Computational Motor control

Assignment 1

Tianchu Zhang

Siqi Zheng

Jian Yang

Lecturer:

Ijspeert Auke

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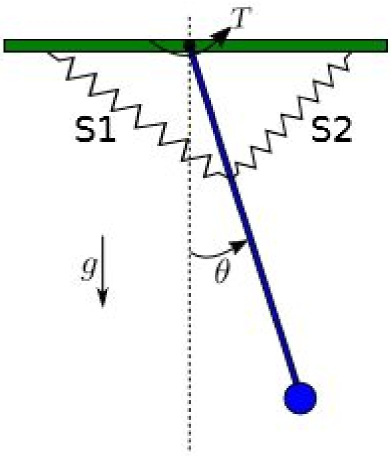
Exercise 1 : Pendulum model with passive element

Explore the pendulum model with two antagonist spring elements

First we develop the movement equations for this simplified pendulum-spring system.



Define the parameters and the direction:

Suppose that θ is the angle formed between the pendulum and the vertical line.

ref_i is defined such that when $\theta = \theta$ ref_i the spring Si is at his relax length (i.e no compression neither elongation)

The difference of the angle is defined as: $\Delta\theta_i = \theta_{refi} - \theta$

In the following context, we assume that the anticlockwise direction is the positive direction for the velocity and the torque.

Movement equation:

In the tangential direction of the movement, we have the following equation:

The torsional force generated by the spring:

$$F_{s1} = k_1 * \Delta\theta_1 \quad when\ \Delta\theta_1 < 0$$

$$F_{s1} = 0 \quad when\ \Delta\theta_1 \geq 0$$

$$F_{s2} = k_2 * \Delta\theta_2 \quad when\ \Delta\theta_2 \geq 0$$

$$F_{s2} = 0 \quad when\ \Delta\theta_2 < 0$$

The torque generated by the gravity:

$$F_g = -mg * sin\theta$$

According to the 3th Newton law, we have:

$$F_{s1} + F_{s2} + F_g = mL\ddot{\theta}$$

$$k_1 \Delta \theta_1 + k_2 \Delta \theta_2 - mgsin\theta = mL\ddot{\theta}$$

$$\frac{k_1}{mL} \Delta \theta_1 + \frac{k_2}{mL} \Delta \theta_2 - \frac{gsin\theta}{L} = \ddot{\theta}$$

Let $K_1 = \frac{k_1}{m}$, $K_2 = \frac{k_2}{m}$

$$\frac{K_1}{L}(\theta_{ref1} - \theta) + \frac{K_2}{L}(\theta_{ref2} - \theta) - \frac{gsin\theta}{L} = \ddot{\theta}$$

<u>The state equation of the system:</u>

Let $x_1 = \theta$, $x_2 = \dot{\theta}$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \frac{K_1}{L}(\theta_{ref1} - x_1) + \frac{K_2}{L}(\theta_{ref2} - x_1) - \frac{g}{L} sinx_1$$

Considering that the we implement this in the modulus lab4_pendulum.py with the ‘if’ conditions:

```
if s_theta_ref1 - theta > 0:
    delta_theta1 = 0
if s_theta_ref2 - theta < 0:
    delta_theta2 = 0
```

When the damper is added, the state equation changes to:

