Parallel \& Distibuted Computing: Lecture 12

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Test Driven Development (TDD)

1. Space Arrangement (Continue from lecture 12)
2. Test Driven Development (TDD)
3. Example: Topological gift-wrapping
4. Julia implementation
5. Test-driven development example

Space Arrangement (Continue from lecture 12)

Definitions

- Arrangement is the decomposition of the d-dimensional affine or projective space,
- into connected and (relatively open) cells of lower dimensions
- induced by an intersection of a finite collection of geometric objects

Formally:

- arrangement $\mathcal{A}(\mathcal{S}) = X$,
- where $X := \bigcup_{k=0}^{d} X_k$,
- X_k is called the k-skeleton of the cellular complex X, usually with $d \in \{2, 3\}$,
- providing a cellular decomposition of the space $\mathbb{E}^d$ where the {underlying space} (point-set) of $\mathcal{S}$ is embedded.

Computational goal

Given the input collection $\mathcal{S}$, how to compute and represent the chain complex

$$C_\bullet := C_3 \underset{\partial_3}{\overset{\delta_2}{\leftrightarrows}} C_2 \underset{\partial_2}{\overset{\delta_1}{\leftrightarrows}} C_1 \underset{\partial_1}{\overset{\delta_0}{\leftrightarrows}} C_0,$$

where C_p $(0 \leq p \leq d)$, with $d \in \{2, 3\}$, is a linear space of p-chains (sets of p-cells with algebraic structure), and where $\delta_{p-1} = \partial_p^\top$.

Arrangement of cellular complexes: a quick tour

Paoluzzi, Shapiro, DiCarlo, 2018 (submitted)

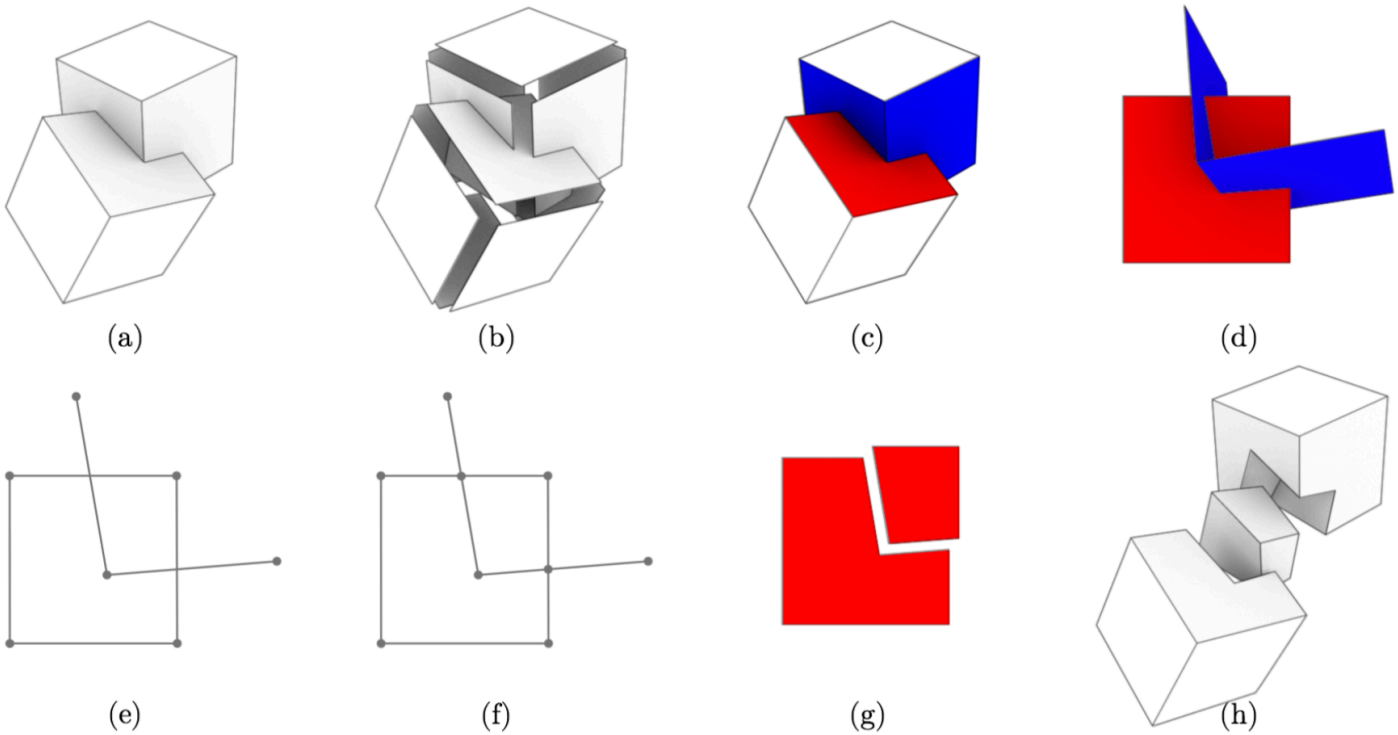


Fig. 1. Cartoon display of the Merge algorithm: (a) the two input solids; (b) the exploded input collection B of 2-cells embedded in $\mathbb{E}^3$; (c) 2-cell σ (red) and the set $\Sigma(\sigma)$ (blue) of possible intersection; (d) affine map of $\sigma \cup \Sigma$ on $z = 0$ plane; (e) reduction to a set of 1D segments in $\mathbb{E}^2$; (f) pairwise intersections; (g) regularized plane arrangement $\mathcal{A}(\sigma \cup \Sigma)$; (h) exploded 3-complex extracted from the 2-complex $X_2(\cup_{\sigma \in B} \mathcal{A}(\sigma \cup \Sigma))$ in $\mathbb{E}^3$. The LAR representation of both the input data (not a complex) and the output data (3-complex with three 3-cells) is given in Appendix A

Test Driven Development (TDD)

