Proof of concept: multidimensional morphological operators

Mathematical morphology (MM)
From Wikipedia

Mathematical morphology (MM) is a theory and technique for the analysis and processing of geometrical structures, based on set theory, lattice theory, topology, and random functions.

MM is also the foundation of morphological image processing, which consists of a set of operators that transform images according to the above characterizations.

Figure : http://en.wikipedia.org/wiki/File:DilationErosion.png

Example

Generation of a random image

with random noise

Let us consider the chain $\gamma \in C_2$ of white pixels (i.e. the cochain $\chi : C_2 \to \mathbb{R} : \chi(\gamma) = 255$)

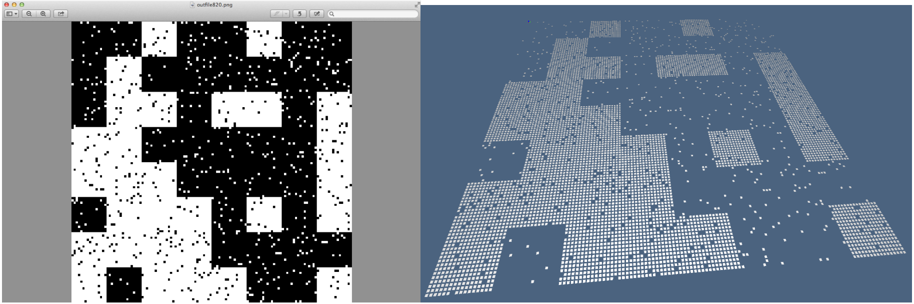
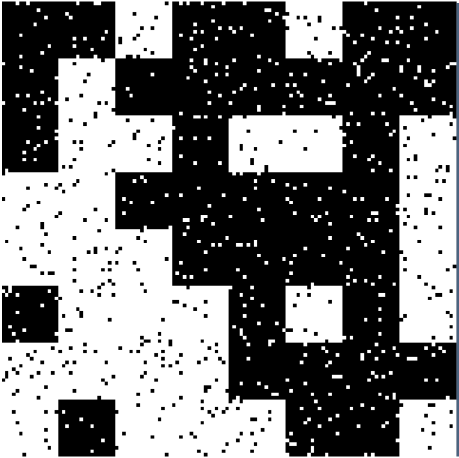


Figure : (a) PNG image; (b) exploded solid model of $\gamma \in \mathbb{E}^3$

Extraction of boundary of a chain

$\beta = \partial_2(\gamma) \in C_1$



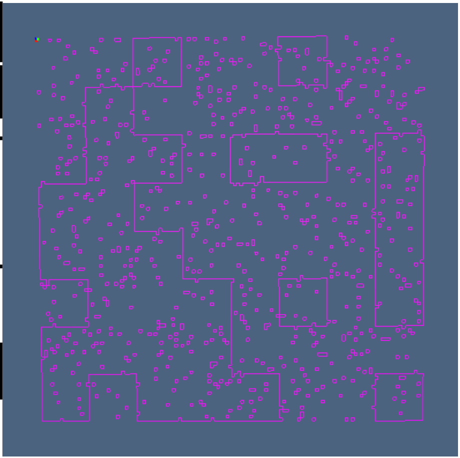


Figure : (a) PNG image; (b) exploded solid model of $\gamma \in \mathbb{E}^3$

Down(boundary)

$\eta = \mathcal{EV}(\partial_2(\gamma)) \in C_0$

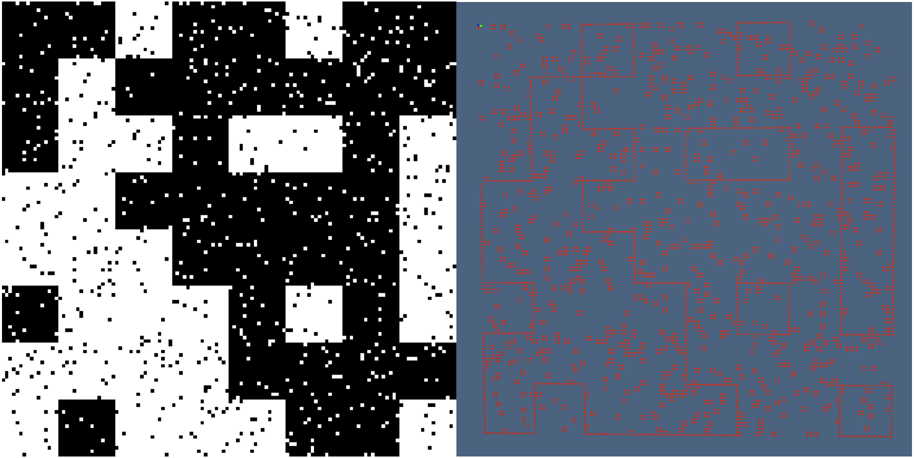


Figure : (a) PNG image; (b) exploded solid model of $\gamma \in \mathbb{E}^3$

Up(up(down(boundary)))