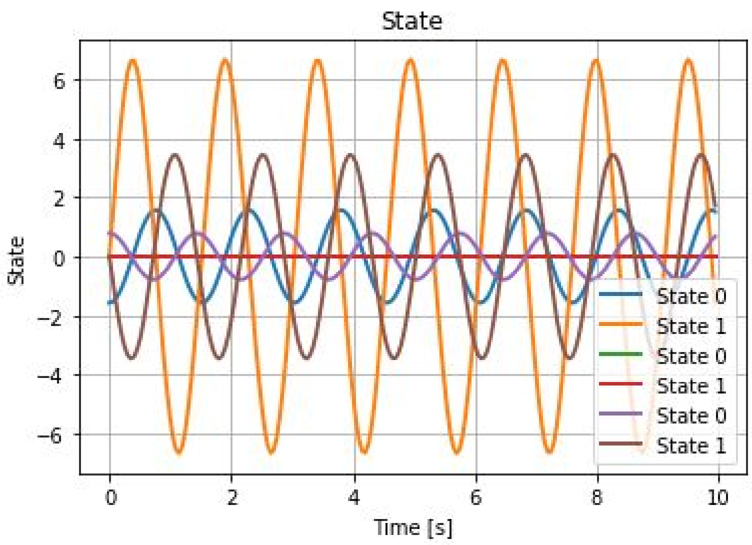
$$\dot{x}_1 = x_2$$

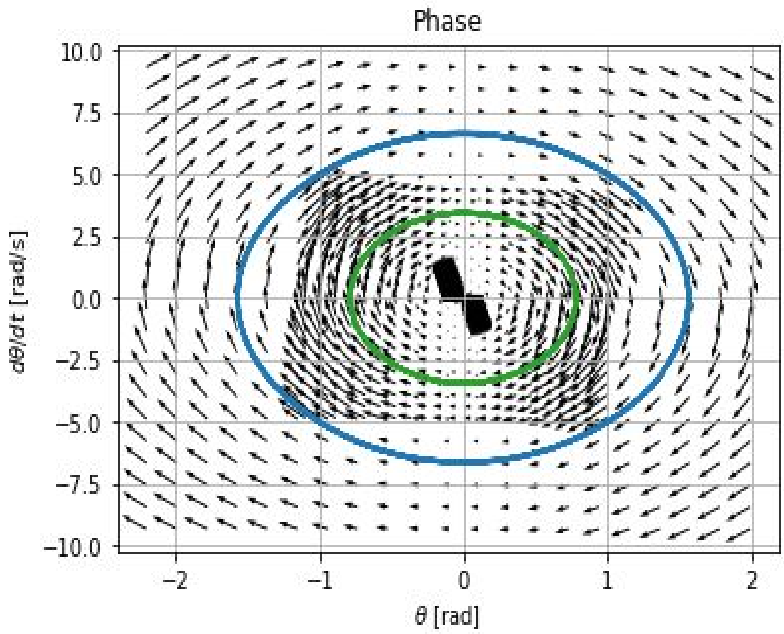
$$\dot{x}_2 = \frac{K_1}{L}(\theta_{ref1} - x_1) - \frac{B_1}{L} x_2 + \frac{K_2}{L}(\theta_{ref2} - x_1) - \frac{B_1}{L} x_2 - \frac{g}{L} sinx_1$$

1.a Does the system have a stable limit cycle behavior? Describe and run an experiment to support your answer.

Yes, the system has a stable limit cycle behavior. No matter at which angle (between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$) and speed we release the pendulum, the energy of the system will vary among 3 forms of energy: the kinetic energy, the potential energy of the springs and the gravitational potential energy. Since there is no energy dissipation, the system will behavior the simple harmonic motion.

Here is the phase plane of the system with different initial conditions





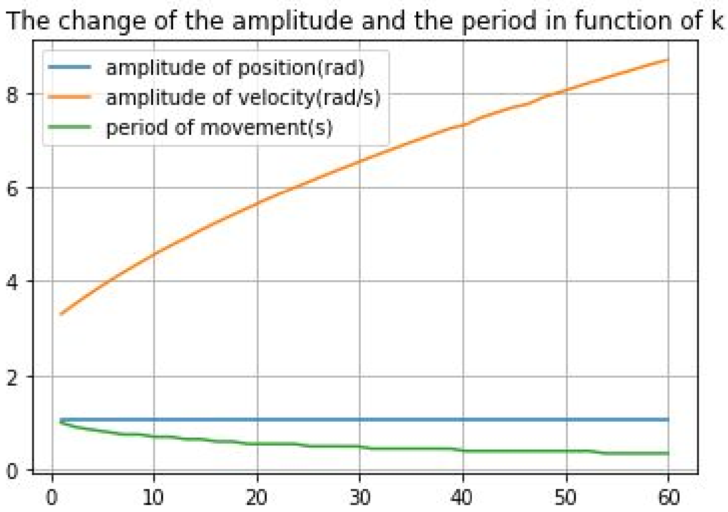
1.b Explore the role of spring constant (K) and spring reference angle (θ_0) in terms of range of motion, amplitude, ... Support your responses with relevant plots

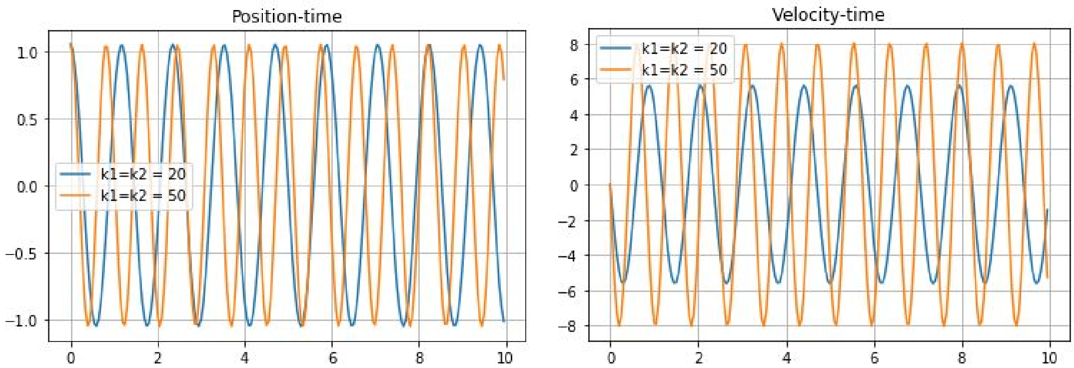
Change K:

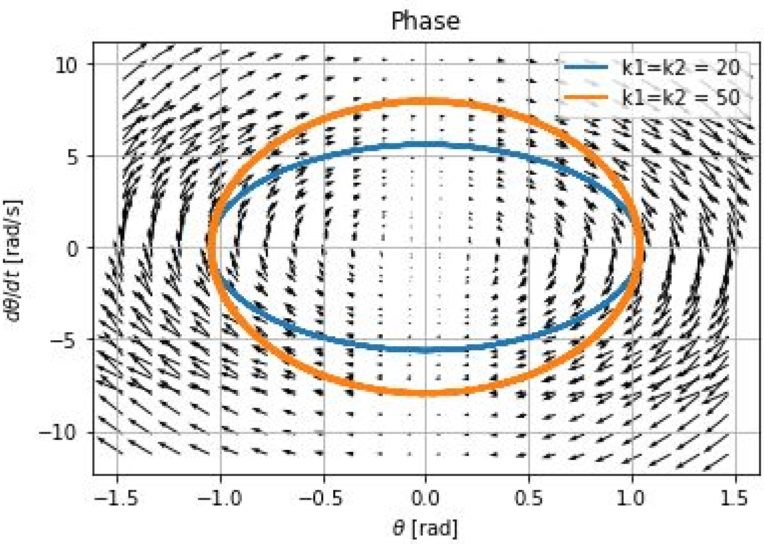
In this case, we still have the same value of the k and $\boldsymbol{\theta}_0$ for the spring 1 and 2. For the certain initial condition and spring reference angle $\theta_0 = 0$, we change the value of spring constant K from 1 to 60. In the case that k changes, the system keeps its symmetricity, so we just need to analyse the maximal position and velocity at the right side, and it's the same case at the left.

