Dynamic bicycle model

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1 Simple dynamic model

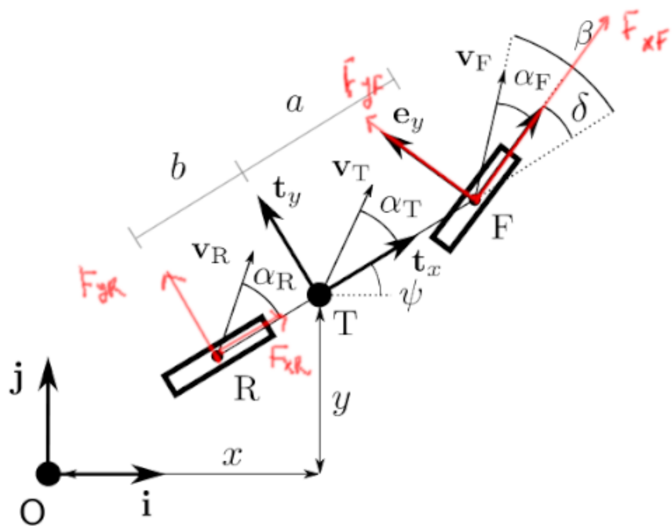

Figure 1: Bicycle model

$$\begin{aligned} m_T\ddot{x} &= F_{xF}cos(\psi+\delta) + F_{xR}cos(\psi) - F_{yF}sin(\psi+\delta) - F_{yR}sin(\psi) \\ m_T\ddot{y} &= F_{xF}sin(\psi+\delta) + F_{xR}sin(\psi) + F_{yF}cos(\psi+\delta) + F_{yR}cos(\psi) \\ I_T\ddot{\psi} &= F_{xF}asin(\delta) + F_{yF}acos(\delta) - F_{yR}b \end{aligned} \tag{1}$$

$$\begin{aligned} \alpha_F &= arctan(\frac{\dot{y} + a\dot{\psi}cos(\psi)}{\dot{x} - a\dot{\psi}sin(\psi)}) - (\delta+\psi) \\ \alpha_R &= arctan(\frac{\dot{y} - b\dot{\psi}cos(\psi)}{\dot{x} + b\dot{\psi}sin(\psi)}) - \psi \end{aligned} \tag{2}$$

At first, as state vector this has been used

$$
\begin{aligned}
z1 &= x \\
z2 &= y \\
z3 &= \psi \\
z4 &= \dot{x} \\
z5 &= \dot{y} \\
z6 &= \dot{\psi}
\end{aligned}
\tag{3}
$$

So that

$$
\begin{aligned}
\dot{z_1} &= z_4 \\
\dot{z_2} &= z_5 \\
\dot{z_3} &= z_6 \\
\dot{z_4} &= \frac{F_{xF}cos(z_3+\delta) + F_{xR}cos(z_3) - F_{yF}sin(z_3+\delta) - F_{yR}sin(z_3)}{m_T} \\
\dot{z_5} &= \frac{F_{xF}sin(z_3+\delta) + F_{xR}sin(z_3) + F_{yF}cos(z_3+\delta) + F_{yR}cos(z_3)}{m_T} \\
\dot{z_6} &= \frac{F_{xF}asin(\delta) + F_{yF}acos(\delta) - F_{yR}b}{I_T}
\end{aligned}
\tag{4}
$$

With slip angles

$$
\begin{aligned}
\alpha_F &= arctan(\frac{z_5 + az_6cos(z_3)}{z_4 - az_6sin(z_3)}) - (\delta + z_3) \\
\alpha_R &= arctan(\frac{z_5 - bz_6cos(z_3)}{z_4 + bz_6sin(z_3)}) - z_3
\end{aligned}
\tag{5}
$$

Now instead of using $\dot{x}$ and $\dot{y}$, v_T and α_T have been used. The trasformations are the following:

$$
\begin{aligned}
\dot{x} &= v_T cos(\psi + \alpha_T) \\
\dot{y} &= v_T sin(\psi + \alpha_T) \\
\\
\ddot{x} &= \dot{v_T} cos(\psi + \alpha_T) - v_T(\dot{\psi} + \dot{\alpha_T})sin(\psi + \alpha_T) \\
\ddot{y} &= \dot{v_T} sin(\psi + \alpha_T) + v_T(\dot{\psi} + \dot{\alpha_T})cos(\psi + \alpha_T)
\end{aligned}
\tag{6}
$$

Substituting and simplyfing with the help of Matlab

$$
\begin{aligned}
\dot{v_T} &= \frac{F_{xF}cos(\alpha_T - \delta) + F_{xR}cos(\alpha_T) + F_{yF}sin(\alpha_T - \delta) + F_{yR}sin(\alpha_T)}{m_T} \\
\dot{\alpha_T} &= \frac{-F_{xF}sin(\alpha_T - \delta) - F_{xR}sin(\alpha_T) + F_{yF}cos(\alpha_T - \delta) + F_{yR}cos(\alpha_T) - m_T v_T \dot{\psi}}{m_T v_T} \\
\ddot{\psi} &= \frac{F_{xF}asin(\delta) + F_{yF}acos(\delta) - F_{yR}b}{I_T}
\end{aligned}
\tag{7}
$$

$$
\begin{aligned}
\alpha_F &= arctan(\frac{v_T sin(\alpha_T) + a\dot{\psi}}{v_T cos(\alpha_T)}) - \delta \\
\alpha_F &= arctan(\frac{v_T sin(\alpha_T) - b\dot{\psi}}{v_T cos(\alpha_T)})
\end{aligned}
\tag{8}
$$