$\beta_2 = \mathcal{FV}(\mathcal{VE}(\partial_2(\gamma))) \in C_2 \equiv (\mathcal{FV} \circ \mathcal{VE} \circ \partial_2)(\gamma)$

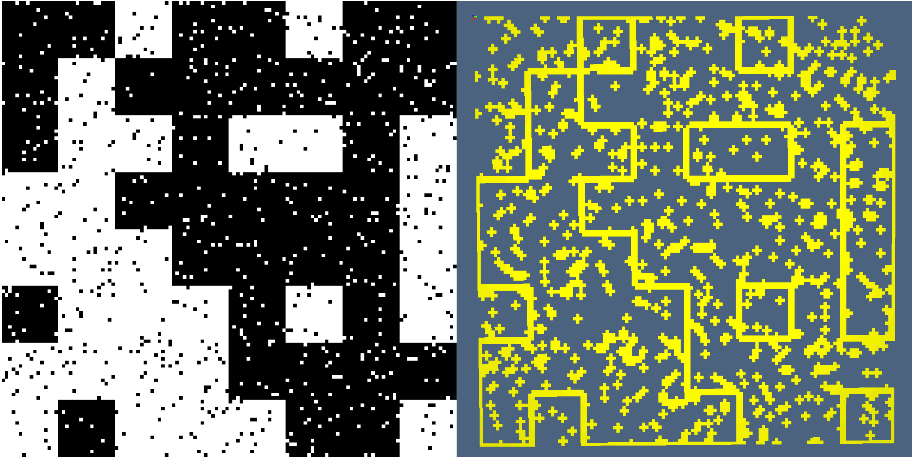


Figure : (a) PNG image; (b) exploded solid model of $\gamma \in \mathbb{E}^3$

Boolean combination of chains

$\beta_2 = (\mathcal{FV} \circ \mathcal{VE} \circ \partial_2)(\gamma)$ Dilation $= \beta_2 - \gamma$ (yellow); Erosion $= \beta_2 \cap \gamma$ (cyan)

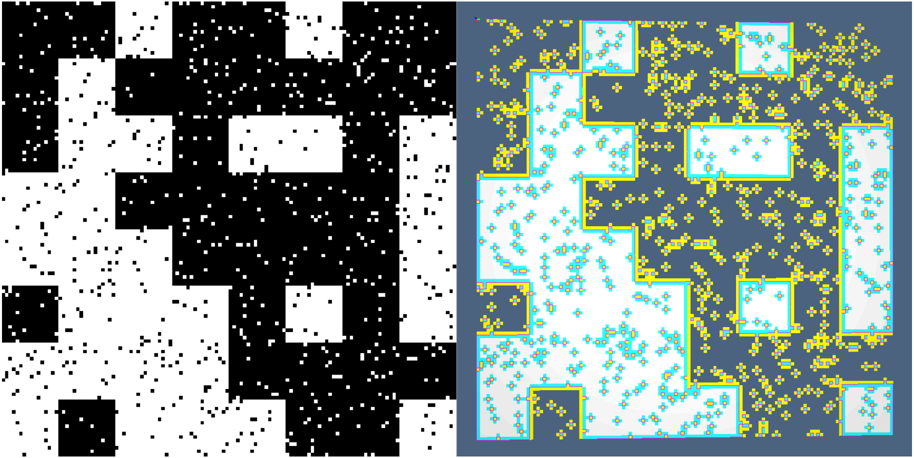
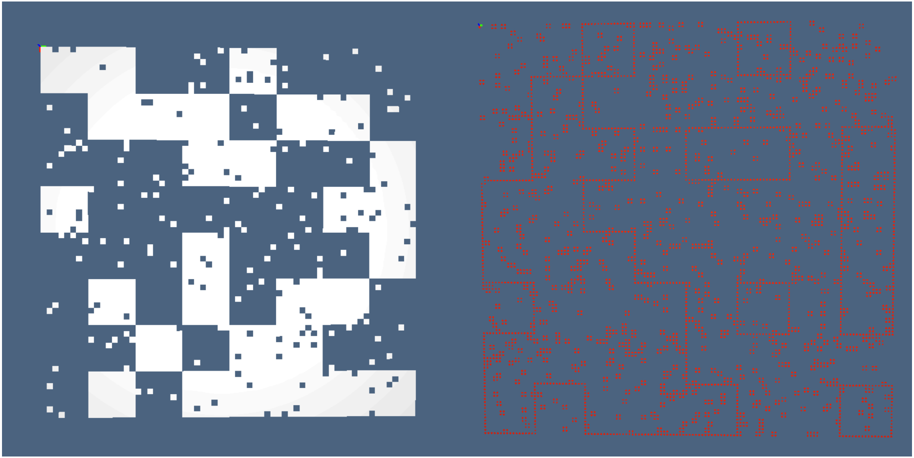