We found that with the growth of K, the amplitude of velocity increase a lot, and the period of movements also have a slight decrease(i.e. the frequency increase). However, the change of K does not have impact on the amplitude of velocity, which remains at 1 rad/s.

The maximal velocity increases since when k increase, the potential energy stored in the spring increases, so the kinetic energy transferred also increases.

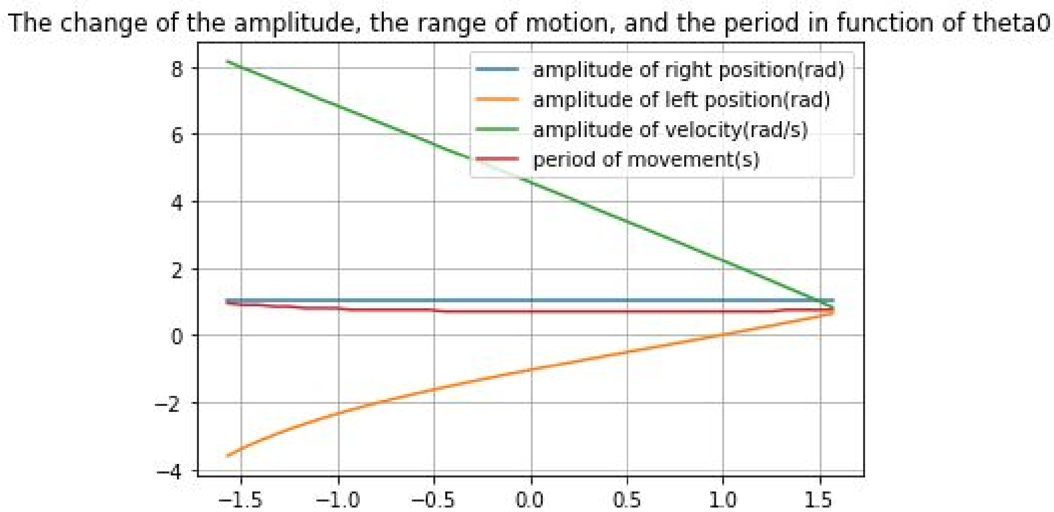

The change of the amplitude and the period in function of k
amplitude of position(rad)
amplitude of velocity(rad/s)
period of movement(s)
0
2
4
6
8
0
10
20
30
40
50
60

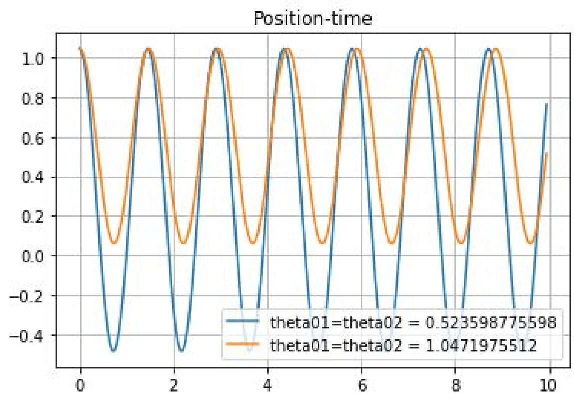

Position-time
k1=k2 = 20
k1=k2 = 50
Velocity-time
k1=k2 = 20
k1=k2 = 50

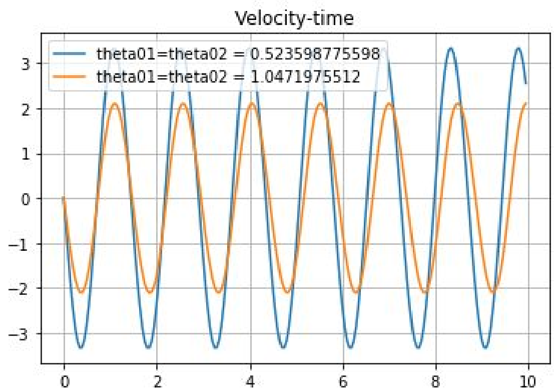

Phase
k1=k2 = 20
k1=k2 = 50
$d\theta/dt$ [rad/s]
θ [rad]

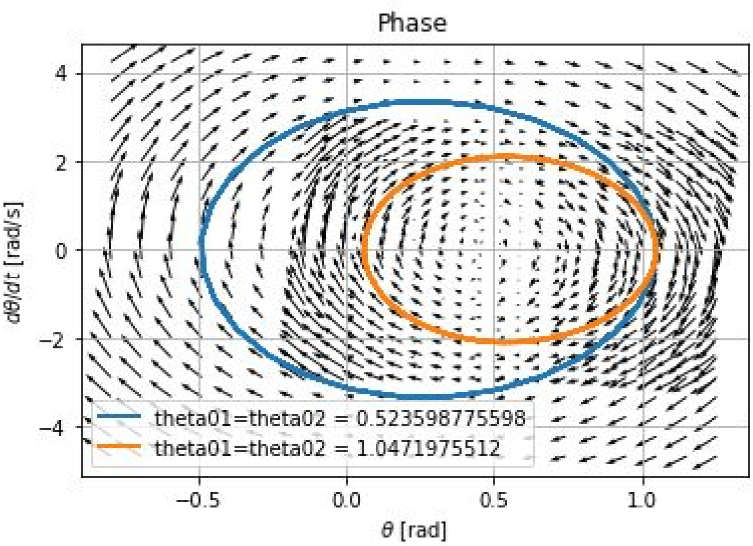
Change θ_0:

For the certain spring constant K = 10, we change the value of the spring reference angle θ_0 (from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$), We found that the maximal velocity decreases and the system lose its symmetricity: the maximal position on the right is always where we release the pendulum: $\boldsymbol{\theta} = \frac{\pi}{3}$, and the maximal position on the left decreases when θ_0 increases.