Definitions

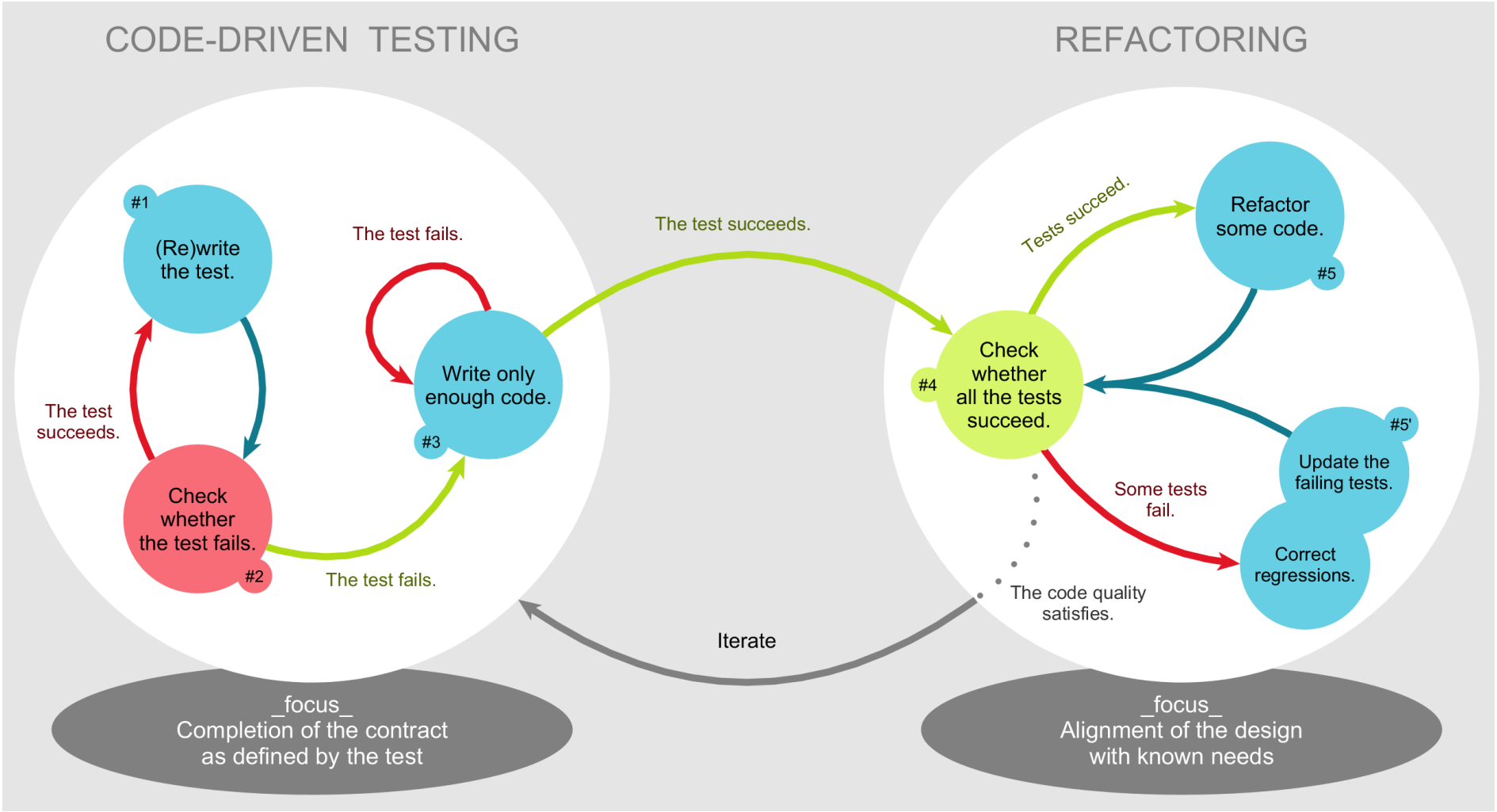
(*Wikipedia**)

Test-driven development (TDD) is a software development process that relies on the repetition of a very short development cycle

- requirements are turned into very specific test cases, then the software is improved to pass the new tests, only
- opposed to software development that allows software to be added that is not proven to meet requirements
- also apply the concept to improving and debugging legacy code developed with older techniques.

State diagram of DDT

(*Wikipedia**)

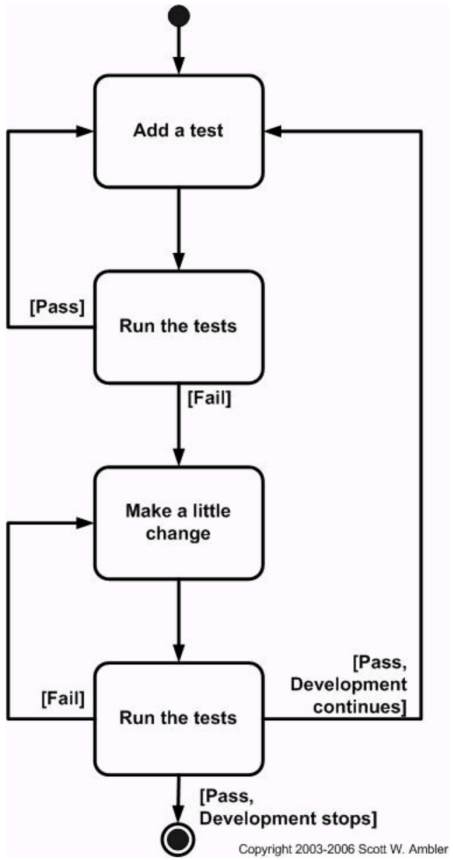


Test-Driven Development workflow

The following sequence is based on the book Beck, K. Test-Driven Development by Example, Addison Wesley - Vaseem, 2003.

See also Test-Driven Development by Example

1. Add a test
2. Run all tests and see if the new test fails
3. Write the code
4. Run tests
5. Refactor code
6. Repeat



Example: Topological gift-wrapping

Arrangement of cellular complexes: a quick tour

Paoluzzi, Shapiro, DiCarlo, 2018 (submitted)

