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# The effect of three point interaction in the stability of criminal organizations

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## 1 Background

- Criminal organizations form dangers to society and rule of law
- Most criminal studies are performed in an observational capacity
- Theories are formed based on biased data with limited validation capacity
- There is a lot of opportunity to use computational models to study the consequences of theories in criminal science

## 2 Aim of the study

Produce a computational model rooted in game theory to study the emergent properties of success supply chain of criminal markets in society.

The goal is to come up with a model of supply chain interactions in criminal markets and slowly introduce more complexities. Initially, we focus the attention on the interaction of a criminal supply chain without competition in a well-mixed population. Later, we can add network effects and effects of rivalry.

Law enforcement is embedded as an apprehension cost and can be represented as heat bath of the system.

## 3 The model

Let  $\sigma_i^X \in \{0, 1\} = \{\text{do not perform criminal act, perform criminal act}\}$  denote the state of agent  $i$  with roles category  $X \in \{\text{Production}, \text{Distribution}, \text{Management}, \text{Civilian}\}$ . Each agent has a connectivity structure according to adjacency matrix  $A$  where a non-zero entry  $a_{ij}$  indicates the connection strength  $a_{ij} \in \mathbb{R}$  between agent  $i$  and agent  $j$ . We first consider symmetrical adjacency matrices and fully connected graphs without self-connections and each edge weight having value 1. The roles of each agent are assigned with probability  $p(x \in X) = \frac{1}{|X|}$ , agent do not change their role within each simulation run. The agent can decide to participate or not in a criminal act based on the state of the local connections.

## 4 The pay-off matrix

We are interested in studying the interaction between successful criminal supply chains and citizens. The maximum pay-off for a criminal organization is to have a functioning supply chain. This implies that there exist a four point interaction between the roles of production, distribution and management; the criminal organization is most effective when the product of interest (drugs, weapons, illegal goods etc) is readily available and can be shipped to and bought by customers (civilians). A civilian can benefit from criminal goods due to a *need* of the product, or obtaining a product at cheaper than market prices.

We therefore delineate the following variables. The variables are listed as benefits ( $b_x$ ) and costs ( $c_x$ ).

- $b_m$  monetary benefit of forming a complete criminal supply chain (production, distribution, management)
- $b_s$  sales benefit of forming a complete criminal supply chain with customers
- $c_a$  detection cost of committing crime

## 5 Evolutionary Dynamics

We evaluate the evolutionary dynamics by using the Fermi update. Let  $x$  denote the flip probability for an agent of moving from strategy  $i \rightarrow j$  or vice versa, then we can define the Fermi update as

$$p(x)_{i \rightarrow j} = \frac{1}{1 + \exp(-\beta(E_j - E_i))} \quad (1)$$

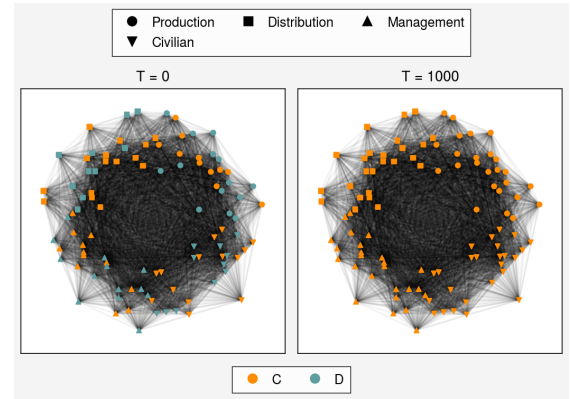
where  $E_i$  is the pay-off or “energy” of the current strategy of an agent and  $E_j$  the energy when using strategy  $j$ . The  $\beta$  parameter denotes the noise. For low values the flip probability approaches one. This represents the conditions by which the agent is highly influenced by its environment and will adopt strategies accordingly. In contrast, high values of  $\beta$  represents scenarios where an agent is making a decision in a highly noisy environment, and their strategy approaches a random choice to defect or conform.

## 6 Simulation

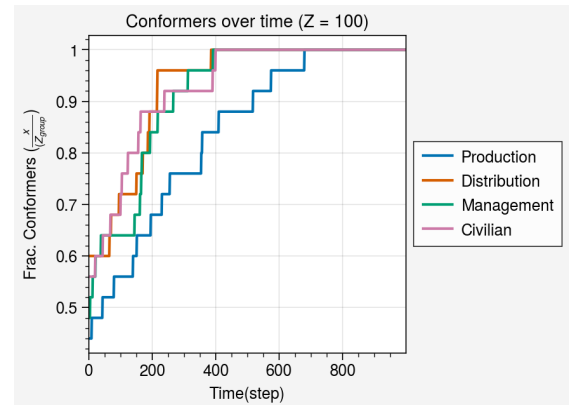
I start by spawning  $Z = 100$  agents using homogeneous mixing. From a network perspective this

is similar to connecting every agent with every other agent (no-self edge). Using update rule in 1, I start by simulating for  $b_s = c_a = 1$  with  $b_m = 0$ . A priori, I would expect that the four point interaction  $CCCC$  has a slight benefit over all other possible states as there is a net negative for most other configurations of the simulation. The conformers over time are visualize in ?? and a snapshot of the state per role is visualized in ??

```
/tmp/ipykernel_163944/166041059.py:35: UserWarning
fig.show()
```



```
/tmp/ipykernel_163944/297734793.py:17: UserWarning
fig.show()
```



**Table 1:** Supply chain pay-off. Note empty cell indicate no pay-off.

Roles				Pay-off			
Production	Distribution	Management	Civilian	Production	Distribution	Management	Civilian
C	C	C	C	$b_s - c_a$	$b_s - c_a$	$b_s + b_m - c_a$	$b_m$
C	D	C	C	$-c_a$		$-c_a$	
C	C	D	C	$-c_a$	$-c_a$		
C	C	C	D	$-c_a$	$-c_a$	$-c_a$	
C	D	D	C	$-c_a$			
C	D	D	D	$-c_a$			
C	D	C	C	$-c_a$		$-c_s$	
C	C	D	D	$-c_a$	$-c_a$		
C	D	C	D			$-c_a$	
D	C	C	C		$-c_a$	$-c_a$	
D	D	C	C			$-c_a$	
D	D	D	C				
D	C	C	D		$-c_a$	$-c_a$	
D	C	D	C		$-c_a$		
D	D	D	D				
D	D	C	D			$c_a$	
D	D	D	D				