Figure : (a) PNG image; (b) exploded solid model of $\gamma \in \mathbb{E}^3$

64×64 example

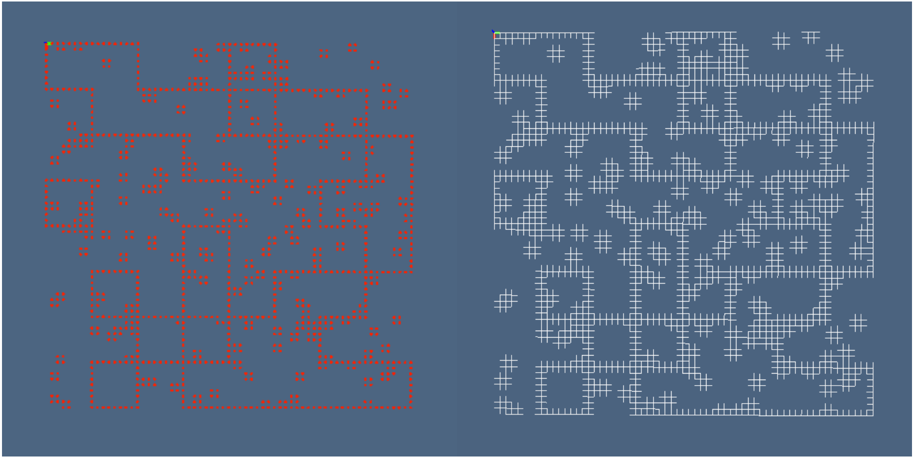
chain $\gamma \in C_2$; chain $\beta = \partial_2(\gamma) \in C_1$



64×64 example

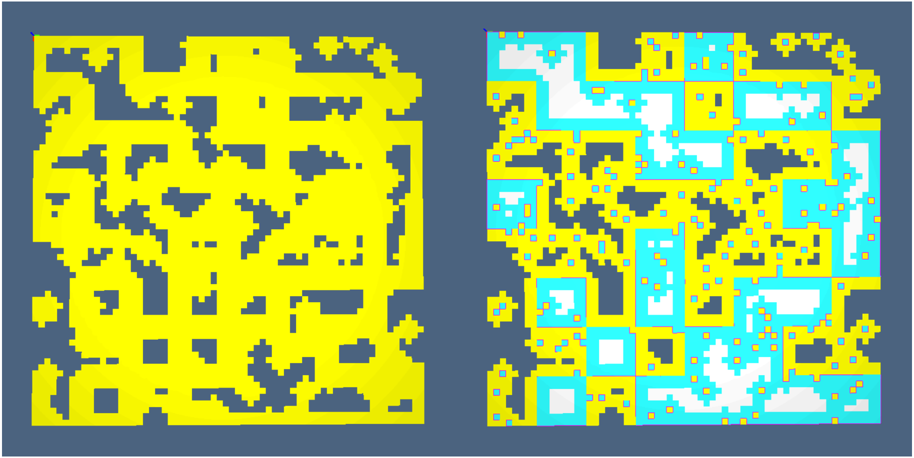
$(\mathcal{VE} \circ \partial_2)(\gamma) \in C_0$

$(\mathcal{EV} \circ \mathcal{VE} \circ \partial_2)(\gamma) \in C_1$



64 × 64 example

$\beta_2 = (\mathcal{FE} \circ \mathcal{EV} \circ \mathcal{VE} \circ \partial_2)(\gamma) \in C_2 \quad \mathcal{DIL}(\beta_2)(\gamma) = \beta_2 - \gamma \quad \mathcal{ERO}(\beta_2)(\gamma) = \beta_2 \cap \gamma$



State of the art

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TODO

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Thanks

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