Michaelis-Menten kinetics [1] describe enzyme-mediated biochemical reaction in Figure 1.

$$\mathrm{E} + \mathrm{S} \underset{\mathrm{k_{-1}}}{\overset{\mathrm{k_1}}{\rightleftharpoons}} \mathrm{ES} \xrightarrow{\mathrm{k_2}} \mathrm{E} + \mathrm{P}$$

where E is an enzyme, S is a substrate, ES is an enzyme-substrate complex and P is a product.

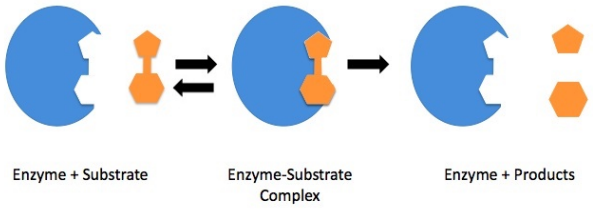

Figure 1: Enzyme-mediated biochemical reaction

According to the Law of Mass Action, the rate of chemical reaction is proportional to the product of the concentrations of the reactants. Thus, the diagram above reads as follows:

- E binds to S to form ES at rate $k_1[E][S]$, where $[E]$ and $[S]$ are the respective concentrations.
- ES breaks into $E + S$ at rate $k_{-1}[ES]$, and to $E + P$ at rate $k_2[ES]$, where $[ES]$ is the concentration of ES.

Note that k_1, k_{-1}, and k_2 are the constants of proportionality, and that the concentrations are functions of time (argument t has been omitted for simplicity). Also note that $[E][S]$ and $[ES]$ are two different quantities.
What are the differential equations describing the change in concentrations $[S]$, $[E]$, $[ES]$ and $[P]$?

References

[1] Leonor Michaelis and Maude L Menten. "The kinetics of the inversion effect". In: *Biochem. Z* 49 (1913), pp. 333–369.