The new state and the state equations are

$$
\begin{aligned}
x1 &= x \\
x2 &= y \\
x3 &= \psi \\
x4 &= v_T \\
x5 &= \alpha_T \\
x6 &= \dot{\psi}
\end{aligned}
$$

$$
\begin{aligned}
\dot{x_1} &= x_4 cos(x_3 + x_5) \\
\dot{x_2} &= x_5 sin(x_3 + x_5) \\
\dot{x_3} &= x_6 \\
\dot{x_4} &= \frac{F_{xF}cos(x_5 - \delta) + F_{xR}cos(x_5) + F_{yF}sin(x_5 - \delta) + F_{yR}sin(x_5)}{m_T} \\
\dot{x_5} &= \frac{-F_{xF}sin(x_5 - \delta) - F_{xR}sin(x_5) + F_{yF}cos(x_5 - \delta) + F_{yR}cos(x_5) - m_T x_4 x_6}{m_T x_4} \\
\dot{x_6} &= \frac{F_{xF}asin(\delta) + F_{yF}acos(\delta) - F_{yR}b}{I_T}
\end{aligned}
\tag{9}
$$

2 Pacejka tyre model

The following Pacejka tyre model (Magic Formula) has been used, taking as inputs the body slip angle α, the vertical load F_z and the lateral friction coefficient

μ_y

$$F_y = -\frac{\mu_y}{\mu_{y0}}(F_{y0} + Sv) \tag{10}$$

With

$$\begin{aligned} F_{y0} = Dsin(Carctan(B\alpha_{eq} - E(B\alpha_{eq} - arctan(B\alpha_{eq})))) \\ \alpha_{eq} = \frac{\mu_{y0}}{\mu_y}(\alpha + S_h) \\ C = a_0 \\ D = (a_1 F_z + a_2) F_z BCD = a_3 sin(2arctan(\frac{F_z}{a_4}))(1 - a_5|\gamma|) \\ B = BCD/CD \\ E = a_6 F_z + a_7 \\ S_h = a_8\gamma + a_9 F_z + a_{10} \\ S_v = a_{11} F_z \gamma + a_{12} F_z + a_{13} \end{aligned} \tag{11}$$

Where a_i, i $\in$ {0,..,13}, are the parameters of the Pacejka model, in particular we have that

$a0$ = Shape factor [-]
$a1$ = Load dependency of lateral friction (*1000) [1/kN]
$a2$ = Lateral friction level (*1000) [-]
$a3$ = Maximum cornering stiffness [N/deg]
$a4$ = Load at maximum cornering stiffness [kN]
$a5$ = Camber sensitivity of cornering stiffness
$a6$ = Load dependency of curvature factor
$a7$ = Curvature factor level
$a8$ = Camber sensitivity of horizontal shift
$a9$ = Load dependency of horizontal shift
$a10$ = Horizontal shift level
$a11$ = Combined load and camber sensitivity of vertical shift
$a12$ = Load dependency of vertical shift
$a13$ = Vertical shift level

3 Aerodynamic force

The following changes have been done in the previous model to take into account for the aerodynamic force $F_A = \frac{1}{2}\rho C_x S v^2$ in the same direction of v_T but in the opposite side, and $F_{Lift} = \frac{1}{2}\rho C_z S v^2$ that "pushes" the vehicle to be sticked on the ground

$$\dot{x_4} = \frac{F_{xF}cos(x_5 - \delta) + F_{xR}cos(x_5) + F_{yF}sin(x_5 - \delta) + F_{yR}sin(x_5) - \frac{1}{2}\rho C_x S v^2}{m_T} \tag{12}$$

$$F_z = mg + \frac{1}{2}\rho C_z S v^2 \tag{13}$$

4 Fuel consumption

The following changes have been done in the previous model to take into account for the fuel consumption. A simplified version has been used, in which the Power is calculated (P_e) and multiplied by a coefficient (C_{fuel}) that accounts for fuel consumption for unit of power erogated. The fuel consumption has as side effect the reduction of the mass over time

$$\begin{aligned} P_e &= (F_{xF} + F_{xR})v_T \\ \dot{m} &= P_e C_{fuel} \end{aligned} \tag{14}$$

5 Tyre wear

The following have been add in the previous model to take into account for wear of the rubber compund of the tyre. The model is called Archard model, and it makes use of the vertical pression ($P_z = \frac{F_z}{Area}$), the longitudinal velocity of the wheels (v_{xF} and v_{xR}) and some parameters (such as $K_{wear}[-]$ and $H[\frac{N}{m^2}]$). The model output is the wear depth over the time ($\dot{h}$), that then in the implementation is converted into mm^3 of wear. Here the Archard model formulation is shown

$$\dot{h} = \frac{K_{wear} P_{load} v_{xi}}{H} \quad \text{where i} \in \{\text{F, R}\} \tag{15}$$