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# Computational tools and file format for virtual interactive indoor mapping

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## Abstract

This paper introduces **FIVE** (Framework for Indoor Virtual Environments) and **HIJSON** (Hierarchical Interactive JSON), respectively a web toolkit for indoor mapping applications and a novel cartographic document format. Client-side **FIVE** applications, entirely based on web technologies, rely on **HIJSON** documents produced server-side using **LAR**, a novel representation scheme for topology and geometry.

An *interactive indoor mapping* environment is a virtual reconstruction of a physical indoor space, where the user may interact with virtual objects, experienced in the actual position they occupy in the real world. Our approach outlines a specialized and evoluted 3D *user interface* giving a glimpse of a section of the real world, that the user can handle intuitively. Furthermore, the virtual indoor environment API provides a platform where many different applications can rely upon. Accessible via web browsers from any kind of device, several applications may coexist on this platform. IoT monitoring, realtime multi-person tracking, and cross-storey user navigation, are already implemented using an automatic search for all valid walkable routes, and taking into account both architectural obstacles and furniture.

The **HIJSON** format is used to represent any geometry of the indoor space of complex buildings, capturing their hierarchical structure, a complete representation of their topology, and all the objects (either smart or not) contained inside. Such textual representation allows the **FIVE** framework to offer a web environment in which the user is presented with 2D or 3D models to navigate. With respect to current cartographic formats, **HIJSON** suggests four major enhancements: (a) exposes a hierarchical structure; (b) uses local metric coordinate systems; (c) may import external geometric models; (d) accepts semantic extensions. The semantic extensions supported by the **FIVE** architecture encapsulate the details about communication protocols, rendering style, and exchanged and displayed information, allowing the **HIJSON** format to be extended with any sort of models of objects, sensors or behaviors.

## 1. Introduction

An *interactive indoor mapping* environment consists of a virtual reconstruction of a physical indoor space, in which the user can move around and interact with virtual objects, that are found in the same position they actually occupy in the real world. Such an interactive indoor mapping can be thought as a specialized and very evoluted *user interface* capable of giving a glimpse of a section of the real world that the user can handle in a natural and intuitive way. Such a reconstructed virtual indoor environment can be considered a general platform where many different applications can rely upon. Both promising and already well explored ICT applications may find in *virtual indoor mapping* the perfect context to be integrated into.

This paper quickly outlines the generation of geometric data of a complex building, to provide both an explicit semantic and a hierarchical model of indoor spaces. **LAR**, a general representation for geometric and solid modeling is used for this purpose. The generated **LAR** structures are exported to **HIJSON** format, extending **GEOJSON** for indoor mapping and the Internet-of-Things. A convenient way to extend the representation capabilities of IoT *smart objects* is also mentioned as semantic extensions, that affects both document format and the web framework, and can be easily collected in a public repository.

In particular, for environments with massive presence of sensor-equipped (or “smart”) objects, which realize the so-called *IoT* (Internet of Things), the interactive indoor mapping represents an ideal integrated interface for IoT monitoring systems. To be specific, it can be the container of indoor navigation systems, giving the user, to be routed across an indoor environment, the opportunity to interact with objects along the suggested paths. Furthermore, in conjunction with the advancements in the field of user indoor location, whose efforts are nowadays focused to realize an integration of positioning systems like GNSS (Global Navigation Satellite system), Wi-Fi, Bluetooth and LTE (Long Term Evolution), to support continuos outdoor/indoor navigation by means of integration of technologies, it represents the most natural interface to perform realtime access monitoring and multi-person tracking.

To enable such an interactive mapping platform it is of

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the utmost importance to set up a descriptive representation of the indoor environment. This description belongs to the field of indoor cartography, which as digital evolution of plain floor plans, has arrived to arouse the interest of big players like Google, that has integrated indoor plans of specific locations of interest [11] into Google Maps. In general, it can be considered “of interest” — such to justify and motivate indoor cartographic applications — both public or commercial places of vast dimensions, as for example airports, train stations, shopping malls, and also private buildings subject to strict access protocols, like warehouses, logistic centers, data centers, etc.

Despite of the growing attention regarding indoor cartography, efforts to specify open formats for indoor representation are few and partial, and certainly not intended to support the interactive indoor mapping, which is conversely the main purpose of this paper.

This work, jointly developed by Sogei S.p.A., an ICT company fully owned by Italian Ministry of Economy and Finance, and the CVDLAB (Computational Visual Design Laboratory) of the “Roma Tre” University, is inspired by the necessities of Sogei itself, which runs one of the largest data center of Europe, so requiring very strict access control policies, which include the recording and the real-time interaction with man/machine maintenance scenarios. Support for this interactive framework, where realtime awareness of the maintainer position inside the data center helps to reduce intervention times and to increase safety and security, has been chosen as case study of interactive indoor mapping, based on the proposed indoor cartographic format.

The remainder of this document is organized as follows. In Section 2 we provide an overview of the state of the art in the field of indoor document standards and related applications. Section 3 is devoted to describe the advances introduced by the novel cartographic document proposed, while section 4 presents the document syntax. Section 5 reports about the toolkit specifically developed to handle the new document format. In Section 6 it is depicted the overall architecture and the implementation of the web based application framework, which is in turn used to achieve the objectives stated above. Section 7 presents a case-study application of the document format discussed in this paper. Finally, Section 8 proposes some conclusive remarks and future developments.

## 2. Related work

Research on the cartographic representation of indoor environments is extensive and heterogeneous with respect to the strategies applied. Different information sources are used, and accuracy of the produced solution depends on the adopted approach. In some cases the information is obtained with automatic or semi-automatic processing of

files that describe the architectural structure of a building, such as BIM (Building Information Modeling) [8] and/or IFC (Industry Foundation Classes) that describe a building project [4]. Image processing is also used to extract topological information from floor plan images [9]. In other works, building information and descriptive parameters are redefined from scratch [12]. Such approaches suffer from the non appropriateness of their representative formats: images contain poor information and CAD files are not designed for this kind of use.

A recurring theme among the use of cartographic information is *indoor navigation* [9, 12, 4]. The proposed approaches are very different in this case too, and based on several strategies with some basic elements in common. An often adopted solution is based on the representation of the routing information as a graph, having a node for each room and an edge for each pair of connected rooms. In some cases the edges are weighted in function of Euclidean distance. The detail level of the graph, and hence the effective usefulness of the calculated paths, can vary depending on the technique and the design choices applied, but in general most of the proposed solutions retrieve information only from architectural structure.

A subject related to navigation is the *location of users*. To locate the exact position of a user inside a building, the currently most applied techniques are based on fingerprinting and triangulation of radio signals (Wi-Fi, Bluetooth, LTE, etc.) flanked by more original solutions based, for example, on image recognition [1]. User tracking issue is faced with solutions that range from the clever utilization of inertial tracking sensors embedded in many smartphones [1] to the adoption of ad hoc devices [9].