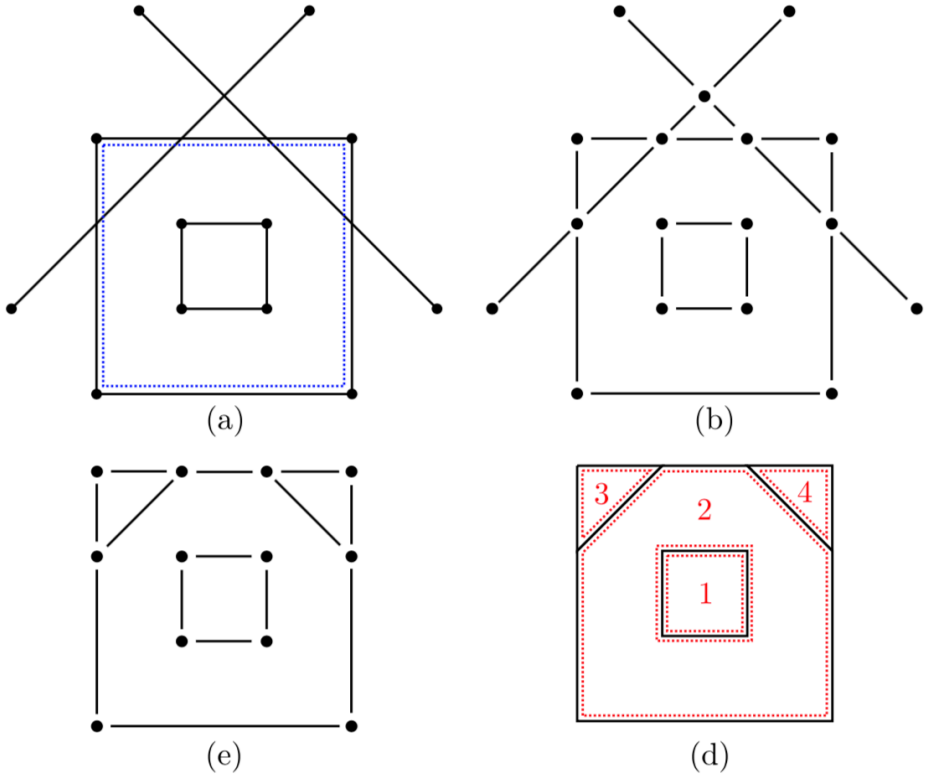
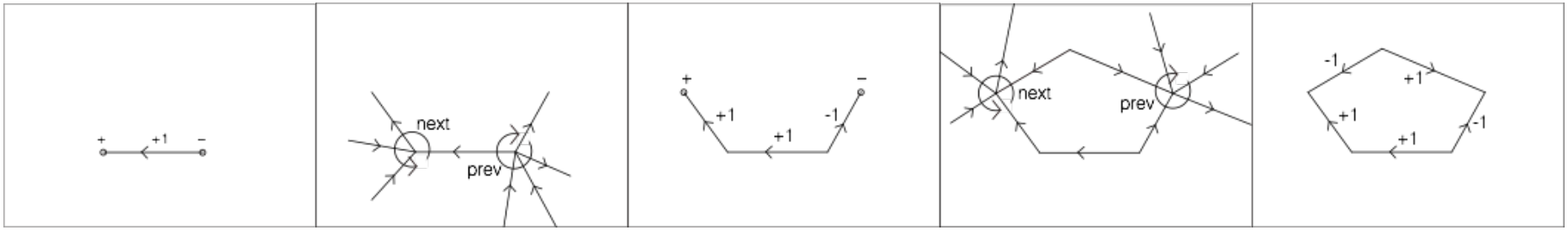


Fig. 2. Basic case: computation of the regularized arrangement of a set of lines in $\mathbb{E}^2$: (a) the input, i.e. the 2-cell σ (blue) and the line segment intersections of $\Sigma(\sigma)$ with $z = 0$; (b) all pairwise intersections; (c) removal of the 1-subcomplex external to σ. To this purpose an efficient and robust point classification algorithm [?] is used, ; (d) interior cells of the regularized 2-complex $X_2 = \mathcal{A}(\sigma \cup \Sigma)$ generated as arrangement of $\mathbb{E}^2$ produced by $\sigma \cup \Sigma$.

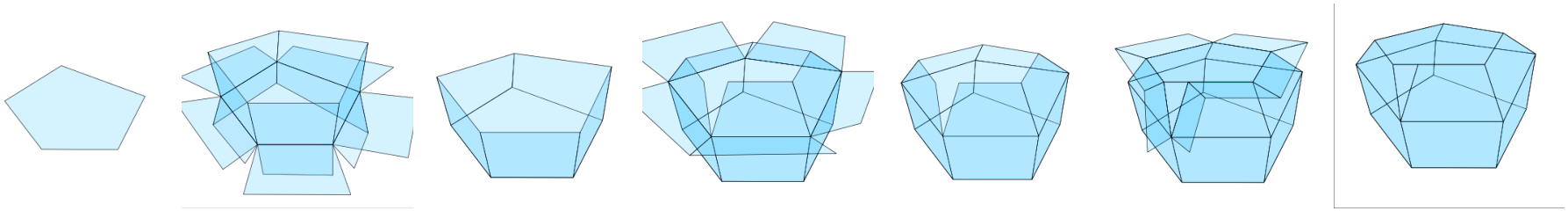
Arrangement of cellular complexes: a quick tour

Paoluzzi, Shapiro, DiCarlo, 2018 (submitted)



(a) (b) (c) (d) (e)

Fig. 11. Extraction of a minimal 1-cycle from $\mathcal{A}(X_1)$: (a) the initial value for $c \in C_1$ and the signs of its oriented boundary; (b) cyclic subgroups on $\delta\partial c$; (c) new (coherently oriented) value of c and ∂c; (d) cyclic subgroups on $\delta\partial c$; (e) final value of c, with $\partial c = \emptyset$.



(a) (b) (c) (d) (e) (f) (g)

Fig. 9. Extraction of a minimal 2-cycle from $\mathcal{A}(X_2)$: (a) 0-th value for $c \in C_2$; (b) cyclic subgroups on $\delta\partial c$; (c) 1-st value of c; (d) cyclic subgroups on $\delta\partial c$; (e) 2-nd value of c; (f) cyclic subgroups on $\delta\partial c$; (g) 3-rd value of c, such that $\partial c = 0$, hence stop.

Arrangement of cellular complexes: a quick tour

Paoluzzi, Shapiro, DiCarlo, 2018 (submitted)

