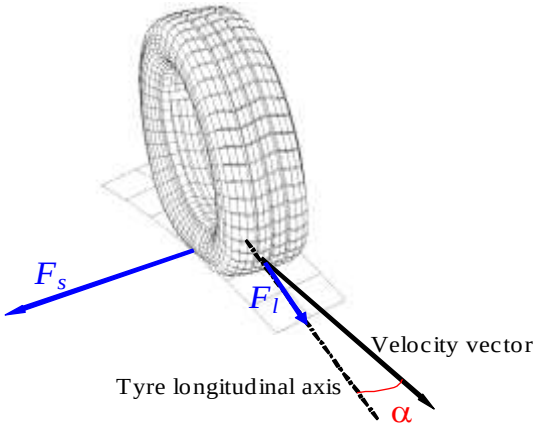
The real behavior is dynamic, due to the transients in tire deformation.

Dynamics (simplified) model:

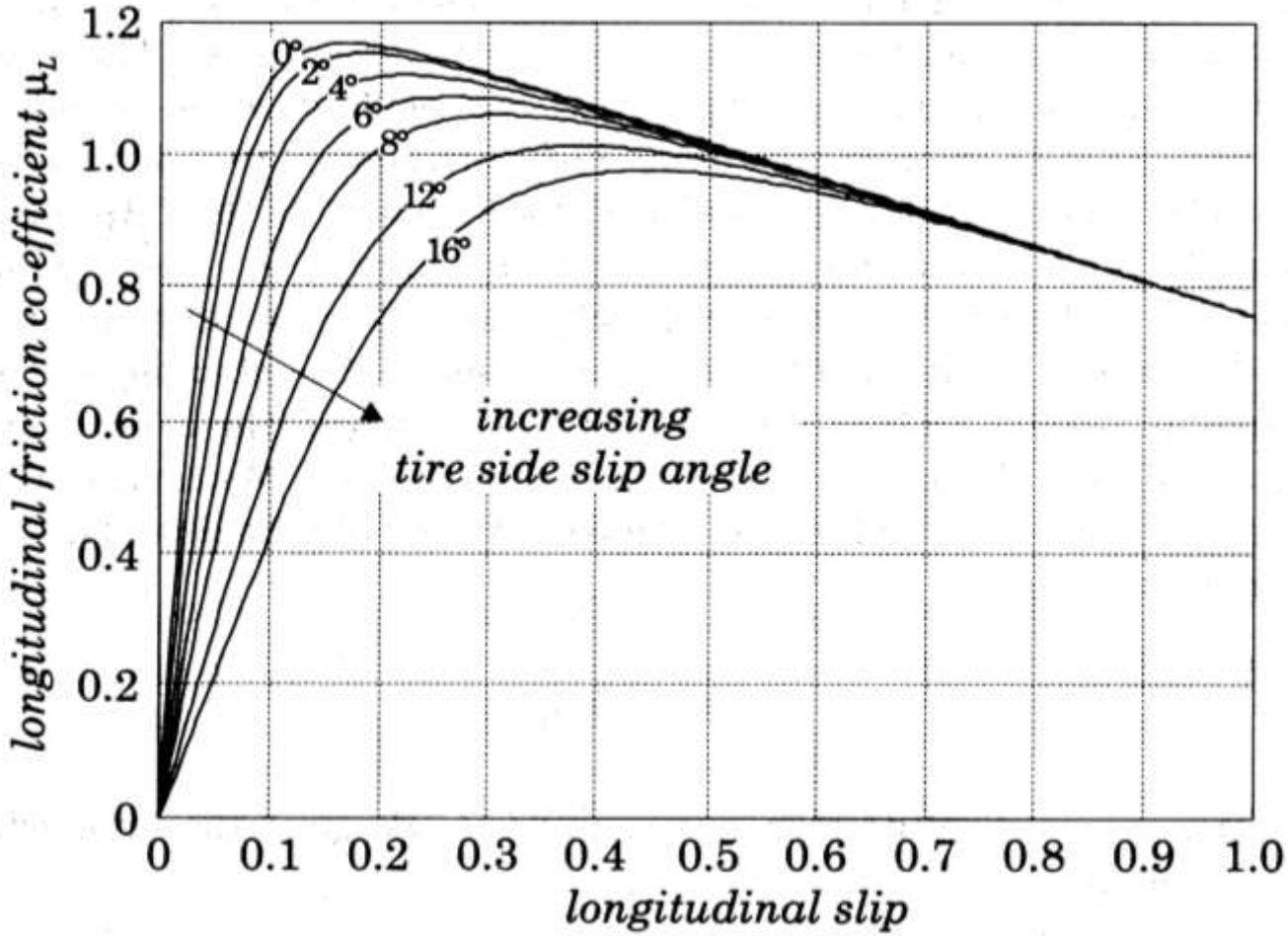
$$\left[\frac{\left(v_w / s_{0l} \right)}{s + \left(v_w / s_{0l} \right)} \right]$$

wheel speed

relaxation length (about 1/2 circumference)



Contact forces: longitudinal



Dry surface
Camber: null

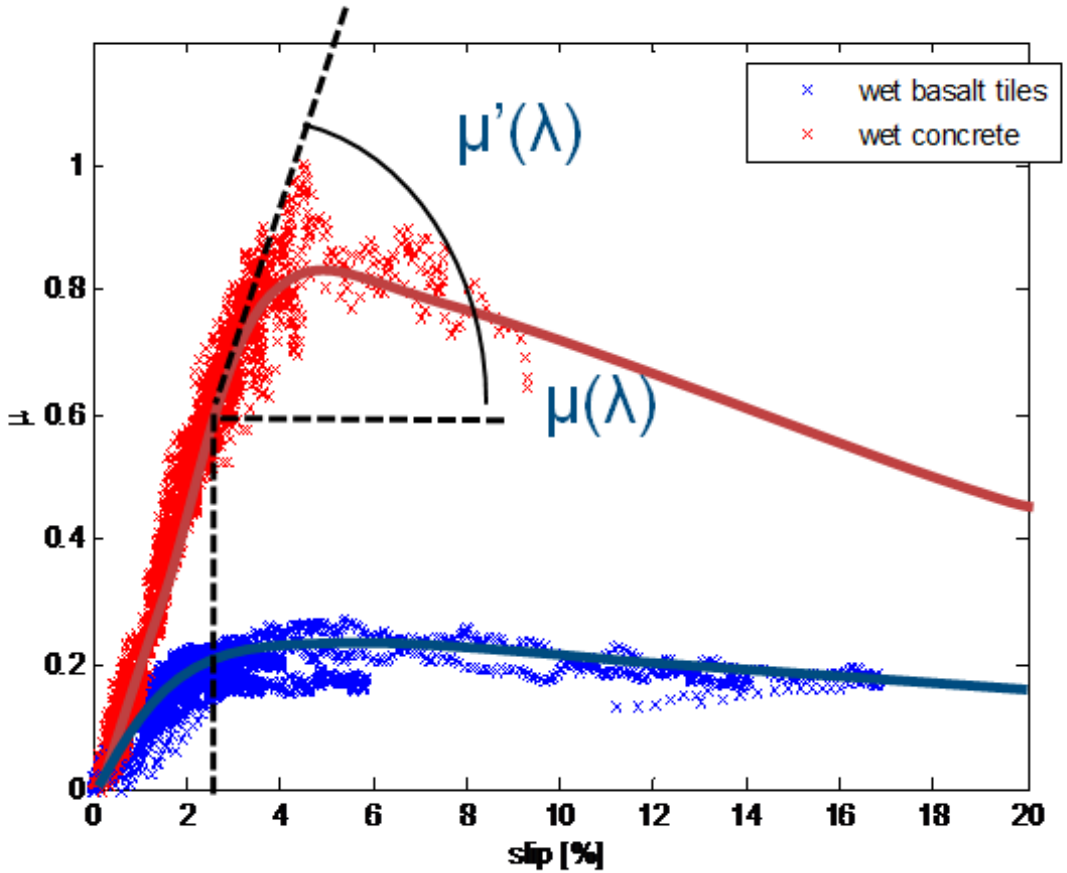
if there is no slip the longitudinal force is zero

force increases linearly up to values of λ of about 0.1-0.15

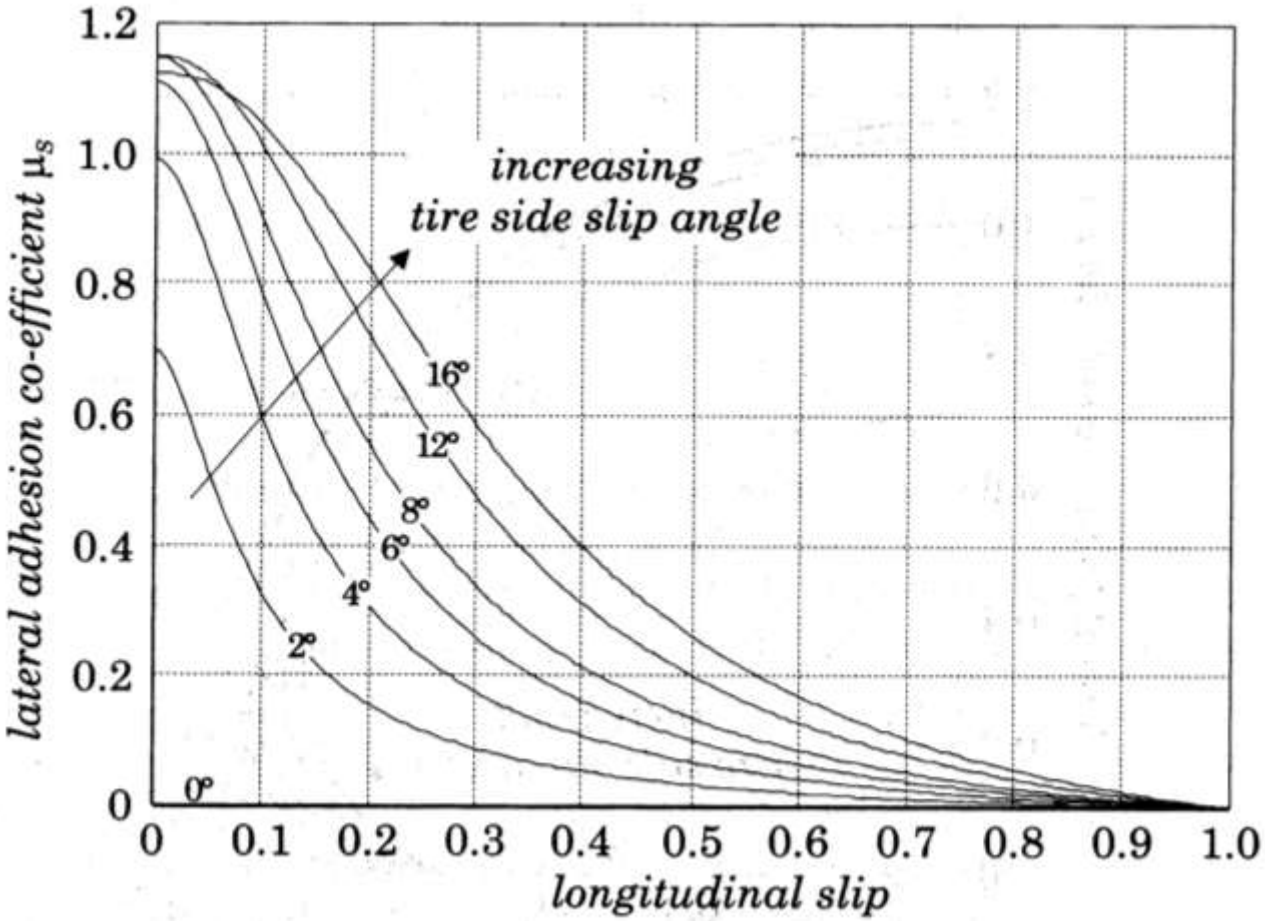
beyond the maximum: slope sign changes

Increasing side-slip angle:
- Reduction of longitudinal force
- The maximum point moves forward

Experimental results (M / C)



Contact forces: lateral



Dry surface
Camber: null

if there is no side-slip angle: lateral force is zero

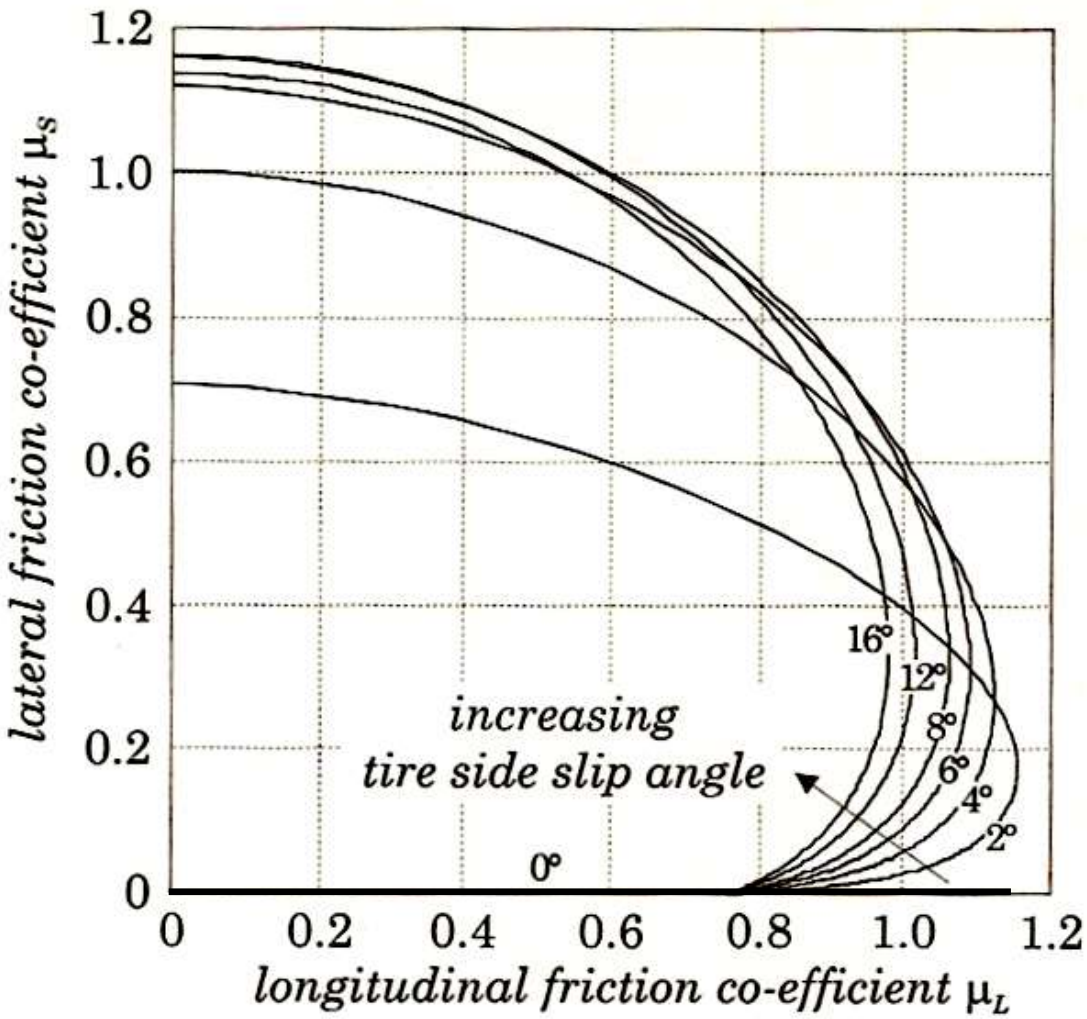
Force increases with increasing side-slip angle

force reaches its maximum at $\lambda = 0$

If wheels locked ($\lambda = 1$) lateral force vanishes (total loss of directionality)

Note: at the same α, varying λ transfers force to/from longitudinal to/from lateral force

Tire-road contact forces

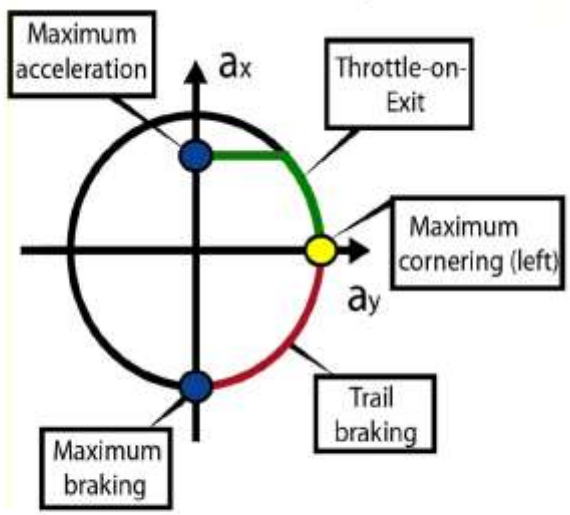


Dry surface
Camber: null

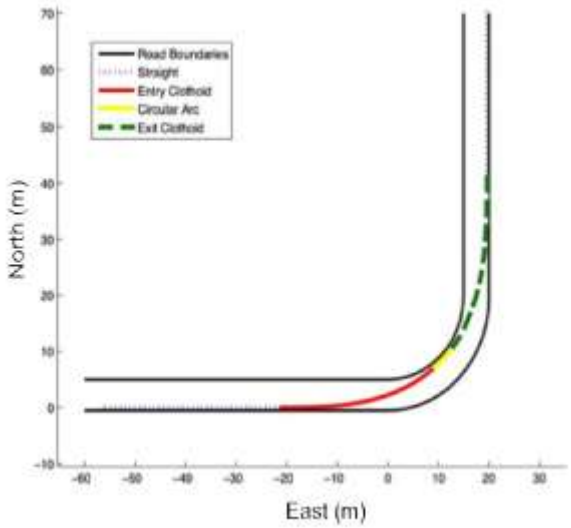
Trade-off lateral-longitudinal

Envelope: almost a circumference (or ellipse)

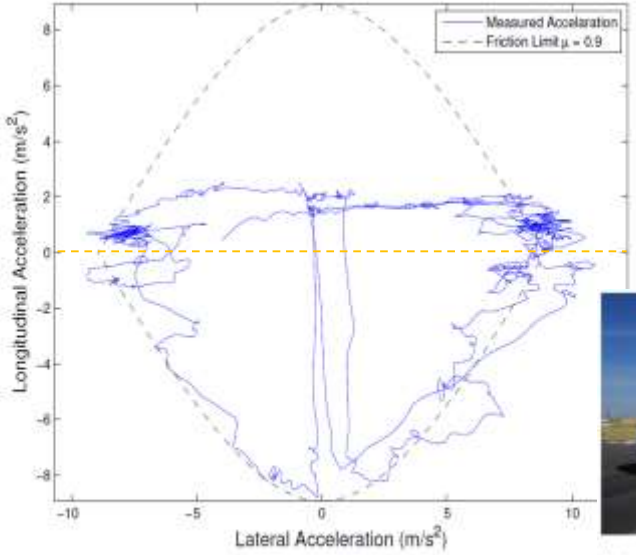
Rermarks: driving at the limit



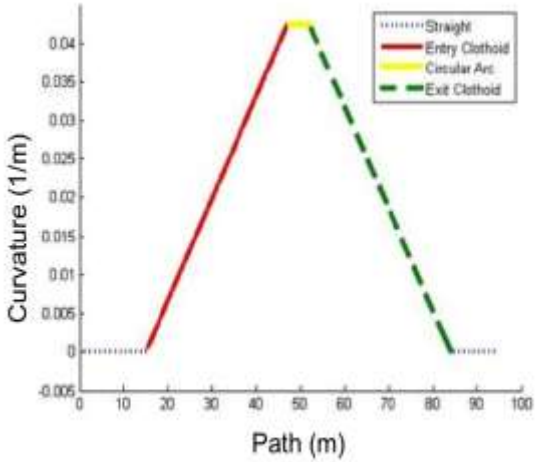
(c) The friction circle model



(a) Racing line curve structure



Autonomous Audi TTs

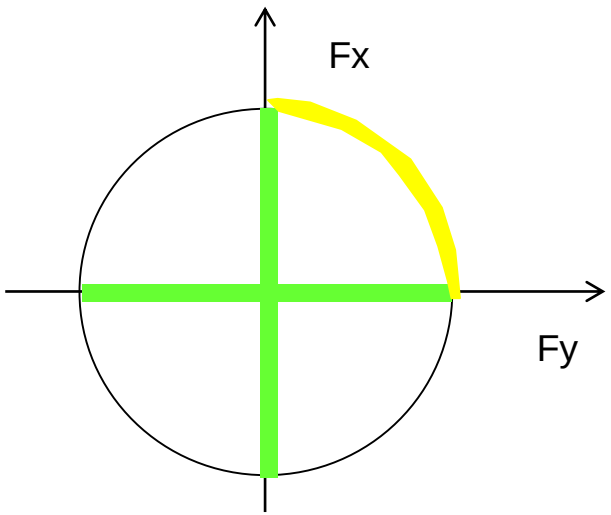


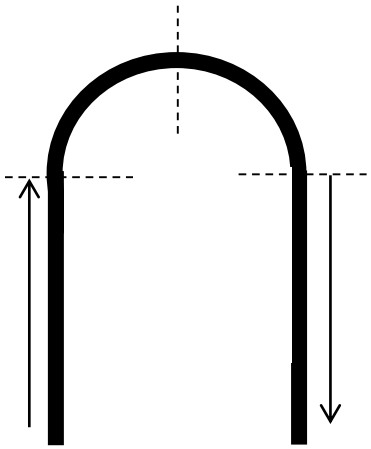
(b) Curvature of the path

An **Euler spiral** is a curve whose curvature changes linearly with its curve length (the curvature of a circular curve is equal to the reciprocal of the radius). Euler spirals are also commonly referred to as **spiros**,**clothoids** or **Cornu spirals**.

(Ref: Theodosis, Gerdes; Path planning…)

Remarks

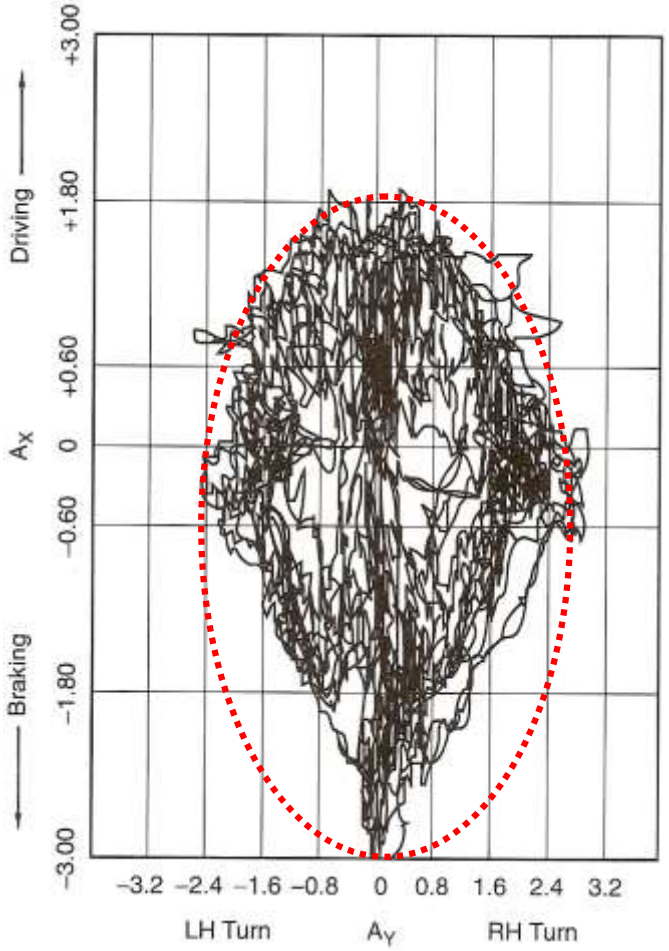




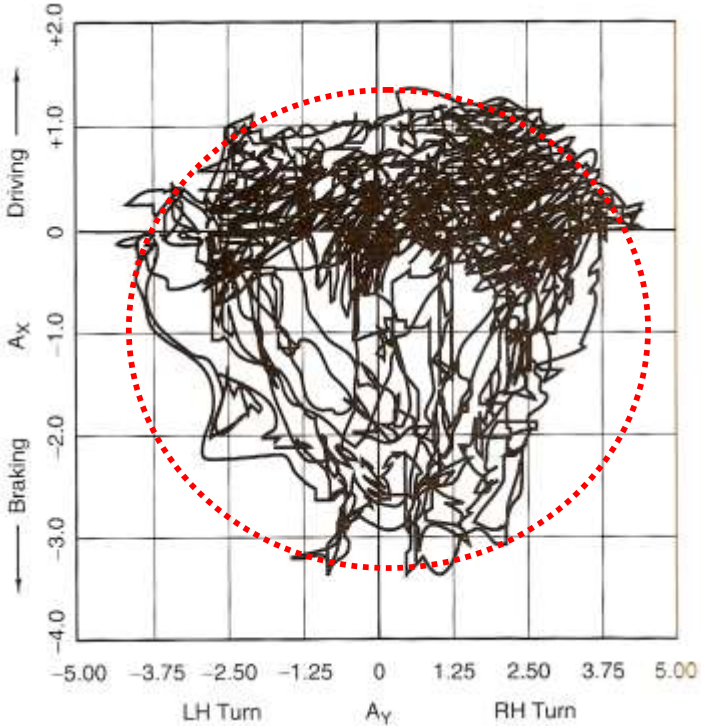
Brake(acceleration) only + curve only (normal driver)

Brake(acceleration) & curve (pro-driver)

g-g plots



Senna, '87, Adelaide



Herbert, 93, Silverstone

Braking and acceleration forces:

$$Brake: \quad Ma_x = -F_{drag}(v) - F_{brake}(v)$$

$$Acceleration: \quad Ma_x = -F_{drag}(v) + F_{traction}(v)$$

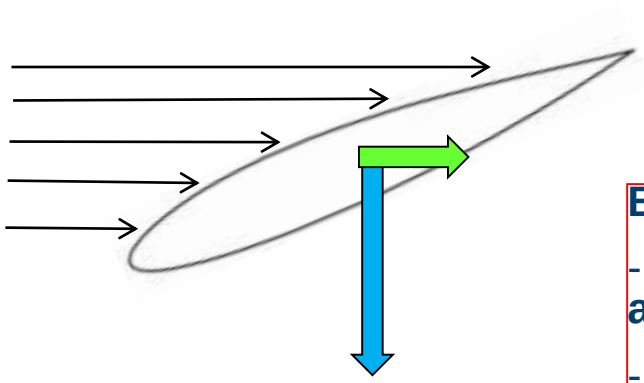
$$F_{drag} = \frac{1}{2}\rho C_x S v^2$$

Maximum braking force:

$$F_{brake}(v) = F_z \mu_{MAX}$$

$$F_z = Mg + F_{lift}$$

$$F_{lift} = \frac{1}{2}\rho C_z S v^2$$

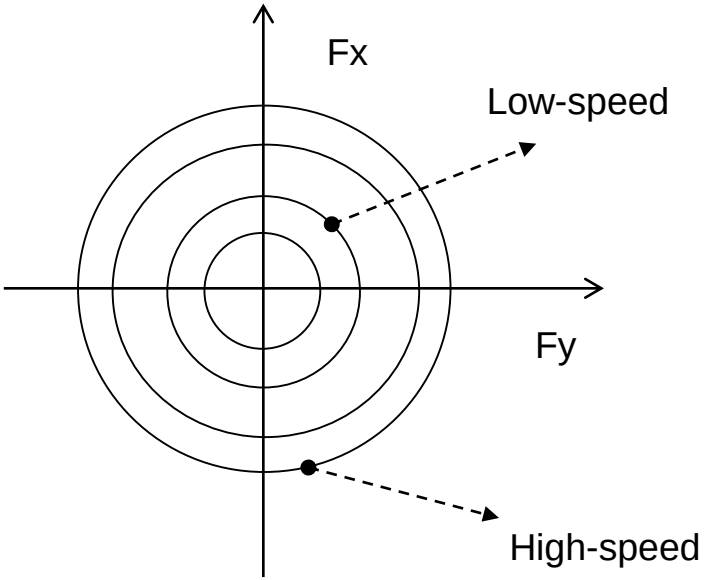


$$efficiency = \frac{C_z}{C_x}$$

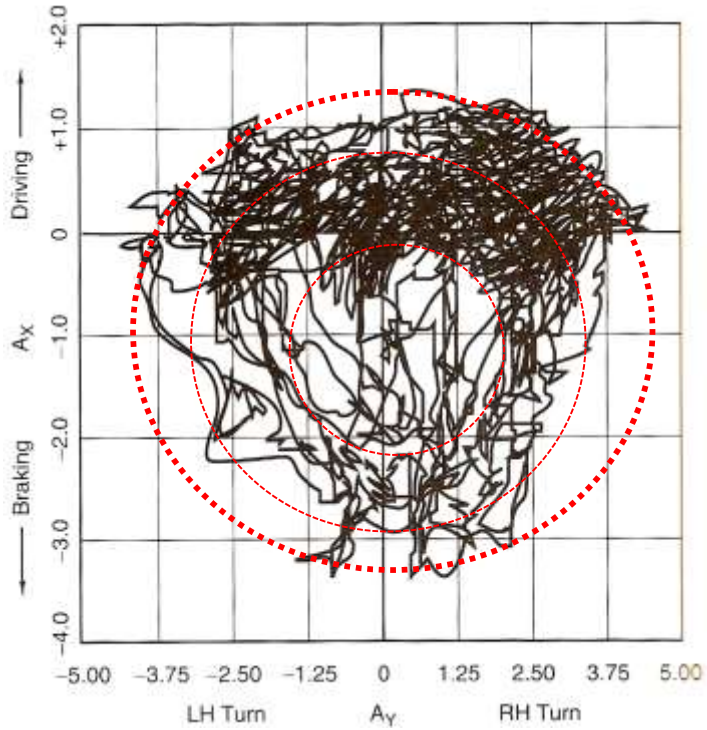
Braking and acceleration forces:

- complex combinantion of road-tire contact forces and aerodynamic effects
- Fz has a "scaling effect" for maximum road-tire forces
- μ_{max} is always the bottleneck for performance limit?

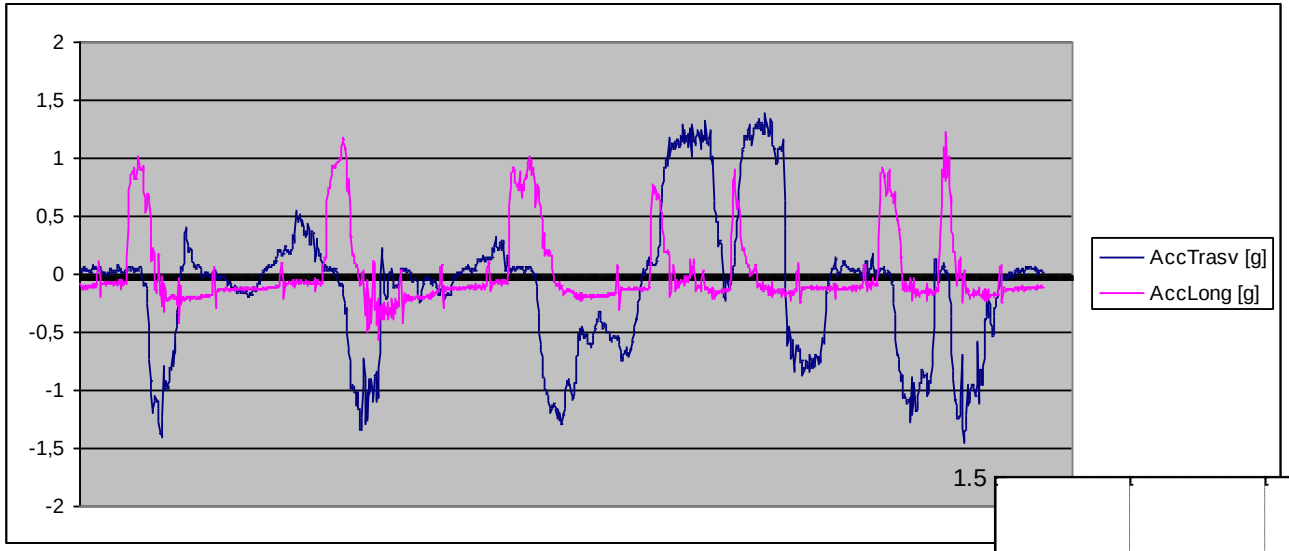
Remarks



For high lift-to-drag vehicles, speed has a scaling effect on maximum longitudinal and lateral forces



g-g plots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1,5
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0,5
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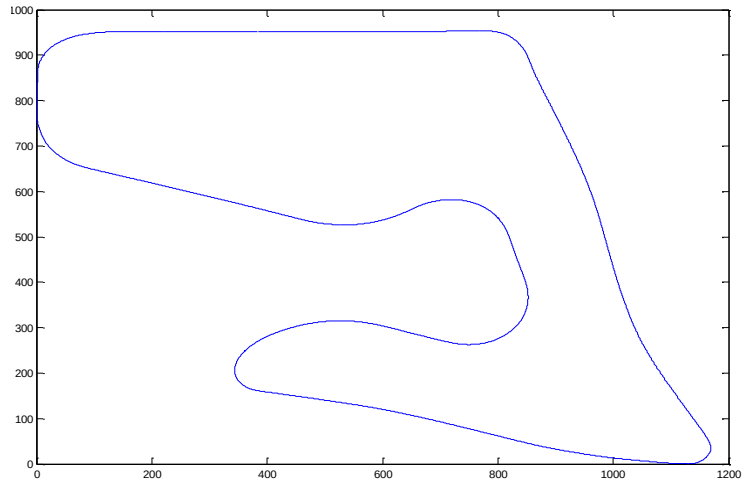
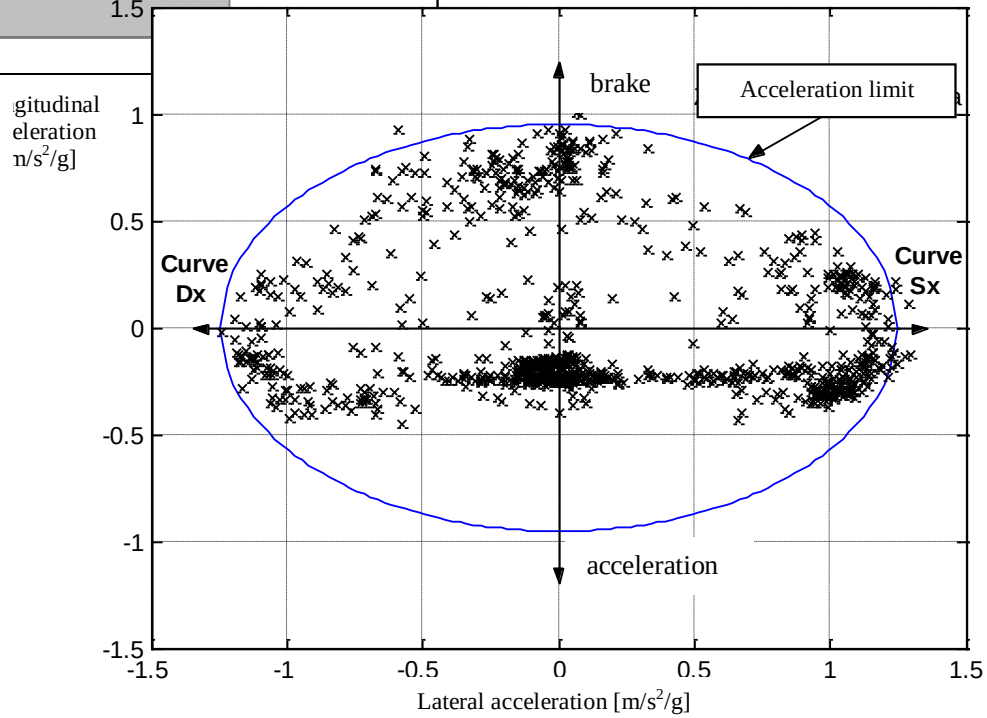
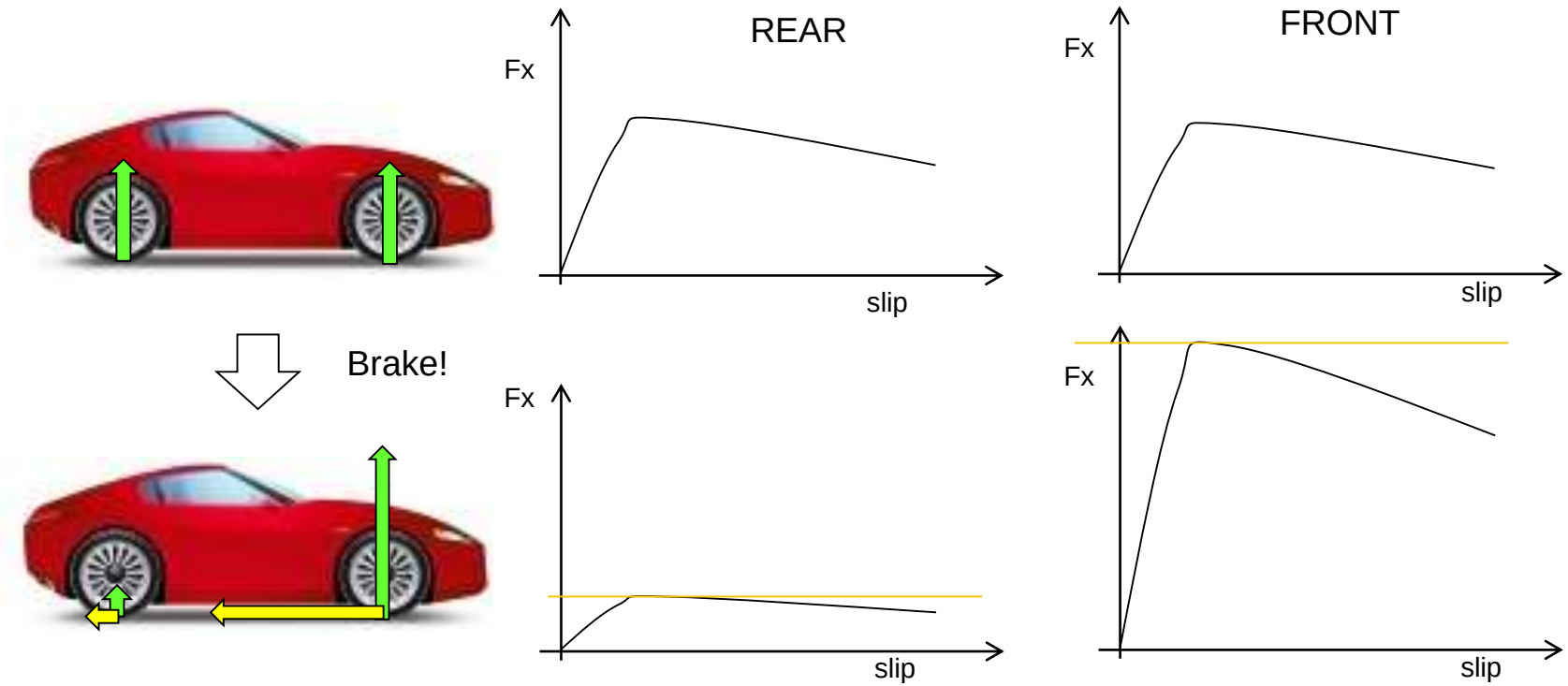
-0,5
-1
-1,5
-2
1.5
AccTrasv [g]
AccLong [g]