The actual “de-facto” standard in terms of geospatial data representation is the *GeoJSON* format [10], which can be easily used for any type of geographical annotation. In some cases it has been slightly adapted to be used in indoor environments: it is the case of the *IndoorJSON* format [13].

### 2.1. The GeoJSON format

*GeoJSON* is a geospatial data interchange format based on JSON, suitable for a geometrical encoding of various geographic data structures. As opposed to GIS formats, *GeoJSON* is an open standard. Positions need to be expressed in geographical coordinates (usually WGS84).

*GeoJSON* supports some geometric primitives, including: Point, LineString, Polygon, MultiPoint, MultiLineString, and MultiPolygon. Lists of geometries are represented by a GeometryCollection. Geometries with additional properties are Feature objects. Lists of Feature are represented by a FeatureCollection.

A single Feature is composed essentially by two mandatory fields: geometry, which describes the object

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217 geometry accordingly to the previously recalled primitives,  
218 and properties which contains additional information  
219 about the Feature.

220 In GeoJSON it is possible to define complex shapes  
221 through the composition of simpler objects. Mainly due  
222 to its simplicity, GeoJSON is widely used and deeply in-  
223 tegrated into several applications and services.

## 224 225 2.2. The IndoorJSON format

226 *IndoorJSON* is a GeoJSON variant defined and used by  
227 *indoor.io*, a Finnish company devoted to indoor environ-  
228 ment mapping. IndoorJSON is compliant with GeoJSON  
229 syntax, and it may consist of any number of Features  
230 and/or FeatureCollections. All Features are in-  
231 terpreted similarly regardless of their grouping into nested  
232 FeatureCollections. IndoorJSON supports all Geo-  
233 JSON geometry types.

234 Some particular properties are used to correctly define  
235 the indoor elements:

- 236 237 238 239 • **level**: describes which storey contains the feature;
- **geomType**: identifies the category of the object, use-  
ful during the visualization process.

240 There are some additional but not mandatories prop-  
241 erties, useful for the indoor representation:

- 242 243 244 245 246 247 248 249 • **accessible**: describes if an element is walkable or  
not;
- **connector**: defines if the element is a connection  
between two storeys;
- **direction**: describes the direction of the connec-  
tion (both ways, only up, only down);

250 A syntax validator is provided by *indoor.io*, but the  
251 commercial nature of this project limits the number of tools  
252 available to deal with this format.

## 254 255 3. Advances on cartographic 256 document formats

257 The focus of this work is the definition of a novel format  
258 of cartographic documents along with the software ecosys-  
259 tem rooted on it. A simple but effective algorithm to find  
260 indoor valid routes is also provided. HIJSON (Hierarchi-  
261 cal Indoor JSON) is the name chosen for the new document  
262 format; it the accompanying software framework, aim to  
263 realize a mapping of real indoor spaces with a virtual in-  
264 teractive web environment. HIJSON is based upon ideas  
265 and design principles collected from previous formats and  
266 identifies four critical improvements with respect to them:  
267 it exposes a *hierarchical structure*, uses *metric local coor-*  
*dinate system*, may import *external geometric models* and  
268 accepts *semantic extensions*.

### 270 3.1. Hierarchical structure

271 The HIJSON format allows for hierarchical description  
272 of indoor spaces. The introduction of a hierarchical struc-  
273 ture establishes a parent-child relation between entities of  
274 the model, reflecting a container-contained relationship.  
275 This directly implies a neater representation than the plain  
276 linear structure adopted by GeoJSON, being a perfect anal-  
277 ogy of objects contained (i.e. placed) into spaces.

278 Therefore, an organized arrangement of spaces is al-  
279 lowed by HIJSON, via logical (or even physical) group-  
280 ing: concepts like building wings, sections, storeys, depart-  
281 ments, etc. can be directly introduced, in order to reflect  
282 into the document structure the actual logical or physical  
283 divisions, categories or relationships among the modelled  
284 spaces.

285 Hierarchical structures are common in computer graph-  
286 ics, where they are used as scene graphs. This accordance  
287 of underlying structures really simplifies 3D rendering algo-  
288 rithms of HIJSON documented environments. Furthermore,  
289 the container-contained relation enables a recurring use of  
290 local reference frames.

### 291 292 3.2. Metric local coordinate system

293 Supported by the hierarchical underlying structure, the  
294 HIJSON document format allows the use of local coordi-  
295 nate systems. Hence the shape of all elements can be con-  
296 veniently modelled using local coordinates, and then placed  
297 in the right position with respect to the position of the par-  
298 ent (or container) element applying a rotation, followed by  
299 a translation transformation.

300 Another substantial advantage is represented by the  
301 adoption of a metric reference frame, consequently simpli-  
302 fying the compilation of the document, either manually gen-  
303 erated or produced by software tools. Just remember that  
304 the GeoJSON coordinates are geographical, a pairs of (ab-  
305 solute) latitude and longitude angles, like the ones provided  
306 by GNSS systems. This kind of coordinates are certainly  
307 not particularly user friendly, when positioning a smart de-  
308 vice or a furniture element within a specific building room.

309 The HIJSON document format is specially designed to  
310 guarantee the user to be routed seamlessly from outdoor to  
311 indoor and vice versa. Even if indoor geometries are entered  
312 in a local metric coordinate system, continuos outdoor/in-  
313 door navigation is ensured through the processing pipeline  
314 detailed below.

### 315 3.3. Hyperlinked geometric models

316 The HIJSON document may further import external ge-  
317 ometric models — either of the buildings themselves or the  
318 interior furniture or devices — that are topologically com-  
319 plete (in the sense of solid modeling [15]) and very com-  
320 pact. Such models, coming from a source outside the docu-  
321 ment, are acquired by hyperlinking JSON files that contain

