eq

cvalore7

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0.1 Introduction

From the linearized model we first neglected the longitudinal dynamics so it remains:

$$x5 = \beta$$
$$x6 = \dot{\psi}$$

$$\dot{x_5} = -\frac{C_F + C_R}{m_T v_{T,const}} x_5 - \frac{m_T v_{T,const} + \frac{aC_F - bC_R}{v_{T,const}}}{m_T v_{T,const}} x_6 + \frac{C_F}{m_T v_{T,const}} \delta \tag{1}$$
$$\dot{x_6} = -\frac{aC_F - bC_R}{I_T} x_5 - \frac{a^2 C_F + b^2 C_R}{I_T v_{T,const}} x_6 + \frac{aC_F}{I_T} \delta$$

Then we do a change in the states, by using the lateral velocity V_y and considering a v_T as constant in the lateral controller design and assuming we are going in a straight path thus $v_T = v_{T0} = V_x$

$$V_y = v_{T0} sin(\beta)$$

small angle assumption of linear model

$$V_y = v_{T0} \beta \tag{2}$$

so:

$$\frac{V_y}{v_{T0}} = \beta$$

in this way we have

$$\dot{V_y} = -\frac{C_F + C_R}{m_T V_x} V_y + (\frac{C_R l_R - C_F l_F}{m_T V_x} - V_x)\dot{\psi} + \frac{C_F}{m_T} \delta$$
$$\ddot{\psi} = \frac{C_R l_R - C_F l_F}{I_T V_x} V_y - \frac{C_F l_F^2 - C_R l_R^2}{I_T V_x} \dot{\psi} + \frac{C_F l_F}{I_T} \delta \tag{3}$$

that in matrix form it is

$$Ax + Bu$$
$$\begin{bmatrix} -\frac{C_F + C_R}{m_T V_x} & \frac{C_R l_R - C_F l_F}{m_T V_x} - V_x \\ \frac{C_R l_R - C_F l_F}{I_T V_x} & -\frac{C_F l_F^2 - C_R l_R^2}{I_T V_x} \end{bmatrix} \begin{bmatrix} V_y \\ \dot{\psi} \end{bmatrix} + \begin{bmatrix} \frac{C_F}{m_T} \\ \frac{C_F l_F}{I_T} \end{bmatrix} [\delta] \tag{4}$$

To this we add the error dynamics, assuming small angles and that the velocity is well tracked we have:

$$\dot{e}_{cg} = V_y + V_x \Delta\psi$$
$$\dot{\Delta\psi} = \dot{\psi} - \rho_t V_x \tag{5}$$

that in matrix form, considering ρ as input, it is:

$$\dot{x} = Ax + Bu$$
$$\dot{x} = \begin{bmatrix} -\frac{C_F + C_R}{m_T V_x} & \frac{C_R l_R - C_F l_F}{m_T V_x} - V_x & 0 & 0 \\ \frac{C_R l_R - C_F l_F}{I_T V_x} & -\frac{C_F l_F^2 - C_R l_R^2}{I_T V_x} & 0 & 0 \\ 1 & 0 & 0 & V_x \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_y \\ \dot{\psi} \\ e_{cg} \\ \Delta\psi \end{bmatrix} + \begin{bmatrix} \frac{C_F}{m_T} & 0 \\ \frac{C_F l_F}{I_T} & 0 \\ 0 & 0 \\ 0 & -V_x \end{bmatrix} \begin{bmatrix} \delta \\ \rho \end{bmatrix} \tag{6}$$

$$y = Cx + Du; D = 0$$
$$y = \begin{bmatrix} 0 & 0 & 1 & d_{la} \end{bmatrix} \begin{bmatrix} V_y \\ \dot{\psi} \\ e_{cg} \\ \Delta\psi \end{bmatrix}$$