Parallel & Distibuted Computing: Lecture 7

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Analysis of Course Project Structure

1. Summary of project
2. The arrangement of space
3. The construction of Boolean Algebras

Summary of project

Project aims

1. Verify the correctness of present implementation
2. Optimize the current code base
3. Parallelize (best way) where possible
4. Document the new codes (~/test/ and ~/docs/)

High-level structure

1. the computation of the chain complex of the $\{\text{partition}\}$ of $\mathbb{E}^d$ induced by the input;
2. the assessment of a finite algebra of sets generated by such partition.

The mapping of terms to their coordinate representation in chain space allows for native binary resolution of every Boolean (CSG) expression between the input terms.

High-level structure

Chain complex of space partition

1. 2-cell partition
2. 2-skeleton in 3D
3. 3-boundary operator

Chains as atoms of Boolean Algebras

1. Algebra of sets
2. Generate-and-test algorithm
3. Binary representation of Boolean terms
4. Bitwise resolution of set algebra expressions
5. Boundary computation

The arrangement of space

Single tasks

Short synthesis of sequential steps of the whole computational pipeline:

Input Facet selection, *i.e.*, construction of the collection $\mathcal{S}_{d-1}$ from $\mathcal{S}_d$, using LAR.
Indexing Spatial index made by intersection of d interval-trees on bounding boxes of $\sigma \in \mathcal{S}_{d-1}$.
Decomposition Pairwise $z = 0$ intersection of line segments in $\sigma \cup \mathcal{I}(\sigma)$, for each $\sigma \in \mathcal{S}_{d-1}$.
Congruence Graded bases of equivalence classes $C_k(U_k)$, with $U_k = X_k/R_k$ for $0 \leq k \leq 2$.
Connection Extraction of $(X^p_{d-1}, \partial^p_{d-1})$, maximal connected components of X_{d-1} $(0 \leq p \leq h)$.
Bases Computation of redundant cycle basis $[\partial^+_d]^p$ for each p-component, via TGW.
Boundaries Accumulation into H += $[o]^p$ (hole-set) of outer boundary cycle from each $[\partial^+_d]^p$.
Containment Computation of antisymmetric containment relation S between $[o]^p$ holes in H.
Reduction Transitive R reduction of S and generation of forest of flat trees $\langle [o_d]^p, [\partial_d]^p \rangle$.
Adjoining of roots $[o_d]^r$ to (unique) outer cell, and non-roots $[\partial^+_d]^q$ to container cells.
Assembling Quasi-block-diagonal assembly of matrices relatives to isolated components $[\partial_d]^p$.
Output Global boundary map $[\partial_d]$ of $\mathcal{A}(\mathcal{S}_{d-1})$, and reconstruction of 0-chains of d-cells in X_d.

Browsing the code base

https://github.com/cvdlab/LinearAlgebraicRepresentation.jl

The construction of Boolean Algebras

Representation of atoms as single points

test and optimize:

- search for internal point of atoms
- efficient search for terms containing a point

The resolution of algebraic formulas

Native bitwise operations between algebraic terms

Design

simple API and mini DSL (Domain Specific Language) for CSG expressions