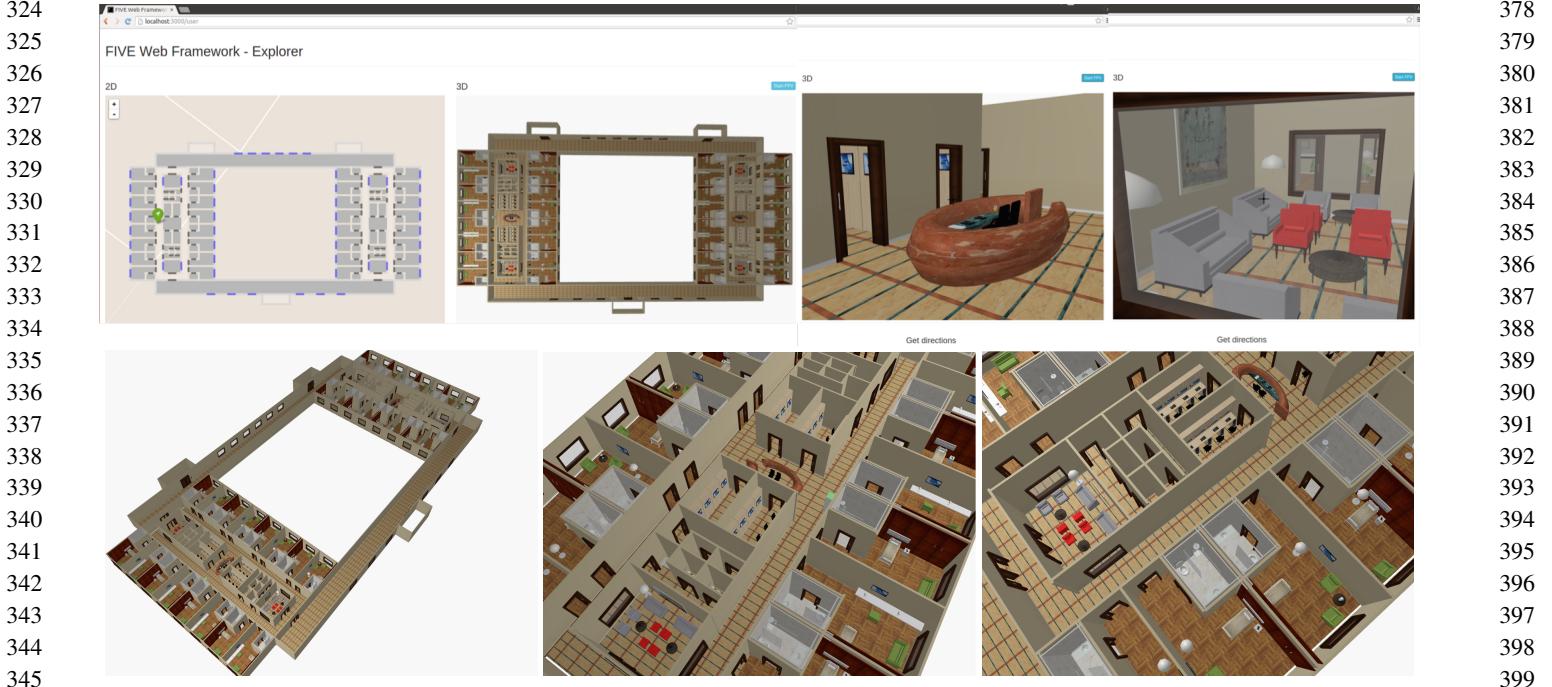


Figure 1: example caption

349 a Linear Algebraic Representation (LAR) of topology and  
 350 geometry, to be expanded for visualization or interaction at  
 351 any useful level of detail.

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 353 The LAR scheme [7] is characterised by a very large  
 354 domain, including architecture, building and construction  
 355 [14], 2D and 3D engineering meshes, non-manifold geo-  
 356 metric and solid models and meshes, and high-resolution  
 357 3D images [6]. This scheme uses the set of *Combinato-*  
 358 *rial Cellular Complexes* (CCC) as mathematical domain  
 359 [2], and various compressed representations of *sparse ma-*  
 360 *trices* [5] as codomain.

361 Since LAR provides a complete representation of the  
 362 topology of the represented space, the matrix  $[\partial_d]$  of the  
 363 boundary operator shall be used to compute the coordinate  
 364 representation  $[c]$  of the *boundary chain* of *any subset c* of  
 365 cells, through *a single* operation of SpMV multiplication [5]  
 366 between the CSR (Compressed Sparse Row) representation  
 367 of  $[\partial]$  and the CSC (Compressed Sparse Column) represen-  
 368 tation of the  $[c]$  chain, resulting in very efficient computa-  
 369 tions on modern hardware, even mobile.

370  
 371 The expansion of a LAR model, to be considered as a  
 372 general-purpose graphic primitive, may be executed on ei-  
 373 ther the server or the supervisor client of the HIJSON Web  
 374 Toolkit architecture (see Section 6.2.1), or even on the *Ex-  
 375 plorer* client, depending on the size and the locality of the  
 376 model to be expanded.

### 3.4. Semantic extensions

380 Semantic extensions make the HIJSON format ex-  
 381 tendsible and customizable, that is able to adequately re-  
 382 spond to any need of objects representation. To define a  
 383 semantic extension means to allow the HIJSON document  
 384 to model an object previously not covered, or even to mod-  
 385 ify the behavior of a comprised one. Semantic extensions  
 386 are to be defined both as HIJSON format syntax and as HI-  
 387 JSON Toolkit source code. In particular it is necessary to  
 388 define respectively a new HIJSON Element and a new HIJ-  
 389 SON Class, as specified below.

## 4. HIJSON structure and syntax

390 The HIJSON document is composed by a  
 391 configuration section, followed by one or more  
 392 FeatureCollections, containing the actual data.

393 Listing 1 shows a simplified HIJSON document, devoid  
 394 of punctual details, to make clear to the reader the overall  
 395 document structure.

```
{
  "config": {
    // ...
  },
  "data": [
    // ...
  {
    "id": "architecture",
    "type": "FeatureCollection",
  }
}
```

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Figure 2: example caption

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456     "features": [
457         // ...
458     ],
459     {
460         "id": "furniture_1",
461         "type": "FeatureCollection",
462         "features": [
463             // ...
464         ],
465     },
466     // ...
467 ]
468 }
```

Listing 1: Example of HIJSON document.

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The configuration includes parameters and settings needed for building representation in the form of a JSON Object. One of the core information in this section is defined by the correspondence between three points of the local coordinate system and three points of the real world, expressed in geographical coordinates. This is needed to ensure a seamlessly passage from local to geographical coordinate system and vice versa.

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After the configuration part, the document includes a list of FeatureCollection. An example of FeatureCollection is given in listing 2.

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```
{
    "id": "architecture",
    "type": "FeatureCollection",
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{
    "type": "Feature",
    "id": "room_0.1",
    "geometry": {
        "type": "Polygon",
        "coordinates": [
            [ [0, 0], [11, 0], [11, 19], [0, 19] ]
        ]
    },
    "properties": {
        "class": "room",
        "parent": "level_0",
        "description": "Office of Mr. Smith",
        "tVector": [10, 20, 0],
        "rVector": [0, 0, 90]
    }
},
// ...
```

Listing 2: Example of FeatureCollection.