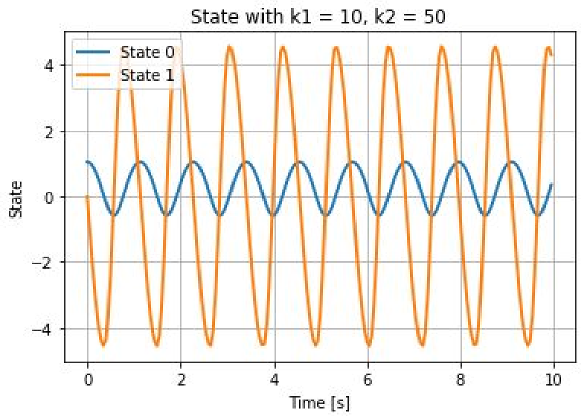


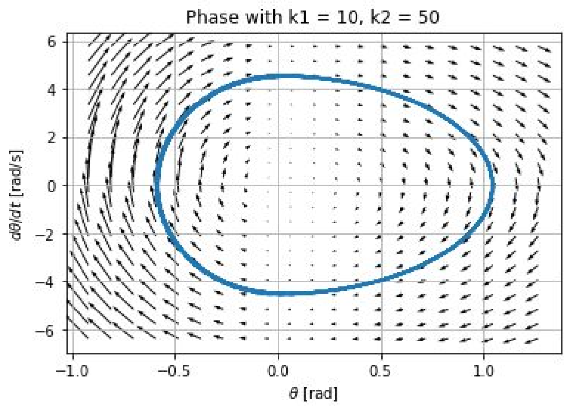
1.c Explain the behavior of the model when you have different spring constants (K) and spring reference angles (θ_{ref}). Support your responses with relevant plots

In this case, the system lose its symmetricity.

Different k:

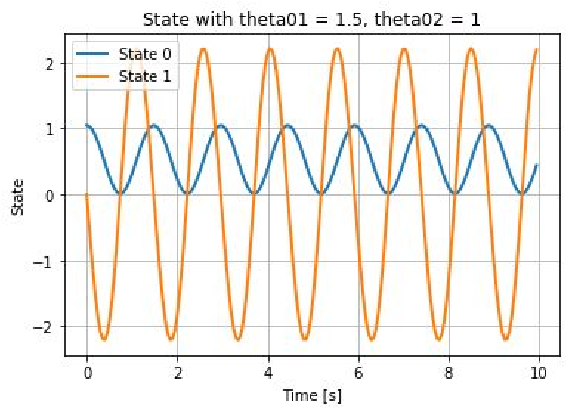
For the certain value of spring reference angle θ_{ref}= 0, we change spring constant K . When K2 increases, we can see that the maximal position at left decreases, while that at right remains the one we release the pendulum.

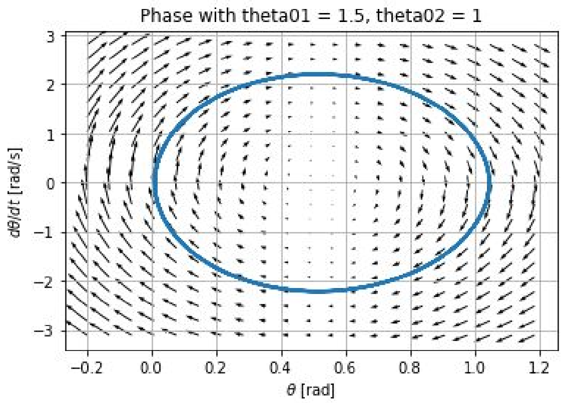




Different theta_ref:

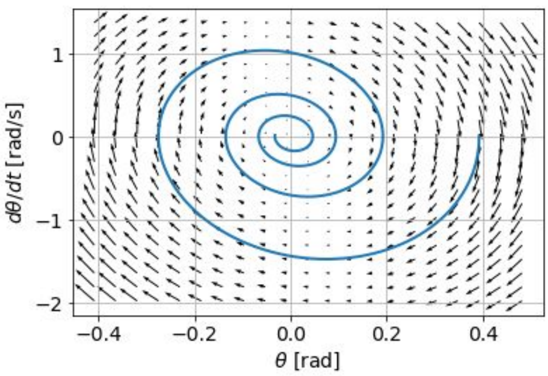
For the certain value of spring constant K = 10, we change spring reference angles θ_{ref}. When θ_{ref} are different, we can see that the maximal position at left changes, while that at right remains the one we release the pendulum.