1000
900
800
700
600
500
400
300
200
100
0
0
200
400
600
800
1000
1200


gitudinal
eleration
n/s²/g]
brake
Acceleration limit
Curve
Dx
Curve
Sx
acceleration
1
0.5
0
-0.5
-1
-1.5
-1.5
-1
-0.5
0
0.5
1
1.5
Lateral acceleration [m/s²/g]

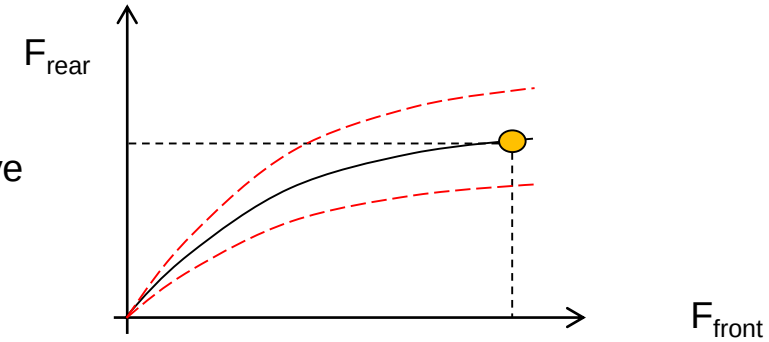
Remark: Electronic Brake Distribution (EBD)



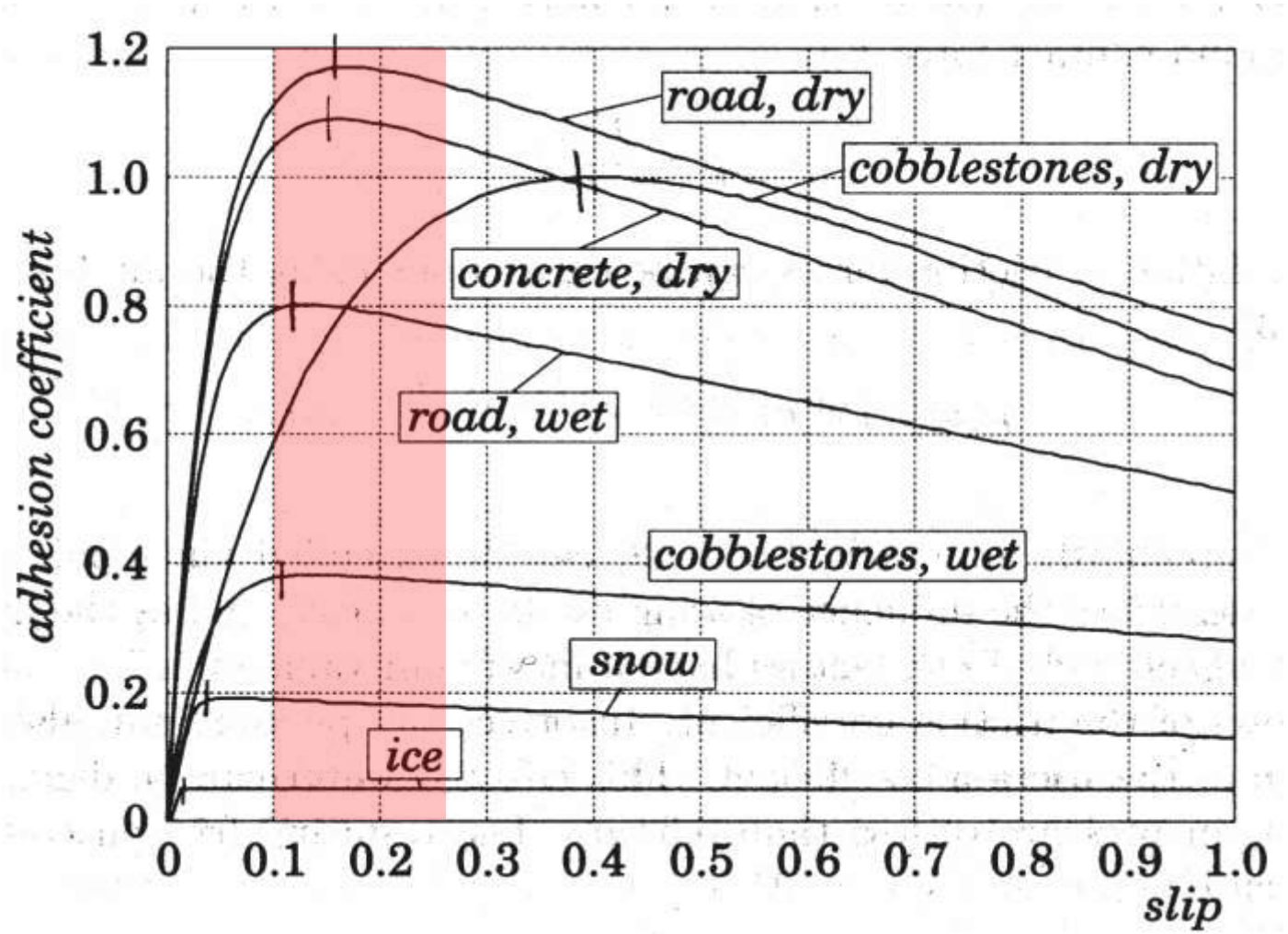
In the results of the previus slide there is NO **E**BD; most probably the mechanical tuning of the BD-curve was wrong; this has limited the maximum braking performance

Brake-dristribution curve

What-if-wrong?



Dependence of the contact forces from the road surface (e.g. longitudinal force)



Variation of the shape

Variation of the peak

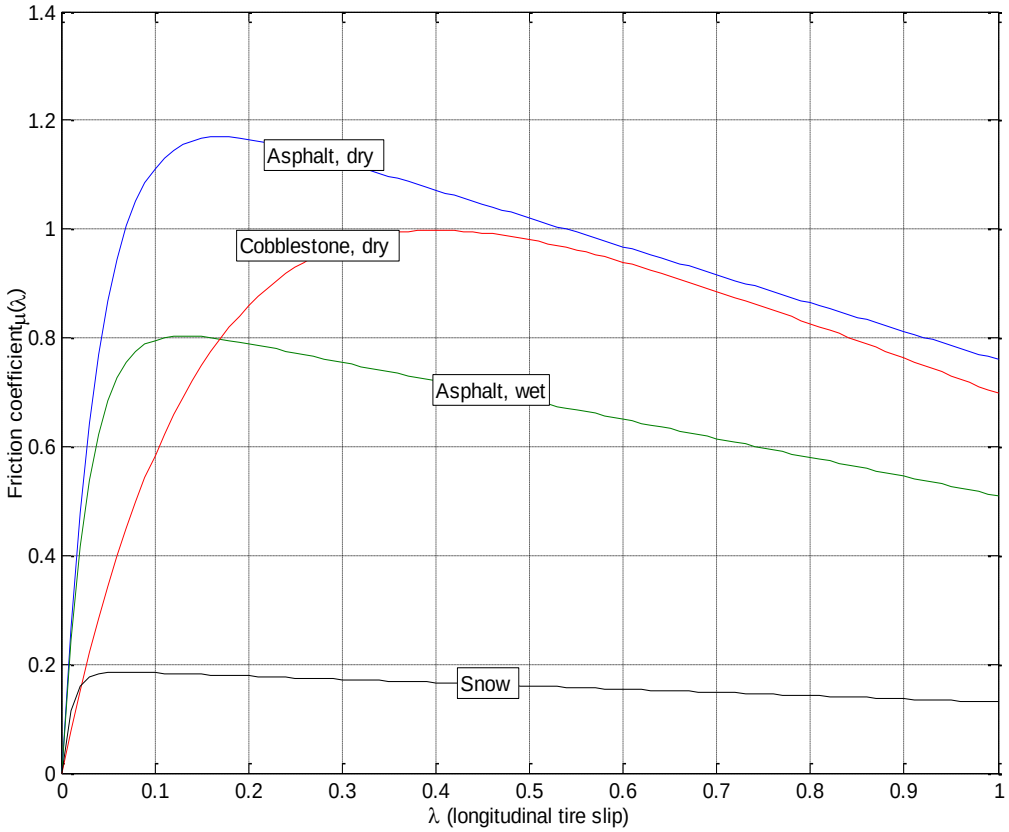
Formulas for modeling tire-road contact forces

Year	Model Name	Model Properties	Features
	Piecewise Linear Model	Empirical	1. Cannot accurately fit curves 2. Easy to identify
1993	Burckhardt Model	Semi-Empirical	1. Can accurately fit curves 2. Has some revised formula
1994	Rill Model	Semi-Empirical	Easy to identify
1987	Magic Formula	Semi-Empirical	1. Can accurately fit curves 2. Has lost of revised formula 3. Can employ different factors
1977	Dahl Model	Analytical	1. Can describe Coulomb friction. 2. Can produces smooth transition around zero velocity
1991	Bliman-Sorine Model	Analytical	Can capture the Stribeck effect in addition to Dahl model
1995	LuGre Model	Analytical	Can combine pre-sliding & sliding in addition to Bliman-Sorine Model

Integrated longitudinal and lateral tire / road friction modeling and monitoring for vehicle motion control, Li Li, Fei-Yue Wang; Qunzhi Zhou, IEEE Transactions on Intelligent Transportation Systems, Volume 7, Issue 1, March 2006 Page (s): 1-19

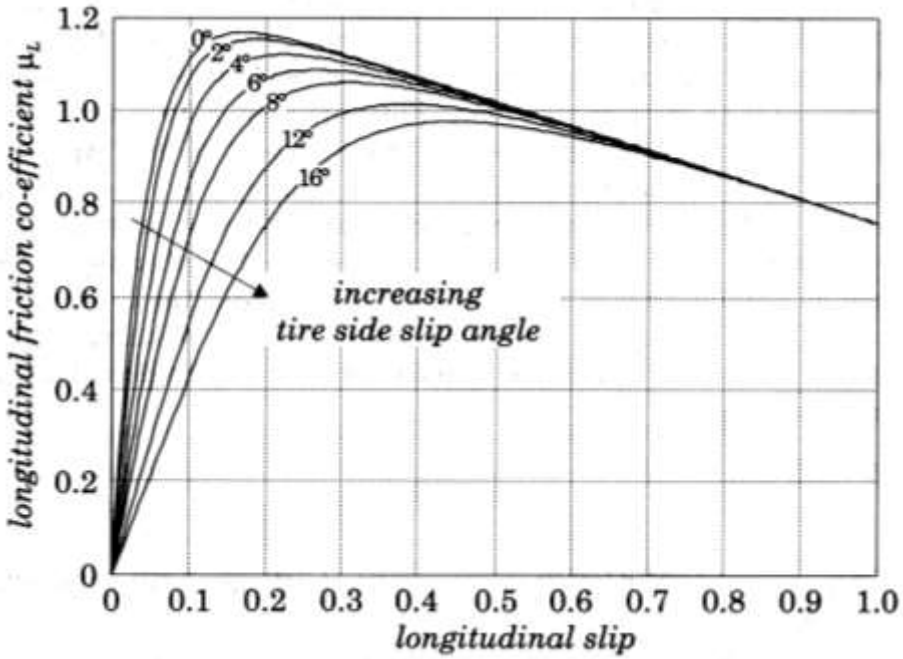
Burckhardt formula for longitudinal force (e.g.)

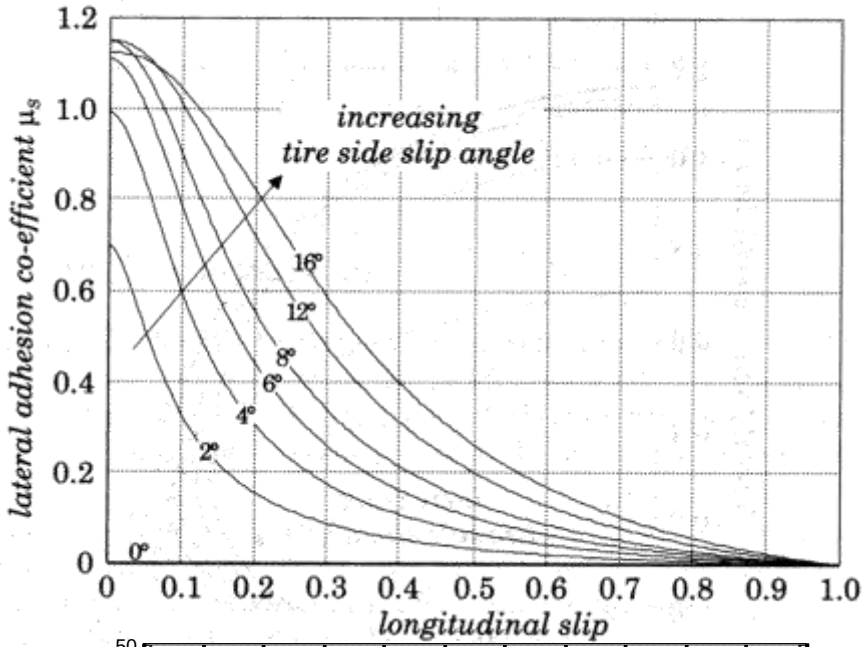
$$\mu(\lambda;\theta_r) = \theta_{r1}\left(1-\exp(-\lambda\theta_{r2})\right) - \lambda\theta_{r3}$$



	θ_{r1}	θ_{r2}	θ_{r3}
Asphalt, dry	28.01	23.99	00:52
Asphalt, wet	0.86	33.82	12:35
Cobblestones, dry	1:37	6:46	0.67
Snow	00:19	94.13	12:06

Problem of brake/traction control: summary





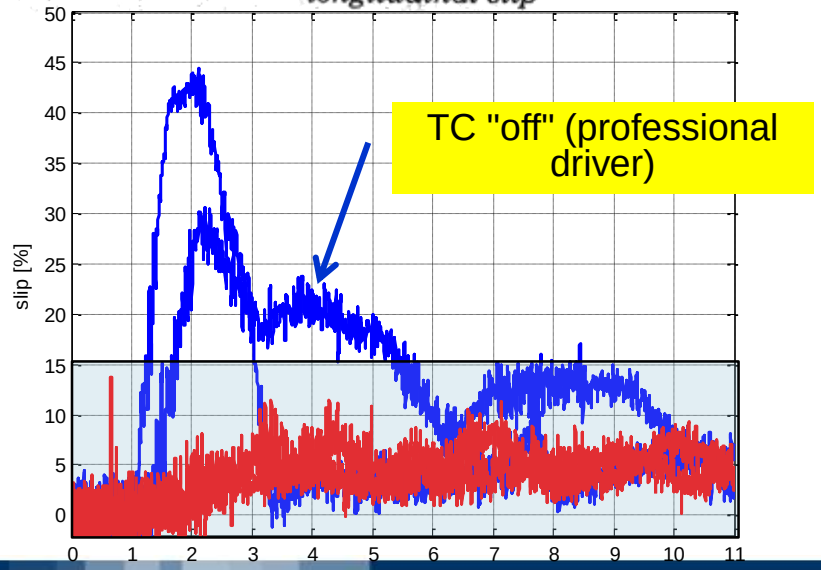
Straight brake / acceleration:
ideal λ is around the peak.

Three issues (ABS/TC motivations):

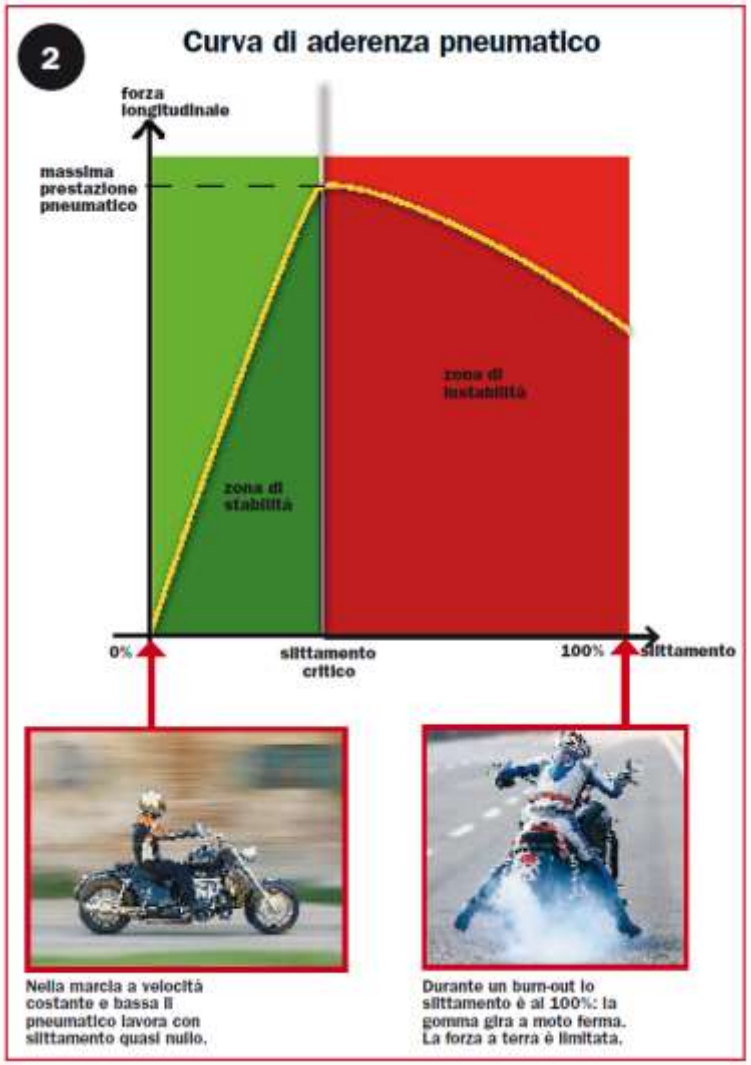
λ=1: loss of brake/traction (- 30%)

λ=1: NO lateral forces

After the peak (λ> 0.15): instability region



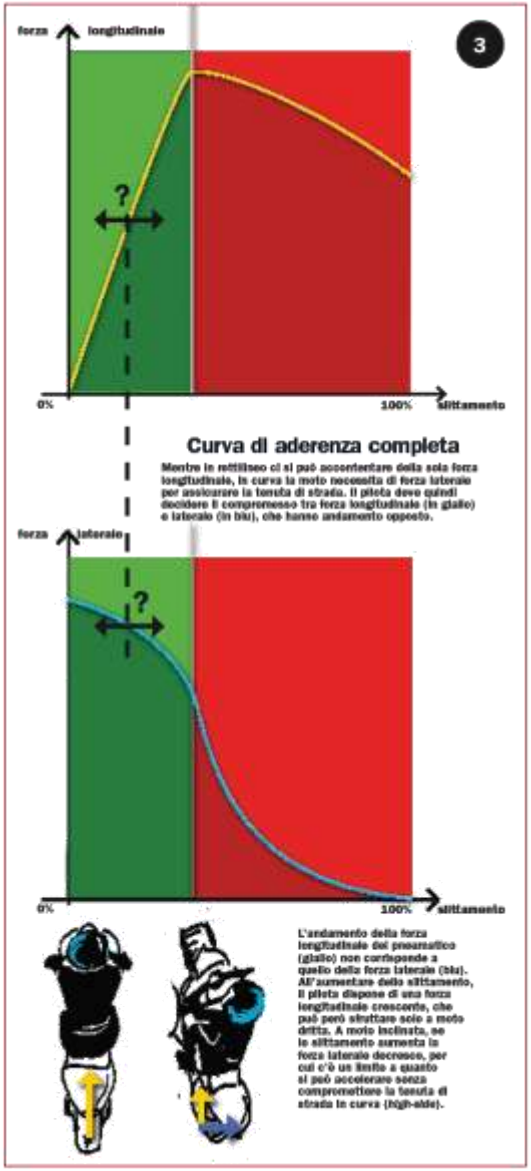
The problem of "TC“: why useful?



Un po' di slittamento non fa mai male
La definizione di slittamento è legata al fatto che la deformazione del pneumatico, necessaria a produrre l'aderenza a terra, tende a tradursi in una velocità periferica superiore a quella di effettivo avanzamento del veicolo. Solo se le due velocità coincidono, lo slittamento è pari a zero; all'estremo opposto, quando la ruota si muove a moto fermo lo slittamento è totale (100%). In uscita di curva (sopra), la situazione è intermedia.

Remark: rear traction

The dilemma of the driver when cornering ...



ABS: History ...

One of the first patents "An Improved Safety Device for Preventing the Jamming of the Running Wheels of Automobiles When Braking ') was filed in Europe in 1932, while a similar result (' Apparatus for Preventing Wheel Sliding ') was obtained in the United States in 1936.

In the railway industry: first application around 1943

The aeronautical field: 1947, B-47 bombers to avoid an excessive increase in braking distance on icy surfaces and damage to the tires.

In 1968: airplanes 'Thunderbirds'.

Ford, Chrysler and Cadillac offered the ABS on a limited number of vehicles. Pneumatic Actuators: very slow and increased stopping distances.

Germany: a joint venture formed by Telefunken and Bendix tried to market a system called ABS Tekline. But the electronic design and manufacturing technologies were too young and unreliable.

1978: Bosch announces "Anti-Blockier System" (hence the acronym ABS).

In the same year, Mercedes Benz offered for the first time the ABS as an option for its high-end (S-class) cars.

Association of European Car Manufacturers (ACEA): equipment required for all new cars registered from 1 July 2004.

ABS will become mandatory for motorcycles in Europe since 2017.

1936
A Robert Bosch viene riconosciuto il brevetto numero 671925 per un dispositivo definito «anti incollaggio dei freni».

1965
Viene avviato lo sviluppo dell'impianto Abs regolato elettronicamente.

1978
Comincia la produzione per il montaggio sulla Mercedes «classe S» e successivamente sulla BMW «serie 7». È un optional molto costoso.

1986
Viene fornito il milionesimo impianto.

1987
Al dispositivo viene aggiunta la funzione Asr, ossia l'antipattinamento in accelerazione.

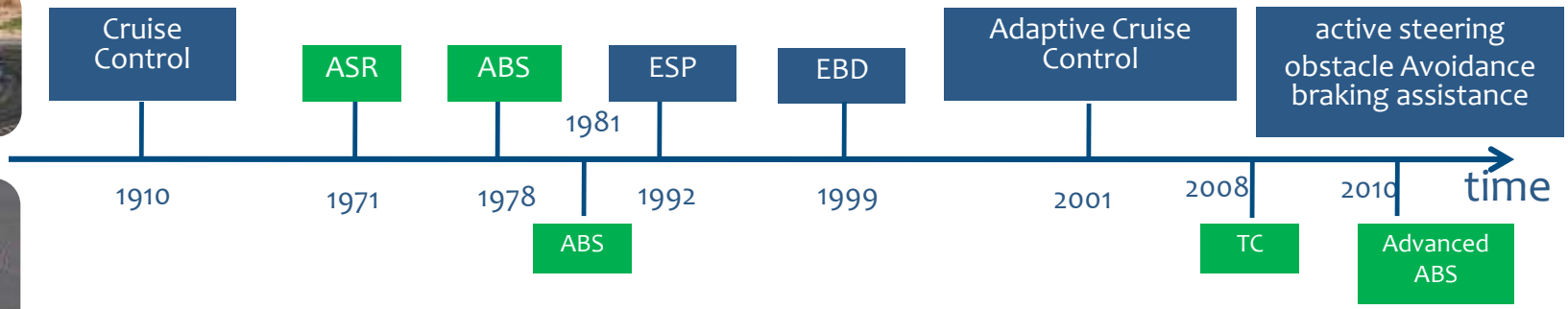
1991
La Bosch presenta l'«Abs/Asr 5», più leggero, compatto e meno costoso, quindi adatto anche alle vetture medie.

1995
Si raggiungono i 20 milioni d'impianti prodotti.

2003
La produzione Bosch tocca la soglia dei 100 milioni di pezzi.

LUGLIO 2004
Entro questa data tutte le auto nuove vendute in Europa dovranno avere di serie l'Abs, come deciso volontariamente dai Costruttori Acea.

Cars Vs. Motorcycles



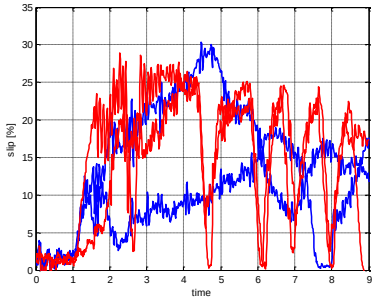
TC generations...

Generation 1

Mainly (only) safety-oriented

No ride-by-wire

Minumum sensors



Generation 2

Performance-oriented!!!

Still no-ride-by-wire

Simple roll-angle detection

Generation 3

Ultimate-performances

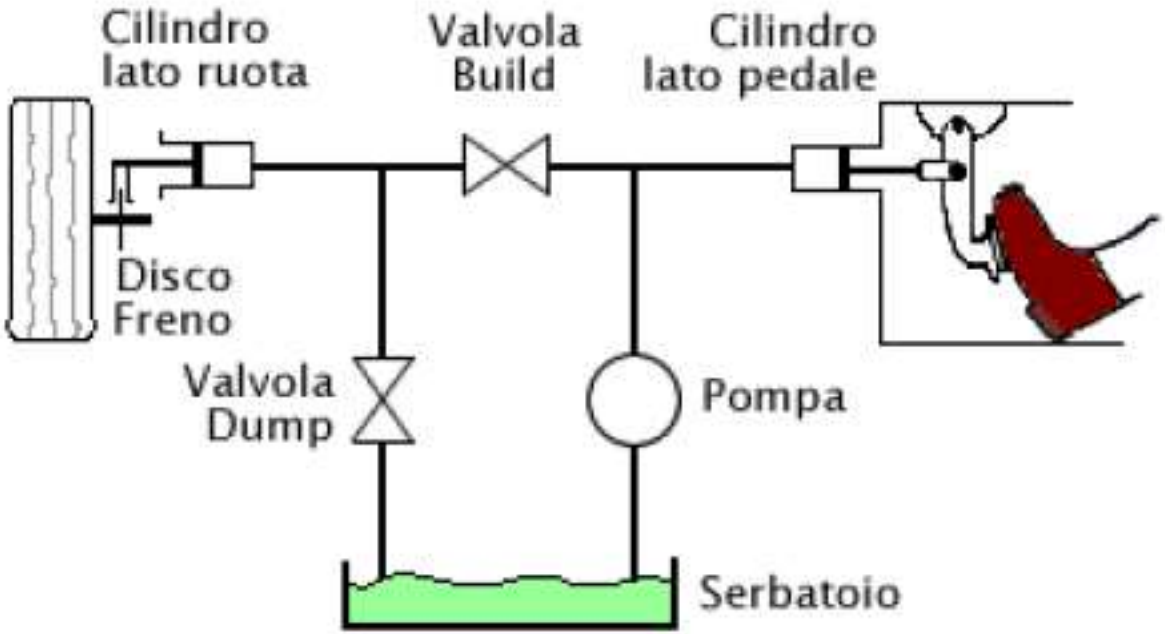
Full roll-angle estimation

Ride-by-wire
Smooth feeling (“traction-control mode”)

+ “Gadgets” (wheelie, launch...)

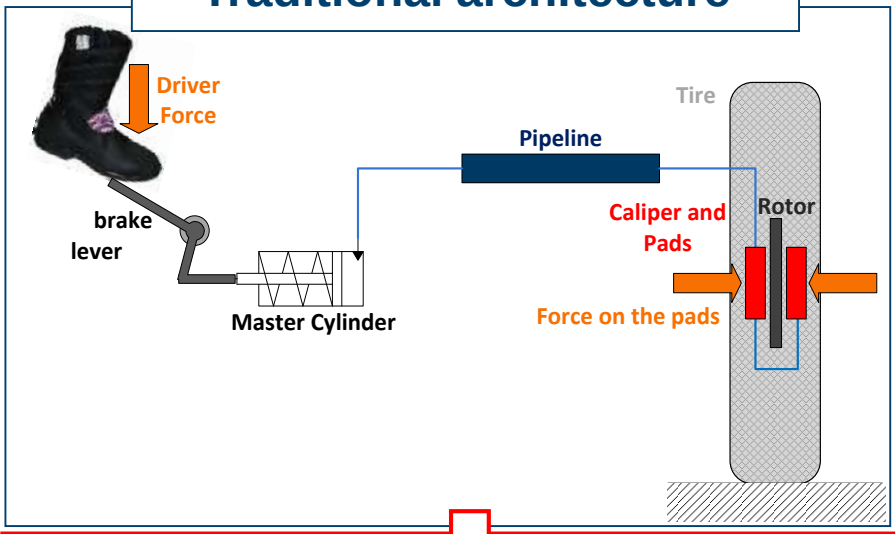
Actuators (traditional layout)

HAB: Hydraulic Actuated Brakes (Increase, hold, decrease)

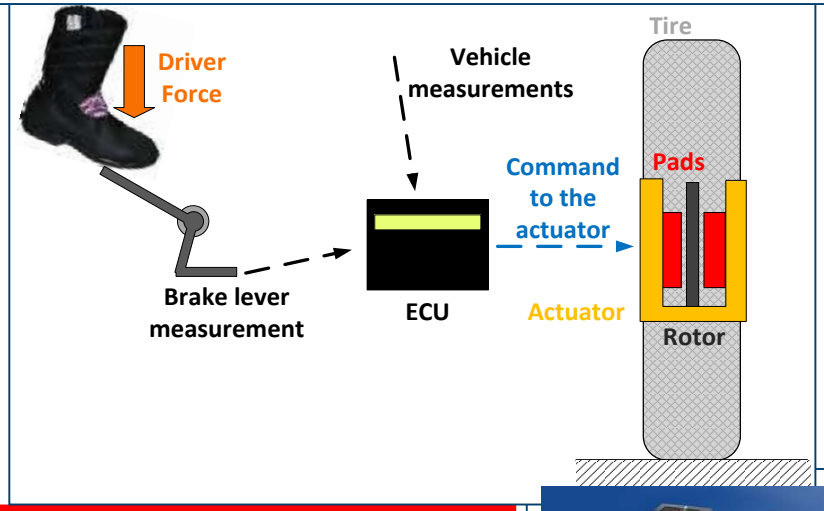


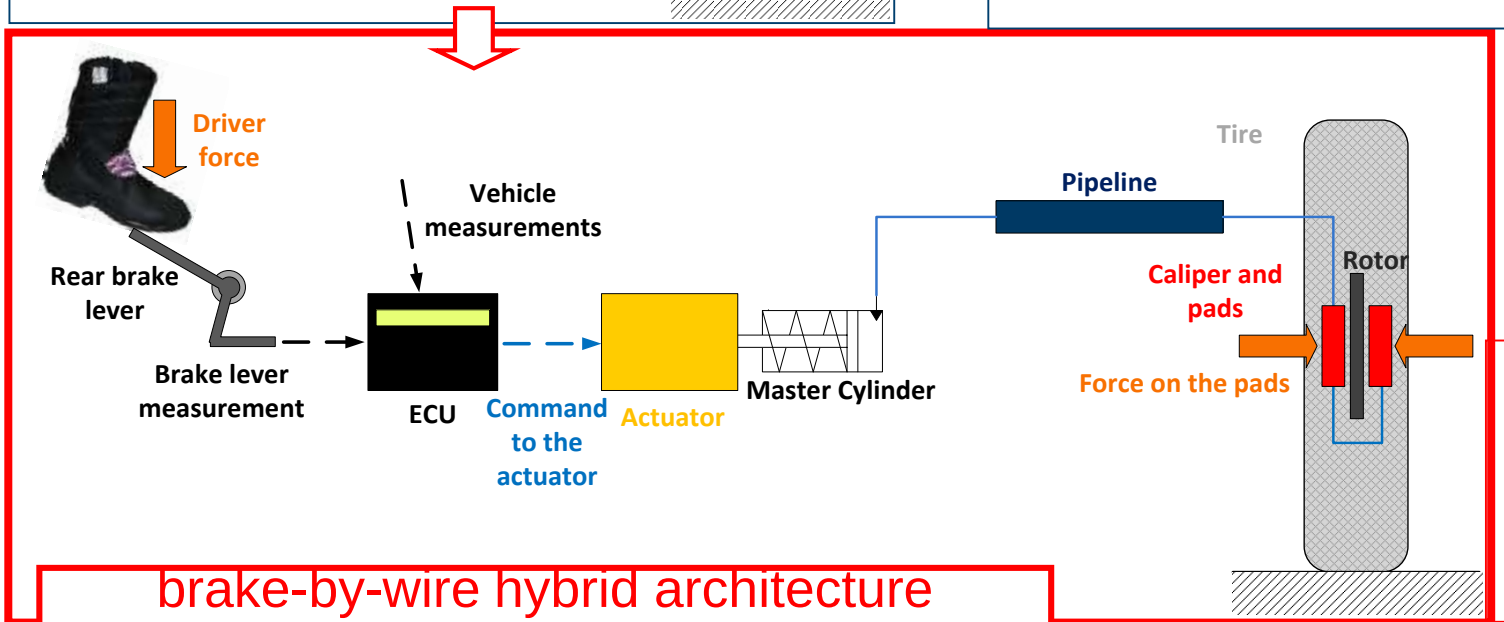
Actuation layout in brake-by-wire architectures

Traditional architecture



Pure brake-by-wire EM architecture

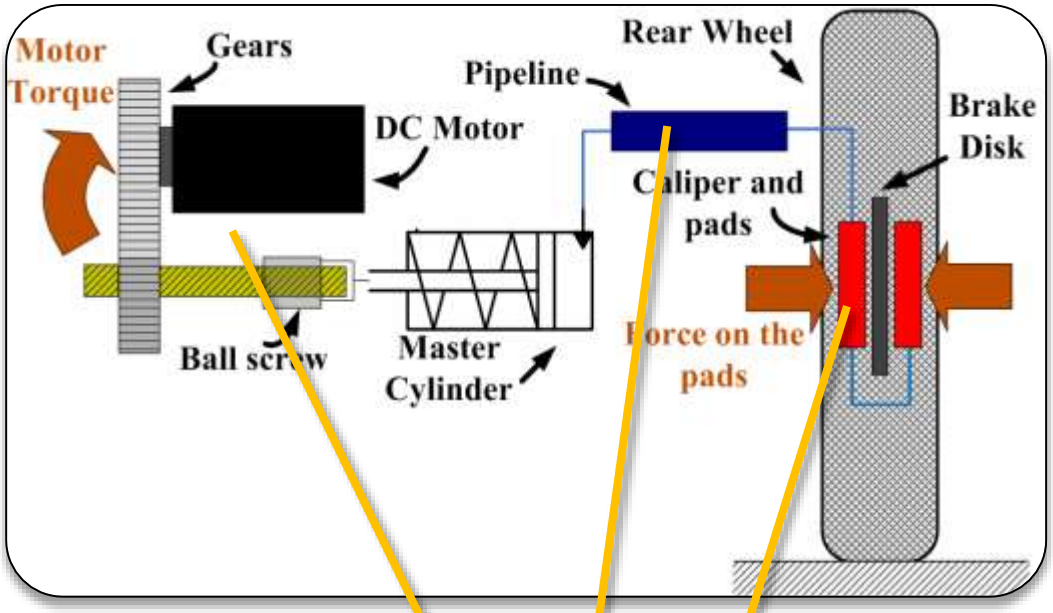


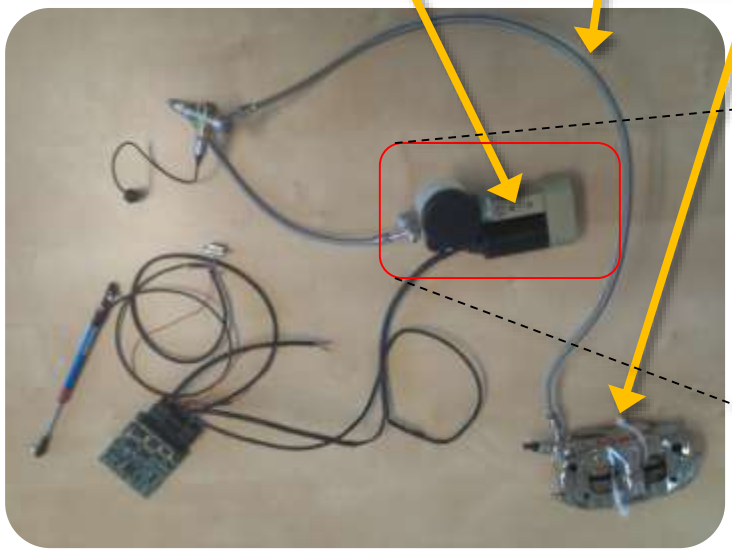


The traditional hydraulic structure is preserved

brake-by-wire hybrid architecture

Hybrid architecture





Master Cyl.