Each element of the list is given in the form of a GeoJSON FeatureCollection, that contains an arbitrary number of HIJSON Elements. Each FeatureCollection imposes a logical relationship that can be used to group together related HIJSON Elements. Since HIJSON Elements adhere to the GeoJ-

540 SON format, each FeatureCollection results compliant  
 541 with GeoJSON syntax and then accepted by any GeoJ-  
 542 SON validator. As detailed below, the HIJSON format  
 543 introduces some additional rules that allow the adoption of  
 544 this format for indoor representation.  
 545

#### 546 4.1. HIJSON Element

547 Dealing with indoor environments, there are essentially  
 548 two classes of objects that is necessary to represent. They  
 549 are (a) architectural elements, like a room, a corridor, a wall,  
 550 etc. and (b) furnishings, intended in a broad sense, such  
 551 as to contain both furniture, like a desk or a chair, and/or  
 552 “smart objects”, like an IP-cam or a connected thermostat.  
 553

554 A HIJSON Element defines a GeoJSON compliant syntax  
 555 to describe both geometry and properties of an object.  
 556 It represents the atomic component of a HIJSON docu-  
 557 ment. It would be a best practice to group together re-  
 558 lated JSON Element using FeatureCollections: sev-  
 559 eral classification strategies can be applied, for example by  
 560 grouping the elements by storey or even by room. Alter-  
 561 natively, since the furnishings are more likely to change  
 562 than the architectural components of a building, these two  
 563 different kinds of elements can be isolated in different  
 564 FeatureCollections.

565 The hierarchical structure of the document gives visible  
 566 form to the capability of HIJSON Elements to have children  
 567 elements. A unique ID is mandatory for every HIJSON El-  
 568 ement.

569 Three Geometry types can be used here: Point,  
 570 LineString and Polygon. The choice of the Geom-  
 571 etry type to be associated to a HIJSON Element implictly  
 572 defines the category of the element: Point is used  
 573 for furnishings, LineString for walls and doors, while  
 574 Polygon may describe levels and rooms.

575 The Geometry coordinates are expressed in metres, by  
 576 convention starting at the bottom-left corner of the element,  
 577 whose position is used to set-up the origin of a local  
 578 coordinate frame. Unlike GeoJSON, where all properties are  
 579 optional, in HIJSON some strict requirements are imposed,  
 580 and some attributes are mandatories:

- 581 • class: represents the element category, used to in-  
 582 stantiate the appropriate *HIJSON Class*;
- 583 • parent: contains the ID of the parent of the element;
- 584 • tVector: represents the translation relative to the  
 585 parent element, expressed in metres;
- 586 • rVector: represents the rotation relative to the par-  
 587 ent element, expressed in nonagesimal degrees.

588 Specific classes may require the mandatory pres-  
 589 ence of other properties. For example, the classes  
 590 internal\_wall and external\_wall that define the  
 591 internal partitions and the external envelope, respectively,

592 require a connections array, containing the IDs of the  
 593 adjacent elements. This information is used by the connec-  
 594 tor children of the element (e.g. doors) to identify the areas  
 595 linked together.

596 Given the nature of the GeoJSON format from which  
 597 HIJSON derives, the elements are represented by their 2D  
 598 shape, like on a planimetry. The property height was in-  
 599 troduced to assign a value to the height of the object, in-  
 600 tended as a third dimension.

601 A description property can provide further infor-  
 602 mation about the element. Arbitrary optional fields can be  
 603 added without restrictions, in order to enrich and extend the  
 604 expressivity of the representation, or simply for the sake of  
 605 documentation.

## 606 5. HIJSON Toolkit

607 The HIJSON Toolkit is a software module that imple-  
 608 ments common operations and transformations on HIJSON  
 609 documents. Written in *JavaScript* language, this software  
 610 module has been built to be deployed in the web environ-  
 611 ment. It is *modular* and entirely *isomorphic*, i.e. can run  
 612 on the server as well as on every client. Working in the  
 613 web environment, the Toolkit benefits of the “fertility” com-  
 614 monly concerning the software development in this field:  
 615 for example, it takes advantage of libraries and frameworks  
 616 such as *React*, “the *JavaScript* library for building user in-  
 617 terfaces” by Facebook, and as *Three.js*, the current de-facto  
 618 standard to deal with *WebGL* technologies.

619 The Toolkit executes the instantiation and extension  
 620 logic of a HIJSON document, and provides a multistage  
 621 transformation pipeline that, according to the requirements,  
 622 can be used either entirely or only in part.

### 623 5.1. Processing pipeline

624 The HIJSON processing pipeline implements the se-  
 625 quence of preliminary transformations that have to be ap-  
 626 plied to a HIJSON document before any further operation.  
 627 It is not strictly required to complete each stage of the  
 628 pipeline: the exit stage depends on the specific use case.

629 The application of the transformation pipeline has a dou-  
 630 ble aim. The first one consists in generating the graph of  
 631 valid paths among all the interesting elements. The second  
 632 objective is the generation of one *GeoJSON* document for  
 633 each storey of the building described by the HIJSON doc-  
 634 ument. In this way a bidimensional layout can be provided  
 635 for every level of the building, and visualized through any  
 636 compliant *GeoJSON* viewer.

637 The HIJSON processing pipeline is composed by six  
 638 elaboration stages, denoted as *validation*, *georeferencing*,  
 639 *parsing*, *graph paths generation*, *2D layers generation*,  
 640 *marshalling*. The pipeline of transformations and the output  
 641 of each stage are shown in Figure 3.

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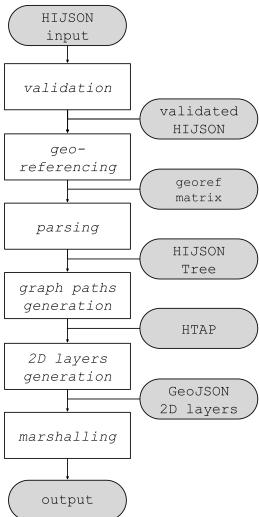


Figure 3: HIJSON processing pipeline

- 665 1. *validation* - The first one is the validation stage.  
666 In order to begin with the effective transformations the  
667 input HIJSON document must be compliant with both  
668 the syntax format and the structural requirements. In  
669 the case the validation stage fails, processing aborts  
670 and does not continue to following stages; instead if this  
671 stage successes, then the output for the next stage is a  
672 validated HIJSON.
- 673 2. *georeferencing* - In the second stage, in order  
674 to allow for continuous outdoor/indoor navigation, the  
675 system needs to compute the georeferencing matrix, a  
676 linear operator able to transform local coordinates into  
677 global coordinates (world coordinate system — latitude  
678 and longitude angles) and vice versa. This task  
679 is accomplished by solving a linear system obtained  
680 from information contained in HIJSON configuration  
681 part and precisely from the correspondence of three  
682 real world points to three points included into the HIJ-  
683 SON document.
- 684 3. *parsing* - The parsing stage takes the validated and  
685 georeferenced HIJSON as its input, that as illustrated  
686 before can be thought of as a list of HIJSON Elements,  
687 parses them and produces an instance of the HIJSON  
688 Tree. The HIJSON Tree is an object in memory rep-  
689 resenting the hierarchical structure of the building de-  
690 scribed by the HIJSON document.
- 691 4. *graph of paths generation* - The fourth  
692 stage is in charge of the generation of the graph of  
693 paths. The algorithm to achieve such a goal is intro-  
694 duced in Section 5.1.1. The graph of paths can be used  
695 to compute valid directions between pairs of points of  
696 interest inside the building model. Once the graph of  
697 paths has been computed, the input HIJSON Tree is

698 augmented with paths information, becoming what has  
699 been called an HTAP (HIJSON Tree Augmented with  
700 Paths). Augmentation always takes place in the form  
701 of an addition of leaf nodes as children of a specific  
702 element (e.g. “room”).