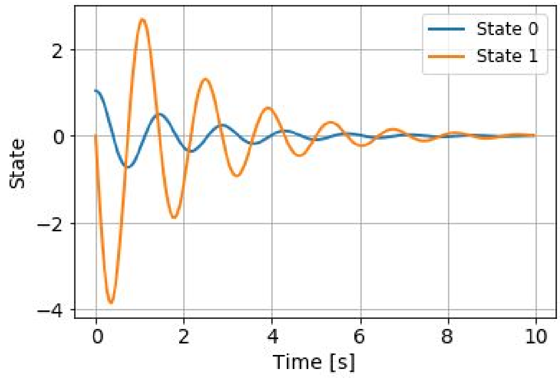




Explore the pendulum model with two antagonist spring and damper elements

1.d How does the behavior now change compared to 1.a. Briefly explain and support your responses with relevant plots

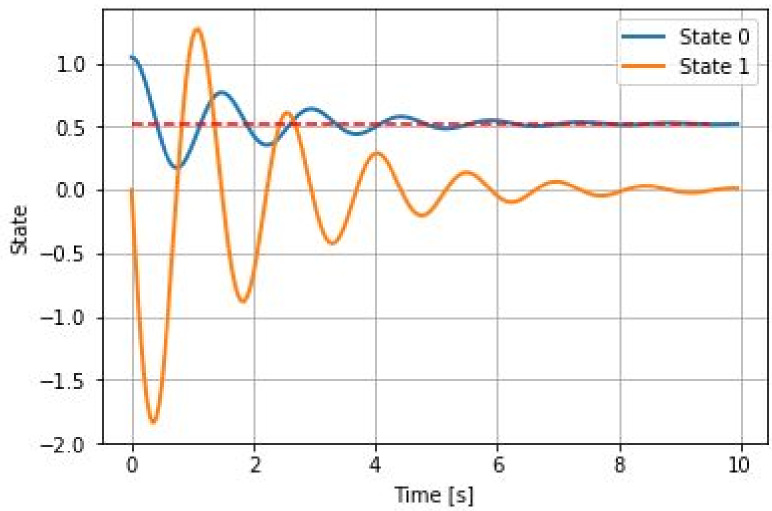




Compare with 1a, an asymptotic stable behaviour can be obtained when the dampers are added to the system. From the phase diagram, it can be seen it converges to (0,0). The amplitude diagram show that in both states, the final value converges to zero as time goes to infinity.

1.e Can you find a combination of spring constant (K), damping constant (B) and spring reference angle (θ ref) that makes the pendulum rest in a stable equilibrium at ($\theta = \pi/6$) radians? Describe the parameters used and support your response with relevant plots.

By using k_1=k_2=10, b_1=b_2=0.5, θ ref_1= θ ref_2=pi/3.1, the figure below is obtained, where the red dotted line is $\pi/6$. Actually, the choice of the damping constant and θ ref_1 does not influence the final position as seen in the simulation, also same for the k_1, as observed from (1.c). When the reference angle is positive, only the values of the reference angles themselves, and the spring constant k_2 will influence the final stable equilibrium position. Thus, arbitrary value can be used for these values to reach a final stable equilibrium of $\pi/6$, except zeros, where the asymptotic stable behaviour is not obtained.



Numerical solution:

From the introduction part, it is obtained the state function is that:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \frac{K_1}{L}(\theta_{ref1} - x_1) - \frac{B_1}{L} x_2 + \frac{K_2}{L}(\theta_{ref2} - x_1) - \frac{B_1}{L} x_2 - \frac{g}{L} sinx_1$$

In order to reach the stable equilibrium at (π/6,0), plug the values of

$$x_1 = \pi/6$$

$$x_2 = 0$$

$$\dot{x}_2 = 0$$

into the second equation above, and

$$0 = \frac{K_1}{L}(\theta_{ref1} - pi/6) \quad + \quad \frac{K_2}{L}(\theta_{ref2} - pi/6) \quad - \frac{g}{2L}$$

Multiply each side with L,

$$0 = \frac{k_1}{m}(\theta_{ref1} - pi/6) \quad + \quad \frac{k_2}{m}(\theta_{ref2} - pi/6) \quad - \frac{g}{2L}$$

rearrange,

$$k_1(\theta_{ref1} - pi/6) \quad + \quad k_2(\theta_{ref2} - pi/6) \quad = \frac{mg}{2L}$$

However, if

$$\theta_{ref1} - pi/6 < 0\,,$$

the force generated by the left spring is zero, and the equation changes to

$$k_2(\theta_{ref2} - pi/6) \quad = \frac{mg}{2L}$$

In the last equation, it can be observed k_1, b_1, b_2, and θ ref_1 disappear, which meets the simulation results observed before.

The expression of θ ref_2 in terms of K_2 is

$$\theta_{ref2} = \frac{g}{2L\,K_2} + pi/6$$

Plug the value of g=9.81, L=1, K_2=10, and

$$\theta_{ref2} = 1.014$$

And the reference angle 2 found in the simulation is π/3.1 = 1.013, which is almost same as the analytical solution above.

1.f What is the missing component between a real muscle and the muscle model with passive components that you just explored? What behaviors do you lack because of this missing component?

The contractile element is the missing component between a real muscle and the muscle model with passive components just being explored. The behavior of reaching a determined stable equilibrium without changing spring and damping component, and the reference angle is not possible due to the missing of the contractile element.