Gearbox

Dc motor

Measurement problem: slip estimation

Control Problem: slip tracking

- with advanced BBW architecture
- with a traditional on-off-valve architecture

Problem goal: slip target

1) Simplified (control-oriented) model
2) Dynamic analysis
3) Controller design

Model "single-corner"

$$\begin{cases} J\dot{\omega} = rF_x - T_b \\ m\dot{v} = -F_x \end{cases}$$

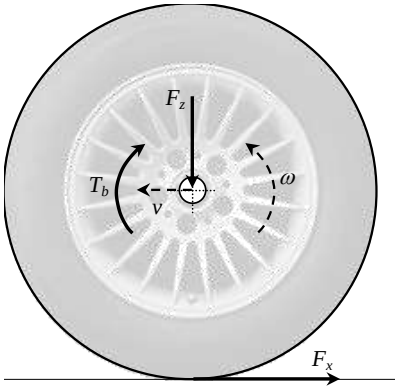
$$F_x = F_z \mu(\lambda, \cancel{\alpha; \theta_r})$$

$$\lambda = (v - \omega r)/v = 1 - r\omega/v$$

$$\begin{cases} J\dot{\omega} = rF_z \mu\left(\dfrac{v - \omega r}{v}\right) - T_b \\ m\dot{v} = -F_z \mu\left(\dfrac{v - \omega r}{v}\right) \end{cases}$$

- ω : angular speed of the wheel ([rad/s]; $\omega > 0$ is assumed);
- v : longitudinal speed of the vehicle body;
- T_b : braking torque (control/input variable);
- F_x : longitudinal road-tire contact force;
- F_z : vertical road-tire contact force;
- J, m and r are the momentum of inertia of the wheel, the quarter-car mass, and the wheel radius

(e.g. $J = 1Kg\,m^2$, $m = 225Kg$, $r = 0.28m$).



SISO system, 2nd order (n= 2), non-linear, time-invariant, strictly proper

Change of state variables

$$\begin{cases} J\dot{\omega} = rF_z\mu\left(\dfrac{v-\omega r}{v}\right) - T_b \\ m\dot{v} = -F_z\mu\left(\dfrac{v-\omega r}{v}\right) \end{cases}$$

$$\dot{\lambda} = -\frac{r}{v}\dot{\omega} + \frac{r\omega}{v^2}\dot{v}$$

$$\omega = \frac{v}{r}(1-\lambda)$$

CHANGE OF STATE VARIABLES

$$\begin{cases} \dot{\lambda} = -\dfrac{1}{v}\left(\dfrac{(1-\lambda)}{m} + \dfrac{r^2}{J}\right)F_z\mu(\lambda) + \dfrac{r}{vJ}T_b \\ m\dot{v} = -F_z\mu(\lambda) \end{cases}$$

SISO system, 2nd order (n= 2), non-linear, time-invariant, strictly proper (or: SISO, non-linear, 1st order, time-varying)

Equilibrium points (I)

$$\begin{cases} \dot{\lambda} = -\dfrac{1}{v}\left(\dfrac{(1-\lambda)}{m} + \dfrac{r^2}{J}\right)F_z\mu(\lambda) + \dfrac{r}{vJ}T_b \\ m\dot{v} = -F_z\mu(\lambda) \end{cases}$$

$$\dot{\lambda} = 0, \quad \eta = \overline{\eta}$$

$$\boxed{\text{not} \quad \dot{v} = 0}$$

$$\overline{T}_b = F_z\left(r + \frac{J}{rm}(1-\overline{\lambda})\right)\mu(\overline{\lambda})$$

Note: DO NOT depend on v.

Equilibrium points

$$\begin{cases} \dot{\lambda} = -\dfrac{1}{v}\left(\dfrac{(1-\lambda)}{m} + \dfrac{r^2}{J}\right)F_z\mu(\lambda) + \dfrac{r}{vJ}T_b \\ m\dot{v} = -F_z\mu(\lambda) \end{cases}$$

Linear, **normalized**, wheel **deceleration**

$$\eta = -\frac{\dot{\omega} r}{g}$$

$$\dot{\lambda} = 0, \quad \eta = \overline{\eta}$$

not $\dot{v} = 0$

$$\lambda = 1 - r\omega / v$$

$$(\dot{\omega} v - \dot{v}\omega) / v^2 = 0$$

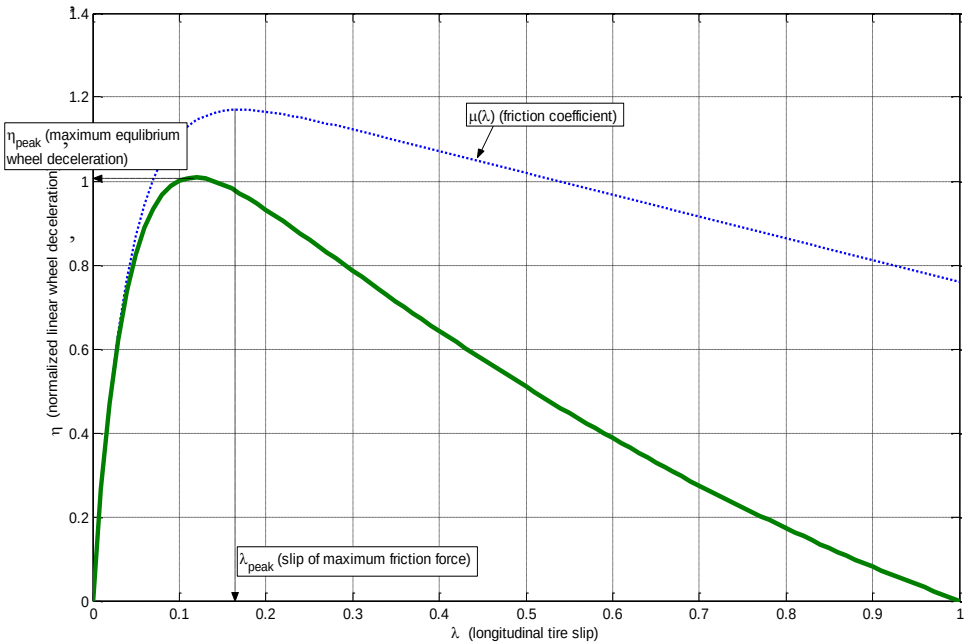
$$\boxed{\dot{\omega} = \omega\dot{v} / v}$$

$$\omega = v(1-\lambda) / r$$

$$m\dot{v} = -F_z\mu(\lambda)$$

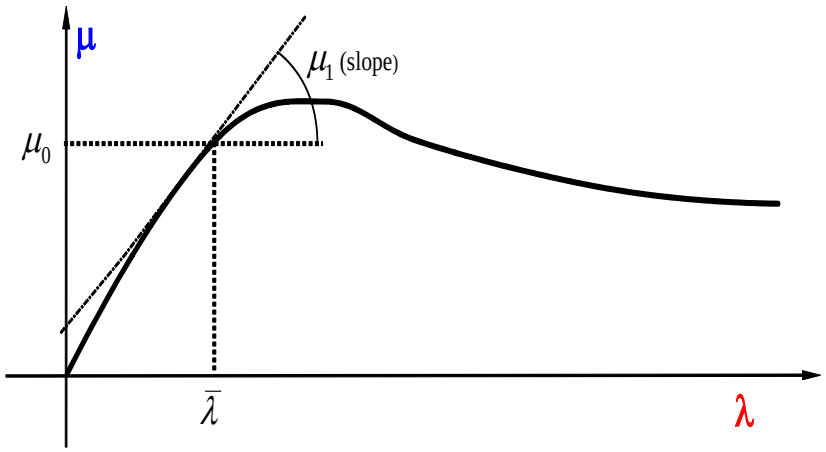
$$\eta = -\dot{\omega} r / g$$

$$\overline{\eta} = (1-\overline{\lambda})\frac{F_z}{mg}\mu(\overline{\lambda})$$



Linearization model (method I)

$$\delta T_b = T_b - \overline{T}_b \qquad \delta\lambda = \lambda - \overline{\lambda}$$



$$\mu_1 = \mu_1(\overline{\lambda}) = \partial\mu(\lambda)/\partial\lambda\big|_{\lambda=\overline{\lambda}}$$

$$\mu_0 = \mu(\overline{\lambda})$$

$$\begin{cases} \dot{\lambda} = -\dfrac{1}{v}\left(\dfrac{(1-\lambda)}{m} + \dfrac{r^2}{J}\right)F_z\mu(\lambda) + \dfrac{r}{vJ}T_b \\ m\dot{v} = -F_z\mu(\lambda) \end{cases}$$

Hypothesis: v dynamics (Body Dynamics) are much slower than λ or ω dynamics (Wheel Dynamics).
<u>Consider *v* slowly variant</u>.

Linearization model (method I)

Equation of λ:

$$\dot{\lambda} = -\frac{1}{v}\left(\frac{(1-\lambda)}{m} + \frac{r^2}{J}\right)F_z\mu(\lambda) + \frac{r}{vJ}T_b$$

$$\dot{\lambda} = -\frac{F_z}{v}\left(\frac{(1-\lambda)}{m}\mu(\lambda) + \frac{r^2}{J}\mu(\lambda)\right) + \frac{r}{vJ}T_b$$

Linearizing around $\bar{\lambda}, \bar{T}_b$

$$\delta\dot{\lambda} = -\frac{F_z}{v}\left(\frac{-1}{m}\mu_0 + \frac{(1-\bar{\lambda})}{m}\mu_1 + \frac{r^2}{J}\mu_1\right)\delta\lambda + \frac{r}{vJ}\delta T_b$$

$$\delta\dot{\lambda} = -\frac{F_z}{v}\left(\left(\frac{(1-\bar{\lambda})}{m} + \frac{r^2}{J}\right)\mu_1 - \frac{\mu_0}{m}\right)\delta\lambda + \frac{r}{vJ}\delta T_b$$

$$G_\lambda(s) = \frac{\left[\frac{r}{vJ}\right]}{s + \left[\frac{F_z}{v}\left(\left(\frac{(1-\bar{\lambda})}{m} + \frac{r^2}{J}\right)\mu_1 - \frac{\mu_0}{m}\right)\right]}$$

Linearization model (method I)

$$J\dot{\omega} = rF_z\mu(\lambda) - T_b \qquad -\frac{r}{g}J\dot{\omega} = -\frac{r}{g}\left(rF_z\mu(\lambda) - T_b\right) \qquad J\eta = -\frac{r^2}{g}F_z\mu(\lambda) + \frac{r}{g}T_b$$

$$\boxed{\eta = -\frac{r^2}{Jg}F_z\mu(\lambda) + \frac{r}{Jg}T_b}$$

$$\delta\eta = -\frac{r^2F_z}{Jg}\mu_1\delta\lambda + \frac{r}{Jg}\delta T_b \qquad G_\eta(s) = -\frac{r^2F_z}{Jg}\mu_1 G_\lambda(s) + \frac{r}{Jg} = \frac{r}{Jg}\left\{1 - \frac{rF_z\mu_1\left[\frac{r}{vJ}\right]}{s + \left[\frac{F_z}{v}\left(\left(\frac{(1-\bar{\lambda})}{m} + \frac{r^2}{J}\right)\mu_1 - \frac{\mu_0}{m}\right)\right]}\right\}$$

$$G_\eta(s) = \frac{r}{Jg}\frac{s + \left[\frac{F_z}{v}\left(\left(\frac{(1-\bar{\lambda})}{m} + \frac{r^2}{J}\right)\mu_1 - \frac{\mu_0}{m}\right)\right] - rF_z\mu_1\left[\frac{r}{vJ}\right]}{s + \left[\frac{F_z}{v}\left(\left(\frac{(1-\bar{\lambda})}{m} + \frac{r^2}{J}\right)\mu_1 - \frac{\mu_0}{m}\right)\right]}$$

$$G_\eta(s) = \frac{r}{Jg}\frac{s + \frac{F_z}{v}\left[\left(\frac{(1-\bar{\lambda})}{m} + \frac{r^2}{J}\right)\mu_1 - \frac{\mu_0}{m} - \mu_1\frac{r^2}{J}\right]}{s + \left[\frac{F_z}{v}\left(\left(\frac{(1-\bar{\lambda})}{m} + \frac{r^2}{J}\right)\mu_1 - \frac{\mu_0}{m}\right)\right]}$$

$$\boxed{G_\eta(s) = \frac{r}{Jg}\frac{s + \frac{F_z}{mv}\left[(1-\bar{\lambda})\mu_1 - \mu_0\right]}{s + \left[\frac{F_z}{v}\left(\left(\frac{(1-\bar{\lambda})}{m} + \frac{r^2}{J}\right)\mu_1 - \frac{\mu_0}{m}\right)\right]}}$$

Linearization model (method II)

Alternative method of linearization

Do not make assumptions of dynamic decoupling between the dynamics of wheel and body. Linearizziamo around a **point (non-equilibrium)** defined by

$$\boxed{\bar{v} \quad \bar{\omega} \quad \bar{\lambda} = (\bar{v} - \bar{\omega} r)/\bar{v}}$$

$$\mu\left(\lambda(v,\omega)\right) = \mu\left(\bar{\lambda}\right) + \left.\frac{\partial \mu}{\partial v}\right|_{\lambda=\bar{\lambda}} (v - \bar{v}) + \left.\frac{\partial \mu}{\partial \omega}\right|_{\lambda=\bar{\lambda}} (\omega - \bar{\omega}) + O((v-\bar{v})^2, (\omega - \bar{\omega})^2)$$

,

$$\mu\left(\lambda(v,\omega)\right) \approx \mu\left(\bar{\lambda}\right) + \left.\left[\frac{\partial \mu}{\partial \lambda}\frac{\partial \lambda}{\partial v}\right]\right|_{\lambda=\bar{\lambda}} (v - \bar{v}) + \left.\left[\frac{\partial \mu}{\partial \lambda}\frac{\partial \lambda}{\partial \omega}\right]\right|_{\lambda=\bar{\lambda}} (\omega - \bar{\omega})$$

$$\mu(\lambda) \approx \mu_0 + \mu_1 \frac{\bar{\omega} r}{\bar{v}^2}\delta v - \mu_1 \frac{r}{\bar{v}}\delta\omega$$

Linearization model (method II)

$$\begin{cases} J\dot{\omega} = rF_z\left[\mu_0 + \mu_1 \dfrac{\overline{\omega} r}{\overline{v}^2}\delta v - \mu_1 \dfrac{r}{\overline{v}}\delta\omega\right] - T_b \\ m\dot{v} = -F_z\left[\mu_0 + \mu_1 \dfrac{\overline{\omega} r}{\overline{v}^2}\delta v - \mu_1 \dfrac{r}{\overline{v}}\delta\omega\right] \end{cases}$$

, $$\begin{cases} J\dot{\overline{\omega}} = rF_z[\mu_0] - \overline{T}_b \\ m\dot{\overline{v}} = -F_z[\mu_0] \end{cases}$$

Linearization point

$$\begin{cases} \delta\dot{\omega} = \left[-\mu_1 \dfrac{r^2 F_z}{\overline{v} J}\right]\delta\omega + \left[\mu_1 \dfrac{\overline{\omega} r^2}{J\overline{v}^2} F_z\right]\delta v - \dfrac{1}{J}\delta T_b \\ \delta\dot{v} = \left[\mu_1 \dfrac{r}{m\overline{v}} F_z\right]\delta\omega + \left[-\mu_1 \dfrac{\overline{\omega} r}{m\overline{v}^2} F_z\right]\delta v \end{cases}$$

The relation we obtain is identical to the previous one ...?

Linearization model (method II)

$$\begin{cases} \delta\dot{\omega} = \left[-\mu_1 \dfrac{r^2 F_z}{\bar{v} J} \right] \delta\omega + \left[\mu_1 \dfrac{\bar{\omega} r^2}{J \bar{v}^2} F_z \right] \delta v - \dfrac{1}{J} \delta T_b \\ \delta\dot{v} = \left[\mu_1 \dfrac{r}{m\bar{v}} F_z \right] \delta\omega + \left[-\mu_1 \dfrac{\bar{\omega} r}{m \bar{v}^2} F_z \right] \delta v \end{cases}$$

```
>> A=sym('[-m1*r^2*Fz/(v*J),m1*w*r^2*Fz/(J*v^2);m1*r*Fz/(v*m),-m1*w*r*Fz/(m*v^2)]');
>> B=sym('[-1/J;0]');
>> C=sym('[1,0;0,1]');
>> I=sym(eye(2));
>> F=C*inv(sym('s')*I-A)*B;
>> pretty(F)

                    [                  2                        ]
                    [             s m v  + m1 w r Fz            ]
                    [- ----------------------------------------]
                    [        2                          2       ]
                    [  s (s v  J m + J m1 w r Fz + m1 r  Fz m v)]
                    [                                           ]
                    [                  m1 r Fz v                ]
                    [- ----------------------------------------]
                    [        2                          2       ]
                    [  s (s v  J m + J m1 w r Fz + m1 r  Fz m v)]
```

$$G_\omega(s) = -\frac{1}{Js} \frac{s + \dfrac{\mu_1(\bar{\lambda}) F_z}{m\bar{v}} (1-\bar{\lambda})}{s + \left[\dfrac{\mu_1(\bar{\lambda}) F_z}{m\bar{v}} \left((1-\bar{\lambda}) + \dfrac{mr^2}{J} \right) \right]} \qquad G_v(s) = -\frac{1}{Js} \frac{\dfrac{\mu_1(\bar{\lambda}) r F_z}{m\bar{v}}}{s + \left[\dfrac{\mu_1(\bar{\lambda}) F_z}{m\bar{v}} \left((1-\bar{\lambda}) + \dfrac{mr^2}{J} \right) \right]}$$

Linearization model (method II)

$$\delta\lambda = -\frac{r}{\bar{v}}\delta\omega + \frac{\bar{\omega} r}{\bar{v}^2}\delta v$$

$$\boxed{G_\lambda(s) = \frac{\left[\dfrac{r}{vJ}\right]}{s + \left[\dfrac{\mu_1(\bar{\lambda})F_z}{m\bar{v}}\left((1-\bar{\lambda}) + \dfrac{mr^2}{J}\right)\right]}}$$

$$\Updownarrow$$

$$G_\lambda(s) = \frac{\left[\dfrac{r}{vJ}\right]}{s + \left[\dfrac{F_z}{v}\left(\left(\dfrac{(1-\bar{\lambda})}{m} + \dfrac{r^2}{J}\right)\mu_1 - \dfrac{\mu_0}{m}\right)\right]}$$

Linearization model

Method I

$$G_\lambda(s) = \frac{\left[\frac{r}{vJ}\right]}{s + \left[\frac{F_z}{v}\left(\left(\frac{(1-\bar{\lambda})}{m} + \frac{r^2}{J}\right)\mu_1 - \frac{\mu_0}{m}\right)\right]}$$

$$G_\eta(s) = \frac{r}{Jg}\,\frac{s + \frac{F_z}{mv}\left[(1-\bar{\lambda})\mu_1 - \mu_0\right]}{s + \left[\frac{F_z}{v}\left(\left(\frac{(1-\bar{\lambda})}{m} + \frac{r^2}{J}\right)\mu_1 - \frac{\mu_0}{m}\right)\right]}$$

Method II

$$G_\lambda(s) = \frac{\left[\frac{r}{vJ}\right]}{s + \left[\frac{\mu_1(\bar{\lambda})F_z}{m\bar{v}}\left((1-\bar{\lambda}) + \frac{mr^2}{J}\right)\right]}$$

$$G_\eta(s) = \frac{r}{Jg}\,\frac{s + \frac{F_z}{m\bar{v}}(1-\bar{\lambda})\mu_1(\bar{\lambda})}{s + \left[\frac{\mu_1(\bar{\lambda})F_z}{m\bar{v}}\left((1-\bar{\lambda}) + \frac{mr^2}{J}\right)\right]}$$

Linearization around an equilibrium point

Assumption of v-slowly-varying

Linearization around a non-equilibrium point

No v-slowly-varying assumption

Analysis: open-loop

$$G_\lambda(s) = \frac{\left[\dfrac{r}{vJ}\right]}{s + \left[\dfrac{\mu_1(\bar{\lambda})F_z}{m\bar{v}}\left((1-\bar{\lambda}) + \dfrac{mr^2}{J}\right)\right]}$$

$$G_\eta(s) = \frac{r}{Jg}\,\frac{s + \dfrac{F_z}{m\bar{v}}(1-\bar{\lambda})\mu_1(\bar{\lambda})}{s + \left[\dfrac{\mu_1(\bar{\lambda})F_z}{m\bar{v}}\left((1-\bar{\lambda}) + \dfrac{mr^2}{J}\right)\right]}$$

Pole (stability):

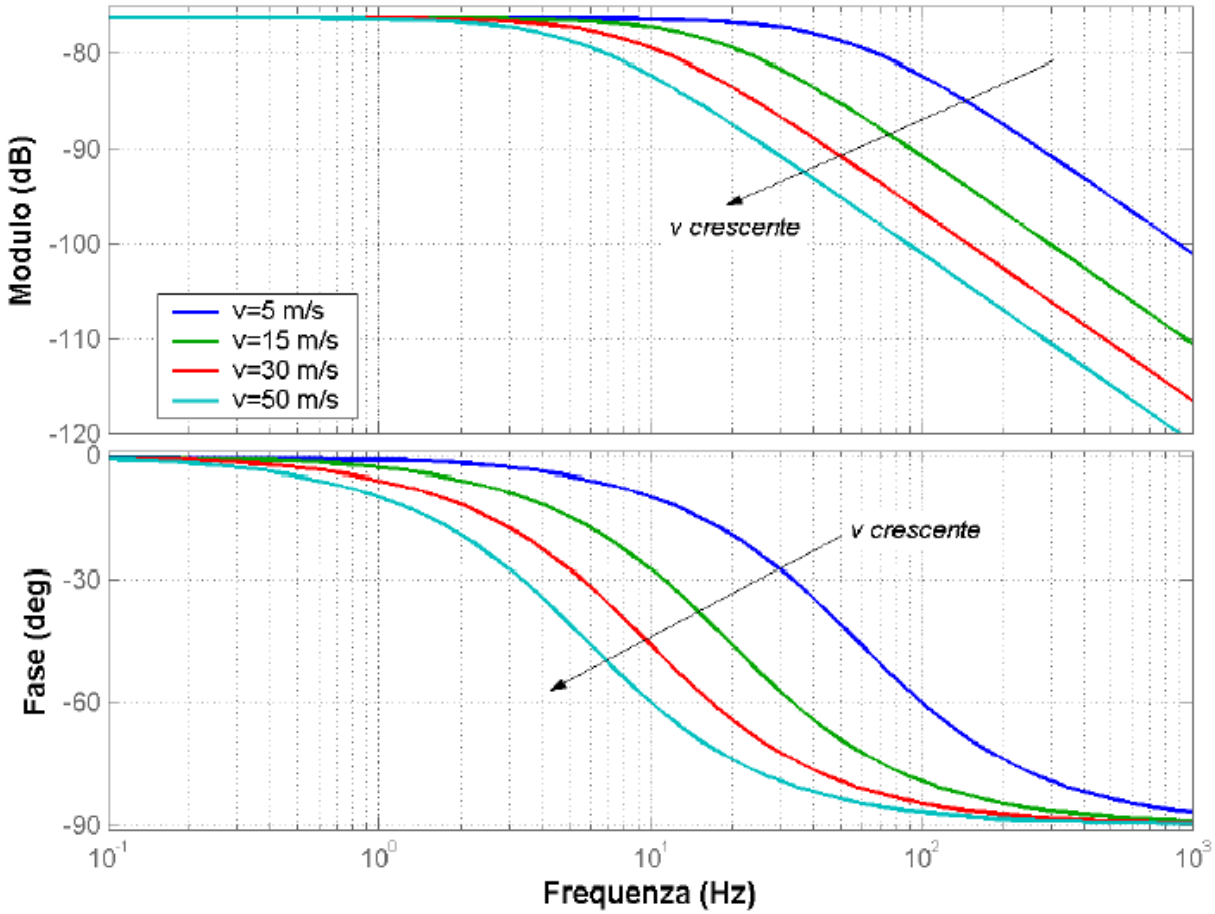
$$\frac{\mu_1(\bar{\lambda})F_z}{m\bar{v}}\left((1-\bar{\lambda}) + \frac{mr^2}{J}\right) > 0 \qquad \mu_1(\bar{\lambda}) > 0$$

Zero (minimum phase):

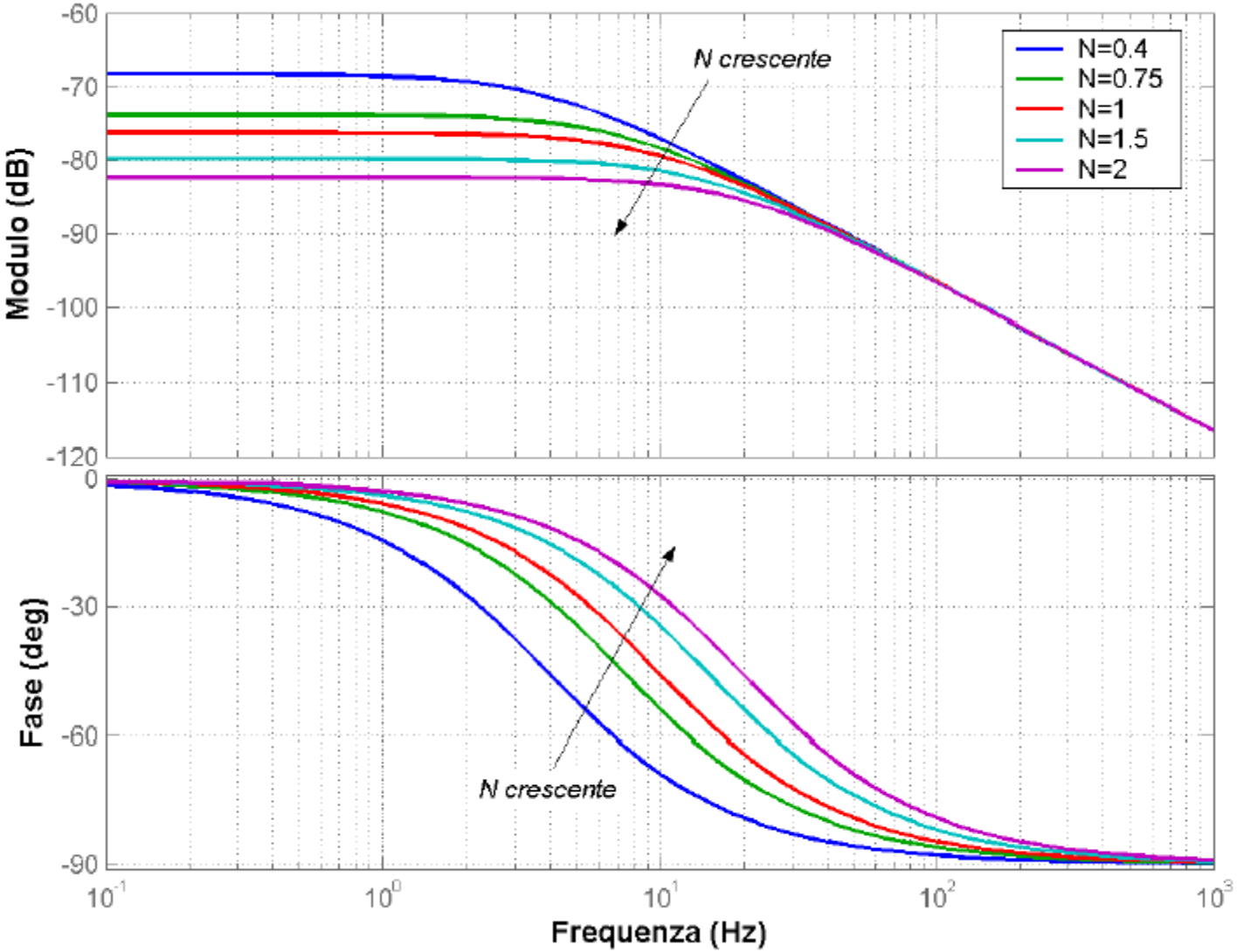
$$\frac{\mu_1(\bar{\lambda})F_z}{m\bar{v}}(1-\bar{\lambda}) > 0 \qquad \mu_1(\bar{\lambda}) > 0$$

Sensitivity with respect to the longitudinal vehicle speed

F.d.t. From T_b to λ (nominal F_z= Mg)

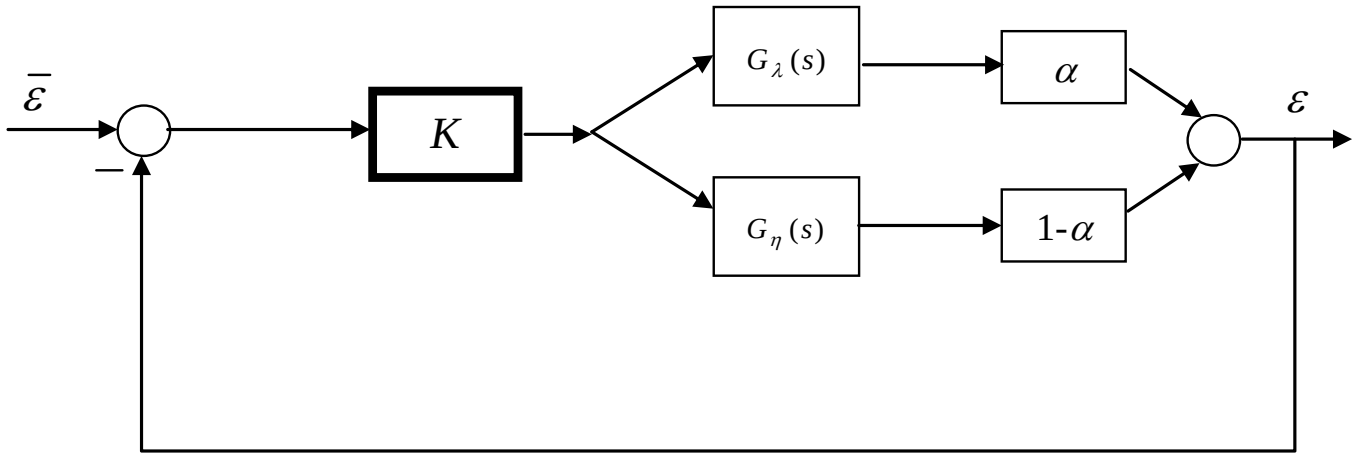


Sensitivity with respect to vertical load



$$N = \frac{F_z}{mg}$$

Control schemes



$$G_\lambda(s) = \frac{\left[\dfrac{r}{vJ}\right]}{s + \left[\dfrac{\mu_1(\bar{\lambda})F_z}{m\bar{v}}\left((1-\bar{\lambda}) + \dfrac{mr^2}{J}\right)\right]}$$

α= 1 (Slip control)
α= 0 (Deceleration control)

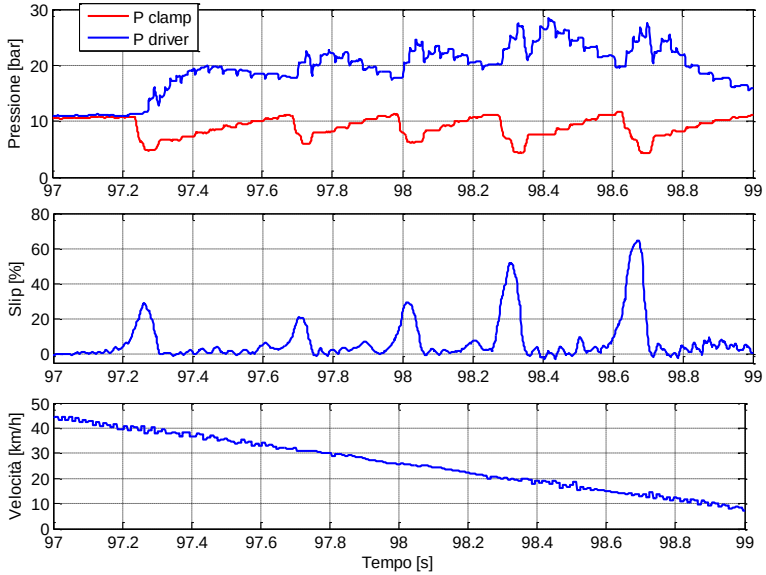
$$G_\eta(s) = \frac{\dfrac{r}{Jg}\left(s + \left[\dfrac{\mu_1(\bar{\lambda})F_z}{m\bar{v}}\left((1-\bar{\lambda})\right)\right]\right)}{s + \left[\dfrac{\mu_1(\bar{\lambda})F_z}{m\bar{v}}\left((1-\bar{\lambda}) + \dfrac{mr^2}{J}\right)\right]}$$

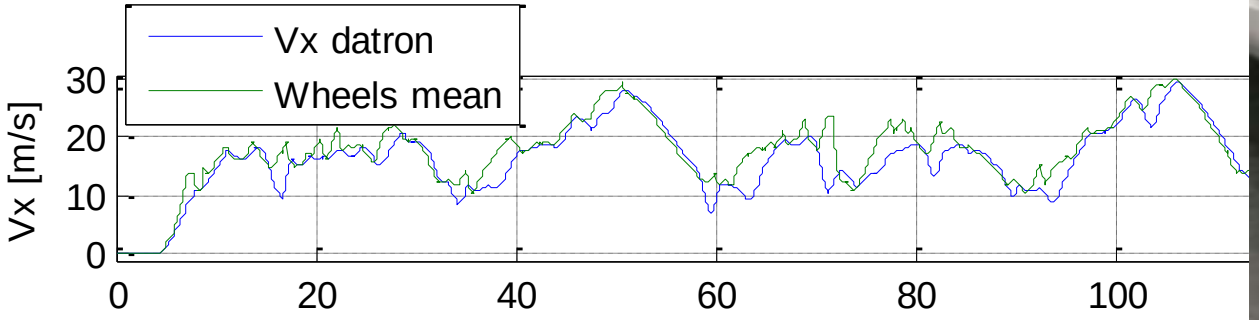
Remark on v-estimation

How to MEASURE v?

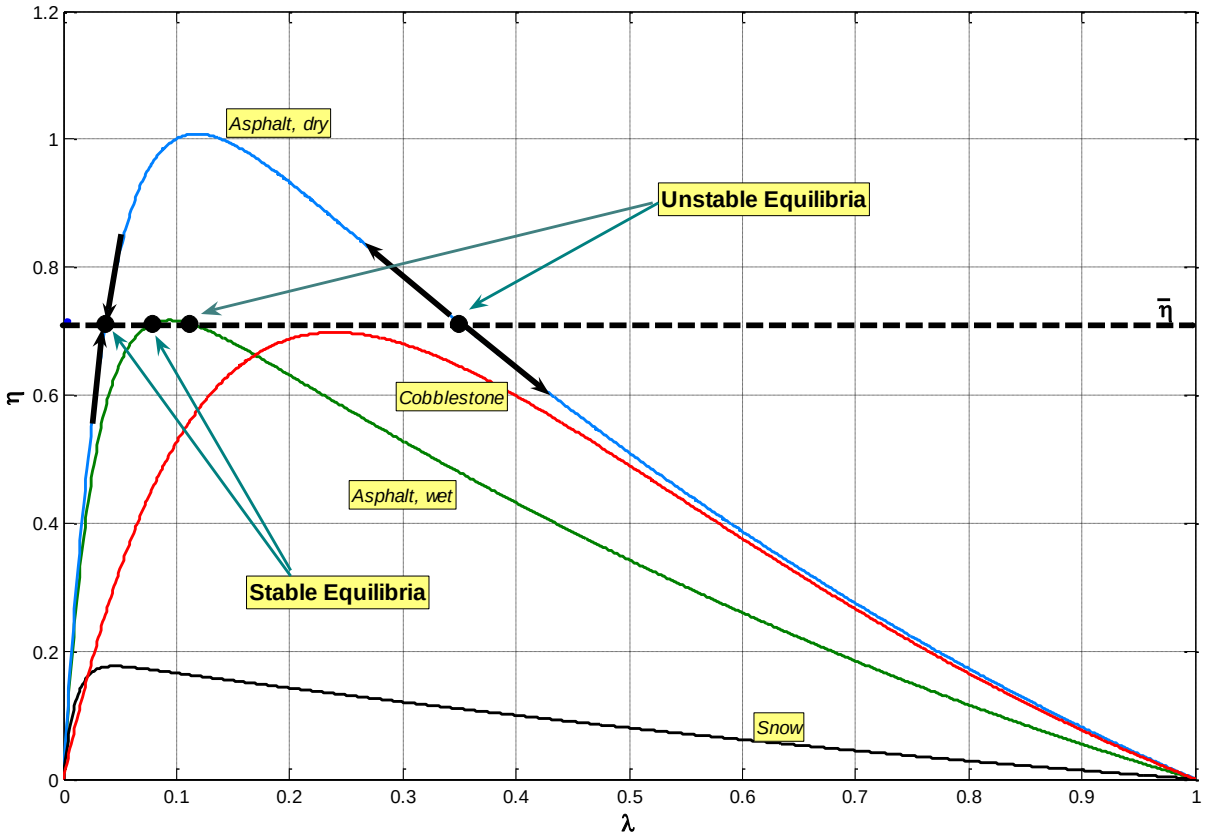
How to ESTIMATE v?