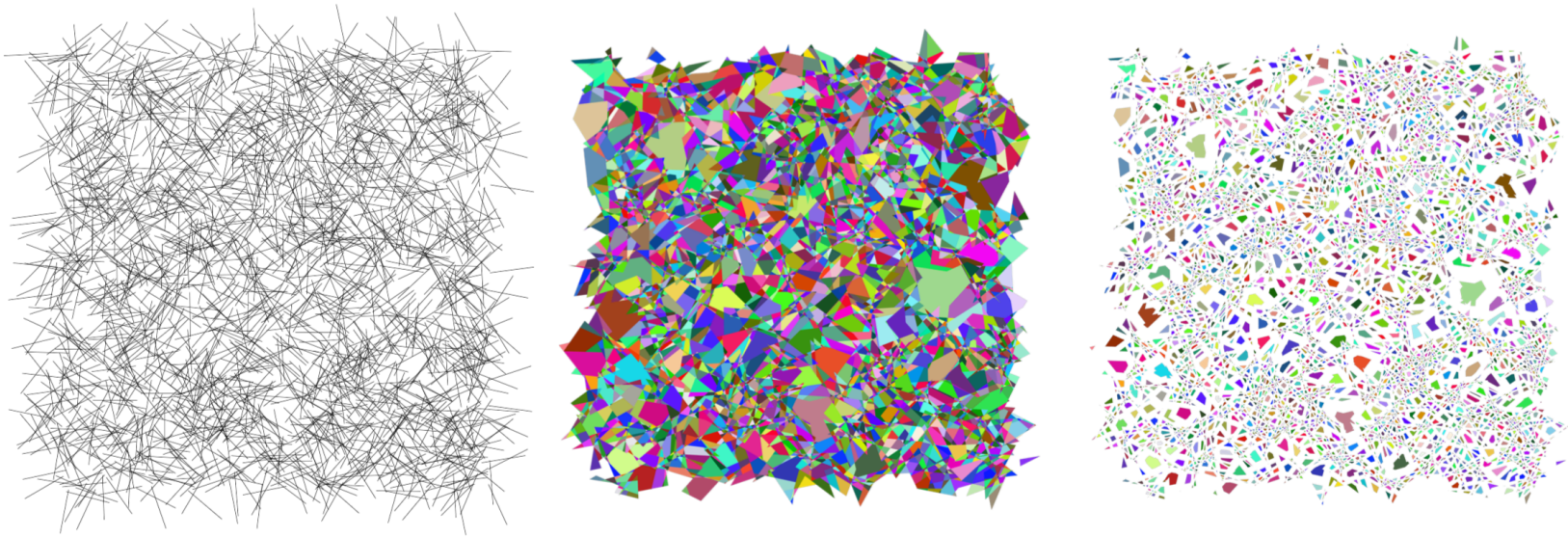


Fig. 5. The regularized 2D arrangement X_2 of the plane generated by a set of random line segments. Note that cells $\sigma \in X_2$ are not necessarily convex. The Euler characteristic is $\chi = \chi_0 - \chi_1 + \chi_2 = 11361 - 20813 + 9454 = 2$.

Arrangement of cellular complexes: a quick tour

Paoluzzi, Shapiro, DiCarlo, 2018 (submitted)

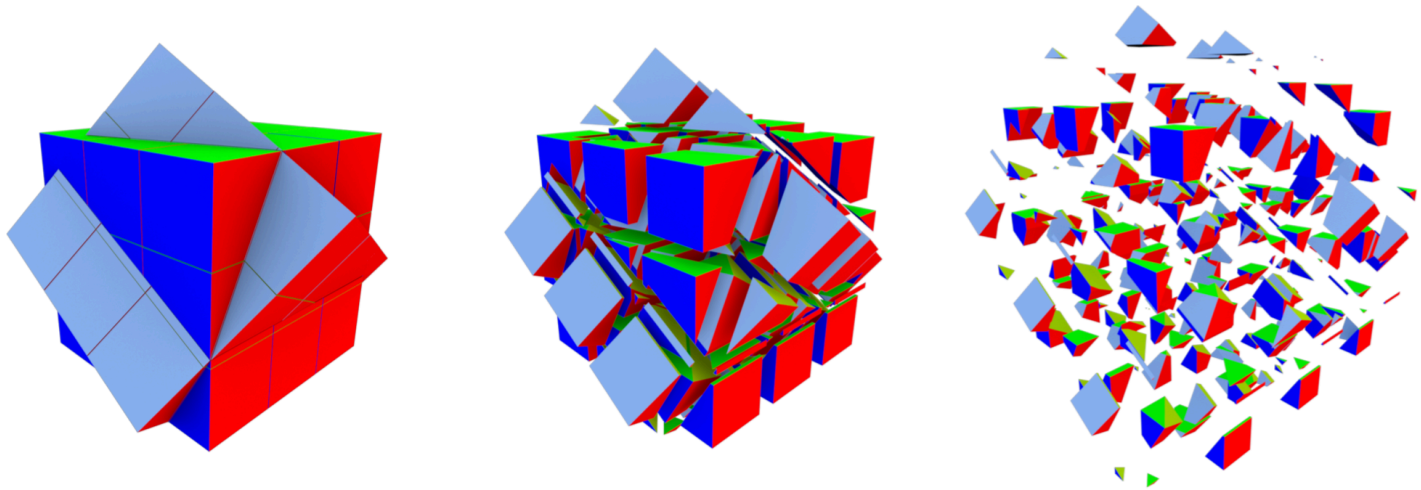


Fig. 6. Merge of two rotated $3 \times 3 \times 3$ cuboidal 3-complexes: (a) the initial positions of the two input 3-complexes, each with 3^3 unit cubic cells; (b) 3-cells of merged complex, with space explosion of small scaling parameter; (c) larger space explosion. Each exploded cell is translated by a scaling-induced movement of its centroid. Output cells are not necessarily convex. In the merged complex we get 236 three-cells and 816 two-cells, both possibly non-convex. Ratio new/old 3-cells is 4.3704.

Arrangement of cellular complexes: a quick tour

Paoluzzi, Shapiro, DiCarlo, 2018 (submitted)

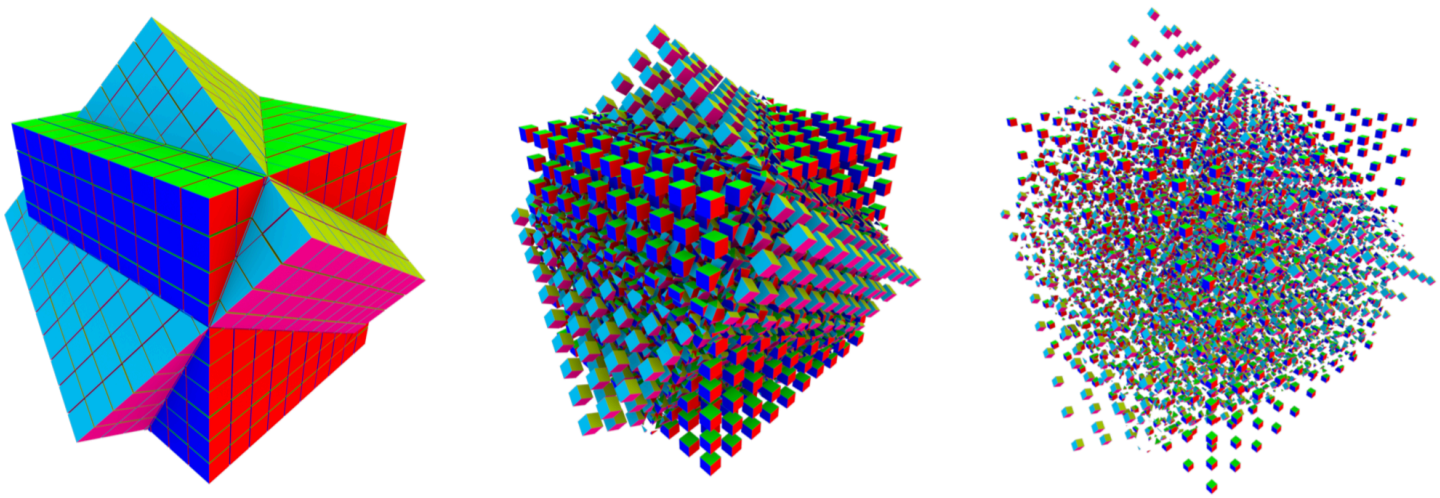


Fig. 7. Merge of two 10^3 3-complexes. In the merged complex we get 8655 3-cells, 26600 2-cells (faces), 26732 1-cells (edges), and 8787 0-cells (vertices). Please note that the Euler characteristic is $\chi = \chi_0 - \chi_1 + \chi_2 - \chi_3 = 8787 - 26732 + 26600 - 8655 = 0$, since the decomposed 3-complex (no its exploded image!) is homeomorphic to the n-sphere, where $\chi = 1 + (-1)^n$, with $n = 3$. Ratio new/old 3-cells is 4.3275.