- 703 5. *2D layers generation* - The fifth stage con-  
704 cerns the generation of GeoJSON *layers*. For each  
705 storey of the building, the Toolkit generates a GeoJ-  
706 JSON layer that can be used for the creation of a 2D  
707 map. Each layer contains only the children of a ‘level’  
708 node of the HIJSON Tree. The presence of a specific  
709 element inside the layer can be finely tuned by means  
710 of a Boolean value. The geographical coordinates of  
711 every elements are calculated by a series of multiplica-  
712 tions between transformation matrices obtained during  
713 the tree traversal to the local coordinates.
- 714 6. *marshalling* - The last stage is responsible for  
715 executing a serialization of the the transformed data.  
716 This stage, in which are performed tasks like breaking  
717 dependency-loops and stringification, is mainly useful  
718 server-side, as the output is there stored ready to be  
719 served to any requiring client.

### 5.1.1 Automatic generation of valid paths

The fourth stage of the processing pipeline is responsible  
727 for the generation of a graph of valid paths through the  
728 entire model represented by the input HIJSON document. The  
729 graph generated according to the algorithm described in the  
730 following, although non optimal, ensures a complete cover-  
731 age of the surface while limiting the number of generated  
732 nodes. The resulting graph is weighted on the edges with  
733 nodes distances. Each graph node may represent either:

- 734 a *standard path node*, i.e. a junction node or possibly  
735 an endpoint of a path;
- 736 b. a *connection node*, used as subproblem composing el-  
737 ement in the divide et impera approach adopted;
- 738 c. an *element node* i.e. HIJSON Element (whose HIJSON  
739 Class explicitly grants his presence in the graph), typi-  
740 cally an endpoint of a path.

The graph of paths allows for calculations of direc-  
744 tions between any two given nodes. Although different  
745 approaches have been explored [3], a very classical solution  
746 has been selected in this case, so directions are actually  
747 computed client-side by applying the Dijkstra algorithm to  
748 the graph.

Taking advantage of the hierarchical structure of the HI-  
750 JSON document, and according to the divide et impera  
751 approach, the problem of paths generation is split in sev-  
752 eral sub-problems, which consist in the computation of the  
753 sub-graphs relative to each individual space, more gener-  
754 ally a single room. The sub-graphs are then linked together

756 through the connection nodes (which in most cases represent doors). The resolution of each sub-problem (as depicted in Figure 4), is composed by four steps, as detailed  
 757 below.  
 758

- 759 1. Computation of the walkable area of the space: this  
 760 task is accomplished by subtracting the shape of the  
 761 obstacles from the area of the space; the result is typi-  
 762 cally a surface with holes.  
 763
- 764 2. Triangulation of the walkable area: the computed sur-  
 765 face is triangulated taking into account the presence of  
 766 holes.  
 767
- 768 3. Identification of graph nodes: for each triangle side  
 769 completely internal to the area, its midpoint is selected  
 770 as standard path node.  
 771
- 772 4. Junction of nodes: nodes relative to the same triangle  
 773 are then linked pairwise; both element nodes and con-  
 774 nection nodes (i.e. doors) are linked to the nearest node  
 775 of the space (i.e. room).  
 776

## 5.2. HIJSON Class definition

To make better use of the possibilities offered by the HIJSON Toolkit and by the HIJSON document format, some custom dynamic behaviors can be described. These behaviors encapsulate the specificities relative to communication protocols with the sensors, as well as to features of user interaction. The interface for such behavior is the HIJSON Class.



789 Figure 5: HIJSON Element/Class/Node relashionship

790 Every HIJSON Element of the input HIJSON document  
 791 has a dynamic counterpart, a running instance called *HIJ-  
 792 SON Node*, instantiated according to the corresponding HIJ-  
 793 SON Class via reflection methods (see Figure 5).

794 To specify a new *HIJSON Class* means to extend the  
 795 Toolkit to deal with a new class of HIJSON Element. To  
 796 extend the toolkit in order to deal with a new class of HIJ-  
 797 SON Element is required to specify a new HIJSON Class,  
 798 by defining the following properties and methods:

- 801 • *in\_graph*: a Boolean value to express if the element  
 802 is an approachable point in the graph of paths;  
 803
- 804 • *in\_2D\_map*: a Boolean value to express if the element  
 805 must be shown in the 2D map;  
 806
- 807 • *get2DStyle()*: a method that returns the 2D map  
 808 appearance of the element, essentially HTML and CSS  
 809 code;

- 810 • *get3DModel()*: a method that returns the 3D model  
 811 appearance of the element, i.e. an instance of *Object3D* of the *THREE.js* framework;  
 812
- 813 • *getWidget()*: a method that returns the information  
 814 widget, a *React* component;  
 815
- 816 • *getProxy()*: a method that returns the server-side  
 817 proxy which encapsulate the IoT sensor communica-  
 818 tion protocol, i.e. a *Node.js* module.  
 819

User's needs for new indoor elements, greatly different sensor equipments, alternative representations of 2D or 3D viewports are accepted by the definition of new HIJSON Classes, that so provide single-point custom extensions of the Toolkit capabilities.

## 6. HIJSON Web Framework

The HIJSON Web Framework responds to the needs of an extendable, customizable, and scalable web framework which provides at the same time IoT monitoring, realtime multi-person tracking and cross-storey user navigation.

Expandability and customizability derive from both design choices and HIJSON inherent characteristics, i.e. the possibility of semantic extensions. Scalability is directly borrowed from technologies used for software development: *JavaScript* language, using *Node.js*, in particular *Express.js* as backend framework, exploiting the power of *WebSocket* protocol through the *Socket.io* library.

Being supported by the *web-as-a-platform*, the framework exposes also an high availability: it is so simple to use as to visit a website, both from desktop or mobile devices, without explicit requirements to install any software package from proprietary stores—access to which is often denied from business devices.

The HIJSON Web Framework deeply relies on HIJSON Toolkit and offers an all-inclusive client/server architecture with a convenient and highly interactive user interface, leaving aside the specific indoor positioning system and the IoT sensors to deal with. A robust application interface is provided and described in the following section.

### 6.1. Applications

The Framework has been designed with focus on two different kind of users: the *Explorer* and the *Supervisor*. They have different requirements and are likely equipped with different devices: while the *Supervisor* monitors the indoor environment through a desktop workstation, the *Explorer* has a smartphone available and needs to be routed across the building.

In both cases, the web platform ensures a perfect alignment with the BYOD (Bring Your Own Device) approach, nowadays often supported by companies that encourage employees to use personal devices.