- **Only with 4 wheels speed**
- **4 wheels and inertial sensing**





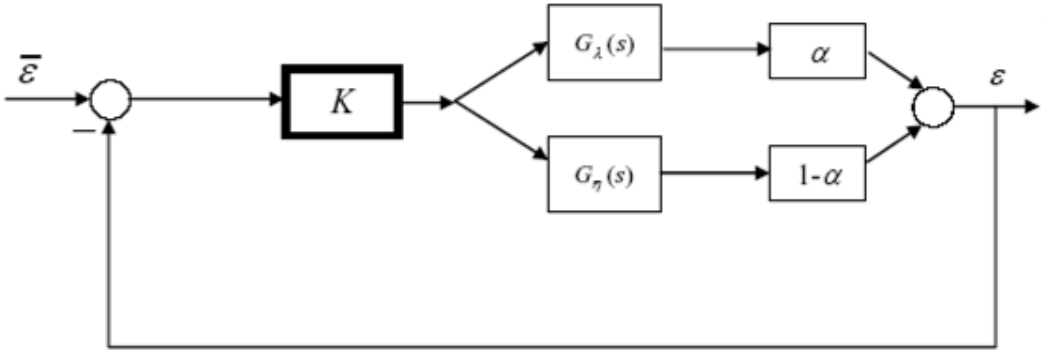
α = 0 (deceleration control)



- No fixed set-point
- No unique equilibrium

- deceleration measurement very simple and robust

α = 0 (deceleration control)



$$G_\eta(s) = \frac{\frac{r}{Jg}\left(s + \left[\frac{\mu_1(\bar{\lambda})F_z}{m\bar{v}}\left((1-\bar{\lambda})\right)\right]\right)}{s + \left[\frac{\mu_1(\bar{\lambda})F_z}{m\bar{v}}\left((1-\bar{\lambda}) + \frac{mr^2}{J}\right)\right]}$$

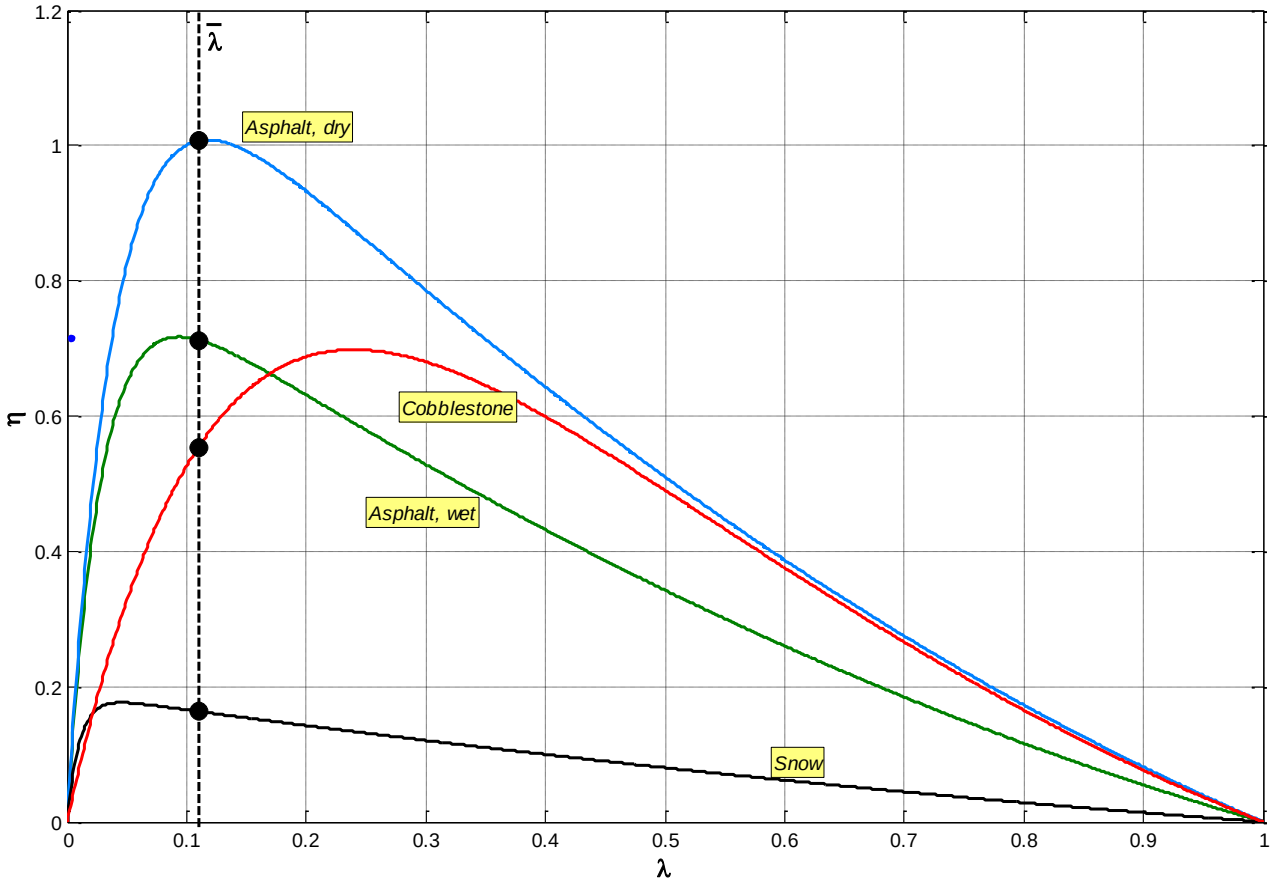
$$\chi_\eta(s) = \left(1 + K\frac{r}{Jg}\right)s + \frac{\mu_1(\bar{\lambda})F_z}{m\bar{v}}\left((1-\bar{\lambda})\left(1 + K\frac{r}{Jg}\right) + \frac{mr^2}{J}\right)$$

$$\Downarrow$$

$$\boxed{\mu_1(\bar{\lambda})\left((1-\bar{\lambda})\left(1 + K\frac{r}{Jg}\right) + \frac{mr^2}{J}\right) > 0}$$

Impossible to find a stabilizing K in all the working conditions

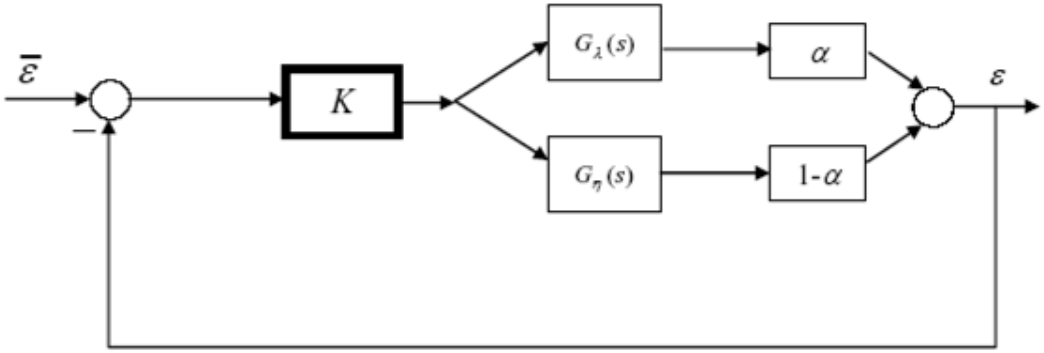
α = 1 (slip control)



- Set-point (almost) fixed
- Unique equilibrium

- Sensitivity to measurement noise (slip measurement is critical)

α = 1 (slip control)



$$G_\lambda(s) = \frac{\left[\dfrac{r}{vJ}\right]}{s + \left[\dfrac{\mu_1(\bar{\lambda})F_z}{m\bar{v}}\left((1-\bar{\lambda}) + \dfrac{mr^2}{J}\right)\right]}$$

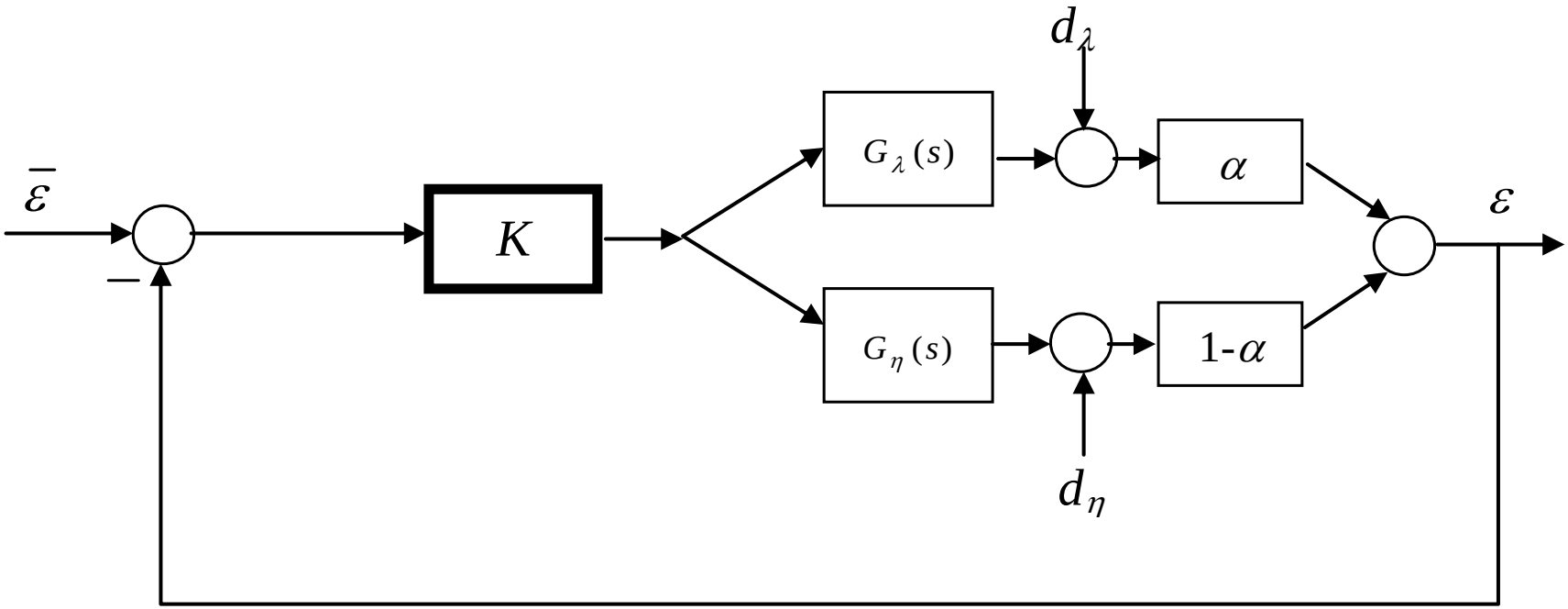
$$\chi_\lambda(s) = s + \frac{1}{\bar{v}}\left[\frac{\mu_1(\bar{\lambda})F_z}{m}\left((1-\bar{\lambda}) + \frac{mr^2}{J}\right) + K\frac{r}{J}\right]$$

$$\Downarrow$$

$$\boxed{K\frac{r}{J} > -\frac{\mu_1(\bar{\lambda})F_z}{m}\left((1-\bar{\lambda}) + \frac{mr^2}{J}\right)}$$

Possible to find a stabilizing K in all the working conditions

Mixed Slip-Deceleration control (MSD-control)

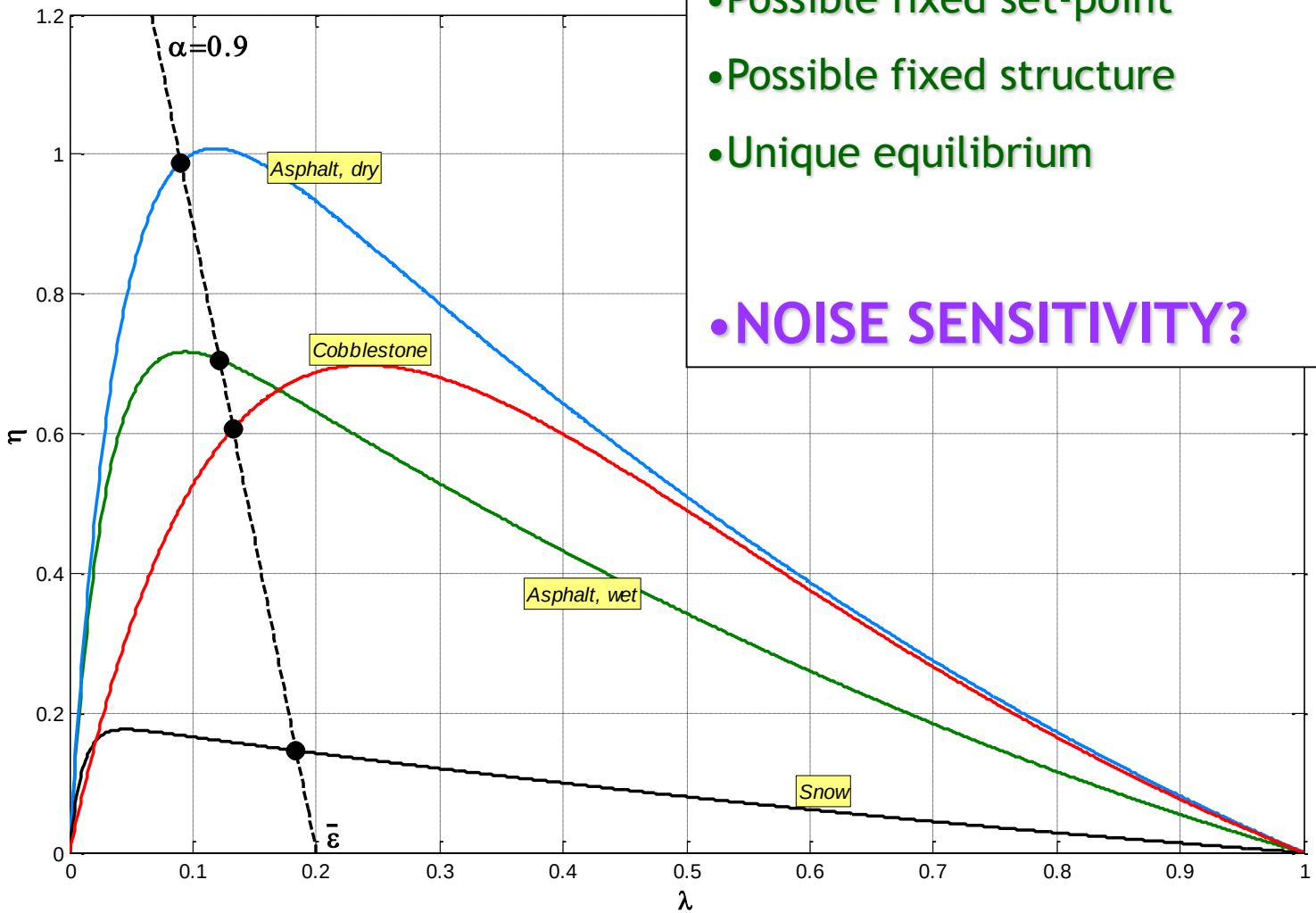


$$\epsilon = \alpha\lambda + (1-\alpha)\eta, \qquad \alpha \in [0,1]$$

Savaresi s.m., M. Tanelli, C. Cantoni (2007). Mixed slip-deceleration control in automotive braking systems. ASME Transactions: Journal of Dynamic Systems, Measurement and Control, Vol.129, No. 1, pp.20-31 [2008 ASME Dynamic Systems and Control Rudolf Kalman Best Paper Award]

PCT/EP2005/050820 (2005). "Sistema di controllo automatico della frenatura", Freni Brembo S.p.A. (Cantoni C., Savaresi S., Charalambakis D., Tanelli M.).

0 <α <1 (mixed slip-deceleration control - MSD-control)



- Possible fixed set-point
- Possible fixed structure
- Unique equilibrium

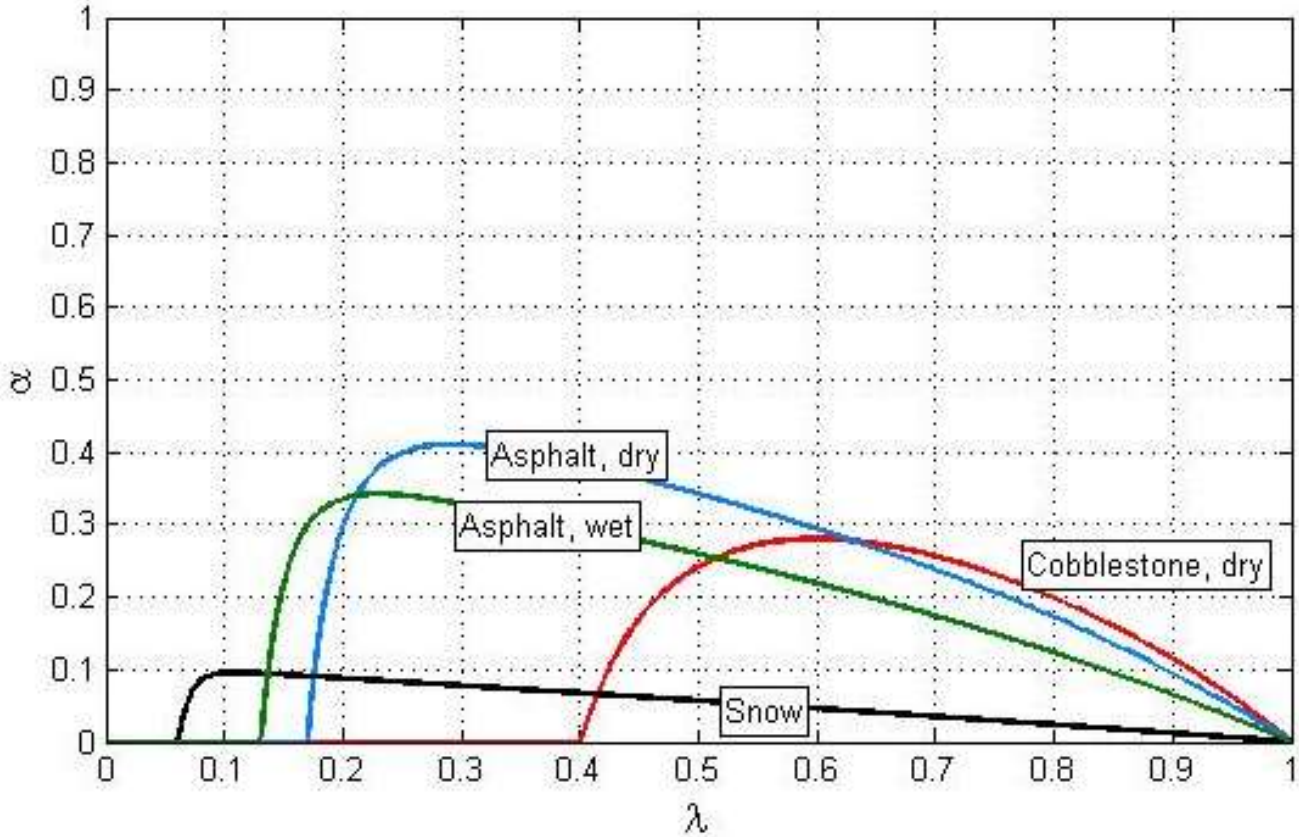
- **NOISE SENSITIVITY?**

Limits in the choice of α

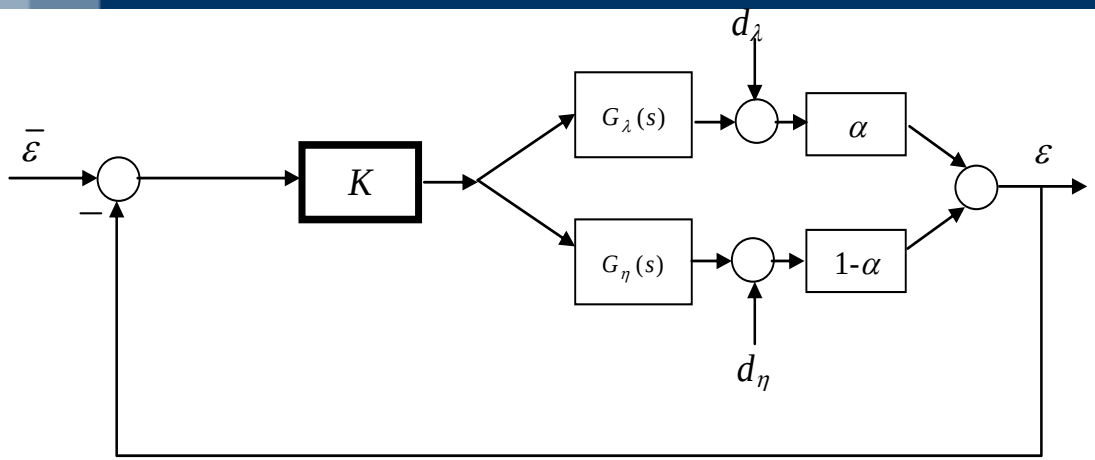
$$\alpha_{min} = \max_{\bar{\lambda},\theta_r} \frac{\mu_1(\bar{\lambda};\theta_r)(\bar{\lambda}-1)\frac{F_z}{mg}}{1+\left[\mu_1(\bar{\lambda};\theta_r)(\bar{\lambda}-1)\right]\frac{F_z}{mg}}$$

$$F_z = m\,g$$

$$\alpha_{min} \approx 0.45$$



Noise sensitivity



$$\sigma_\lambda^2 > \sigma_\eta^2$$

$$D_\varepsilon(s;\alpha) = \left(\alpha D_\lambda(s) + (1-\alpha) D_\eta(s)\right) S_\varepsilon(s;\alpha)$$

$$\gamma(\alpha) = \frac{\mathrm{var}\left[d_\varepsilon(\alpha)\right]}{\mathrm{var}\left[d_\varepsilon(1)\right]}$$

Noise sensitivity

$$\gamma(\alpha) = \frac{\mathrm{var}\left[d_\varepsilon(\alpha)\right]}{\mathrm{var}\left[d_\varepsilon(1)\right]} = \Psi(\alpha)\Phi(\alpha)$$

$$\Psi(\alpha) = \frac{\alpha^2\sigma_\lambda^2 + (1-\alpha)^2\sigma_\eta^2}{\sigma_\lambda^2}$$

$$0.80 \le \alpha \le 0.95$$

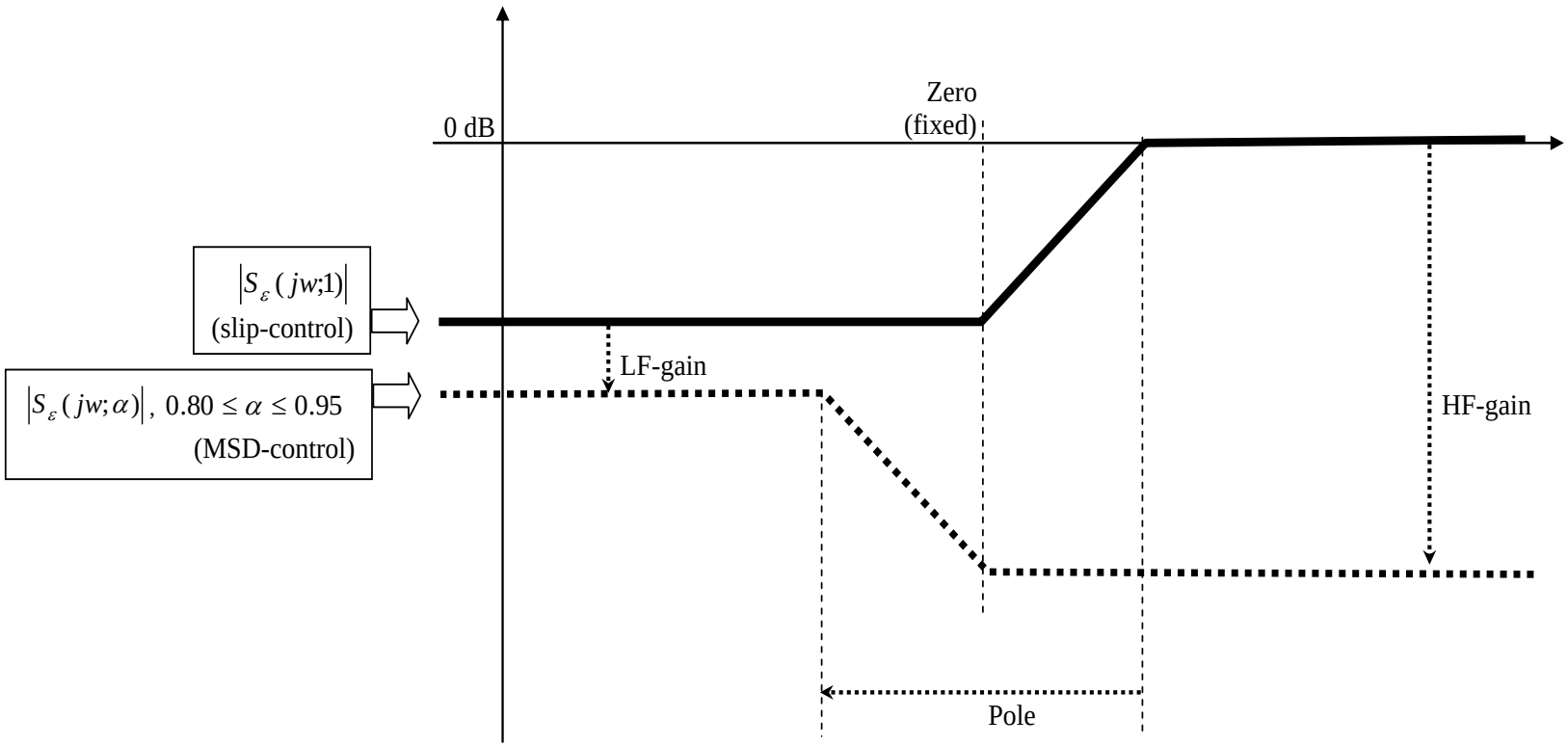
$$\sigma_\lambda^2 > \sigma_\eta^2$$

$$\Psi(\alpha) \approx \alpha^2$$

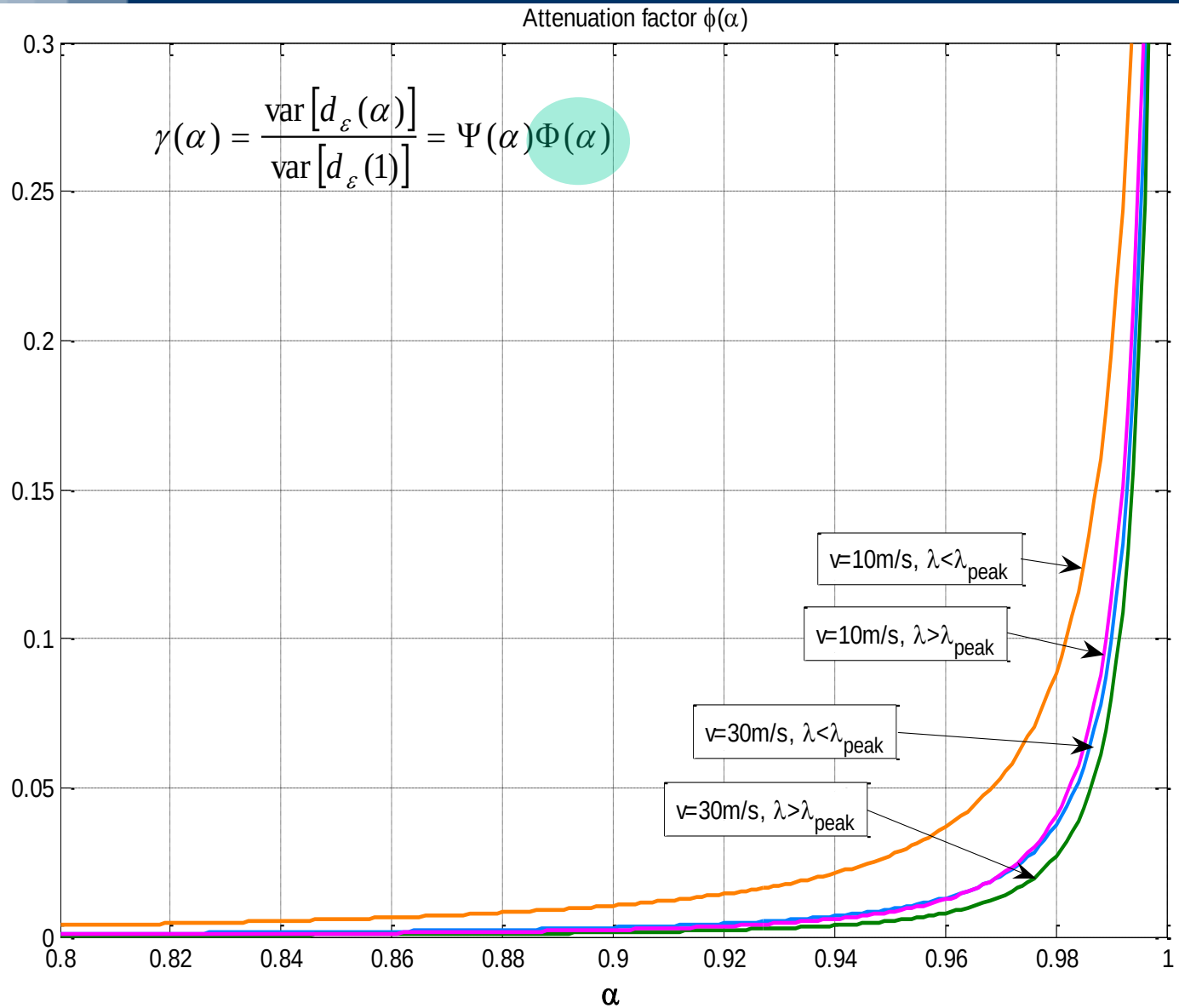
$$\Phi(\alpha) = \frac{\int_{w=0}^{\Omega_N} \left|S_\varepsilon(jw;\alpha)\right|^2 dw}{\int_{w=0}^{\Omega_N} \left|S_\varepsilon(jw;1)\right|^2 dw}$$

Noise sensitivity

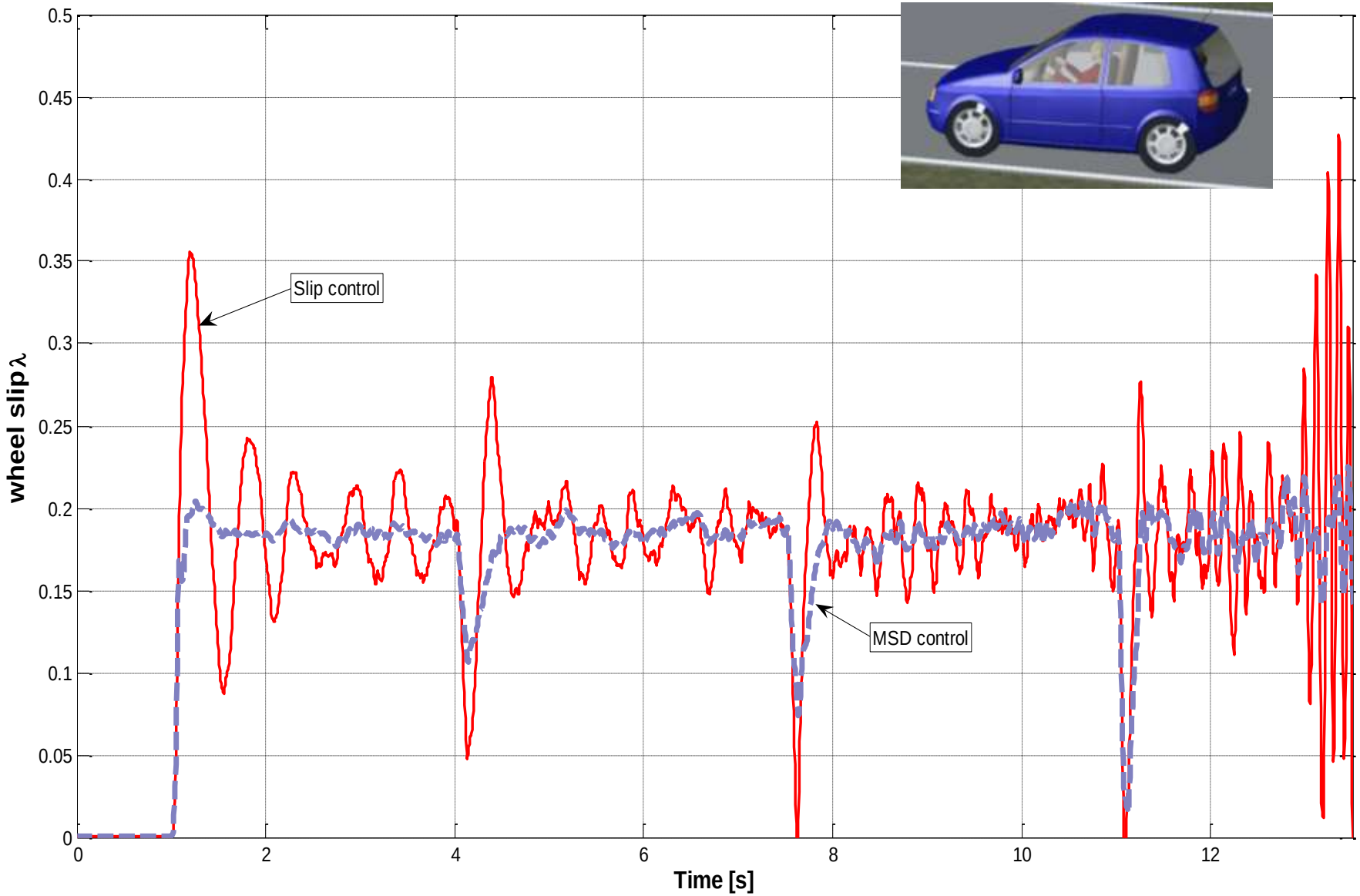
$$S_\varepsilon(s;\alpha) = \frac{s + \dfrac{\mu_1(\bar{\lambda})F_z}{m\bar{v}}\left((1-\bar{\lambda}) + \dfrac{mr^2}{J}\right)}{s\left[1 + K\dfrac{r}{Jg}(1-\alpha)\right] + \dfrac{1}{\bar{v}}\left[\dfrac{\mu_1(\bar{\lambda})F_z}{m}\left((1-\bar{\lambda})\left(1 + K\dfrac{r}{Jg}(1-\alpha)\right) + \dfrac{mr^2}{J}\right) + K\alpha\dfrac{r}{J}\right]}$$



Noise sensitivity



Slip Vs. MSD (α = 0.9)



Slip-control: stability analysis Bode / Nyquist

$[m=225, J=1, r=0.28]$

$$G_\lambda(s) = \frac{\left[\frac{r}{vJ}\right]}{s + \left[\frac{\mu_1(\bar{\lambda})F_z}{m\bar{v}}\left((1-\bar{\lambda}) + \frac{mr^2}{J}\right)\right]} \rightarrow$$

$$G_\lambda(s) = \frac{\frac{0.28}{\bar{v}}}{s + \frac{1}{\bar{v}}\left[\mu_1(\bar{\lambda})9.8\left((1-\bar{\lambda}) + 17.6\right)\right]}$$

Slip-control: Stability analysis (Bode/Nyquist)

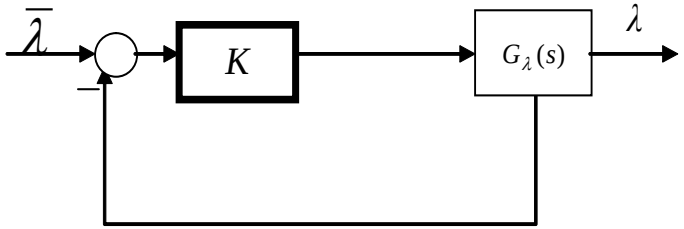
$$1000\left[\frac{\frac{0.28}{30}}{s+\frac{60}{30}}\right]$$

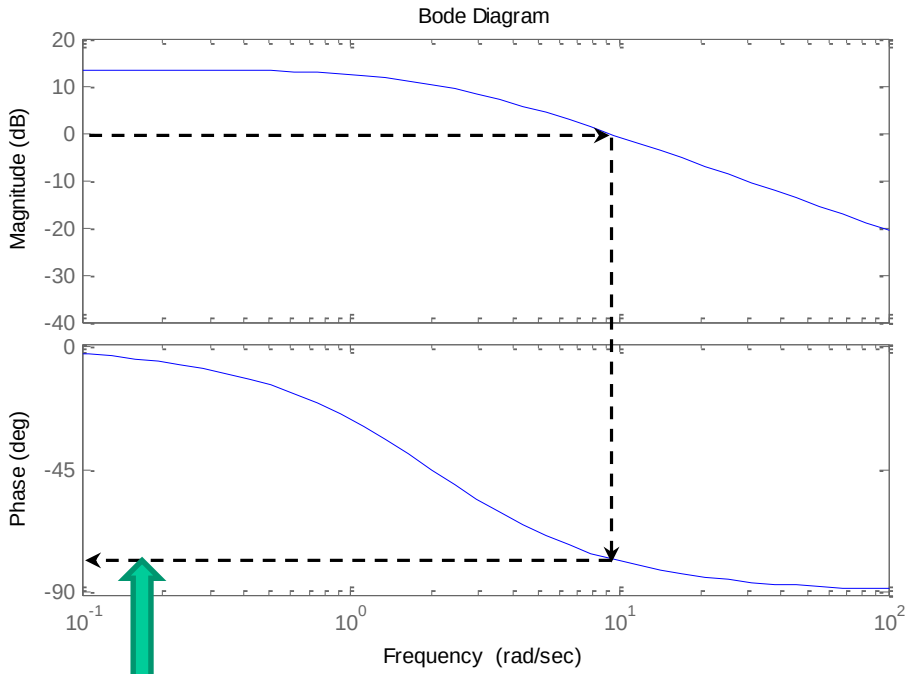
No unstable open-loop pole

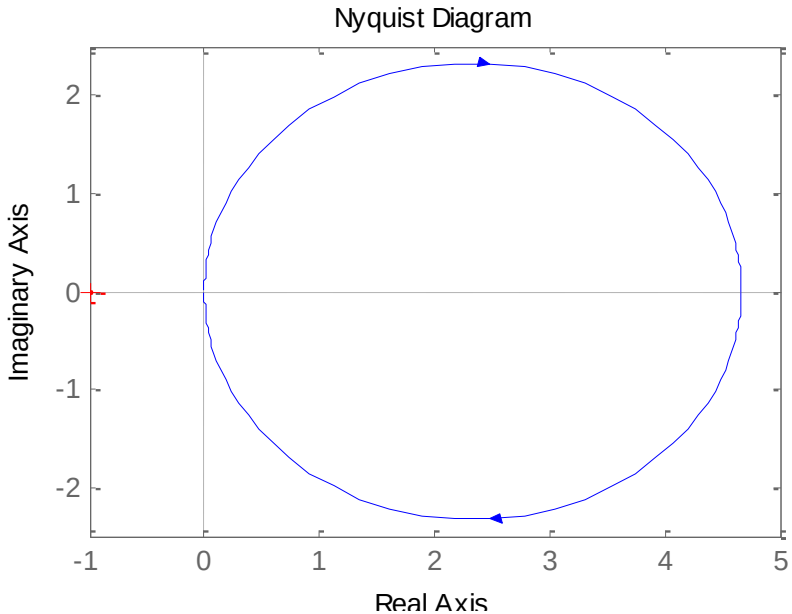
No (-1,0) (counter-clock-wise) encircling