Arrangement of cellular complexes: a quick tour

Paoluzzi, Shapiro, DiCarlo, 2018 (submitted)

ALGORITHM 3: *Fragmentation of 2-cells*

Input: $\mathcal{S}_2 \subset \mathcal{S}_{d-1}$ # *collection* of all 2-cells from $\mathcal{S}_{d-1}$ input in $\mathbb{E}^d$
Output: $[\partial_2]$ # CSC signed matrix
$\widetilde{\mathcal{S}_2} = \emptyset$ # initialisation of collection of local fragments
for $\sigma \in \mathcal{S}_2$ **do** # for each 2-cell σ in the input set
  $M = SubManifoldMap(\sigma)$ # affine transform s.t. $\sigma \mapsto x_3 = 0$ subspace
  $\Sigma = M\,\mathcal{I}(\sigma)$ # apply the transformation to (possible) incidencies to σ
  $\mathcal{S}_1(\sigma) = \emptyset$ # collection of line segments in $x_3 = 0$
  for $\tau \in \Sigma$ **do** # for each 2-cell τ in Σ
    $\mathcal{P}(\tau), \mathcal{L}(\tau) = \emptyset, \emptyset$ # intersection points and int. segment(s) with $x_3 = 0$
    for $\lambda \in X_1(\tau)$ **do** # for each 1-cell λ in $X_1(\tau)$
      if $\lambda \not\subset \{\mathbf{q} \,|\, x_3(\mathbf{q}) = 0\}$ **then** $\mathcal{P}(\tau)$ += $\{\mathbf{p}\}$ # append the intersection point of λ with $x_3 = 0$
    end
    $\mathcal{L}(\tau) = Points2Segments(\mathcal{P}(\tau))$ # Compute a set of collinear intersection segments
    $\mathcal{S}_1(\sigma)$ += $\mathcal{L}(\tau)$ # accumulate intersection segments with σ generated by τ
  end
  $X_2(\sigma) = \mathcal{A}(\mathcal{S}_1(\sigma))$ # arrangement of σ space induced by a soup of 1-complexes
  $\widetilde{\mathcal{S}_2}$ += $M^{-1}\,X_2$ # accumulate local fragments, back transformed in $\mathbb{E}^d$
end
$[\partial_1] = QuotientBases(\widetilde{\mathcal{S}_2})$ # identification of 0- and 1-cells using *kd*-trees and canonical LAR
$[\partial_2] = Algorithm\_1([\partial_1])$ # output computation GRANDE STRONZATA
return $[\partial_2]$

Arrangement of cellular complexes: a quick tour

Paoluzzi, Shapiro, DiCarlo, 2018 (submitted)

ALGORITHM 1: *Computation of signed* $[\partial_d^+]$ *matrix*

/* **Pre-condition:** d equal to space dimension, s.t. $(d-1)$-cells are shared by *two* d-cells */
/* */
Input: $[\partial_{d-1}]$ # CSC signed matrix
Output: $[\partial_d^+]$ # CSC signed matrix
$m, n = [\partial_{d-1}].size$; $marks = Zeros(n)$ # initializations
while Sum$(marks) < 2n$ **do**
  $\sigma = Choose(marks)$ # select the $(d-1)$-cell seed of the column extraction
  if $marks[\sigma] == 0$ **then** $[c_{d-1}] = [\sigma]$
  else if $marks[\sigma] == 1$ **then** $[c_{d-1}] = [-\sigma]$
  $[c_{d-2}] = [\partial_{d-1}]\,[c_{d-1}]$ # compute boundary c_{d-2} of seed cell
  while $[c_{d-2}] \neq []$ **do** # loop until boundary becomes empty
    $corolla = []$
    for $\tau \in c_{d-2}$ **do** # for each hinge τ cell
      $[b_{d-1}] = [\tau]^t[\partial_{d-1}]$ # compute the τ coboundary
      $pivot = \{|b_{d-1}|\} \cap \{|c_{d-1}|\}$ # compute the τ support
      if $\tau > 0$ **then** $adj = Next(pivot, Ord(b_{d-1}))$ # compute the new adj cell
      else if $\tau < 0$ **then** $adj = Prev(pivot, Ord(b_{d-1}))$
      if $\partial_{d-1}[\tau, adj] \neq \partial_{d-1}[\tau, pivot]$ **then** $corolla[adj] = c_{d-1}[pivot]$ # orient adj
      else $corolla[adj] = -(c_{d-1}[pivot])$
    end
    $[c_{d-1}]$ += $corolla$ # insert *corolla* cells in current c_{d-1}
    $[c_{d-2}] = [\partial_{d-1}]\,[c_{d-1}]$ # compute again the boundary of c_{d-1}
  end
  for $\sigma \in c_{d-1}$ **do** $marks[\sigma]$ += 1 # update the counters of used cells
  $[\partial_d^+]$ += $[c_{d-1}]$ # append a new column to $[\partial_d^+]$
end
return $[\partial_d^+]$

Julia implementation

Julia implementation (in the works)

https://github.com/cvdlab/LinearAlgebraicRepresentation.jl

Test-driven development example

Start with a flexible test suite

Create a branch in the repo ...