Exercise 2: Hill muscle mode

Muscle Force-Length Relationship

Two-layer loops:

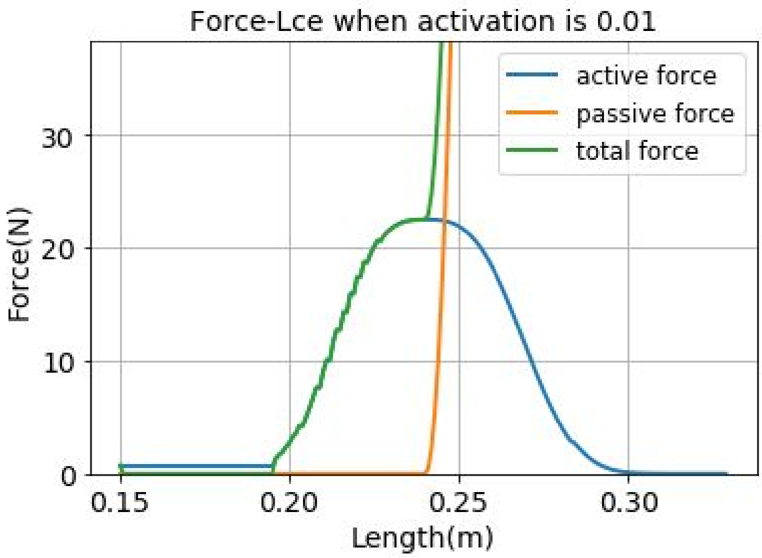
Outer loop: stretch increase

Stretch equals deltalength of mtc, it decides the length of mtu. Setting stretch as an array, it varies from -0.06~0.05 by step of 0.001. In every isometric muscle condition of different length, we get the maximum activeforce and maximum passiveforce during the time of constant stimulations.

Inner loop: time increase

When in isometric condition, we increase time and update the activation to realize the stimulation of muscle. Setting time as an array, it varies from 0.0~0.3 by step of 0.001. In every stimulation, we build the muscle model by function muscle_integrate, record the activeforce and passiveforce every time.

2.a For a given stimulation, explore the relationship between active and passive muscle forces and the length of the contractile element. Plot the force-length relationship curve. Discuss the different regions in the plot.

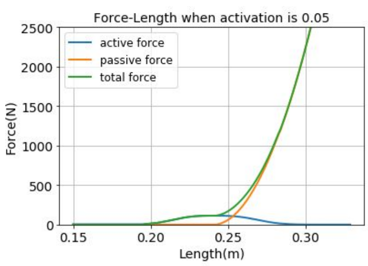


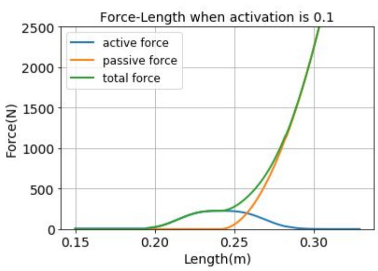
When the length of of muscle is chosen, the hill model approximate the time evolution of tension, and the muscle can vary between passive and fully active state in isometric conditions. The orange and blue curves represent the maximum passive and active force created by the muscle at different length during given constant stimulation. The green one represents the total force. When the length of muscle is close to an ideal length 0.24m ($l_{slack}+l_{opt}$), the maximum active force is the biggest.

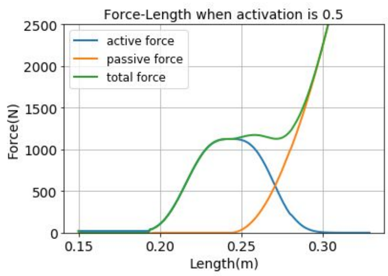
If length of muscle is set smaller than 0.24m($l_{mtu}<l_{slack}+l_{opt}$), the contractile element is in contraction. The passive force equals 0. The total force equals active force. The maximum active force increases with the growth of length of muscle.

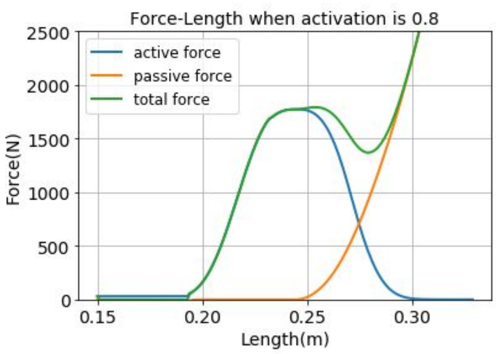
If length of muscle is set bigger than 0.24m($l_{mtu}>l_{slack}+l_{opt}$), the contractile element is in stretching. The passive force increase with length of muscle exponentially. The maximum active force decreases with length of muscle. The passive force begins to contribute more to total force than active force.

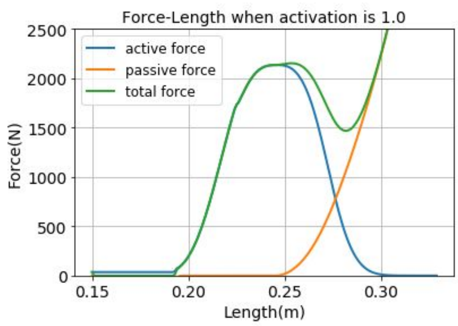
2.b In (2.a), you explored the muscle force-length relationship for a given stimulation. What happens to the relationship when the stimulation is varied between [0 - 1]? Support your response with relevant plots.