Asymptotic stability

Bode criterion is applicable





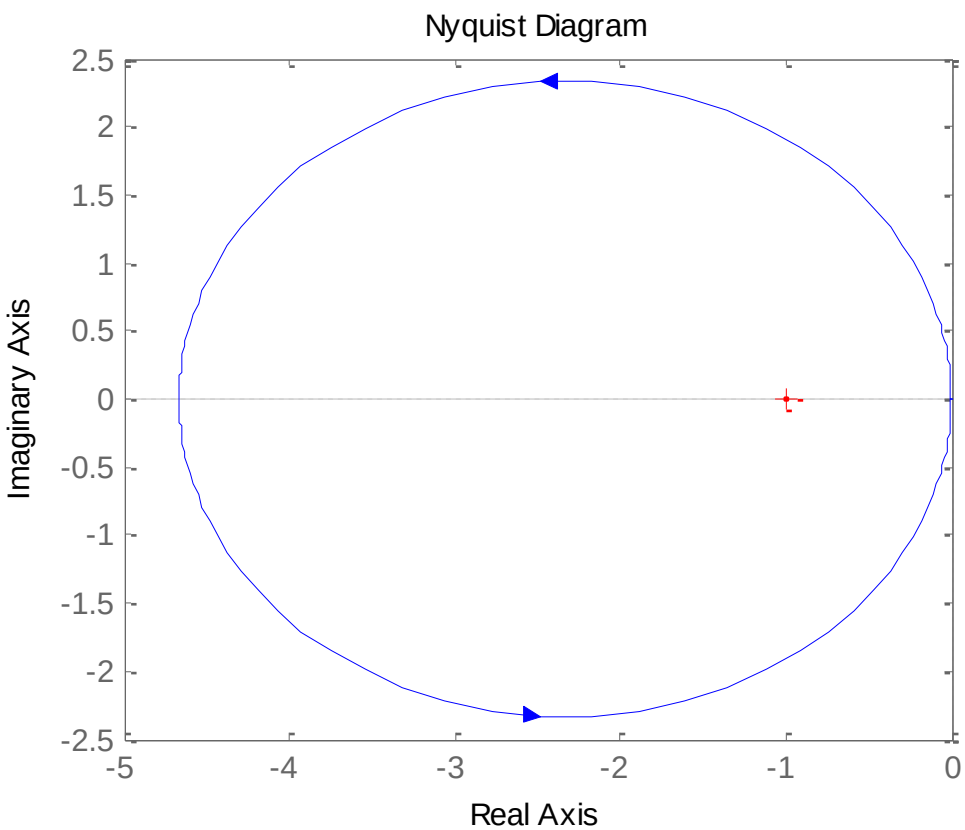


Phase margin = distance to “-180°”

Slip-control: stability analysis Bode / Nyquist

$$1000\left[\frac{\frac{0.28}{30}}{s-\frac{60}{30}}\right]$$

```
>> P=tf([0 0.28/30],[1 -60/30]);
>> nyquist(P*1000)
```



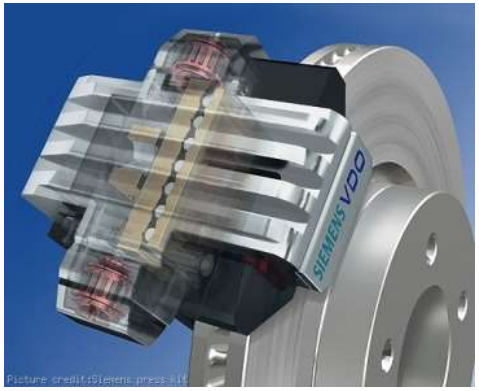
1 unstable open-loop pole

1 (-1,0) (counter-clock-wise) encircling

Asymptotic stability

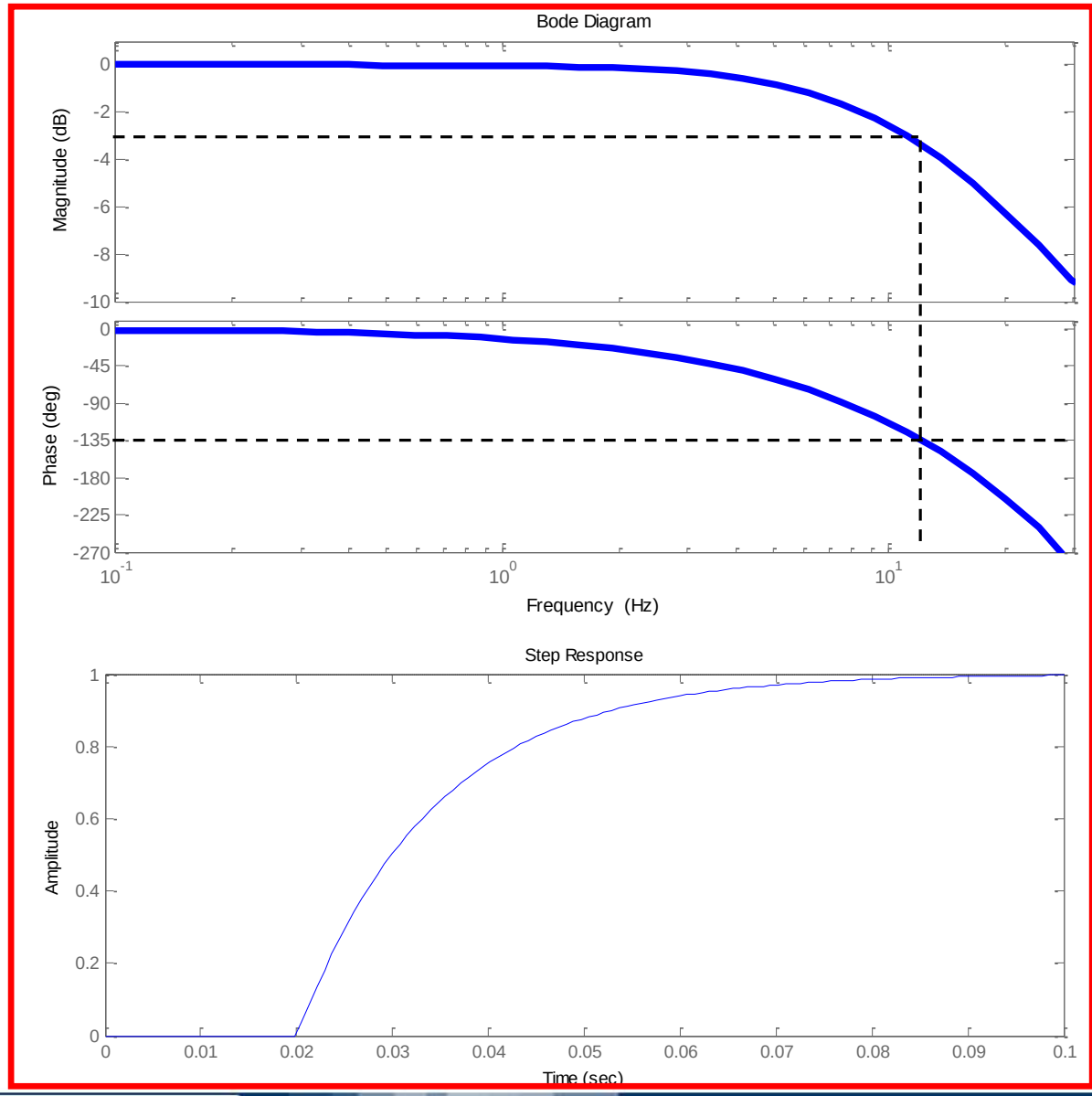
Bode criterion is NOT applicable

Actuator dynamics



Added dynamics (example)

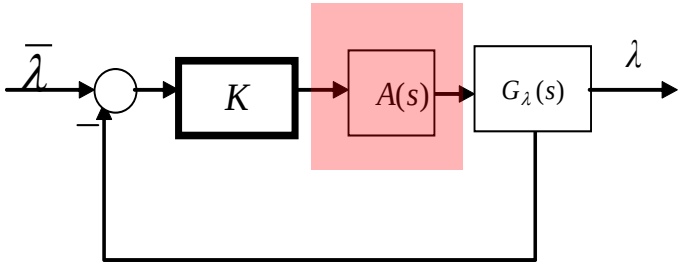
$$F_{caliper}(s) = \frac{70}{s+70} e^{-(0.02)s}$$

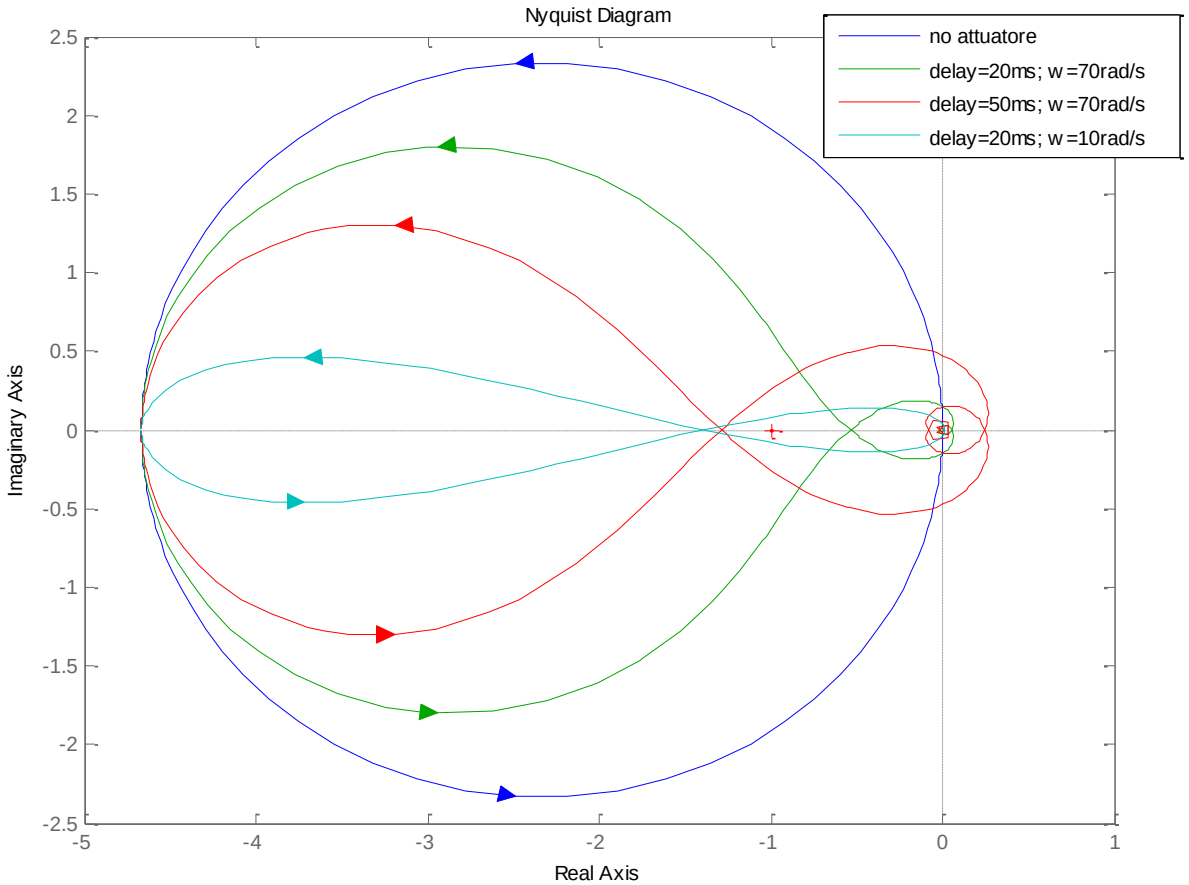


Example - effect (on stability) of actuator dynamics

$$1000\left[\frac{\frac{0.28}{10}}{s-\frac{60}{10}}\right]\left[\frac{70}{s+70}\right]\left[e^{-(0.02)s}\right]$$

Worst case (beyond the peak)





Slip-control: example of a design with PID

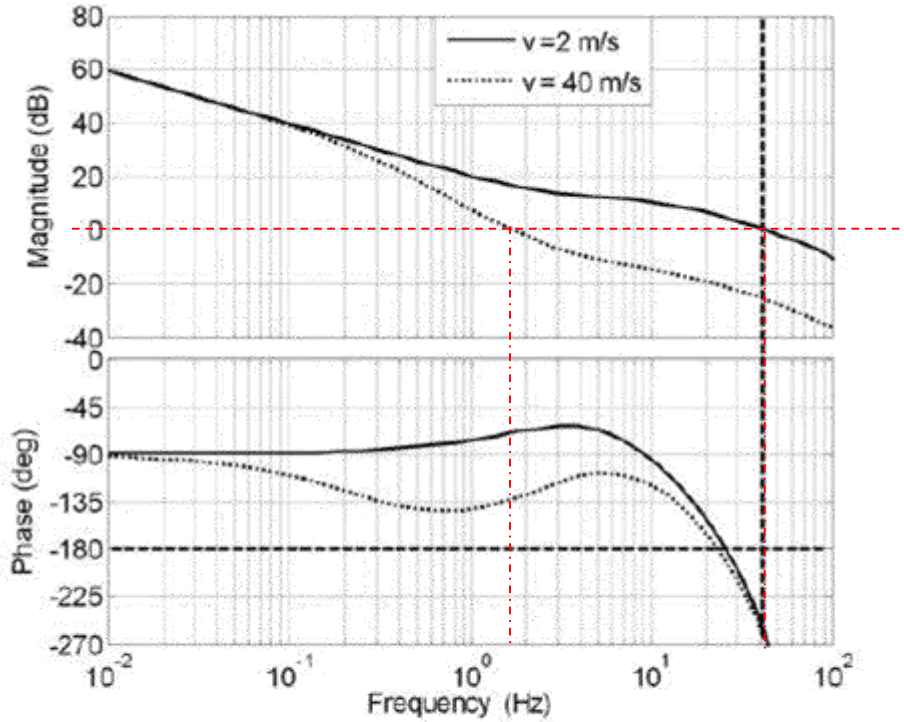


Figure 3.8 Magnitude and phase Bode diagrams of the frequency response associated with the loop transfer function $L_{2\lambda}(s)$ for $\overline{v} = 40$ m/s (*dotted line*) and $\overline{v} = 2$ m/s (*solid line*)

$$L_{2\lambda}(s) = R_\lambda(s) D^{\mathrm{st}}(s),$$

$$D^{st}(s) = G_{\mathrm{caliper}}(s) G_\lambda^{\mathrm{st}}(s)$$

$$G_\lambda^{\mathrm{st}}(s) = \frac{0.3/\overline{v}}{s + 60/\overline{v}},$$

$$R_\lambda(s) = 12000 \frac{(1 + \frac{1}{20}s)^2}{s(1 + \frac{1}{500}s)}.$$

Cut-off frequency: between 2 and 40Hz about

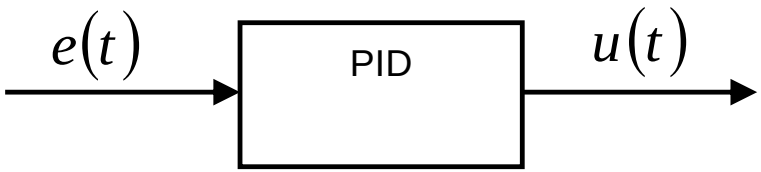
At 2m/s becomes unstable

$$G_{\mathrm{caliper}}(s) = \frac{\omega_{\mathrm{act}}}{s + \omega_{\mathrm{act}}} e^{-s\tau}, \qquad (1.4)$$

with $\omega_{\mathrm{act}} = 70$ rad/s and $\tau = 10$ ms.

Recall: “Ideal” PID

Controllers **PID** → action **P**roportional,**I**ntegral, **D**erivative.



$$u(t) = K_p e(t) + K_i \int_0^t e(\tau)d\tau + K_d \frac{de(t)}{dt}$$

Coeff. Proportional

Coeff. Integral Action

Coeff. Derivative Action

$$u(t) = K_p \left[e(t) + \frac{1}{T_i} \int_0^t e(\tau)d\tau + T_d \dot{e}(t) \right]$$

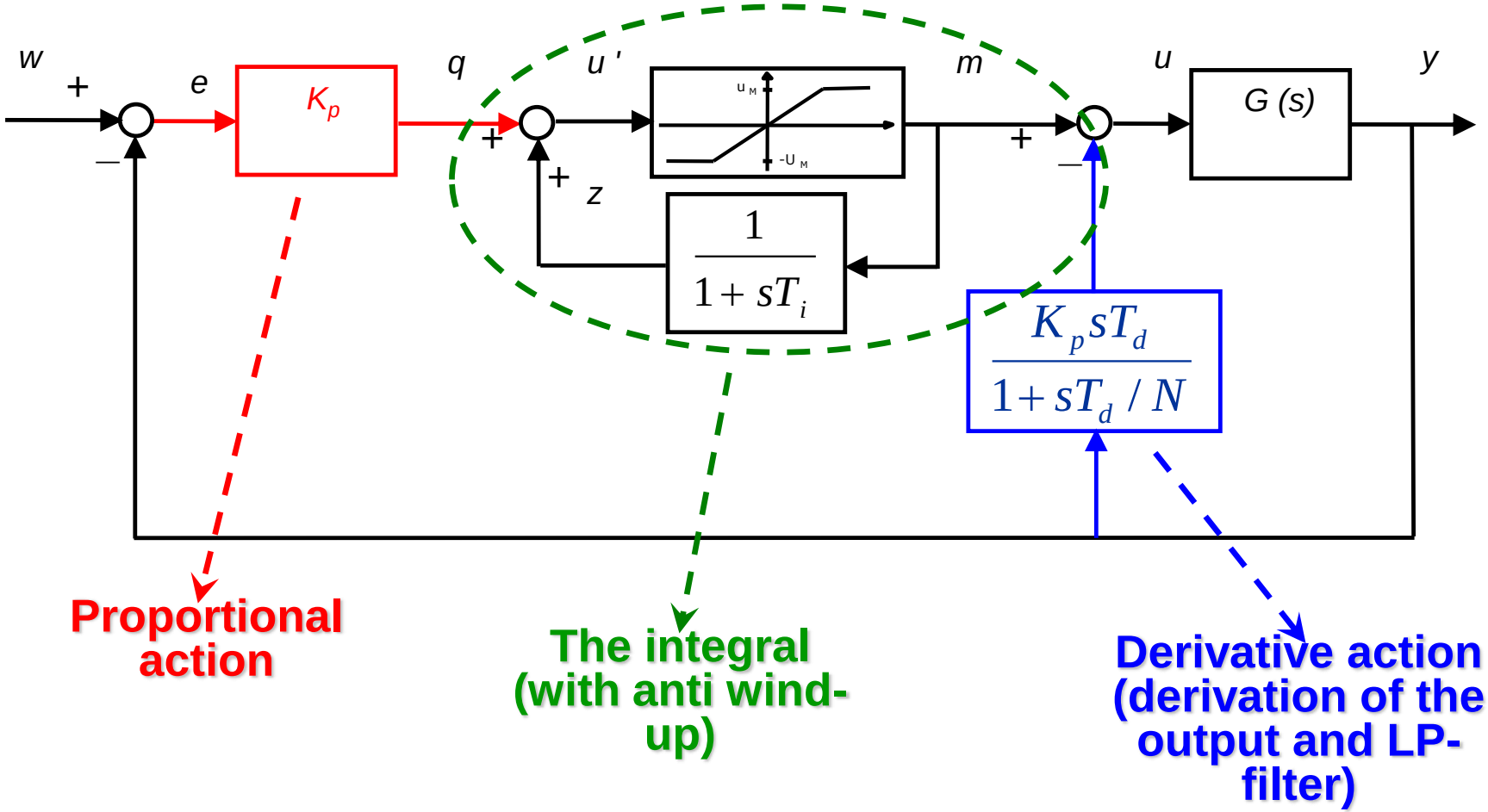
Integral action time

Derivative action time

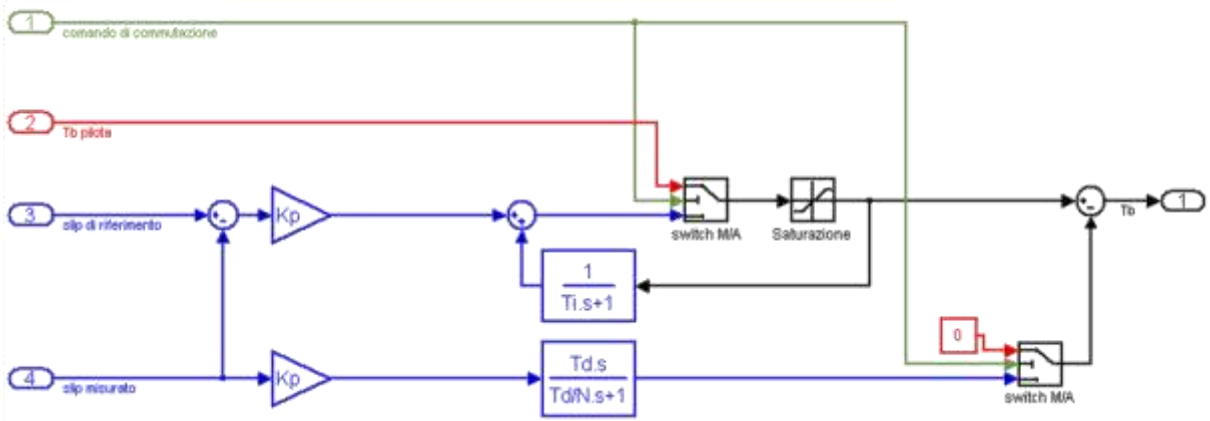
$$T_i = \frac{K_p}{K_i}$$

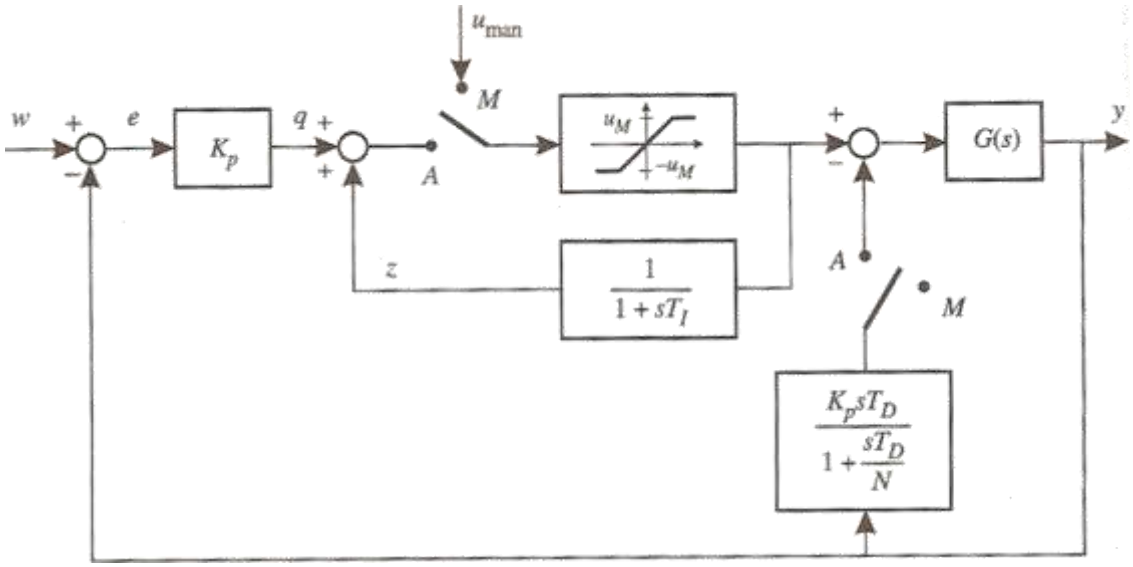
$$T_d = \frac{K_d}{K_p}$$

Recall: “real” PID



Logic activation-deactivation on a PID



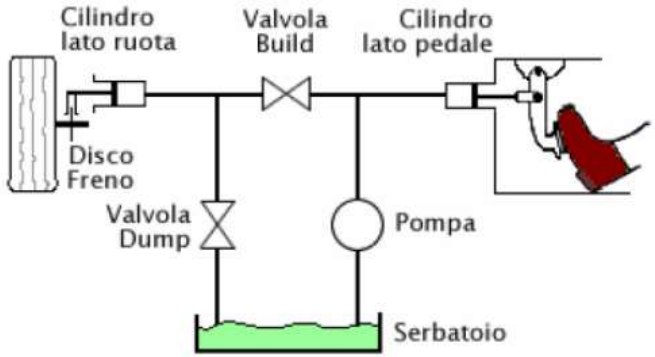


activation and deactivation variable(s)?

slip-control algorithms (ABS) with three-states-actuator

Actuator

HAB: Hydraulic Actuated Brakes (increase, hold, decrease)



Constraints:

1. Rate limiter $\frac{dT_b}{dt} = k$
2. Control action: u in {K, 0,-k}

Control actions:

Open Build - Close dump → Increase pressure (u = k)

Close Build - Open dump → decrease pressure (u =-k)

Build Close - Close dump → hold pressure (u = 0)

Model (recall)

$$\begin{cases} \dot{\lambda} = -\frac{1}{v}\left(\frac{(1-\lambda)}{m} + \frac{r^2}{J}\right) F_z \mu(\lambda) + \frac{r}{vJ} T_b \\ m\dot{v} = -F_z \mu(\lambda) \end{cases}$$

Hp: v is a parameter ("slowly variant")

$$\dot{\lambda} = -\frac{1-\lambda}{\omega r}\left(\frac{(1-\lambda)}{m} + \frac{r^2}{J}\right) F_z \mu(\lambda) + \frac{r}{J}\frac{1-\lambda}{\omega r} T_b$$

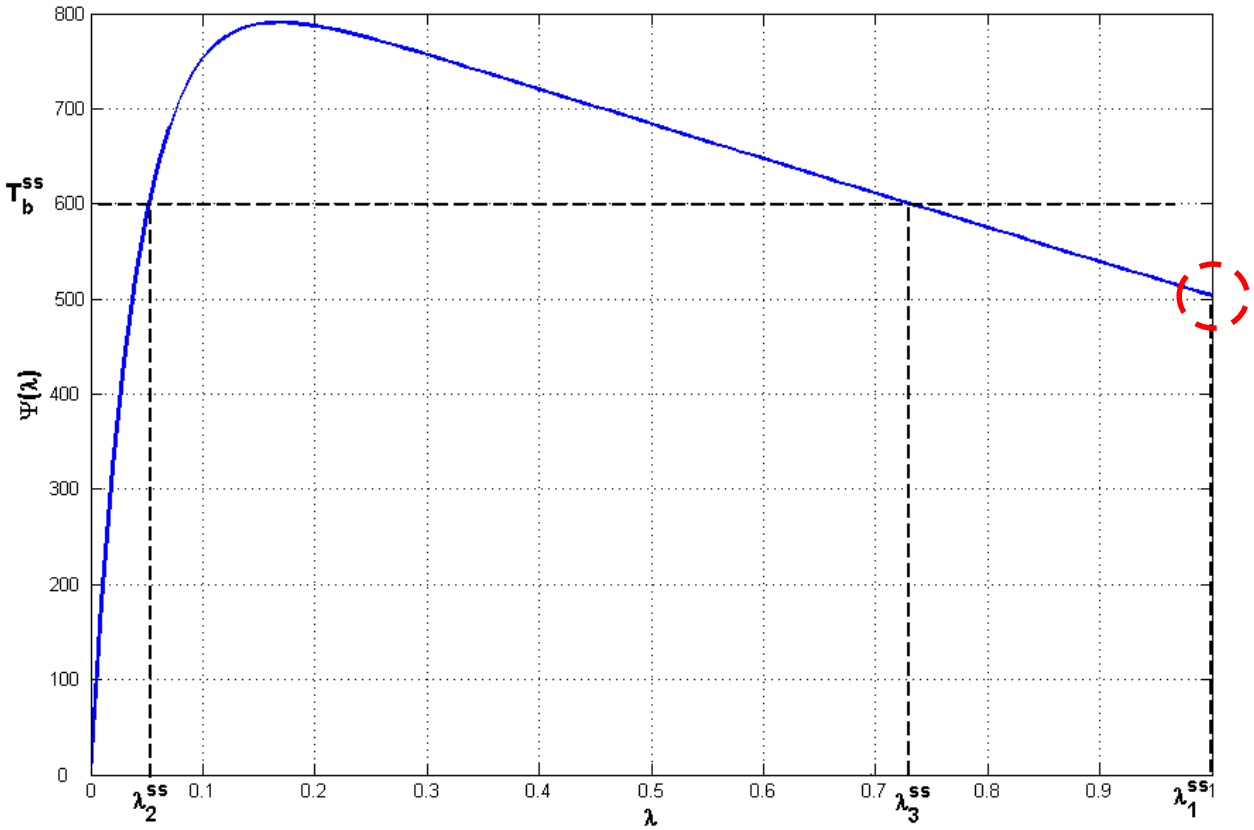
Equilibria (open loop)

$$\dot{\lambda} = -\frac{1-\lambda}{\omega r}\left(\frac{(1-\lambda)}{m} + \frac{r^2}{J}\right) F_z \mu(\lambda) + \frac{r}{J}\frac{1-\lambda}{\omega r} T_b$$

$$\boxed{\begin{aligned} &\lambda = \lambda^{ss} \\ &T_b = T_b^{ss} = \Psi(\lambda) = F_z \mu(\lambda)\left(r + \frac{J}{rm}(1-\lambda)\right) \end{aligned}}$$

Equilibria (open loop)

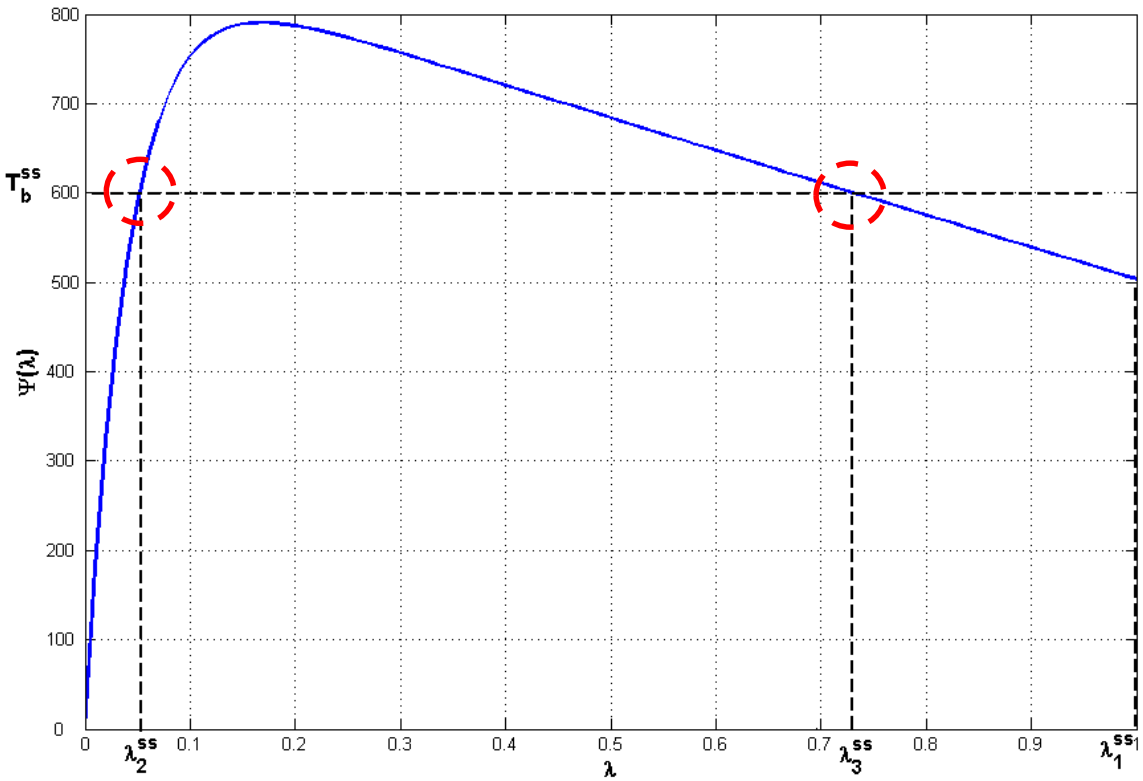
$$T_b^{ss} > \max_{\lambda} \Psi(\lambda) \rightarrow \lambda_1^{ss} = 1$$



Equilibria (open loop)

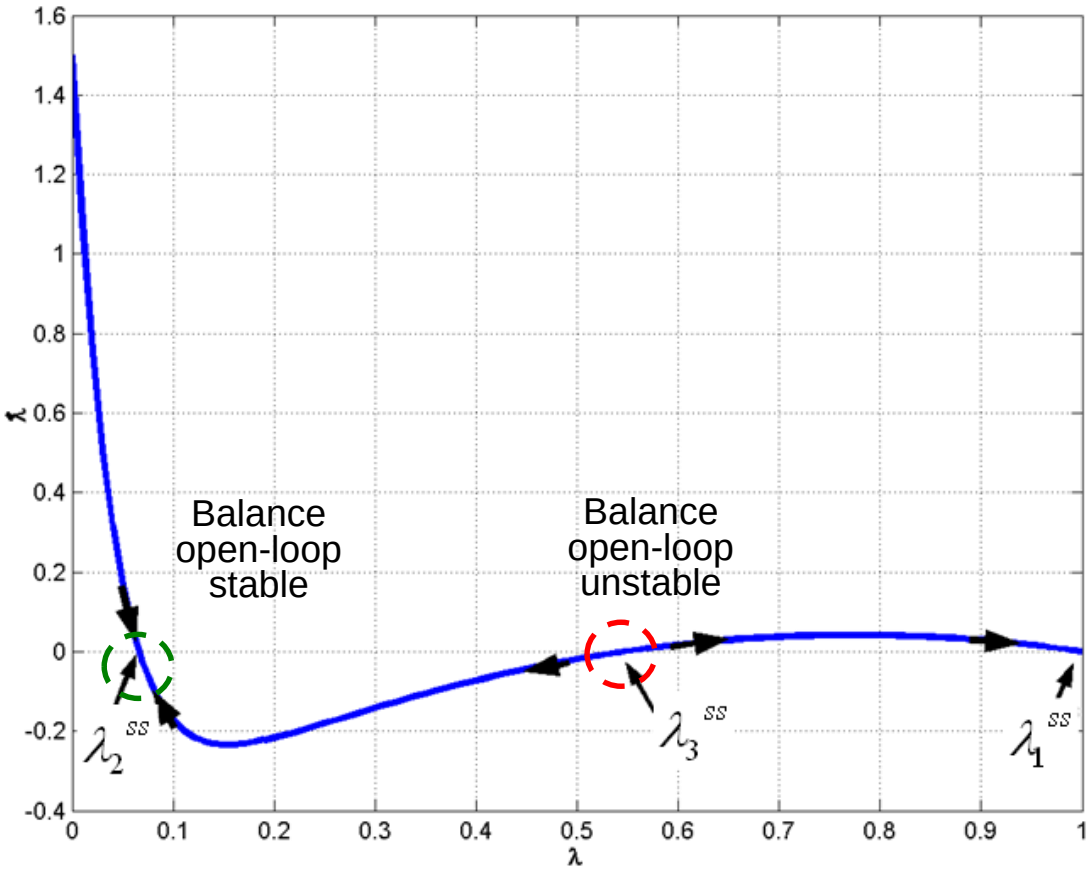
$$T_b^{ss} \leq \max_{\lambda} \Psi(\lambda)$$