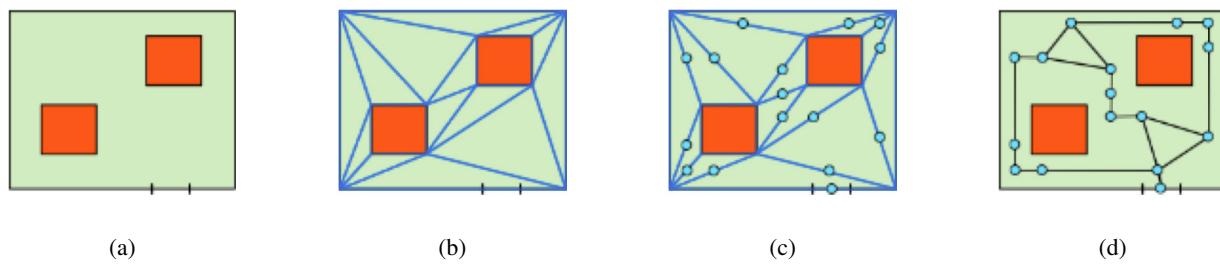
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Figure 4: graph of paths generation: (a) detection of obstacles and computation of walkable area; (b) triangulation of walkable area; (c) identification of graph nodes; (d) junction of nodes.

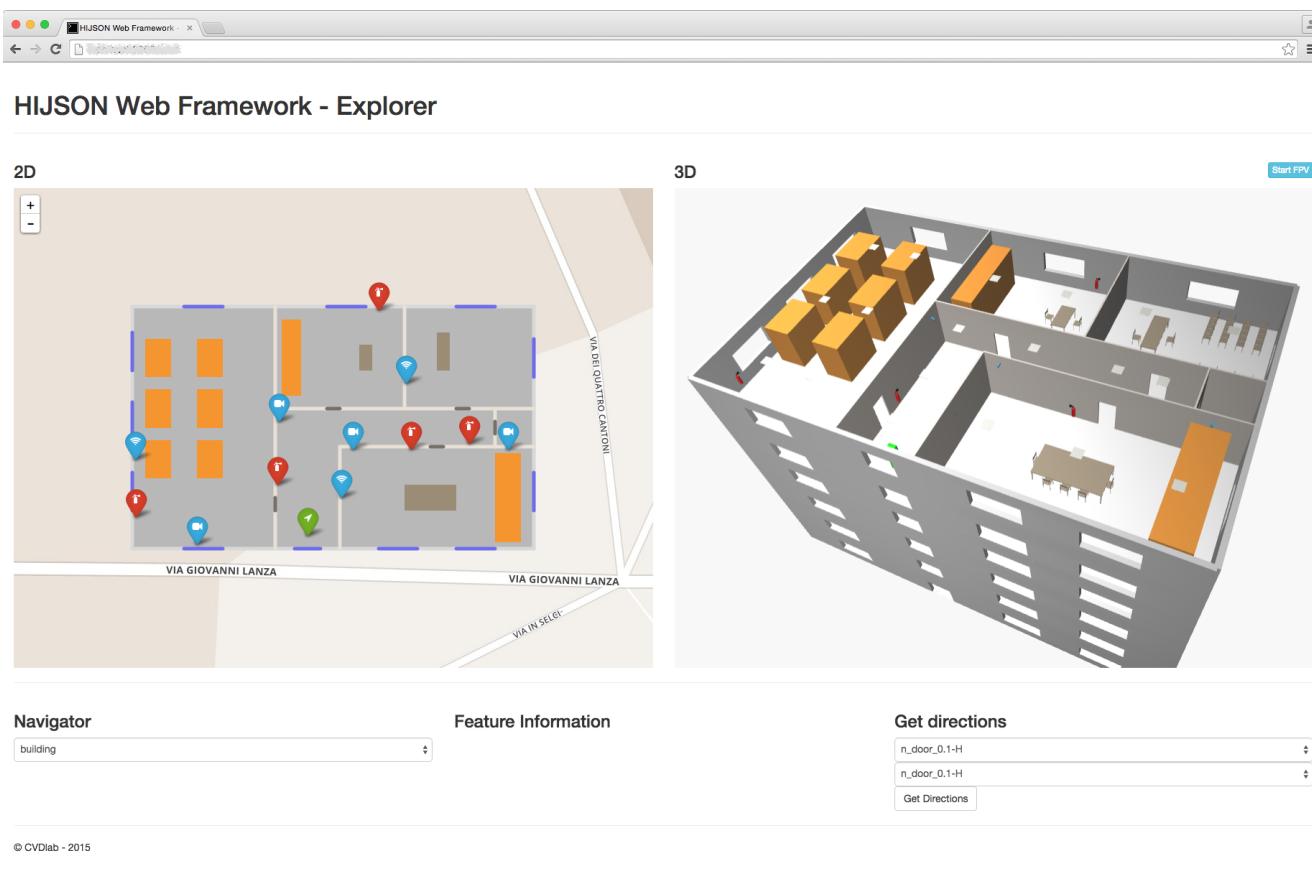
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Figure 6: HIJSON Web Framework UI

### 6.1.1 IoT monitoring

An *IoT monitoring application* consists of an interface showing to the user, in a single, integrated and centralized way, the information collected from all the smart objects modelled in the HIJSON document. IoT monitoring application provides bidirectional communication, since the interface let the user receive information coming from smart objects while allowing him to send commands to them.

As the name itself may suggest, it is an activity specif-

ically performed by a *Supervisor* user, but it can be also suitable to be deployed for the *Explorer* user, since she can take advantage of the interactive information coming from the surroundings objects while she moves across the indoor environment.

Monitoring different smart objects may require different ways to visualize and/or send data and commands. Modularity and extensibility of the application respond superbly to these requirements, by providing for each class of ob-

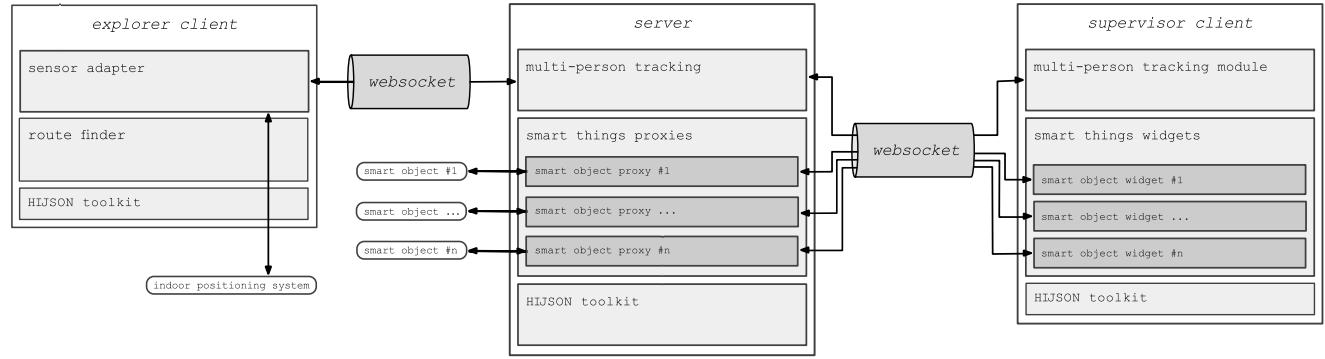


Figure 7: HIJSON Web Toolkit architecture

jects a different interface of visualization and interaction, as a result of the polymorphism principles introduced by the HIJSON Class. In particular, the user interface is characterized by a dual-display mode, that allows the user to see at the same time a 2D map that gives an overall glance in a simplified plan, and a 3D virtual environment to navigate into, as shown in Figure 6.