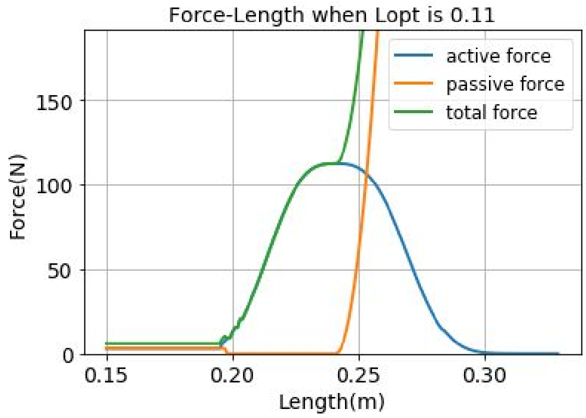


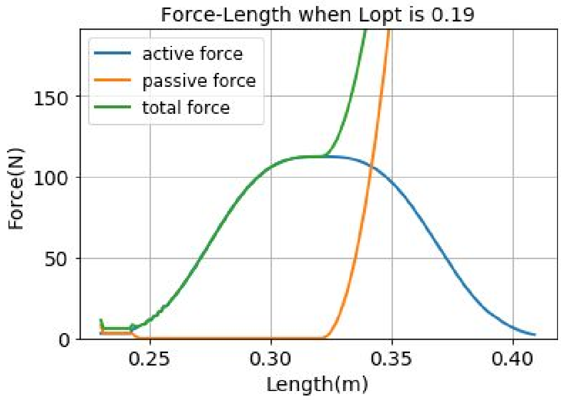


When the simulation increases, the passive force curve does not change, but the maximum of the active force curve increases a lot. Because the active force(F_{ce}) is proportional to the value of stimulation(A), and the passive force which equals F_{pe}-F_{be} is only relative to the length of different element in the muscle.

2.c Describe how the fiber length (l_{opt}) influences the force-length curve. (Compare a muscle comprised of short muscle fibers to a muscle comprised on long muscle fibers.)

For a certain stimulation = 0.05 ,





Compare to muscle with the short fiber muscle, the force-length curve of long fiber muscle moves forward as a whole along the length-axis. And we also found the shape of curve of long fiber muscle is more flat than that of the short fiber muscle. Because the muscle with long fiber is more flexible and elastic.

Muscle Velocity-Tension Relationship

In this test, we attach different load on the muscle and launch the muscle at a certain initial length (here we choose [-0,0] as our initial state). After releasing, the mass system will have a dynamical behavior, with the dynamical equation implemented in the lab4_mass.py:

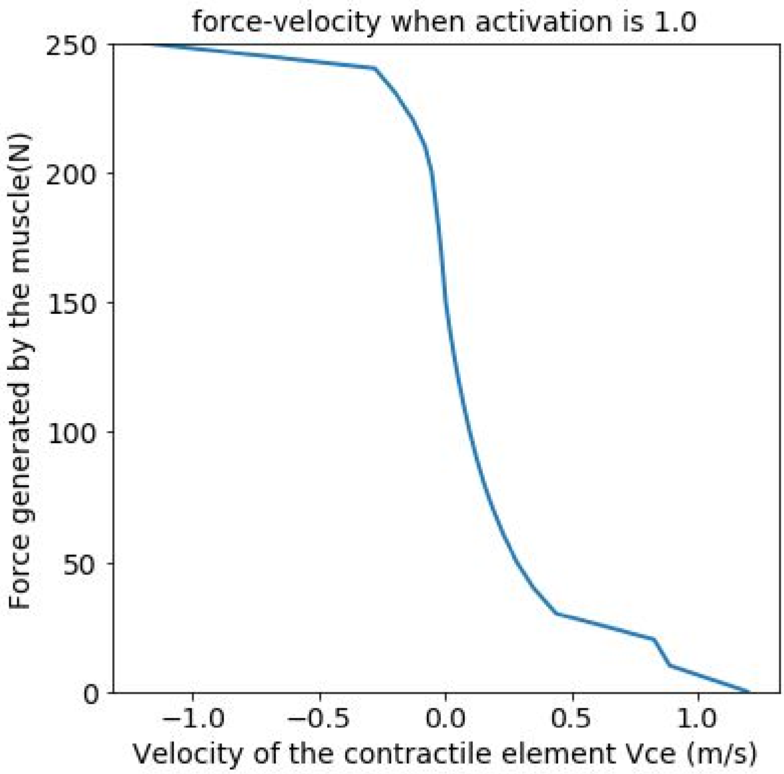
```
d2x = -force/mass_params.mass + mass_params.g
```

By integrating this equation, we can have the state(position,velocity) at each time step. Then we can have the maximal velocity of the mass, which correspond to the time that the acceleration of the mass is 0. The 0 acceleration means that, the force generated by the muscle is equal to the gravity of the load. By this indirect way, we can obtain a combination of velocity and force for each hanged load. What we need to do is just to vary the load and obtain a serie of this combination, then plot them.

Analyse the code:

In the function ***isotonic_contraction*** , we have 2 layers of the for loop. The outer loop for increasing the load, while the inner loop contains 2 small loops: the first inner loop is before releasing the muscle, the muscle should be integrated by the activation in a certain time, the second loop is after releasing the muscle, we integrate the dynamic equation to find out the position and the velocity.

2.d For a stimulation of 1.0 and starting at optimal muscle length, explore the relationship between contractile element velocity and external load. Plot the Velocity Tension relationship curve. Include shortening and lengthening regions



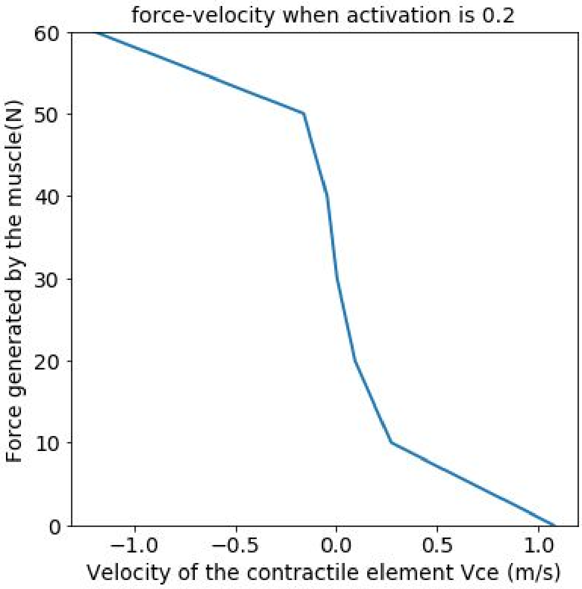
The velocity < 0 corresponds that the muscle is lengthening, the velocity > 0 corresponds that the muscle is shortening.