Use the bash-git-prompt ...

```
  GNU nano 2.0.6                File: .bash_profile

# Git Bash Completion
# if [ -f $(brew --prefix)/etc/bash_completion ]; then
#   . $(brew --prefix)/etc/bash_completion
# fi

# Git Prompt
# GIT_PS1_SHOWDIRTYSTATE=true
# export PS1='[\u@mbp \w$(__git_ps1)]\$ '

# bash-git-prompt
if [ -f "$(brew --prefix)/opt/bash-git-prompt/share/gitprompt.sh" ]; then
  __GIT_PROMPT_DIR=$(brew --prefix)/opt/bash-git-prompt/share
  source "$(brew --prefix)/opt/bash-git-prompt/share/gitprompt.sh"
fi
```

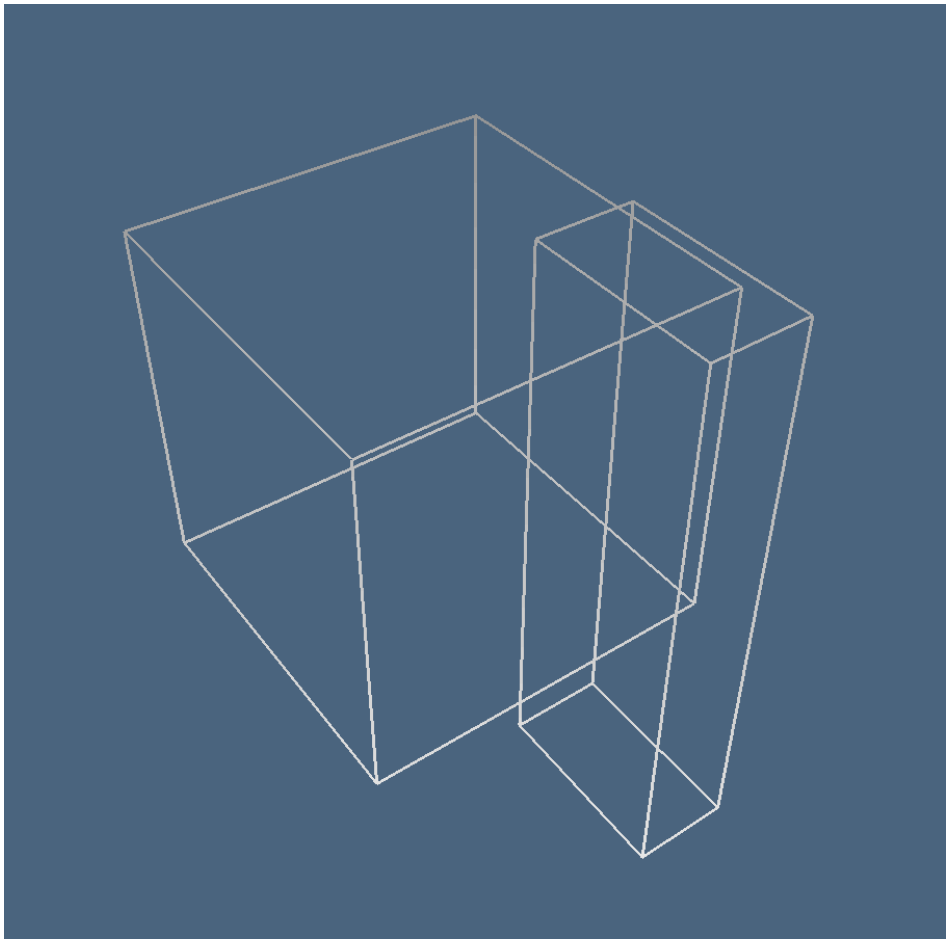
Test suite of data configurations, and get errors

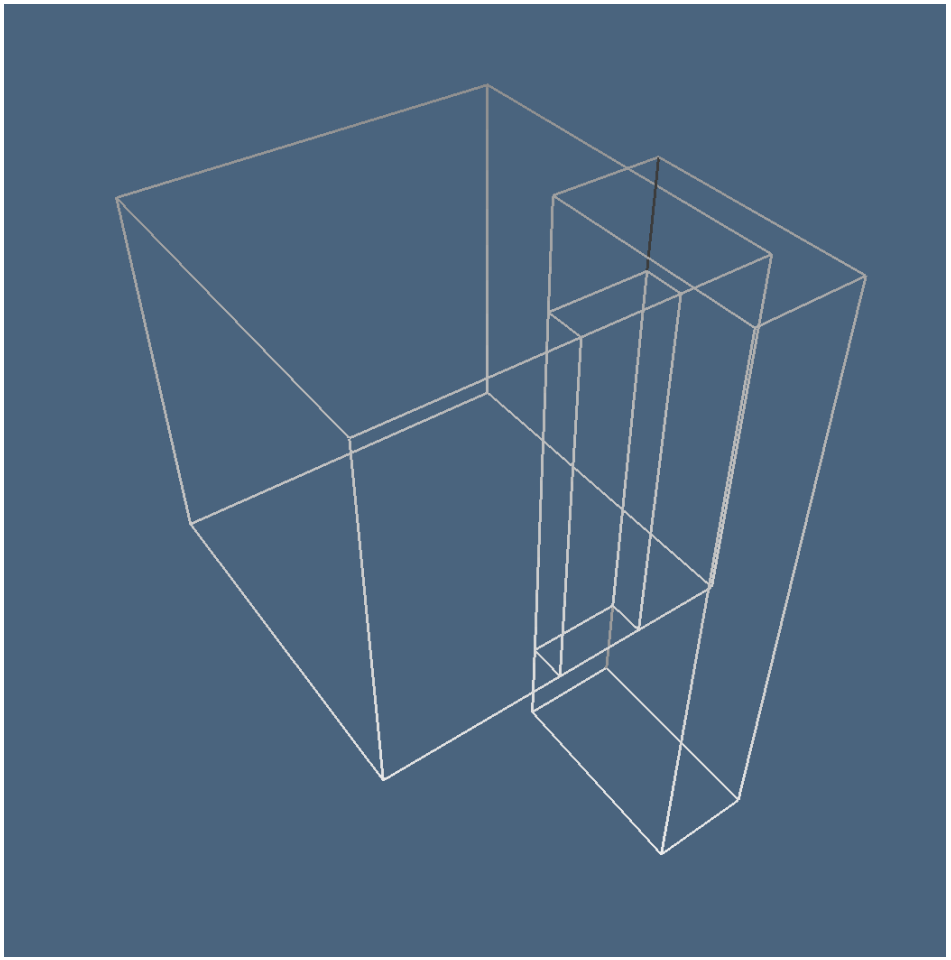
```
using LinearAlgebraicRepresentation
Lar = LinearAlgebraicRepresentation
using Plasm

#translate = Lar.t(0,0,0) #OK
#translate = Lar.t(0.25,0,0) #OK
#translate = Lar.t(0.25,0,0.25) #OK
#translate = Lar.t(0.25,0,-0.25) #OK
#translate = Lar.t(.25,.25,1.0) #OK
#translate = Lar.t(.25,1,-.25) #OK
translate = Lar.t(.25,0.9,-.25) #OK
#translate = Lar.t(.25,.25,-.25) #KO

V1,(_,EV1,FV1,CV1) = Lar.cuboid([1,1,1],true)
V2,(_,EV2,FV2,CV2) = Lar.cuboid([.25,.5,1.5],true)
assembly = Lar.Struct([(V1,FV1,EV1),translate,(V2,FV2,EV2)])
W,FW,EW = Lar.struct2lar(assembly)
Plasm.view(W,EW)
V,(EV,FV,CV),(copEV,copFE,copCF) = Lar.chaincomplex( W,FW,EW )
Plasm.view(V,EV)
```

Test suite of data configurations, and get errors





This example works . . .