Alongside with typical smart objects, suitable to deal with like thermostats, where the user can read the room temperature and turn the heating on/off, other kinds of objects, that are not properly considered “smart”, can be integrated into the HIJSON environment. It is the case, for example, of fire extinguishers, that are able to show the date of their last check, stored in a database.

### 6.1.2 Realtime multi-person tracking

Realtime multi-person tracking allows a *Supervisor* to monitor the current real position of people inside the building. This kind of task can be useful for several reasons, including security, logistics or to supervise composite operative workflows. Each device equipped with the *Explorer* application is in charge of locating itself, interacting with the indoor positioning system, and notifying the current position in continuos mode. Evidence of the people position is given to the *Supervisor* both into a 2D map and an immersive 3D virtual environment (see Figure 6).

### 6.1.3 Cross-storey user navigation

The HIJSON Framework also provides the capability to give directions to *Explorer* users that must move across the indoor environment. The user specifies a starting and an ending point and the system provides him with a valid connection path. This feature strongly rely on the graph of paths generated by the Toolkit, so starting and ending points must be nodes of the graph. *Connection nodes* are introduced to represent stairs or elevators, enabling cross-storey paths to be computed. Since paths can span more than one

storey, the most effective way to display them to the user is to show the connection nodes visualized in one or more 2D maps.

## 6.2. Architecture

Like the vast majority of the web based applications, the Framework exposes an overall architecture that is inherently *client/server*. In particular, two different types of possible clients are identifiable, one for each different kind of users: the *Supervisor* client and the *Explorer* client. Both of them connect to the same server.

The indoor space described by the input HIJSON document is processed by the server via the processing pipeline. After that, any connecting *Explorer* client, presumably via a mobile device, will be provided with the information to perform cross-storey navigation of the building, while reporting the user position to the server. The server will feed any connecting *Supervisor* client with users positions, along with data from sensor-equipped objects present in the environment, achieving both IoT monitoring and realtime multi-person tracking.

### 6.2.1 Server Architecture

An architectural scheme of the framework is provided in Figure 7. A web server module is responsible for listening to connecting clients. Each client connection is handled by the web server module providing all the required resources and then by opening a WebSocket channel, in order to have both *Explorer* and/or *Supervisor* communication protocol data flow within. In particular, the multi-person tracking module receives position data from *Explorer* clients. It aggregates and sends these information to connected *Supervisor* clients through the WebSocket channel, using a simple but reliable protocol described later. Independence from particular IoT sensor equipment communication protocols is achieved introducing a smart object proxy module. This one is defined in the HI-

1080 JSON Class and is obtained via the `getProxy()` method  
 1081 for each smart object modelled as previously described.  
 1082

### 1084 6.2.2 Explorer client architecture

1085 The *Explorer* client architecture is generally deployed on  
 1086 a mobile device, which is usually supplied to a user who  
 1087 needs to be routed across the environment described by the  
 1088 HIJSON document. The sensor adapter module en-  
 1089 capsulates the communication logic with the indoor po-  
 1090 sitioning system. The presence of this module ensures inde-  
 1091 pendence from particular technologies, so allowing the *Ex-  
 1092 plorer* client to rely on different indoor positioning systems  
 1093 (Wi-Fi, Bluetooth, LTE, etc.).

1094 Every time a sensor adapter observes a perceptible  
 1095 modification in user position, it sends the new position  
 1096 information to the server through the single opened Web-  
 1097 Socket, spawning a simple message with the syntax de-  
 1098 scribed in listing 3.

```
1099 currentPosition = {  

1100   coordinates: [x, y],  

1101   levelId: level-ID  

1102 }  

1103
```

1104 Listing 3: Example of message sent by the *Explorer* client  
 1105 to the server.

1106 Relevant information includes, beside current coor-  
 1107 dinates, the indication of the storey of the possibly multilevel  
 1108 building the user is in.

1109 It is to remark that, when not outflanked by the introduc-  
 1110 tion of an external server, the problem of communication  
 1111 between positioning system and the low level APIs of the  
 1112 browser is left to positioning system itself or to whom is in  
 1113 charge of specific deployments of the HIJSON Web Frame-  
 1114 work. The smart object widget module, being in  
 1115 common with the *Supervisor* client, will be discussed in the  
 1116 next section.

### 1117 6.2.3 Supervisor client architecture

1118 The architecture of *Supervisor* client includes two modules.  
 1119 The first one, named multi-person tracking mod-  
 1120 ule, is responsible to receive through the WebSocket, from  
 1121 the server information about *explorers* of the environment,  
 1122 showing them in the user interface. The second module,  
 1123 named smart object widget, communicates with  
 1124 the server to propose the user realtime information about  
 1125 sensor-equipped objects in the environment. Data passes  
 1126 through the single WebSocket opened between the server  
 1127 and every *Supervisor* client. Relying on a naive but ef-  
 1128 fective communication protocol, each smart object  
 1129 widget exchanges data only with its corresponding  
 1130 smart object proxy on the server. To ensure the

1131 data are posted only when the user requires the informa-  
 1132 tion relative to a specific smart object, a *widget lifecycle*  
 1133 *protocol* is implemented. This one is based on 4 event trig-  
 1134 gers `on_before_show`, `on_show`, `on_before_hide`,  
 1135 `on_hide`, as suggested by their names. When the user re-  
 1136 quires information about a smart object, its widget has to  
 1137 be rendered, and `on_before_show` the server is notified  
 1138 to connect via relative proxy to the sensor. Once connected,  
 1139 the server begin to send data via WebSocket. Received data  
 1140 are shown through the widget to the user. When done, the  
 1141 `on_before_hide` event of the widget is triggered, a no-  
 1142 tification is sent to the server announcing to stop sending  
 1143 data, and the proxy closes the connection to the sensor.  
 1144 Widget lifecycle protocol ensures that only required data  
 1145 are sent from the server to the client.

## 1146 7. Case study

1147 As case study of the discussed approach, we have taken  
 1148 into account the need of Sogei S.p.A. to support its main-  
 1149 tenance service workflow, since its data center, one of biggest  
 1150 data centers in Europe, is subject to very strict access con-  
 1151 trol policies.