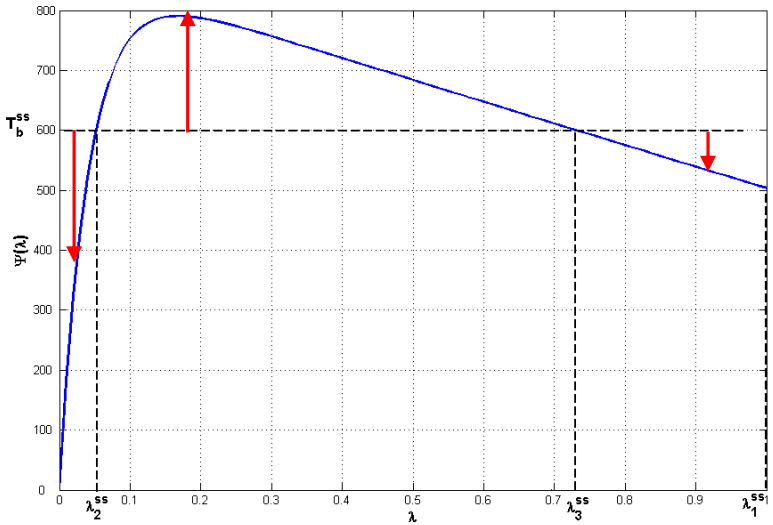
$$\lambda_2^{ss} = \bar{\lambda}_2, \qquad \lambda_3^{ss} = \bar{\lambda}_3$$



Phase diagrams (open-loop)

$$\dot{\lambda} = -\frac{(1-\lambda)}{J\omega}\left(\Psi(\lambda) - T_b\right)$$





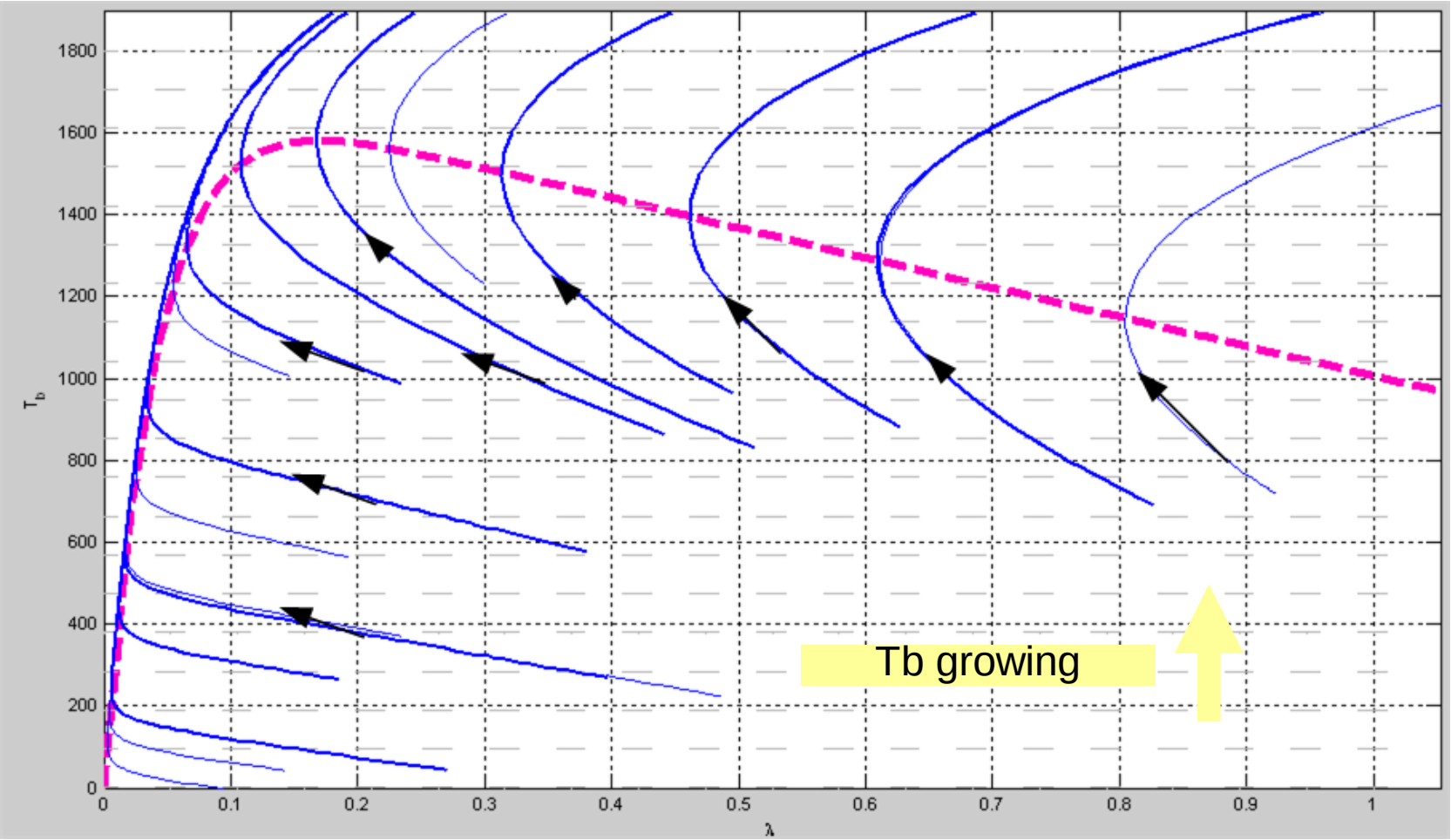
Analysis of closed loop

$$\begin{cases} \dot{\lambda} = -\frac{1-\lambda}{\omega r}\left(\frac{(1-\lambda)}{m} + \frac{r^2}{J}\right) F_z \mu(\lambda) + \frac{r}{J}\frac{1-\lambda}{\omega r} T_b \\ \dot{T}_b = u \end{cases}$$

We analyze the phase diagrams: u in {K, 0,-k}

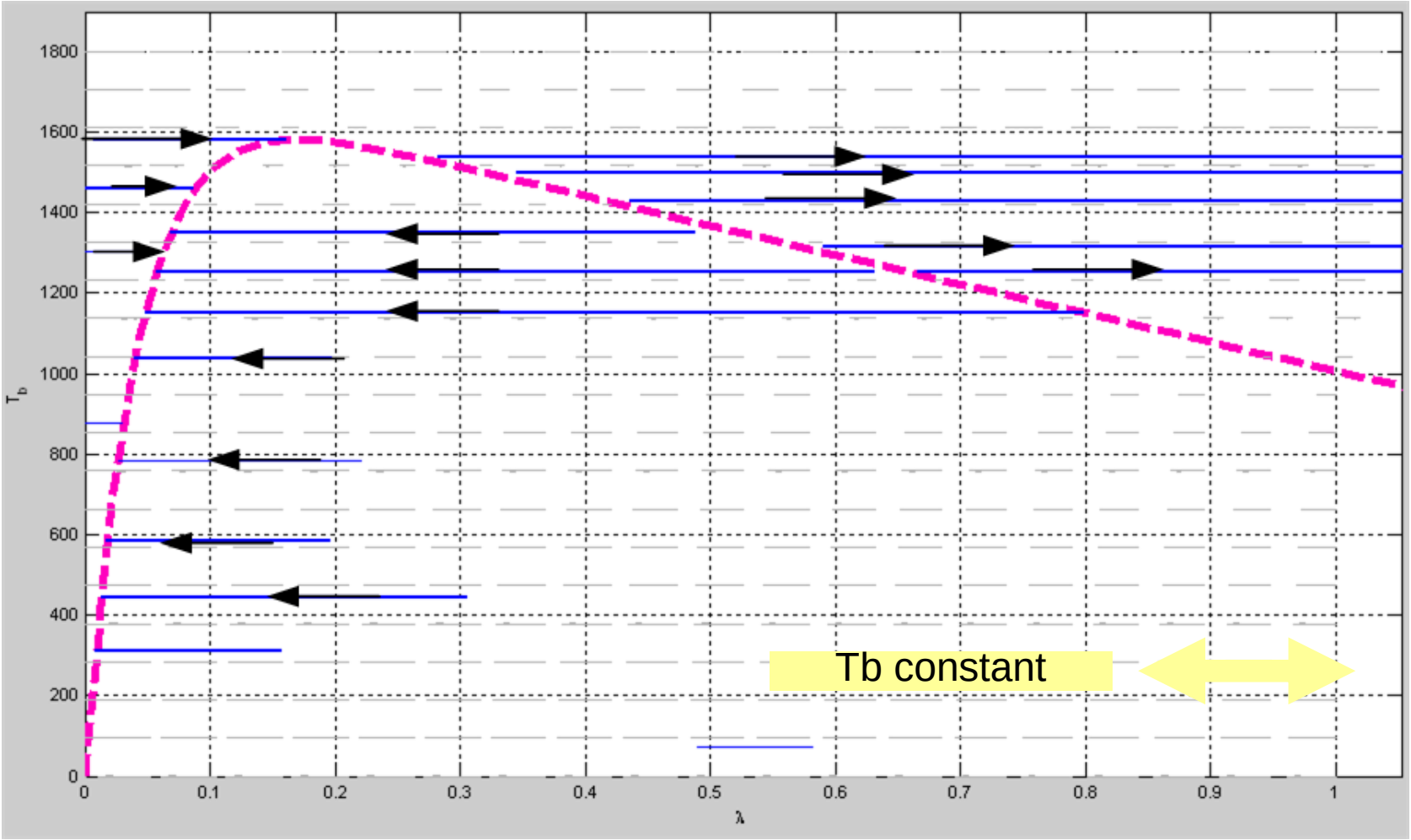
Increase: *u* = *k*

$$\dot{\lambda} = -\frac{(1-\lambda)}{J\omega}\left(\Psi(\lambda) - T_b\right)$$



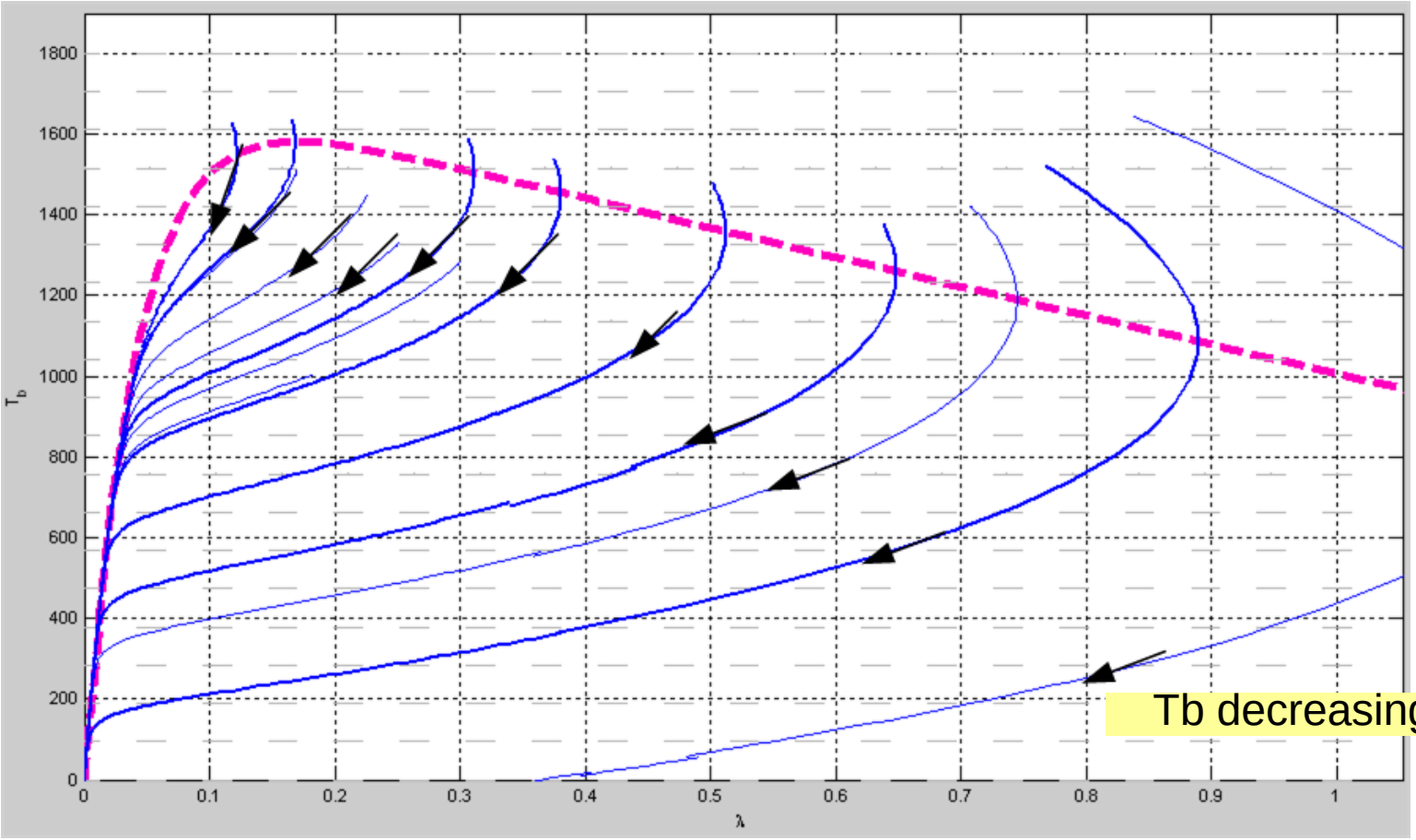
Hold: *u = 0*

$$\dot{\lambda} = -\frac{(1-\lambda)}{J\omega}\left(\Psi(\lambda) - T_b\right)$$



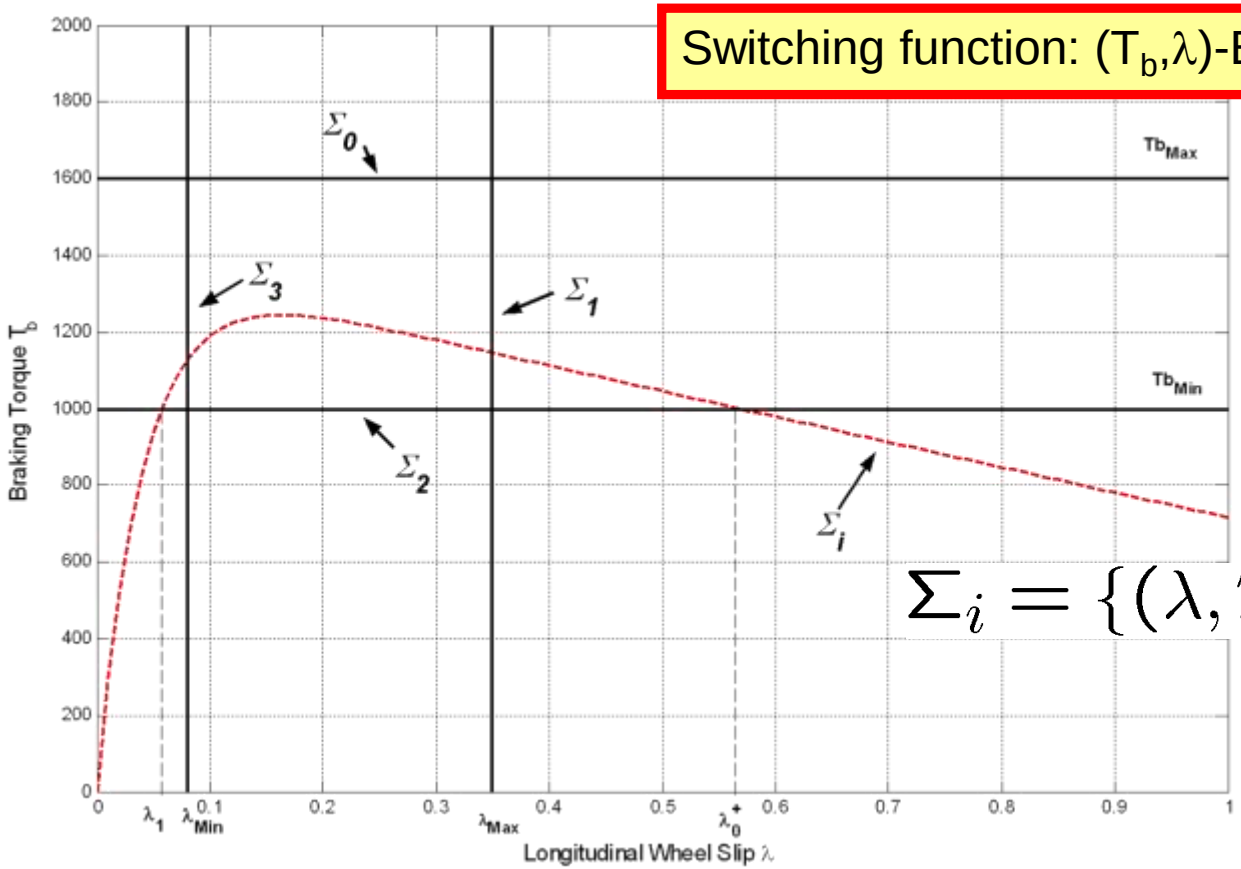
Decrease: *u =-k*

$$\dot{\lambda} = -\frac{(1-\lambda)}{J\omega}\left(\Psi(\lambda) - T_b\right)$$



Switching Control Logic

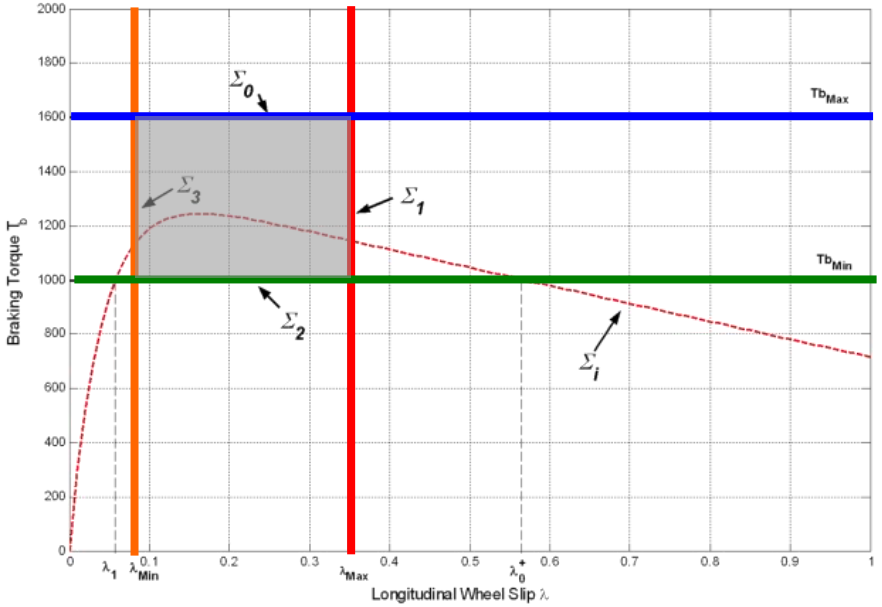
Switching function: (T_b,λ)-Box

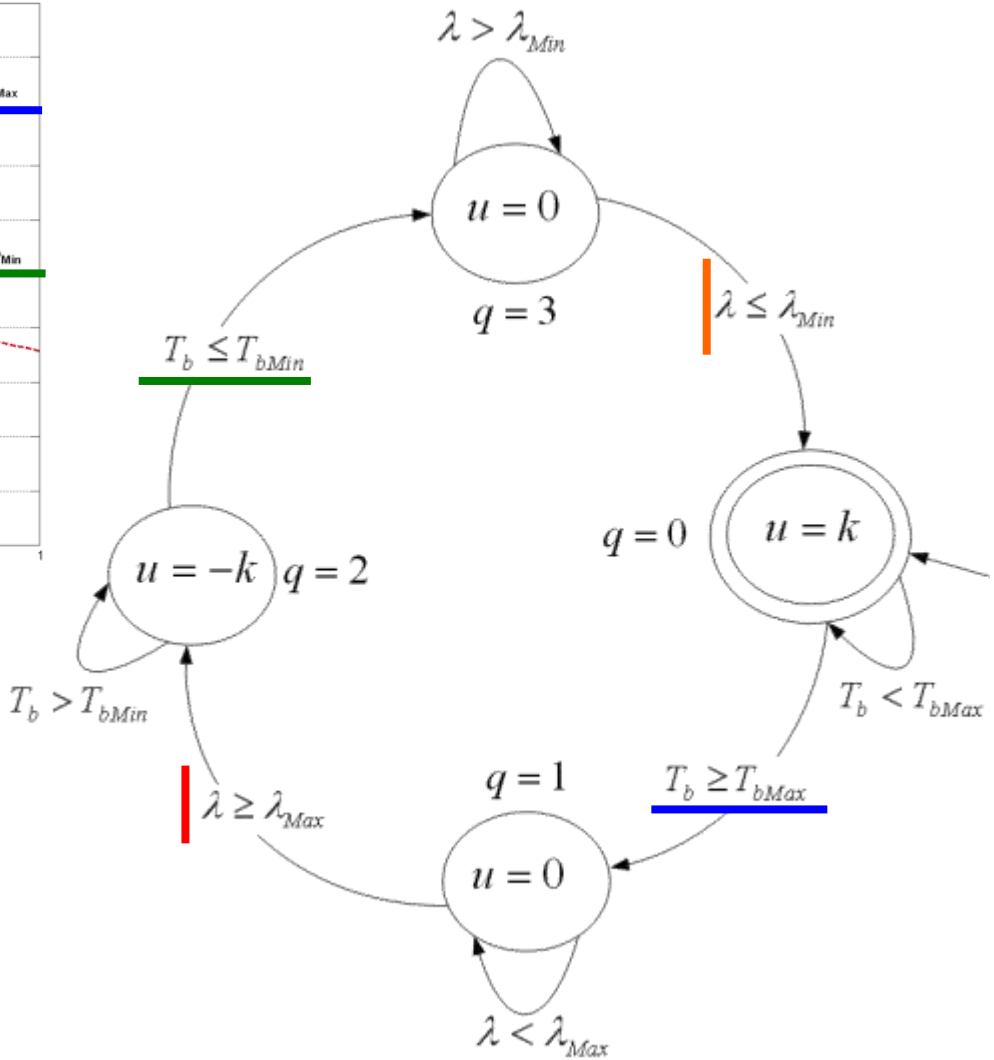


$$\Sigma_i = \{(\lambda, T_b) \ : \ T_b = \Psi(\lambda)\}$$

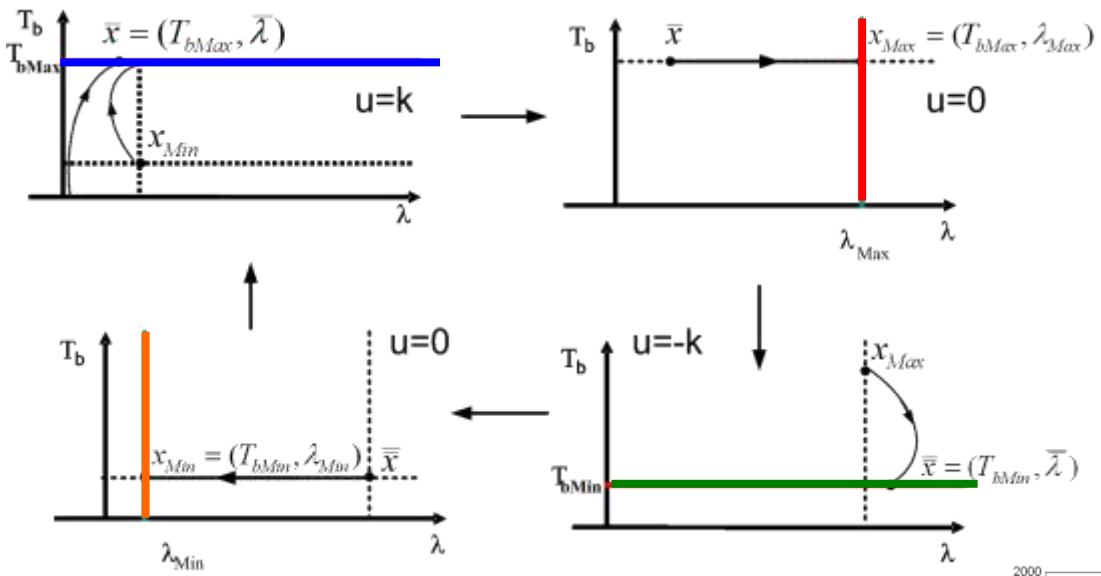
$$\Sigma_0 = \{(\lambda, T_b) \ : \ H_0(\lambda, T_b) := T_b - T_{bMax} = 0\},$$
$$\Sigma_1 = \{(\lambda, T_b) \ : \ H_1(\lambda, T_b) := \lambda - \lambda_{Max} = 0\},$$
$$\Sigma_2 = \{(\lambda, T_b) \ : \ H_2(\lambda, T_b) := T_b - T_{bMin} = 0\},$$
$$\Sigma_3 = \{(\lambda, T_b) \ : \ H_3(\lambda, T_b) := \lambda - \lambda_{Min} = 0\}.$$

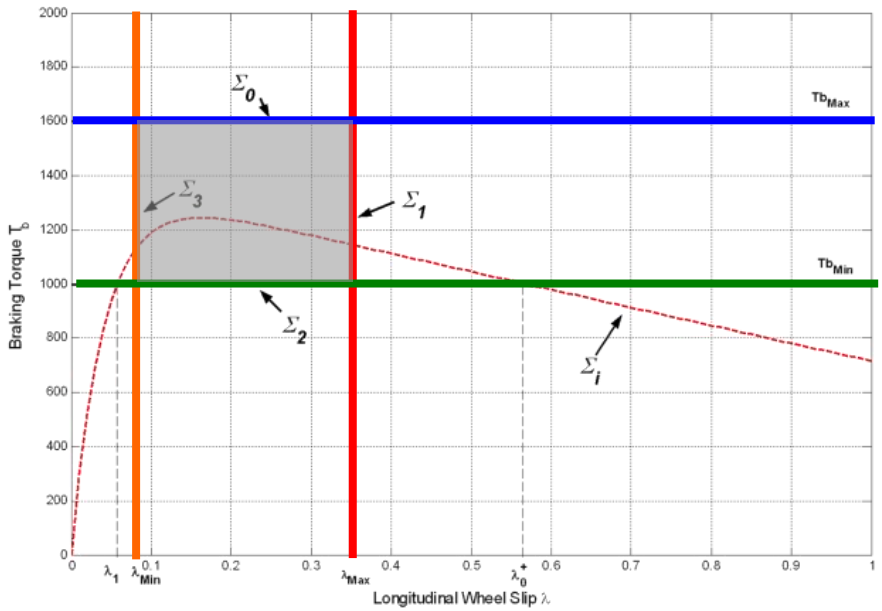
Switching Control Logic



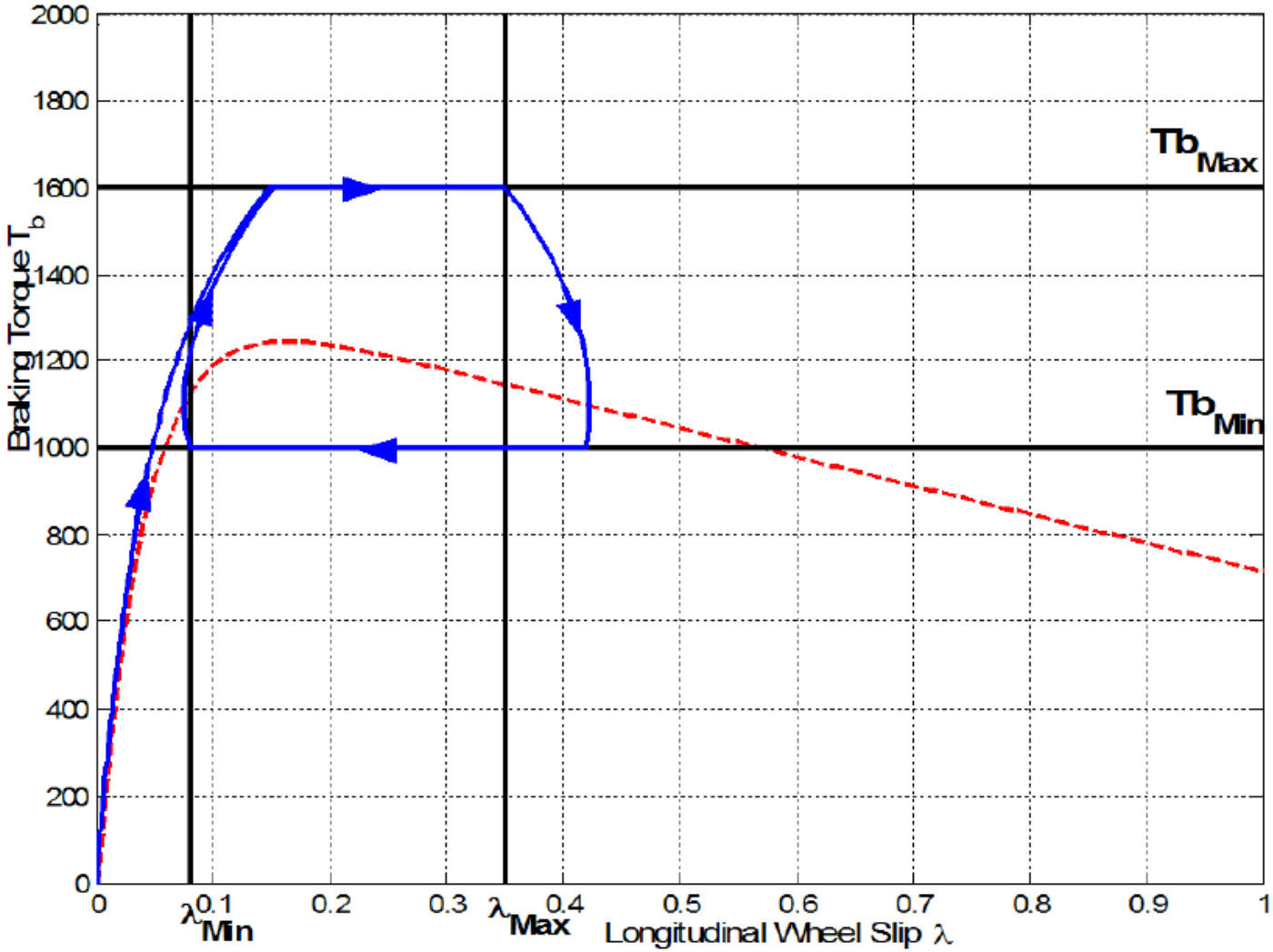


Limit-cycle: analysis

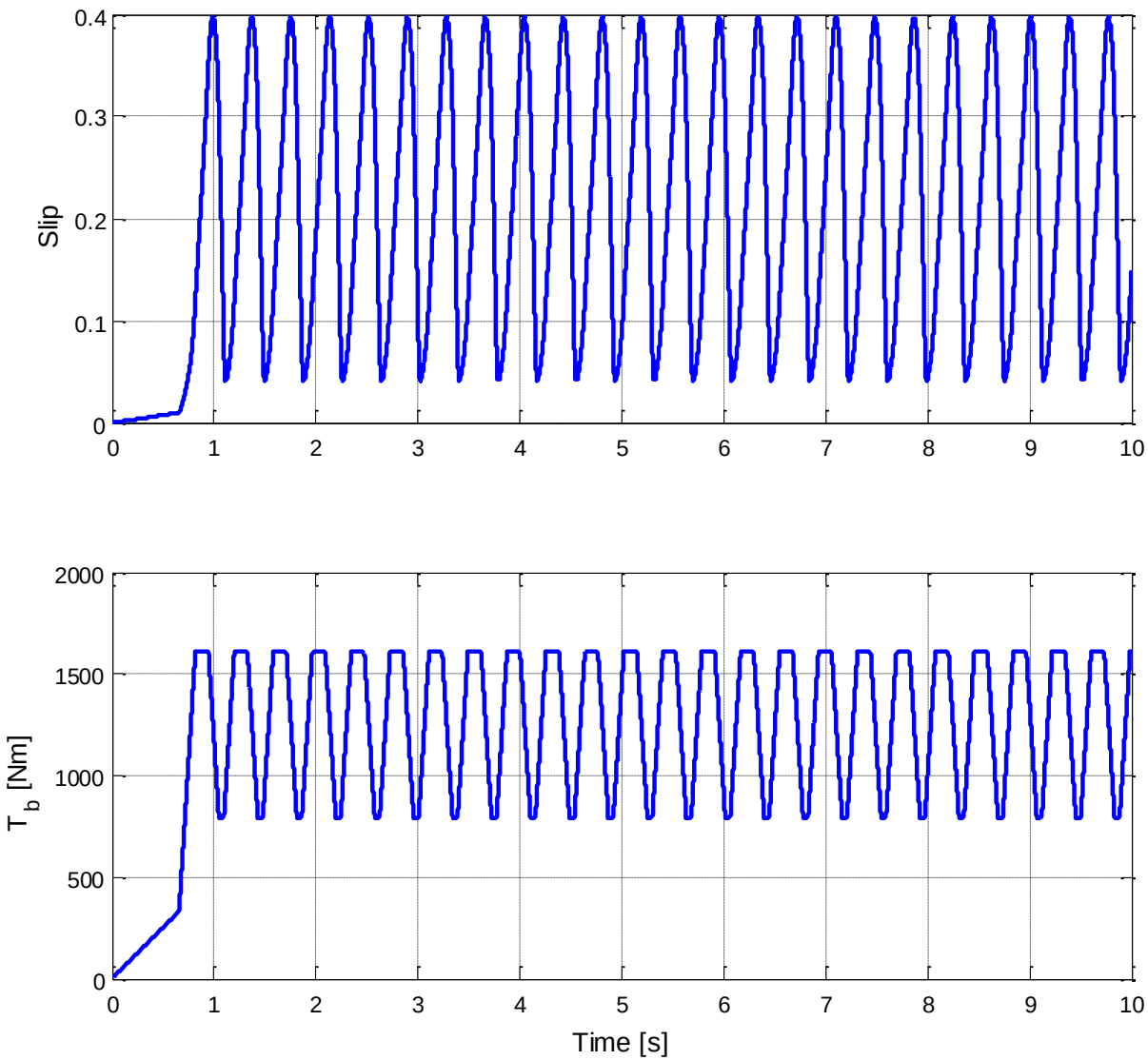




Limit-cycle: analysis



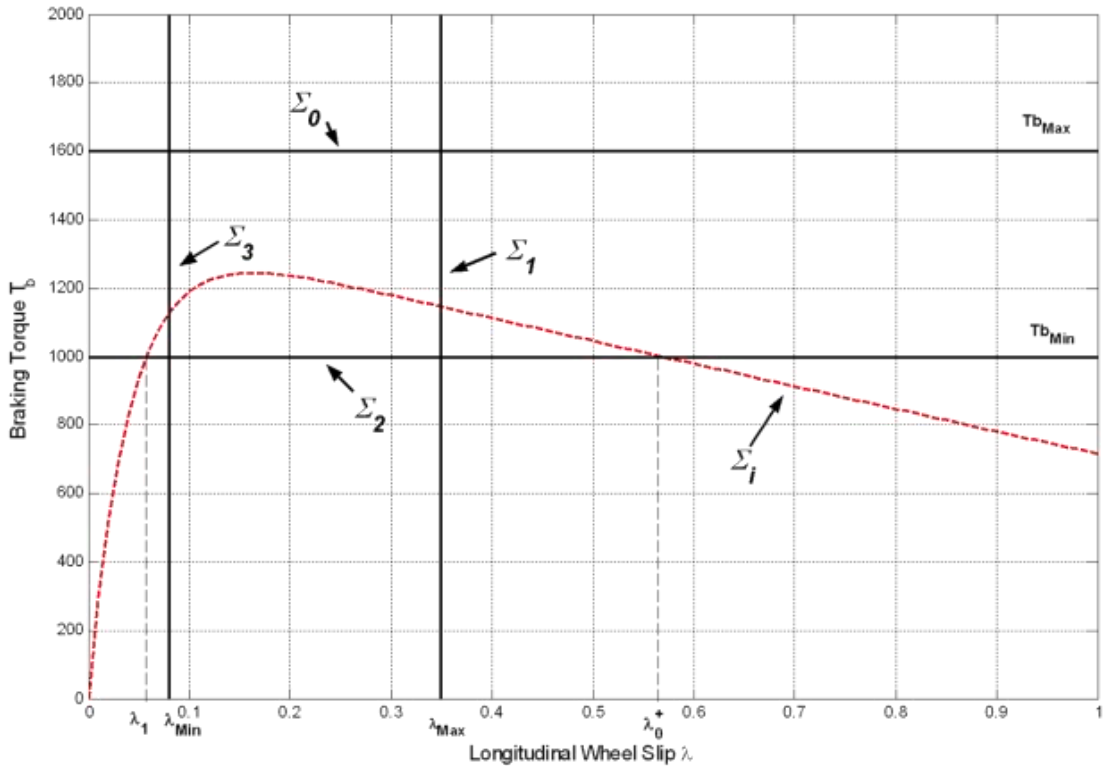
Limit-cycle: analysis



Existence of limit cycle: necessary conditions

$$\Sigma_0 \cap \Sigma_i = \emptyset$$

$$\Sigma_2 \cap \Sigma_i \neq \emptyset$$



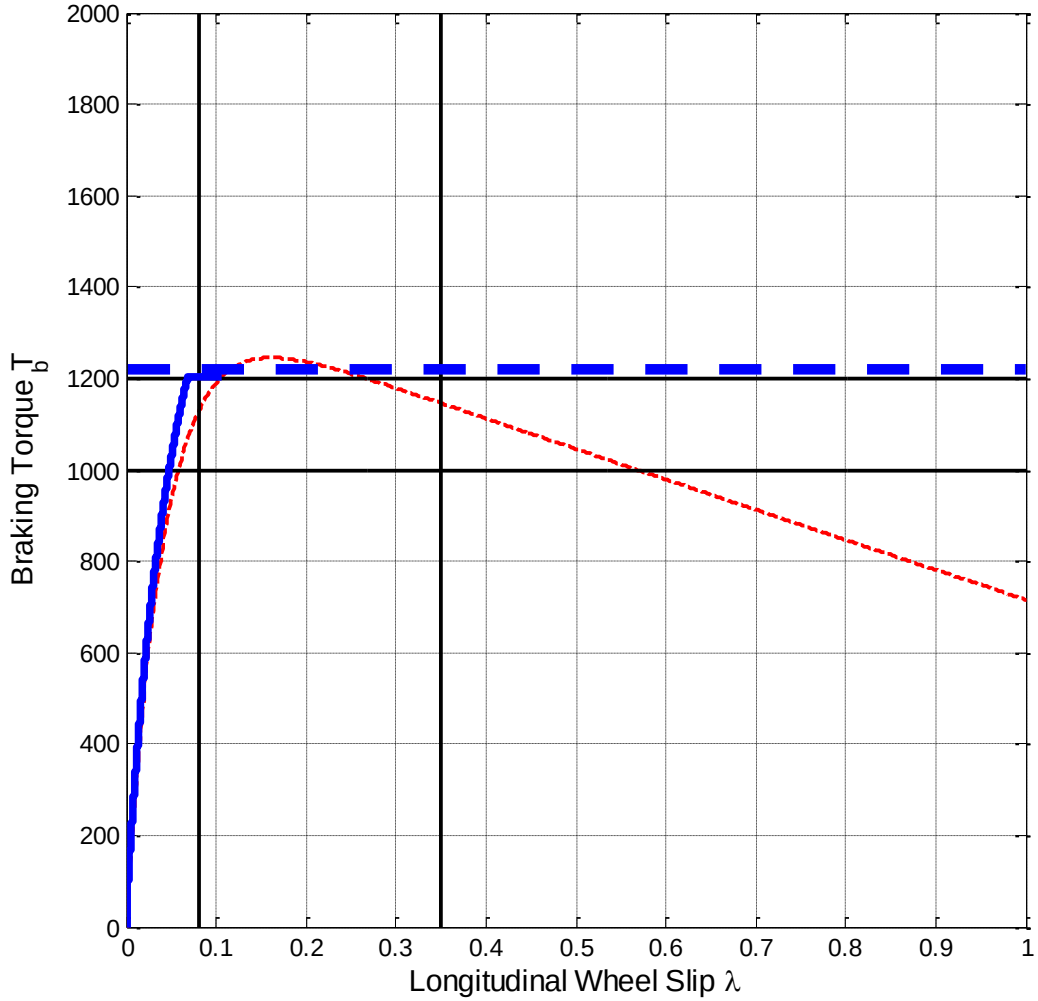
if $\Sigma_2 \cap \Sigma_i = \{(T_{bMin}, \lambda_1), (T_{bMin}, \lambda_0^+)\}, \ \lambda_1 < \lambda_0^+$