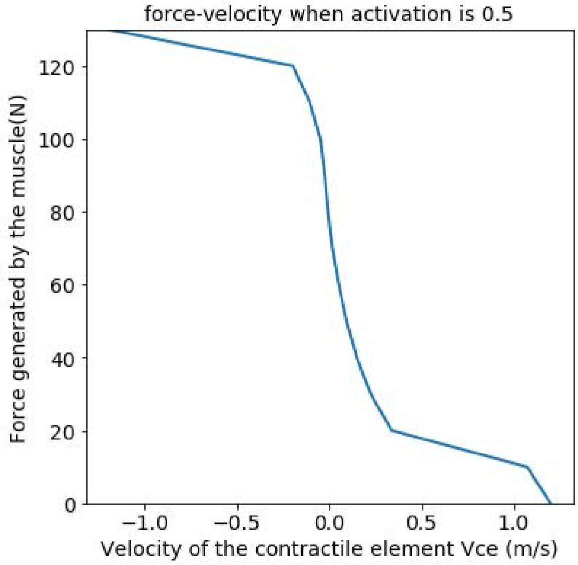
2.e For the muscle force-velocity relationship, why is the lengthening force greater than the force output during shortening?

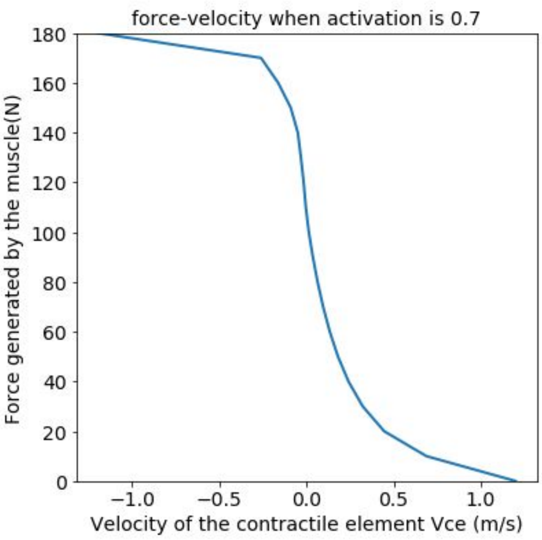
During concentric contraction (shortening), muscles generate less force compared to isometric contractions, but consume greater amounts of energy as shortening velocity increases. Conversely, more force is generated and less energy is consumed during eccentric activation (lengthening), the muscle force is proportional to the velocity of the muscle.

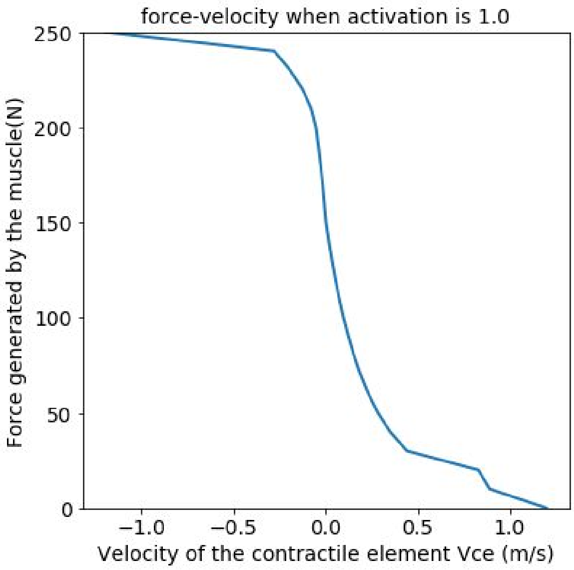
2.f How does the parameter muscle activation influence the force-velocity relationship. Show and explain the behavior for multiple muscle activation

When starting at the optimal muscle length, we have the following results:


force-velocity when activation is 0.2
Force generated by the muscle(N)
Velocity of the contractile element Vce (m/s)


force-velocity when activation is 0.5
Force generated by the muscle(N)
Velocity of the contractile element Vce (m/s)


force-velocity when activation is 0.7
Force generated by the muscle(N)
Velocity of the contractile element Vce (m/s)


force-velocity when activation is 1.0
Force generated by the muscle(N)
Velocity of the contractile element Vce (m/s)

When the activation increases, the force during the lengthening also increases, however the slope of the linear phase of curve does not change . Because linear damper behavior $T = M + B\dot{x}$, B remains the same value, T increases with the growth of M.

Appendix

A1: Explanations of some codes

Function muscle_integrate :

integrate the muscle model by calling function *step* from object muscle, and create the dictionary res which stores the property parameters of object.

Function step :

integrate the muscle model by calling function updateActivation to compute the updated muscle activation, calling function computeMuscleTendonLength to compute the updated length of mtc.

calling functions _F_PE_star, _F_BE, _F_SE, _f_l to compute the different forces by inputting initialized length of SE, CE. (initialization is done in function InitializeMuscleLength), then calling functions _f_v, _v_CE to compute the velocity of CE.

Computing length of SE, CE again according to the velocity of CE.