Test suite of data configurations, and get errors

This one enters an infinite loop . . . WHY ?

ANSWER: because loops in faces were not considered (error due to the naif implementation)

```
#translate = Lar.t(0,0,0) #OK
#translate = Lar.t(0.25,0,0) #OK
#translate = Lar.t(0.25,0,0.25) #OK
#translate = Lar.t(0.25,0,-0.25) #OK
#translate = Lar.t(.25,.25,1.0) #OK
#translate = Lar.t(.25,1,-.25) #OK
#translate = Lar.t(.25,0.9,-.25) #OK
translate = Lar.t(.25,.25,-.25) #KO

V1,(_,EV1,FV1,CV1) = Lar.cuboid([1,1,1],true)
V2,(_,EV2,FV2,CV2) = Lar.cuboid([.25,.5,1.5],true)
assembly = Lar.Struct([(V1,FV1,EV1),translate,(V2,FV2,EV2)])
W,FW,EW = Lar.struct2lar(assembly)
Plasm.view(W,EW)
V,(EV,FV,CV),(copEV,copFE,copCF) = Lar.chaincomplex( W,FW,EW )
```

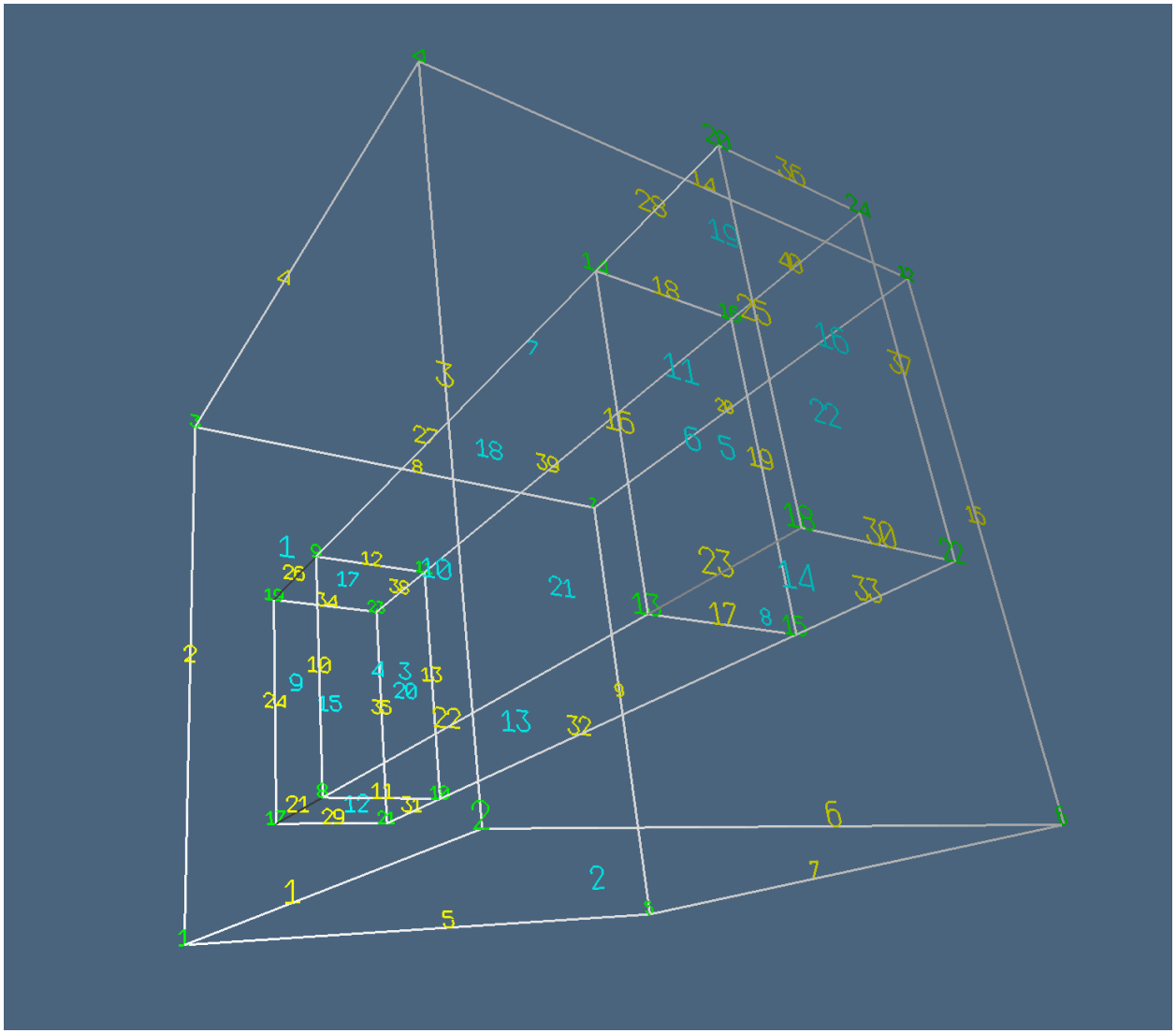
Create more direct test data (w.r.t. error)

```
(V, EV, FV) = ([0.0 0.0 0.0 0.0 1.0 1.0 1.0 0.25 0.25 0.5
0.5 1.0 0.25 0.25 0.5 0.5 0.25 0.25 0.25 0.25 0.5 0.5 0.5
0.5; 0.0 0.0 1.0 1.0 0.0 0.0 1.0 0.25 0.75 0.25 0.75 1.0
0.25 0.75 0.25 0.75 0.25 0.25 0.75 0.75 0.25 0.25 0.75 0.75;
0.0 1.0 0.0 1.0 0.0 1.0 0.0 0.0 0.0 0.0 0.0 1.0 1.0 1.0 1.0
1.0 -0.25 1.25 -0.25 1.25 -0.25 1.25 -0.25 1.25],
Array{Int64,1}[[1,2],[1,3],[2,4],[3,4],[1,5],[2,6],[5,6],
[3,7],[5,7],[8,9],[8,10],[9,11],[10,11],[4,12],[6,12],[13,
14],[13,15],[14,16],[15,16],[7,12],[8,17],[8,13],[13,18],
[17,19],[18,20],[9,19],[9,14],[14,20],[17,21],[18,22],[10,
21],[10,15],[15,22],[19,23],[21,23],[20,24],[22,24],[11,23],
[11,16],[16,24]],#,[4,14],[3,9]],
Array{Int64,1}[[4,2,3,1],[2,5,6,1],[7,9,10,3,11,5,8,1],[9,
10,11,8],[4,13,14,2,16,15,6,12],[13,14,16,15],[7,4,3,12],[7,
5,6,12],[9,19,17,8],[9,13,14,8],[20,13,14,18],[10,17,8,21],
[13,10,8,15],[13,15,22,18],[23,19,17,21],[20,24,22,18],[23,
9,19,11],[9,14,16,11],[24,14,16,20],[23,10,11,21],[10,16,11,
15],[16,15,22,24]])

Plasm.view(Plasm.numbering(.25)((V,[[[k] for k=1:size(V,2)],EV,FV])))

#V,(EV,FV,CV),(copEV,copFE,copCF) = Lar.chaincomplex(V,FV,EV)
```