1152 The overall state of the data center can be monitored  
 1153 through the *Supervisor* client. In this case the considered  
 1154 smart objects belong to a range of different devices, going  
 1155 from webcams, that provide on request the captured video  
 1156 streams, by way of alarm and antifire systems, till to indi-  
 1157 vidual servers, that can be monitored along several dimen-  
 1158 sions, including operating temperature, workload, etc.

1159 The most common maintenance scenario consists of an  
 1160 intervention by a technician that have to move across the  
 1161 environment, and locate within a huge data center the  
 1162 machine on which to operate, a not trivial task due to the pres-  
 1163 ence of thousands of similar-looking machine racks. Thus  
 1164 the operator will be equipped with an *Explorer* client, which  
 1165 will drive him to the target machine on which operate, while  
 1166 continuously notifying to a security *Supervisor* his position,  
 1167 obtained by interacting with the indoor positioning system.  
 1168 The maintenance workflow supervisor, using the *Supervi-*  
 1169 *sor* client, is able to monitor the operator position within the  
 1170 data center, verifying that he does not deviate on unautho-  
 1171 rized paths, triggering some console alarm if this happens.

1172 The purpose is to support the process of those in  
 1173 charge of carrying out the “ticket-maintenance” activities,  
 1174 as quickly as possible and without error. The “maintenance  
 1175 man” will be guided to the right sub-system among thou-  
 1176 sandes of racks. The real-time awareness of the relative po-  
 1177 sitions between the “maintenance man” and the rack — con-  
 1178 taining the sub-system — will help to reduce intervention  
 1179 times and to increase safety. By knowing when the main-  
 1180 tenance process starts, the system can automatically move, in  
 1181 real time, services, which are hosted on virtual machines, to  
 1182 other systems, thus maintaining the continuity of services

1188  
 1189 and, at the same time, reducing the global risk factor. When  
 1190 the “ticket-maintenance” is over, and the technician goes  
 1191 away, it is possible to immediately restore the pre-existing  
 1192 conditions of services after a complete test has been per-  
 1193 formed.

## 1194 8. Conclusions 1195

1196 In this paper a novel document format, named HIJSON,  
 1197 for indoor cartographical descriptions has been introduced.  
 1198 Utilization of local metric coordinate system, avoiding the  
 1199 manipulation of global geographical coordinates, really in-  
 1200 convenient when dealing with indoor spaces and objects,  
 1201 greatly enhances the modeling and rendering of the doc-  
 1202 ument content. Currently, we produce the HIJSON document  
 1203 from a python script using two libraries for geometric com-  
 1204 puting (`pyplasm` and `larcc` [7, 14, 6]). The modeling  
 1205 process can be further improved by implementing a LAR-  
 1206 based graphical editor to assist the user during the descrip-  
 1207 tion of the indoor space. The realization of such an editor is  
 1208 already in our plans.

1209 The HIJSON format focuses on a hierarchical represen-  
 1210 tation of the indoor spaces that allows for completely cap-  
 1211 turing their topology. On the basis of this representation a  
 1212 virtual web environment can be rebuilt working as a unify-  
 1213 ing platform to run a bunch of different applications. The  
 1214 reference architecture of such a platform has been also im-  
 1215 plemented and described in this work.

1216 The architecture supports a whole range of applications:  
 1217 IoT monitoring, realtime multi-person tracking and user  
 1218 cross-storey navigation are already implemented and de-  
 1219 scribed. A very convenient way to extend the representation  
 1220 capabilities of smart objects is also mentioned as semantic  
 1221 extensions. These extensions, which affects both document  
 1222 format and its web framework, might be easily collected  
 1223 in a public repository. Community could both use public  
 1224 available extensions or contribute by mapping new (smart)  
 1225 objects inside the HIJSON document format.

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## A. Appendix

### A.1. Short summary of the LAR scheme

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1312 V = [[5.,0.],[7.,1.],[9.,0.],[13.,2.],[15.,4.],[17.,8.],[14.,9.],[13.,10.],[11.,11.],[9.,10.],[5.,9.],[7.,
1313 9.],[3.,8.],[0.,6.],[2.,3.],[2.,1.],[8.,3.],[10.,2.],[13.,4.],[14.,6.],[13.,7.],[12.,10.],[11.,9.],[9.,7.],
1314 [7.,7.],[4.,7.],[2.,6.],[3.,5.],[4.,2.],[6.,3.],[11.,4.],[12.,6.],[12.,7.],[10.,5.],[8.,5.],[7.,6.],[5.,5.]]
1315 FV = [[0,1,16,28,29],[0,15,28],[1,2,17],[1,16,17,33],[2,3,17],[3,4,18,19],[3,17,18,30],[4,5,19],[5,6,19],
1316 [6,7,20,21,22,32],[6,19,20],[7,8,21],[8,9,21,22],[9,11,23,24],[9,22,23],[10,11,24,25],[10,12,25],
1317 [12,13,25],[13,14,27],[13,26,27],[14,15,28],[14,27,28,29,36],[16,29,34],[16,33,34],[17,30,33],[18,19,31],[18,30,
1318 [31],[19,20,31,32],[22,23,32,33],[23,24,34,35],[23,33,34],[24,25,27,36],[24,35,36],[25,26,27],[29,34,35],[29,
1319 35,36],[30,31,32,33]]

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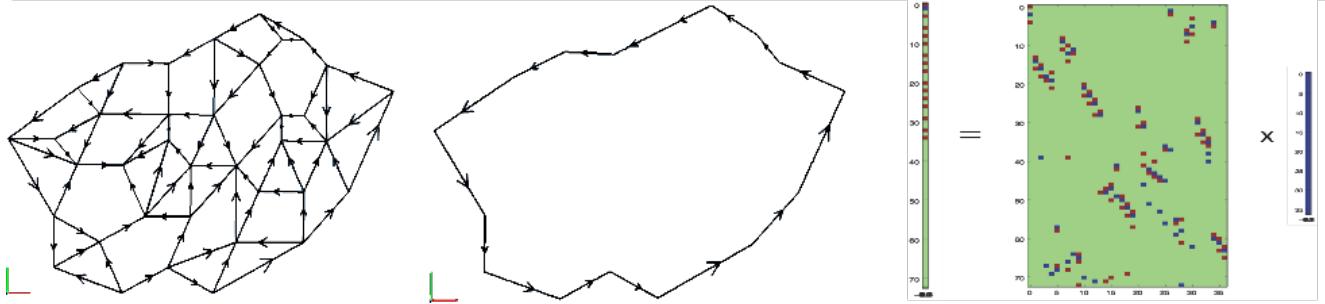


Figure 8: A toy example of the LAR scheme: (a) the bare minimum of data with *complete* information about topology; (b) the extracted boundary; (c) the extraction method  $[e] = [\partial][f]$  giving the coordinate representation (in the discrete basis of the 1-cells) of the boundary edges  $[e]$  by product of the sparse boundary operator matrix  $[\partial]$  times the coordinate representation  $[f]$  of the 2-cells (faces), in the discrete basis of the 2-cells.

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