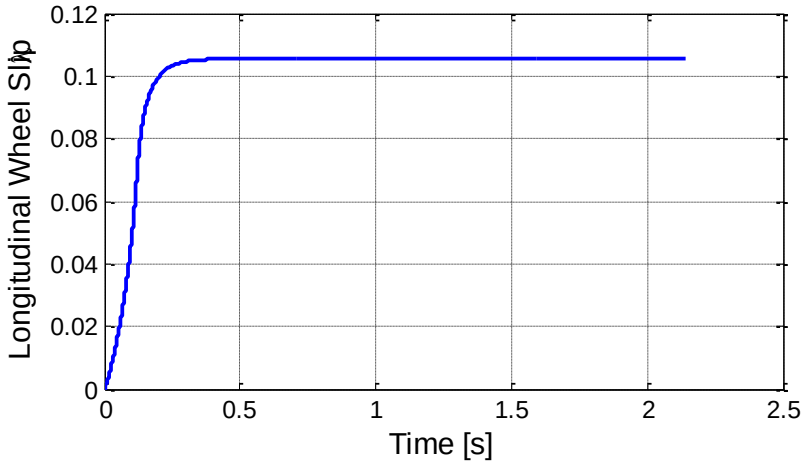
then

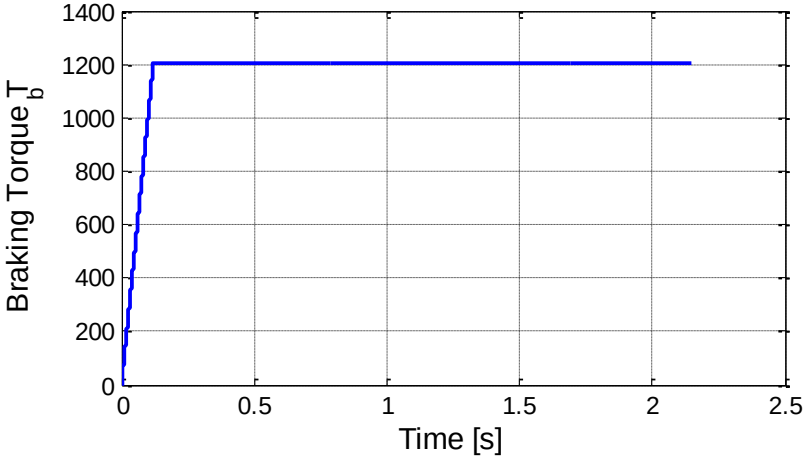
$$\exists t \ : \ \Phi(t - t_0, \lambda_{Max}, T_{bMax}; q = 2) \cap \Sigma_2 = \{(T_{bMin}, \overline{\overline{\lambda}})\}, \text{ s.t. } \left(\overline{\overline{\lambda}} < \lambda_0^+\right)$$

Limit-cycle: analysis

First condition violated: NOT a limit cycle but a steady-state condition

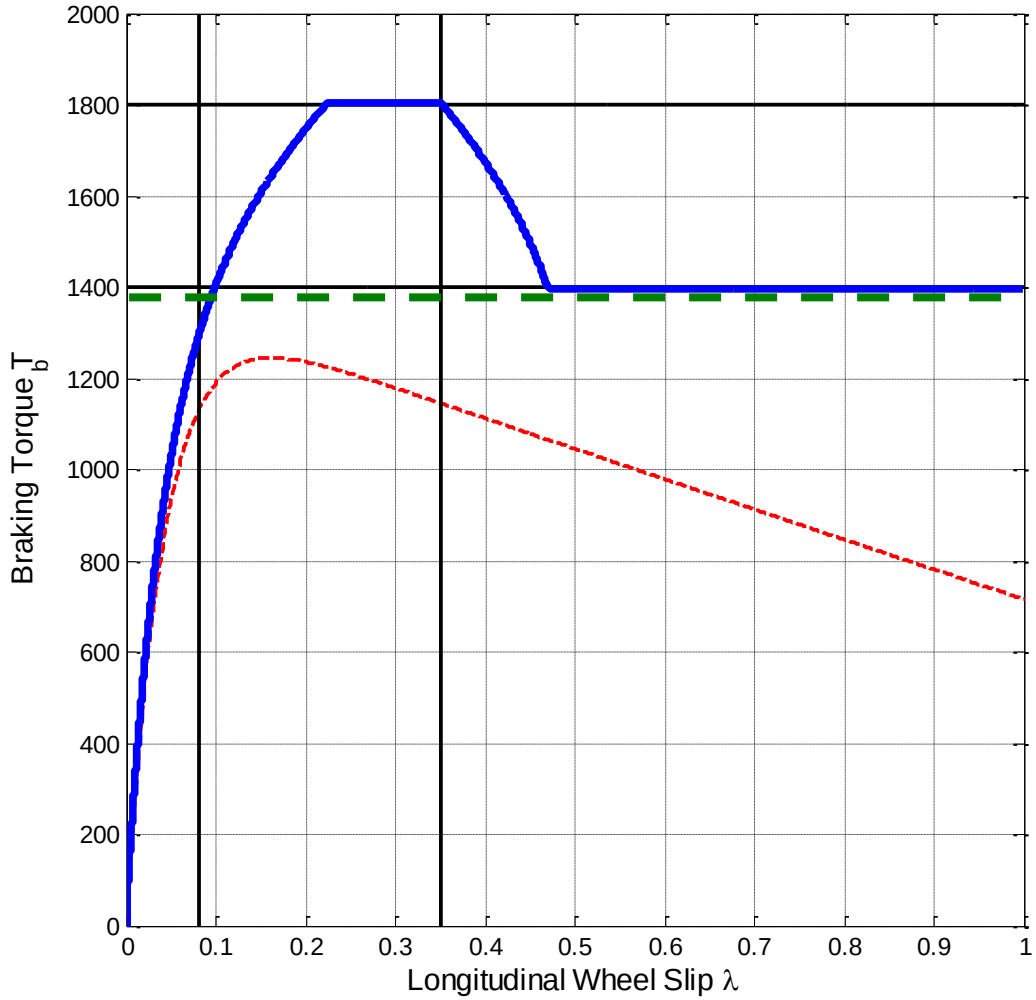


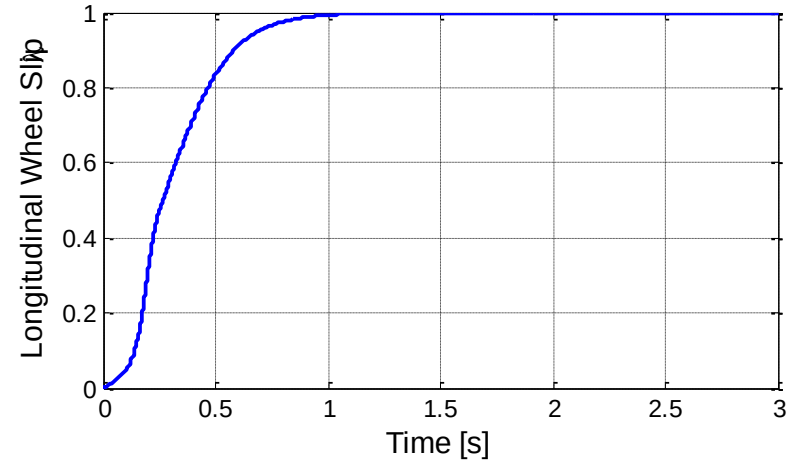


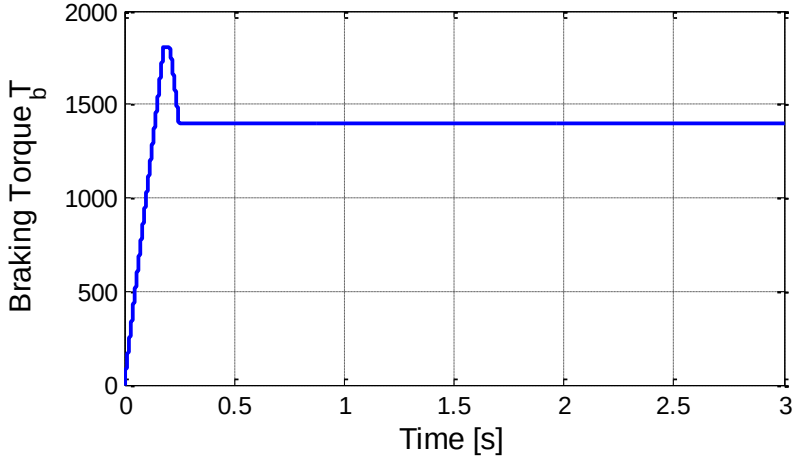


Limit-cycle: analysis

The second condition is violated: NOT a limit cycle but wheel locks

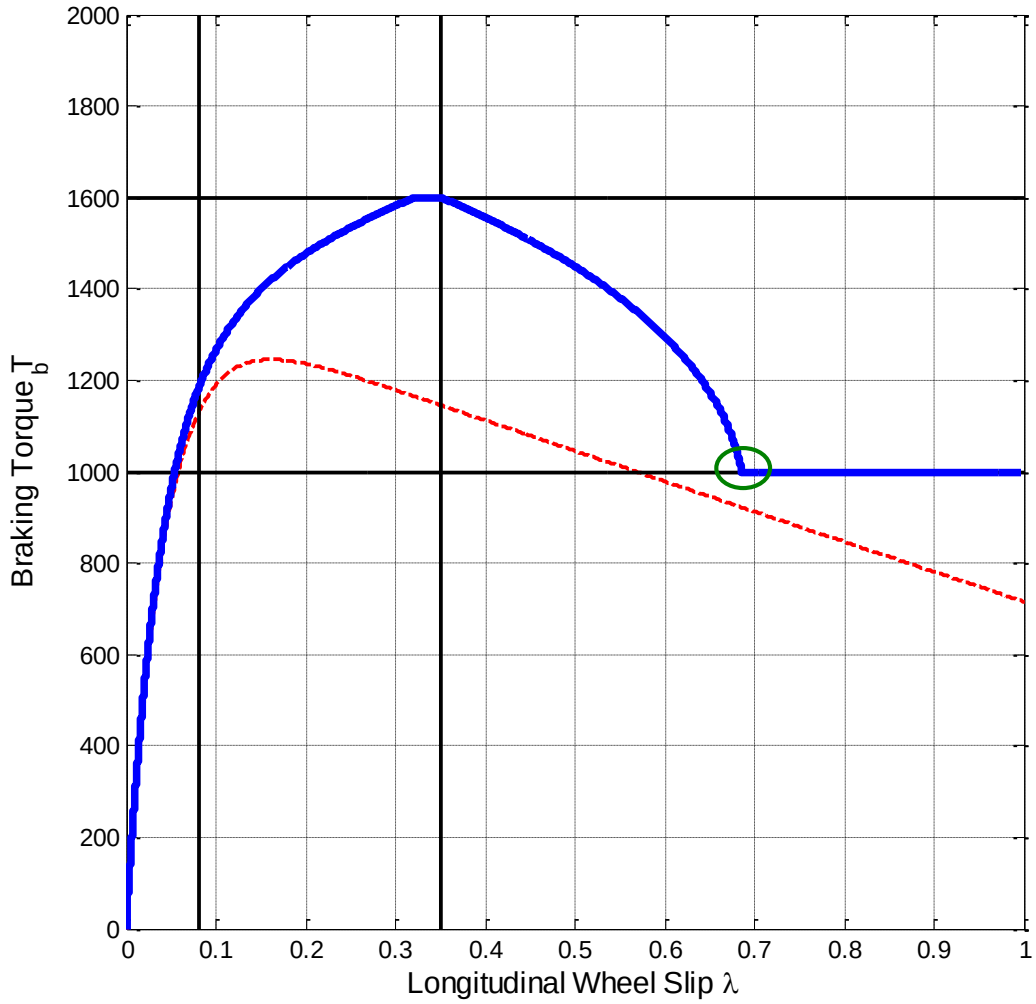


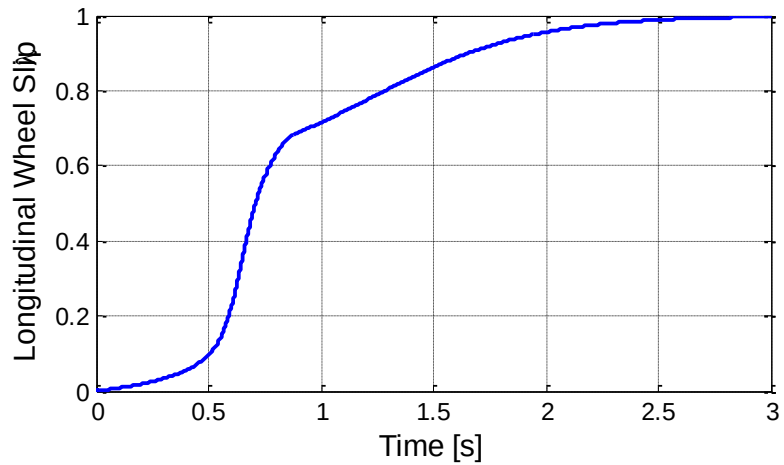


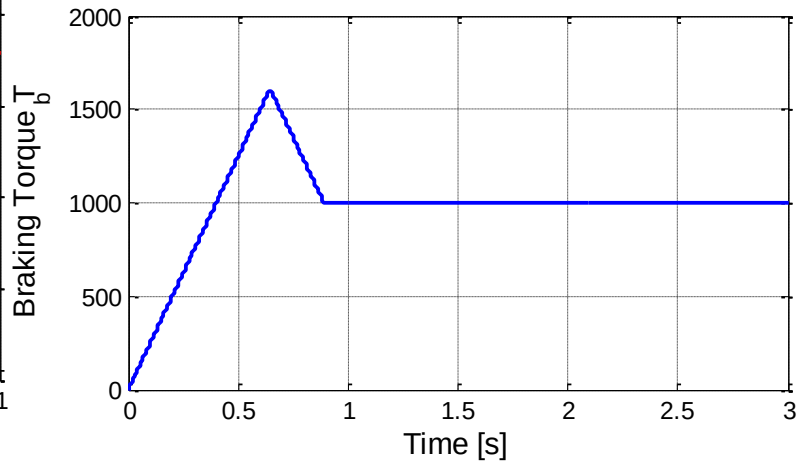


Limit-cycle: analysis

Third condition violated: NOT a limit cycle but wheel locks

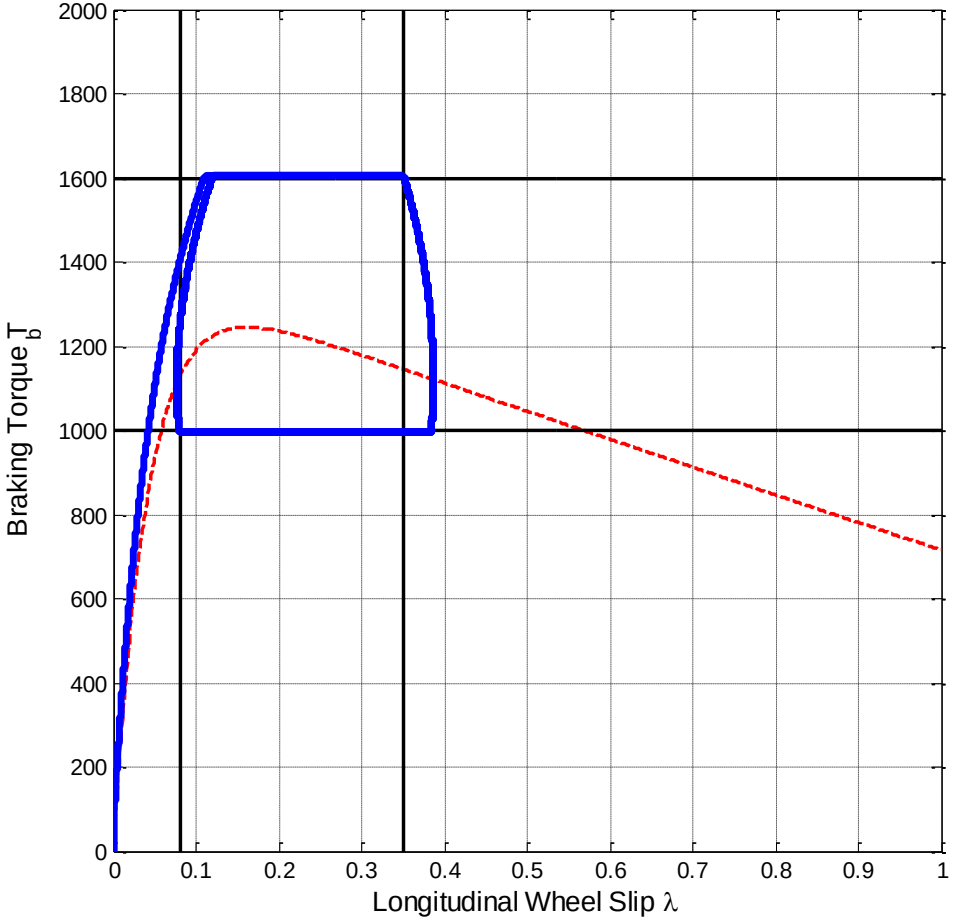




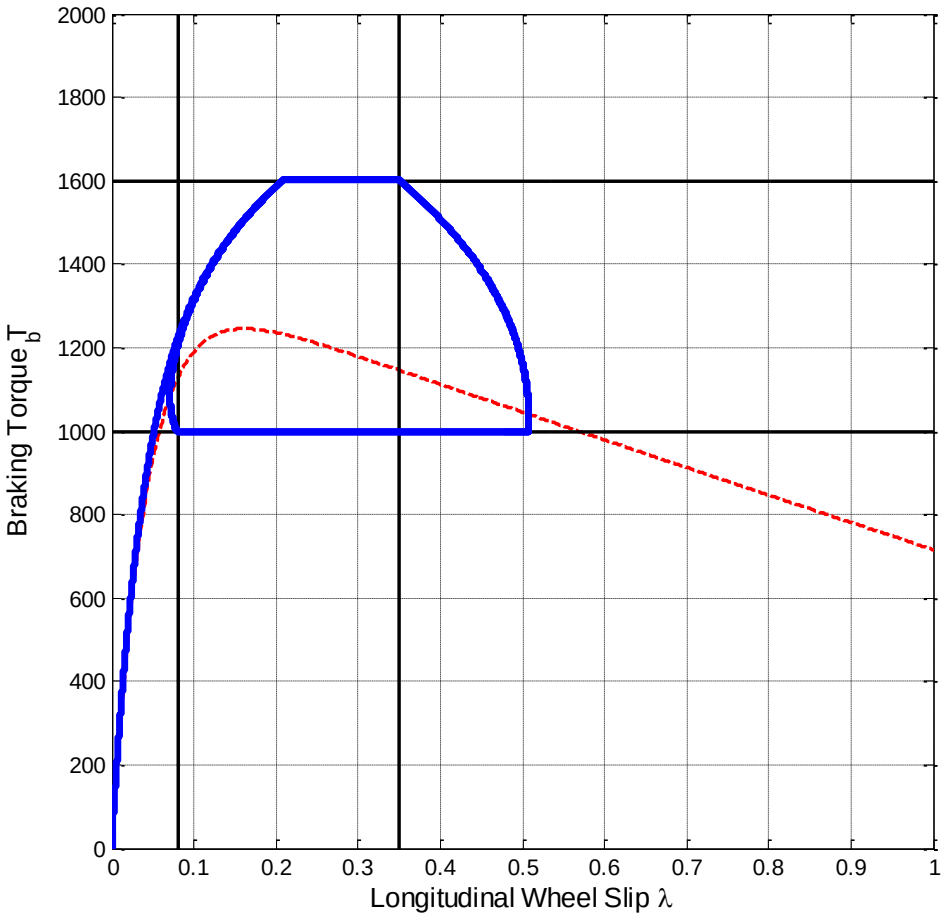


Limit-cycle: analysis

Influence K large / small



K large



K small

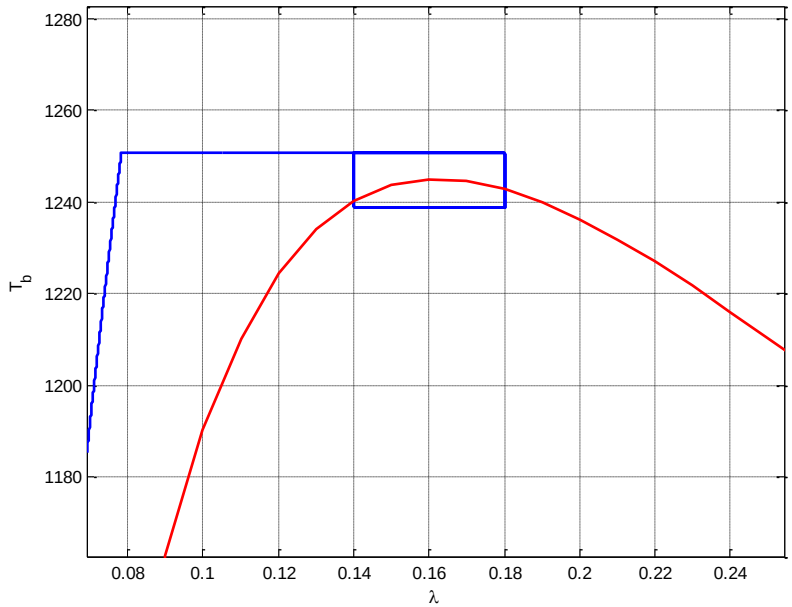
Comments

Ideal behavior:

- "Box" very tight around the peak of the curve of friction
- Transitions very fast (high-k)

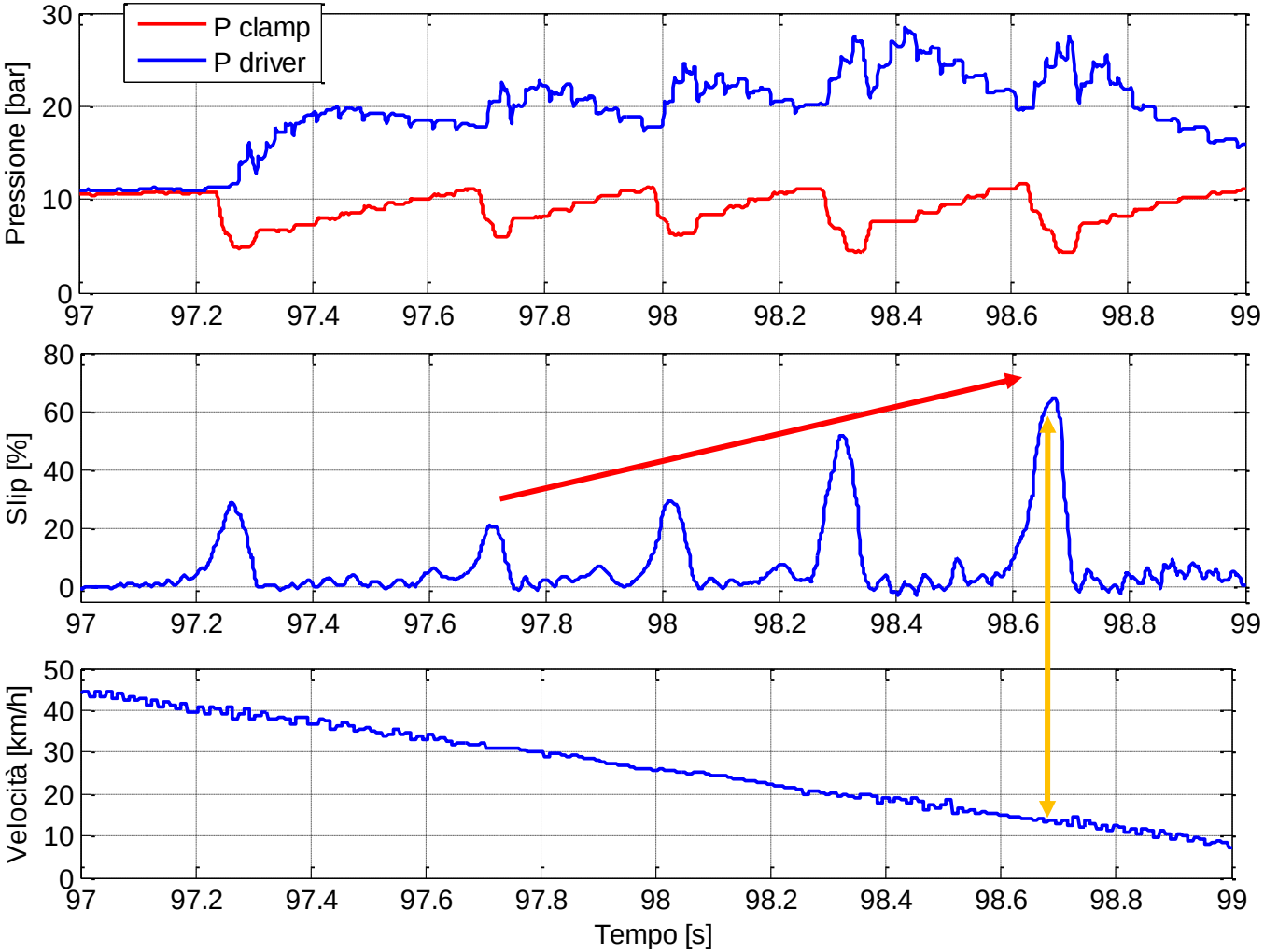
In practice we have to find a compromise between:

- K small / large
- Box small (performance) / large (robustness)
- Robustness (fixed) & Adaptation (variant) Box



Appendix 1: experimental results (ABS)

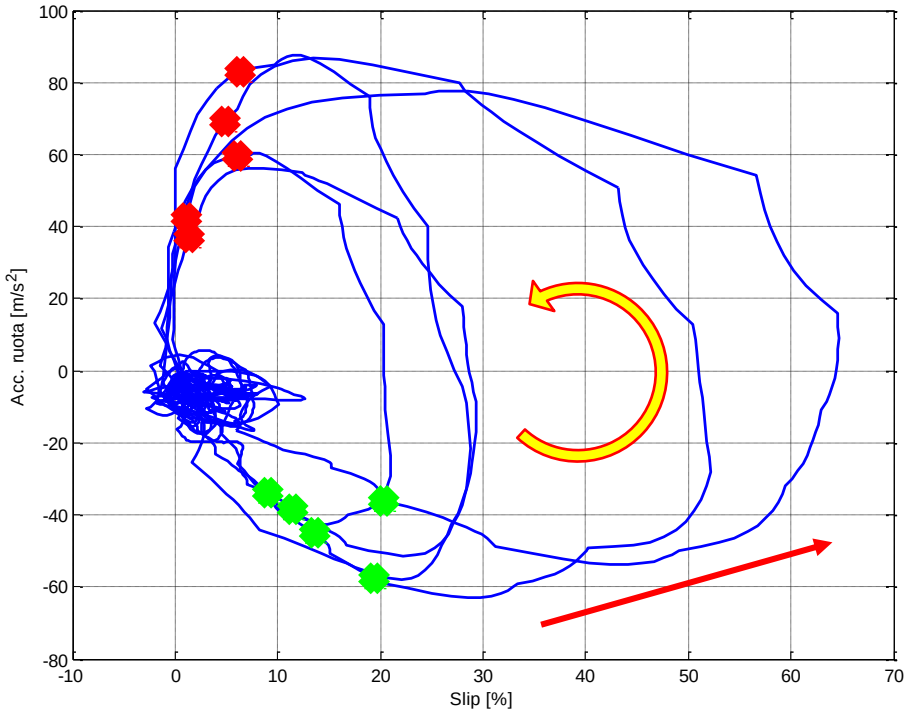
Application example (ABS standard)
ABS active

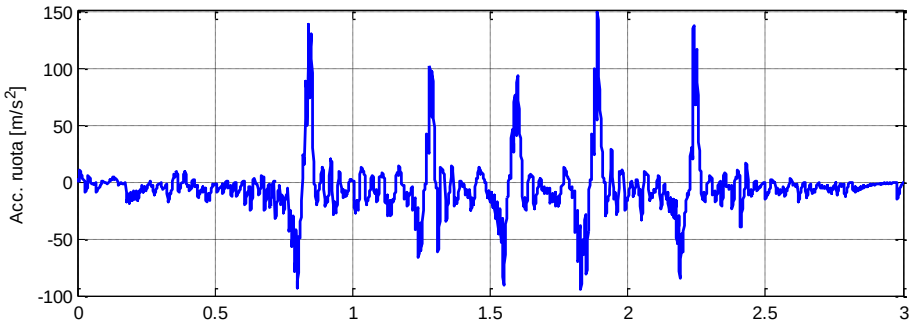


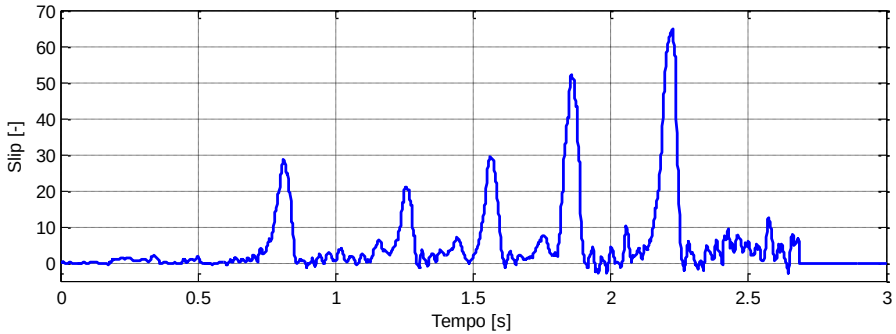
Deceleration average:
0:49 g

Application example (ABS standard)
ABS active

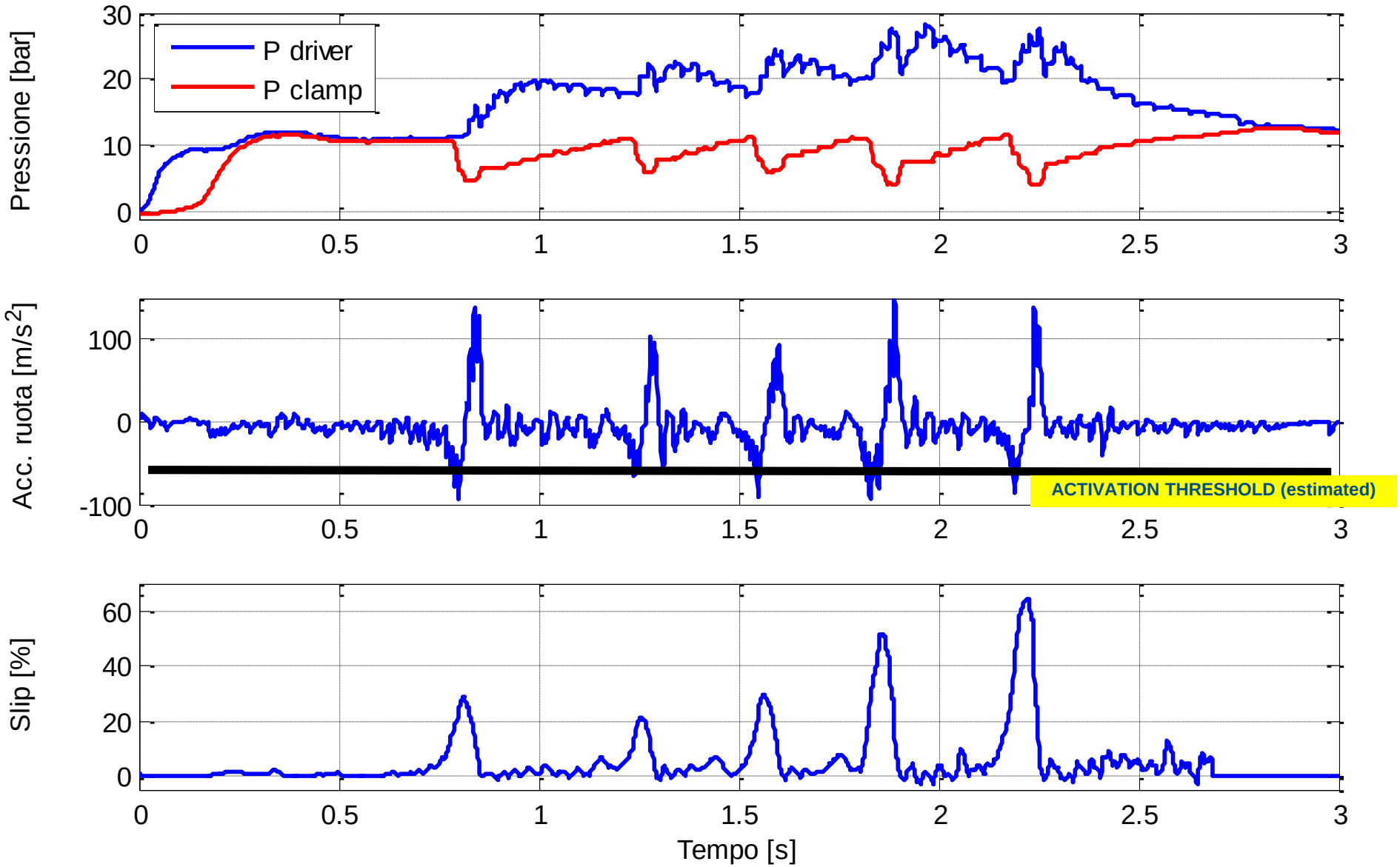
Braking: phase plane and time course



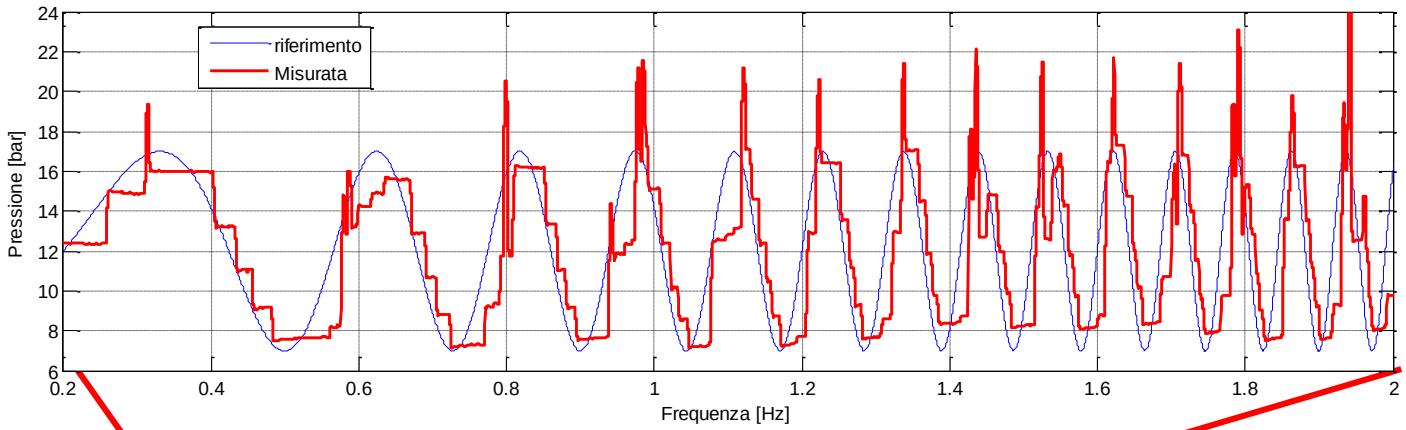




Application example (ABS standard)
ABS active

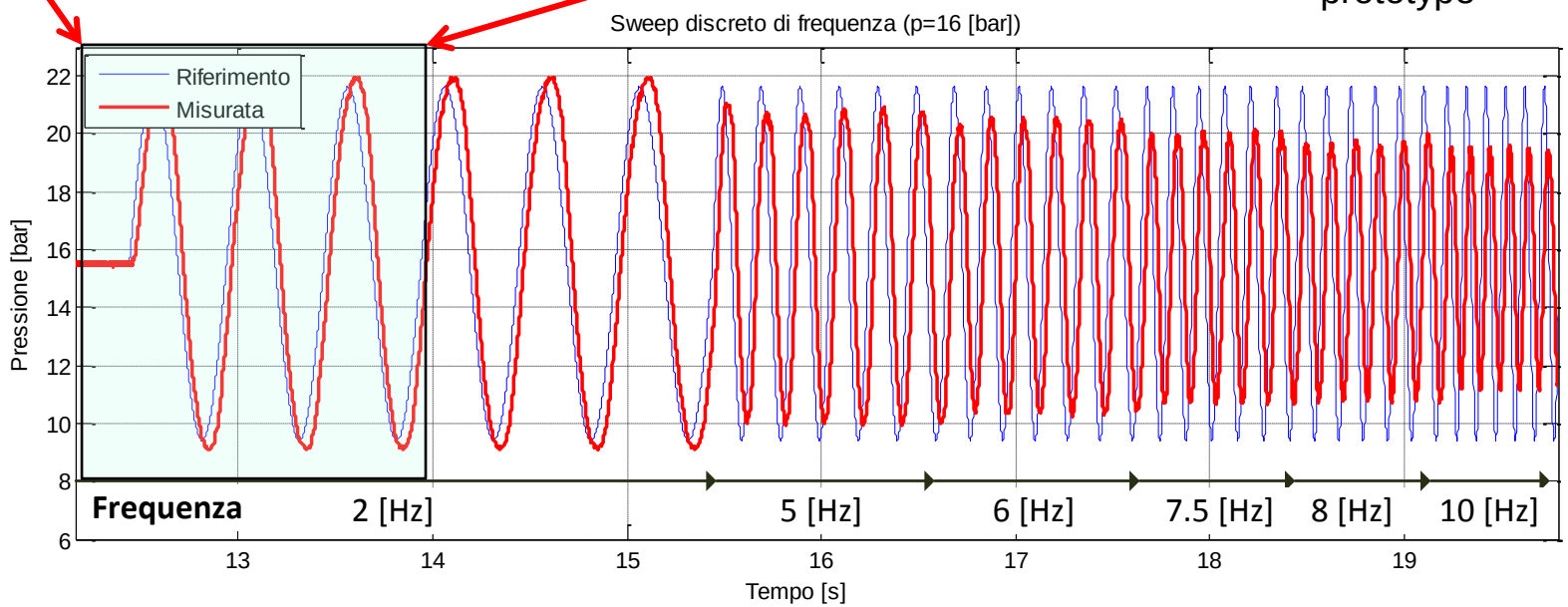


Comparison of actuators
Pressure-tracking results (comparison)