Use the simplest data to test step-by-step the TDD



Find the first coded algorithm in error, and rewrite it

if extensive tests do not find what is wrong

Actually, in the previous implementation one did not take care of loops in the faces

Restart from pseudocode, if any !!

ALGORITHM 1: *Computation of signed* $[\partial_d^+]$ *matrix*

/* **Pre-condition:** d equal to space dimension, s.t. $(d-1)$-cells are shared by *two* d-cells */
/* */
Input: $[\partial_{d-1}]$ # CSC signed matrix
Output: $[\partial_d^+]$ # CSC signed matrix
$m, n = [\partial_{d-1}].size;\ marks = Zeros(n)$ # initializations
while Sum$(marks) < 2n$ **do**
  $\sigma = Choose(marks)$ # select the $(d-1)$-cell seed of the column extraction
  if $marks[\sigma] == 0$ **then** $[c_{d-1}] = [\sigma]$
  else if $marks[\sigma] == 1$ **then** $[c_{d-1}] = [-\sigma]$
  $[c_{d-2}] = [\partial_{d-1}]\,[c_{d-1}]$ # compute boundary c_{d-2} of seed cell
  while $[c_{d-2}] \neq []$ **do** # loop until boundary becomes empty
    $corolla = []$
    for $\tau \in c_{d-2}$ **do** # for each hinge τ cell
      $[b_{d-1}] = [\tau]^t[\partial_{d-1}]$ # compute the τ coboundary
      $pivot = \{|b_{d-1}|\} \cap \{|c_{d-1}|\}$ # compute the τ support
      if $\tau > 0$ **then** $adj = Next(pivot, Ord(b_{d-1}))$ # compute the new adj cell
      else if $\tau < 0$ **then** $adj = Prev(pivot, Ord(b_{d-1}))$
      if $\partial_{d-1}[\tau, adj] \neq \partial_{d-1}[\tau, pivot]$ **then** $corolla[adj] = c_{d-1}[pivot]$ # orient adj
      else $corolla[adj] = -(c_{d-1}[pivot])$
    end
    $[c_{d-1}]$ += $corolla$ # insert *corolla* cells in current c_{d-1}
    $[c_{d-2}] = [\partial_{d-1}]\,[c_{d-1}]$ # compute again the boundary of c_{d-1}
  end
  for $\sigma \in c_{d-1}$ **do** $marks[\sigma]$ += 1 # update the counters of used cells
  $[\partial_d^+]$ += $[c_{d-1}]$ # append a new column to $[\partial_d^+]$
end
return $[\partial_d^+]$

Start from comments, and expand them

```
"""
#   Input:[∂d-1] #   Compressed Sparse Column (CSC) signed matrix (aij), where aij ∈ {-1,0,1}
#   Output: [∂d+] #  CSC signed matrix
#
#   [∂d+] = [] ; m,n = [∂d-1].shape ; marks = Zeros(n) #     initializations
#   while Sum(marks) < 2n do
#       σ = Choose(marks) #  select the (d - 1)-cell seed of the column extraction
#       if marks[σ] == 0 then [cd-1] = [σ]
#       else if marks[σ] == 1 then [cd-1] = [-σ]
#       [cd-2] = [∂d-1] [cd-1] #     compute boundary cd-2 of seed cell
#       while [cd-2] ≠ [] do #   loop until boundary becomes empty
#           corolla = []
#           for τ ∈ cd-2 do #    for each "hinge" τ cell
#               [bd-1] = [τ]^t [∂d-1] # compute the τ coboundary
#               pivot = {|bd-1|} ∩ {|cd-1|} #    compute the τ support
#               if τ > 0 then adj = Next(pivot,Ord(bd-1)) #  compute the new adj cell
#               else if τ < 0 then adj = Prev(pivot,Ord(bd-1))
#               if ∂d-1[τ,adj] ≠ ∂d-1[τ,pivot] then corolla[adj] = cd-1[pivot] #     orient adj
#               else corolla[adj] = -(cd-1[pivot])
#           end
#           [cd-1] += corolla #  insert corolla cells in current cd-1
#           [cd-2] = [∂d-1][cd-1] #  compute again the boundary of cd-1
#       end
#       for σ ∈ cd-1 do marks[σ] += 1 #  update the counters of used cells
#       [∂d+] += [cd-1] #    append a new column to [∂d+]
#   end
#   return [∂d+]
"""
```

Look at the debugged code

./algorithm-one.jl

Look at the output: $[\partial_3^+]$, here called copFC

```
copFC = algorithm_one(V,FV,EV,copFE)
Matrix(copFC)
```

```
julia> Matrix(copFC)
22×5 Array{Int8,2}:
  1  -1   0   0   0
  1  -1   0   0   0
 -1   1   0   0   0
  0   0   1  -1   0
  1  -1   0   0   0
  0   0   1   0  -1
 -1   1   0   0   0
 -1   1   0   0   0
  1   0   0  -1   0
  0   1  -1   0   0
 -1   0   0   0   1
 -1   0   0   1   0
  0  -1   1   0   0
  1   0   0   0  -1
 -1   0   0   1   0
 -1   0   0   0   1
 -1   0   0   1   0
  0   1  -1   0   0
 -1   0   0   0   1
 -1   0   0   1   0
  0  -1   1   0   0
 -1   0   0   0   1
```

Let interpret the matrix of the boundary transformation $[\partial_3^+]$

- The transformation $[\partial_3$ goes from C_3 (3-chains) to C_2 (2-chains)
- incidence from cells to faces is by row
- incidence from faces to cells is by columns
- there are five 3-cells (including the exterior space) in the arrangement (3-space partition) induced by the input 2-complex X, whose 2-skeleton X_2 is given by the set of 2-faces.

Topological gift-wrapping algorithm

Notice that 3-cells where unknown before the algorithm execution