Traditional electro-hydrualic system (ABS - in production)

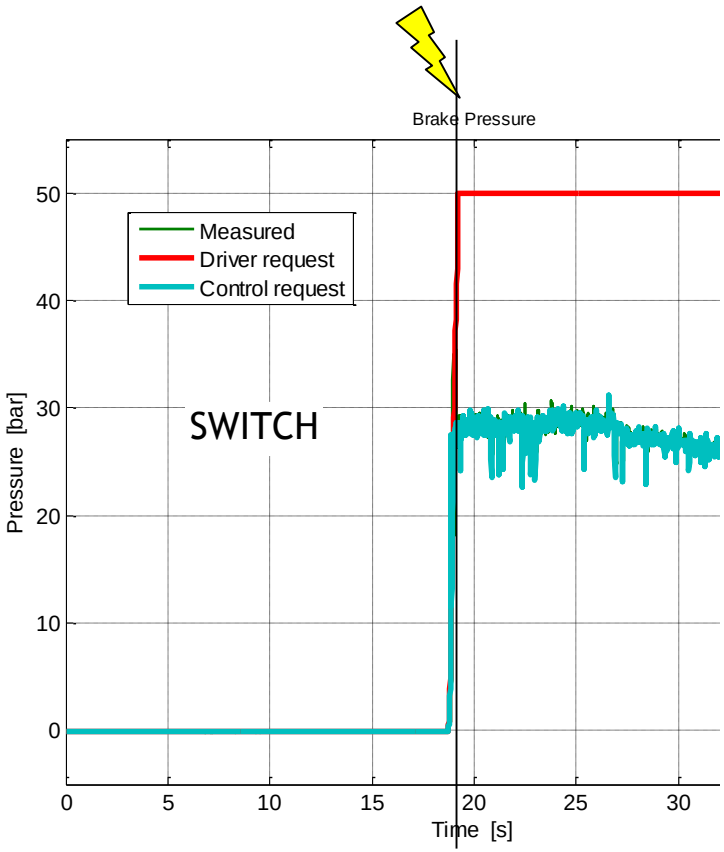
BBW prototype

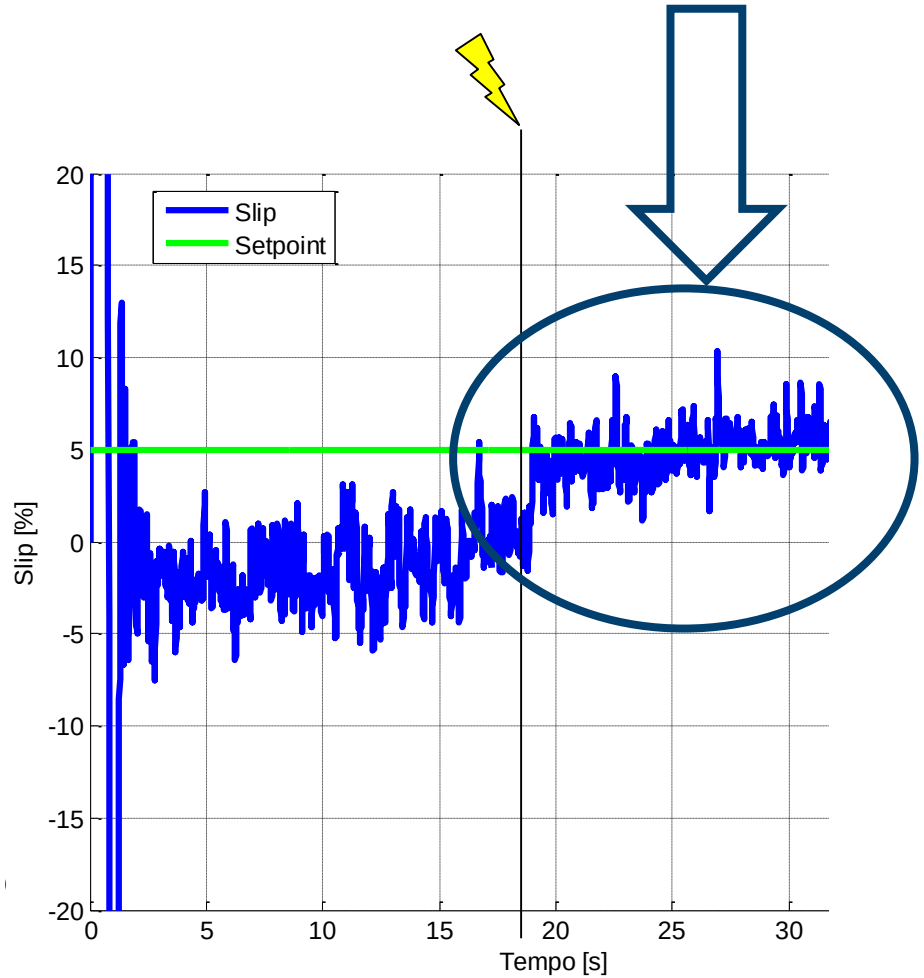


Slip-control results (example – advanced BBW actuator)

Panic-brake experiment with target slip at 5%

Very accurate slip-tracking

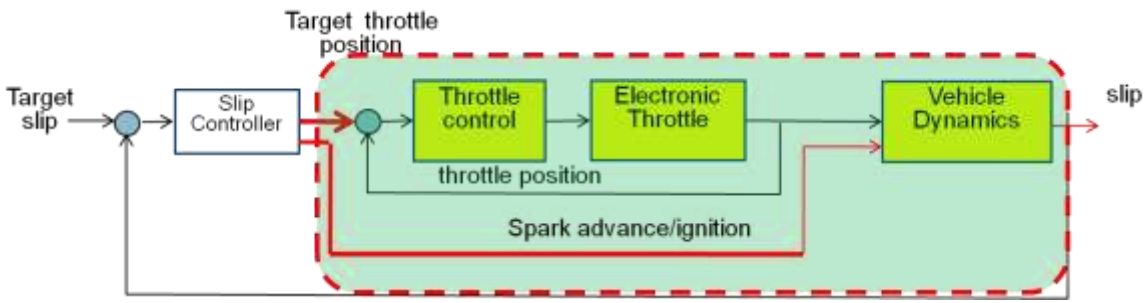




Appendix 2: Traction Control

Black-box identification of engine-to-slip dynamics for motorcycle traction control

Horn, M. and Savaresi, S.M.: Experimental Identification of Engine-to-Slip Dynamics for Traction Control Applications in a Sport Motorbike. European journal of control, vol.16, no.1, p. 88-108, 2010.



The control variables taken into account are the **throttle September point** and **spark advance**
The controlled variable is the **rear wheel slip**

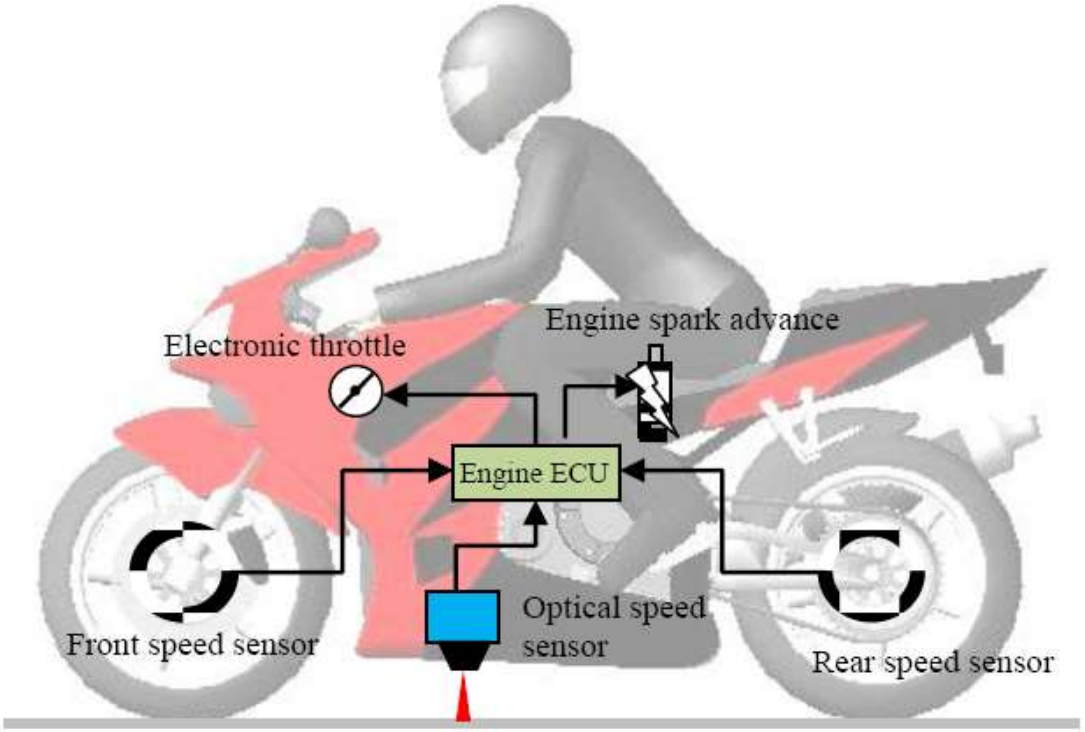
Developing a first principles model of the dynamics would be very demanding as the dynamic is very complex (**engine, transmission, wheel dynamics, pitch and vertical dynamics ....**).

Approach: black-box modeling

Experimental set-up (I / O)

The experiments on performend Have Been a World-championship SBK motorcycle (1000cc 4-stroke 4-cylinder engine) That can deliver blackberries than 200 HP. The motorbike is equipped with:

- an Electronic Throttle Body (ETB)
- an Electronic Control Unit (ECU) to control the throttle position and the engine spark-advance;
- two wheel encoders to measure the rotational speed of the wheels;
- an optical velocity sensor to measure the real longitudinal velocity of the vehicle chassis with respect to the road.

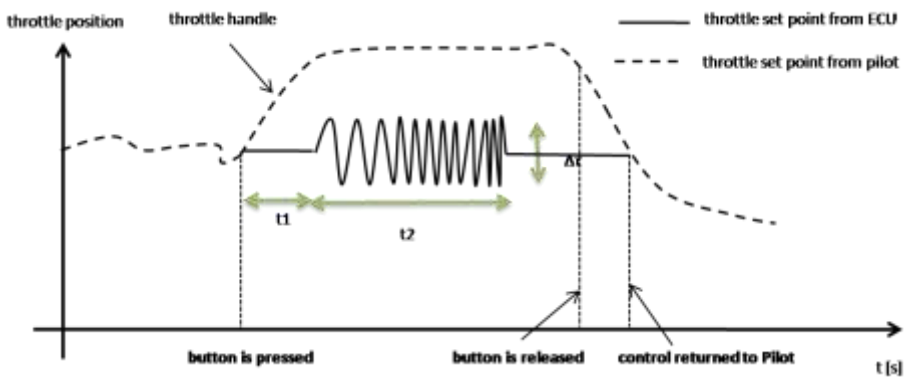


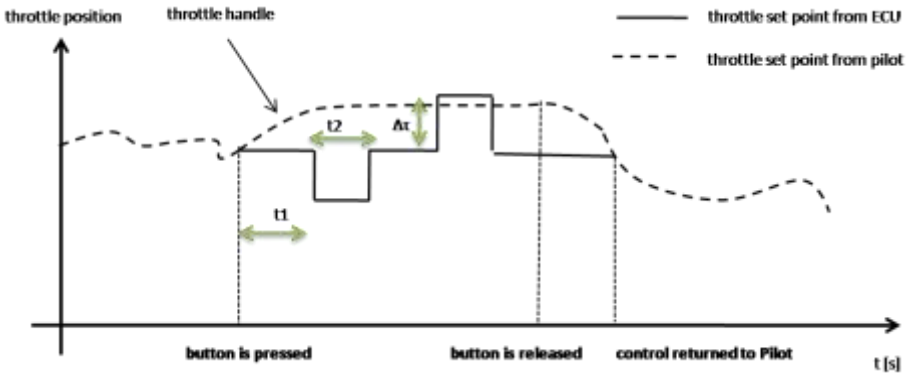
Design of experiment ....

Tests on a long (3.5 km) straight dry asphalt patch:

1) the rider is asked to Trim the motorcycle to constant speed
2) the test is trigged by the driver with a button: At that point the ECU completely command overrides the driver, and the excitation signal is applied around the neighborhood of the initial condition.

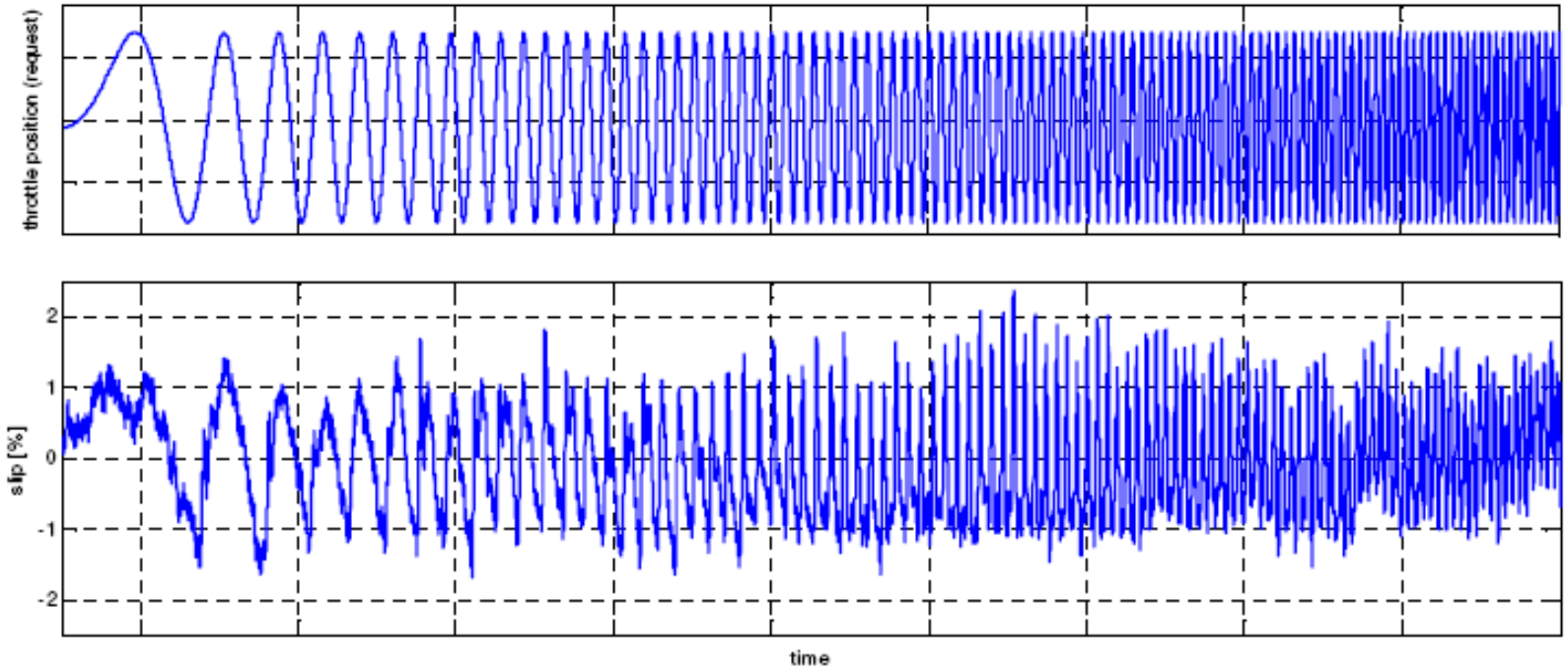
This experiment is non-trivial, but has the major advantage of **being repeatable and providing the real dynamic behavior of the motorbike** (And rider), on a real test-track (**Whereas test-rig Typically experiments are flawed by non-realistic conditions)**.





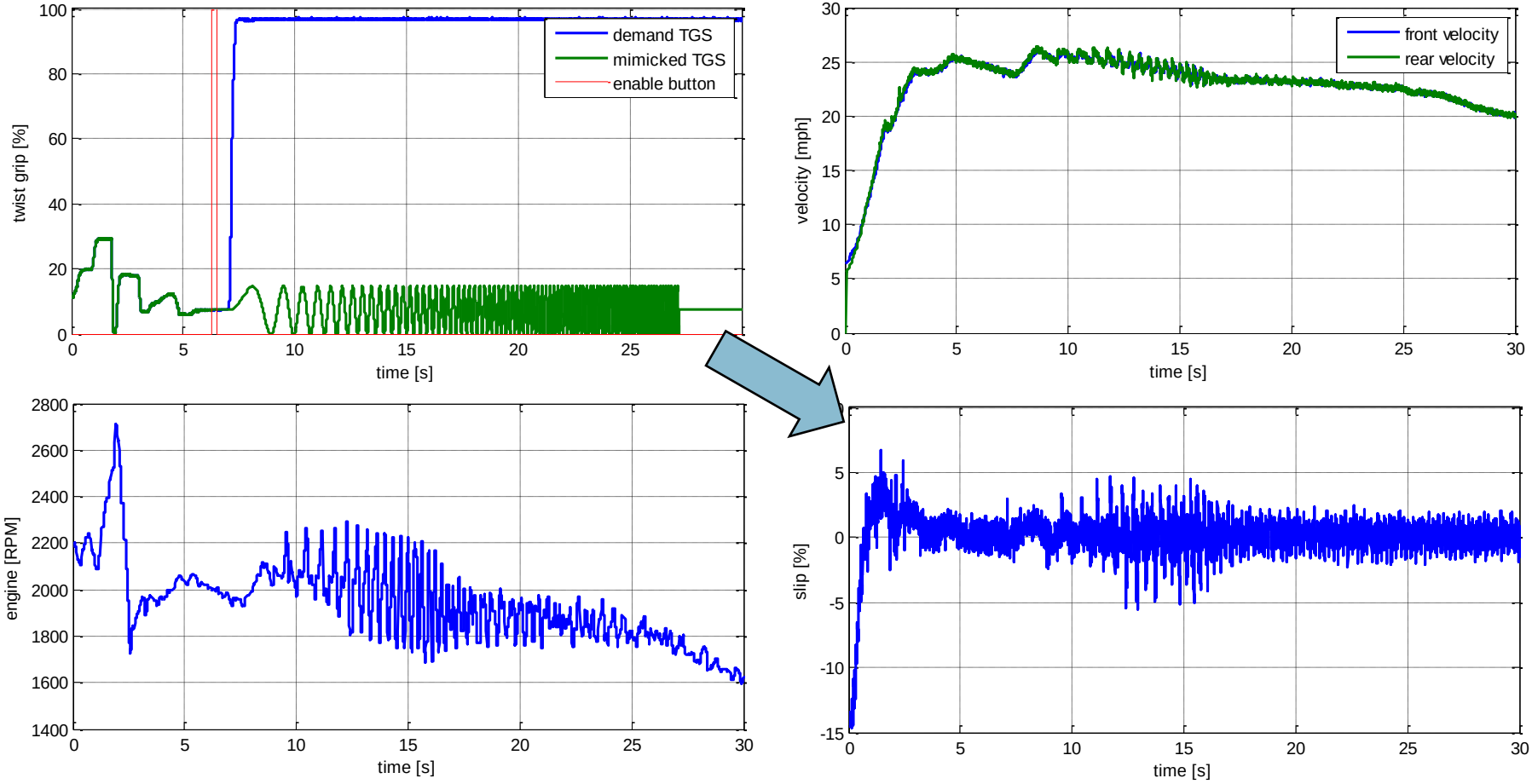
Two different excitation are Considered: 1) sine sweep
2) step like

Tests Protocol - example



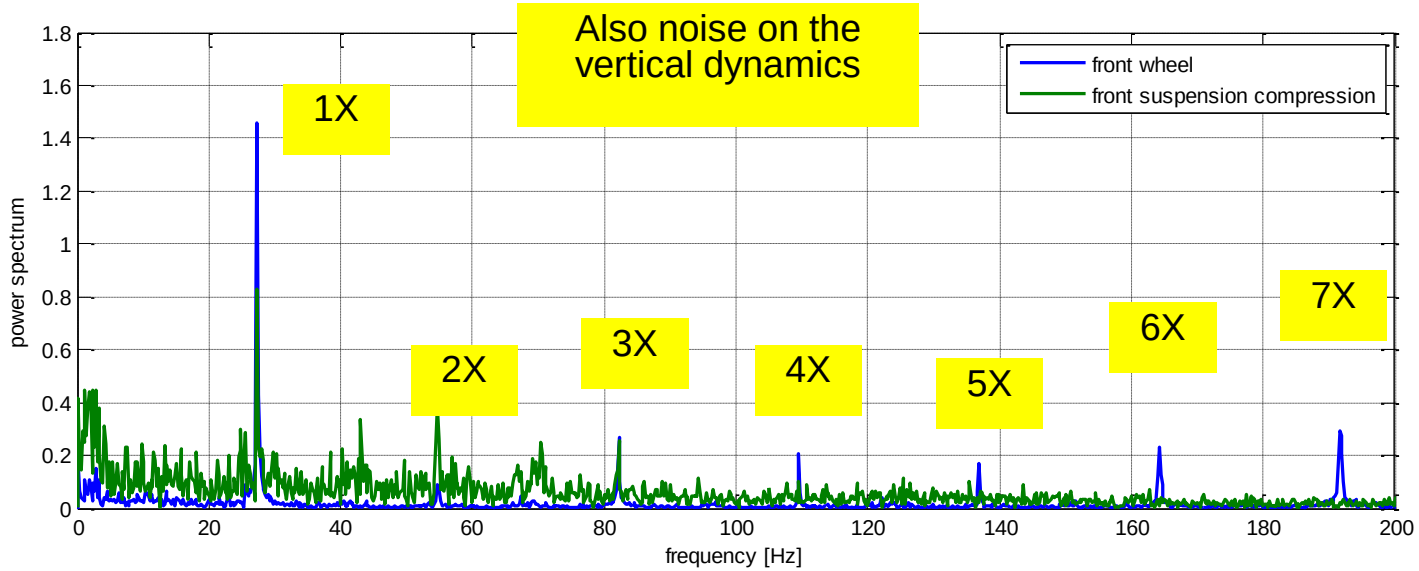
Example of a throttle sweep experiment. Top: requested throttle position; bottom: measured wheel slip.

Twist Grip Slip to Dynamics - Experiment "Sweep" (Example)



Estimated slip - disturbances

Front Wheel

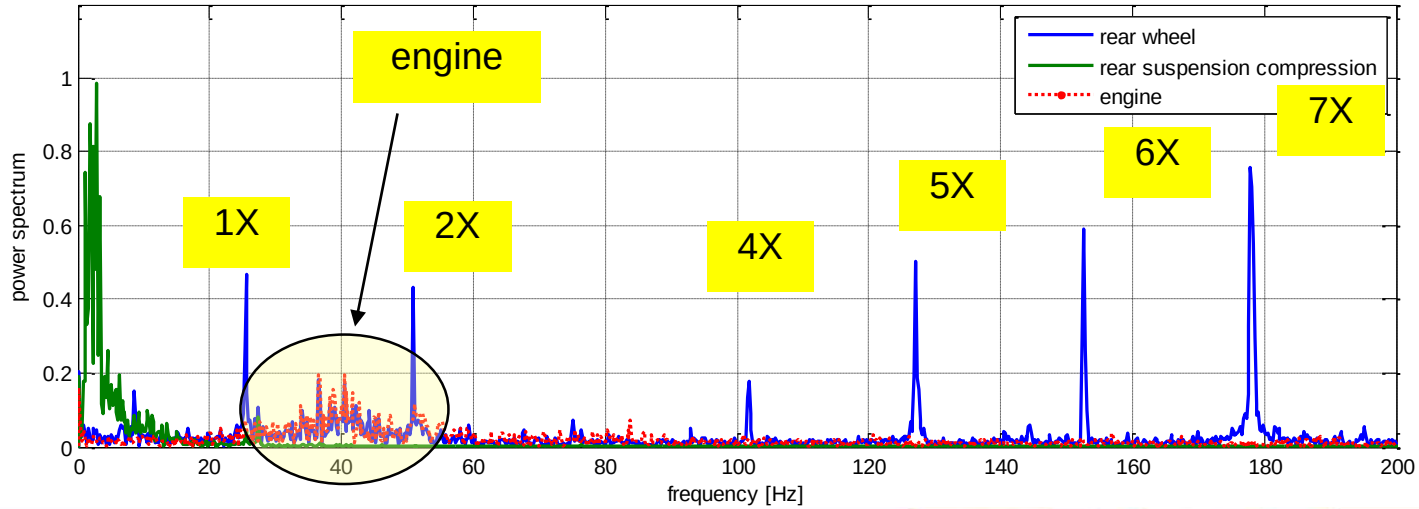


1X @ 182 km / h:

-27.4 Hz front

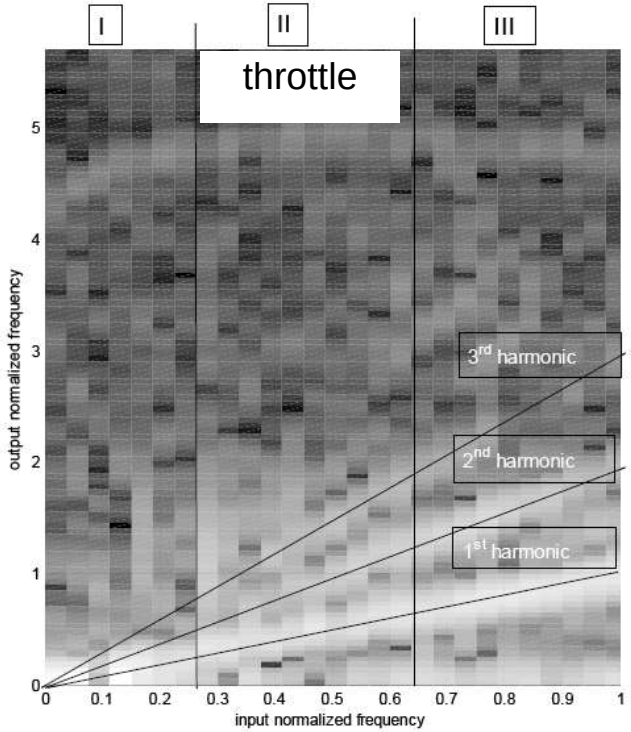
-25.5 Hz rear

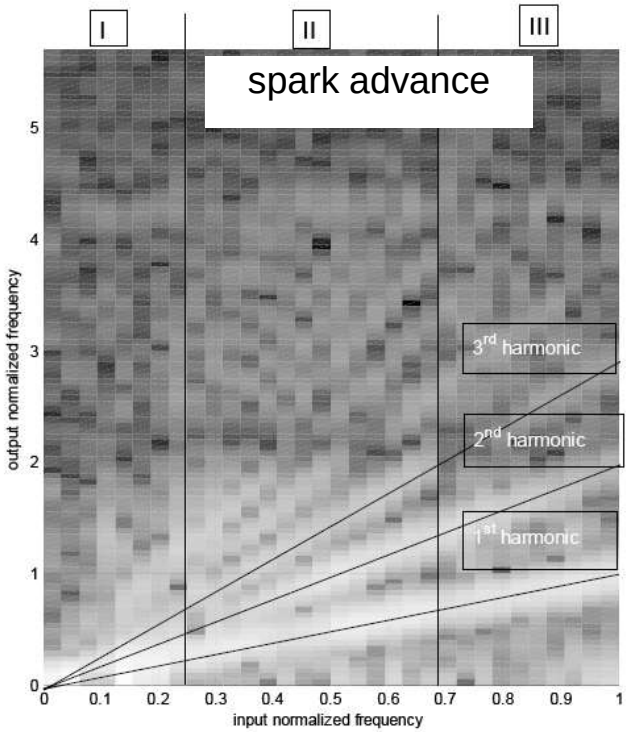
Rear Wheel



Black-Box Identification - spectrograms

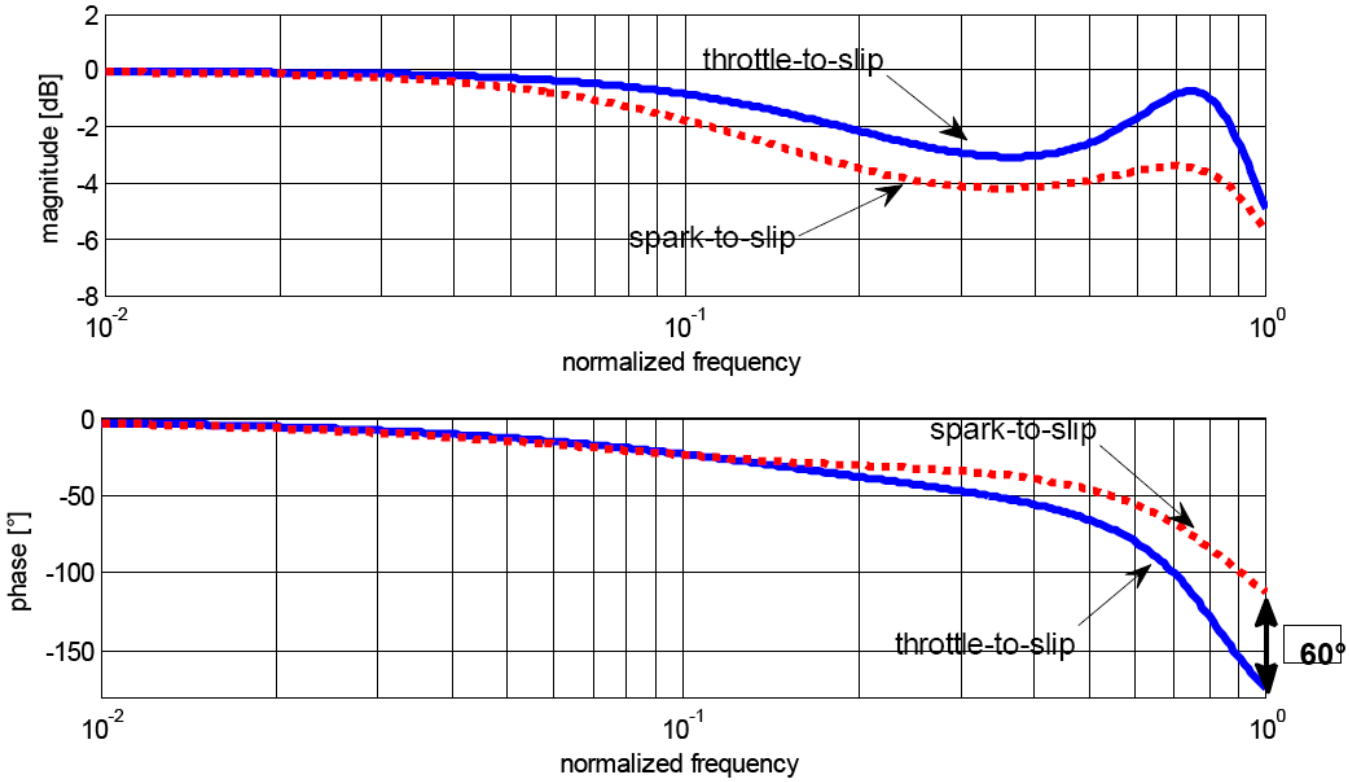
Sine sweeps provide an immediate way to quantify the system nonlinearities.





The system is strongly nonlinear

Black-Box Identification - Conclusions

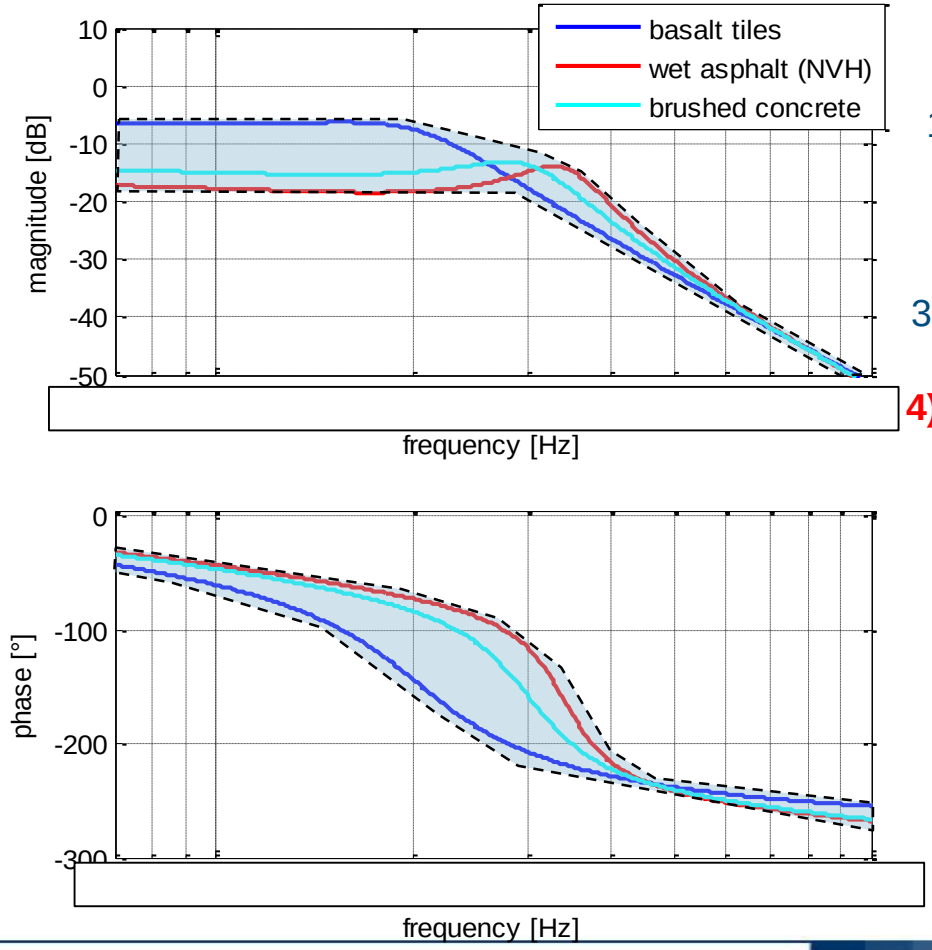


traction control via throttle is doable.

Using ALSO spark-advance increases the bandwidth of the traction-control loop

Remark
Longitudinal Wheel Slip Dynamics – sensitivity to road surface

Necessary to design ROBUST or ADAPTIVE controller



1) There is to considerable spread in terms of **gain** (10 dB)

2) There is to considerable spread in term of **phase.**

3) Open loop bandwidth (φ = 180 °) varies from **xHz →y Hz**

4) **Worst** conditions are Those with **low friction** surfaces (worst phase loss)

5) Low-friction condition however have the largest gain