ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

MODEL PREDICTIVE CONTROL

ME-425 Final Project

ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Zhe Chen
Tianchu Zhang

07.06.2018

1. Introduction

In this project, we implemented a MPC controller for a quadrotor, which is an aircraft driven by four independently controlled rotors. We tested our controller on nonlinear model and the corresponding linearized one, the quadrotor achieved good performance in both cases.

2. Nonlinear model and linearization

Question: *Interpretation of the structure of matrices Ac and Bc. Explain the nonzero numbers in rows 4 and 5 of Ac and the nonzero rows of Bc in connection to the nonlinear dynamics.*

The Ac and Bc represent the following state-space description:

$$\ddot{\boldsymbol{X}} = A_c\boldsymbol{X} + B_c\boldsymbol{U}$$

Where $\boldsymbol{X}$ is the 12-dimension state and the U is the difference between the actual input and the input at the equilibrium, where the model is linearized.

The α, β, γ are the rotation around the 3 axes of the intrinsic coordinate *($\boldsymbol{X_B}$ $\boldsymbol{Y_B}$ $\boldsymbol{Z_B}$),* while x, y, z is the position of the center of the mass in the world coordinate.

If we project the $\boldsymbol{Z_B}$ in the world coordinate *($\boldsymbol{X_0}$ $\boldsymbol{Y_0}$ $\boldsymbol{Z_0}$)* by the Euler angles, we get:

$$\boldsymbol{Z_B} = \begin{bmatrix} sin\alpha \cdot sin\gamma + cos\alpha \cdot sin\beta \cdot cos\gamma \\ -sin\alpha \cdot cos\gamma + cos\alpha \cdot sin\beta \cdot sin\gamma \\ cos\alpha \cdot cos\beta \end{bmatrix} \begin{bmatrix} \boldsymbol{X_0} \\ \boldsymbol{Y_0} \\ \boldsymbol{Z_0} \end{bmatrix}$$

If we take the first order of Taylor expansion around 0, we get $sinx \cong x, cosx \cong 1,$ $sinx \cdot siny \cong 0$, so:

$$\boldsymbol{Z_B} \cong \begin{bmatrix} \beta \\ -\alpha \\ 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{X_0} \\ \boldsymbol{Y_0} \\ \boldsymbol{Z_0} \end{bmatrix}$$

By the given dynamic equation:

$$\ddot{\boldsymbol{O}}_{\boldsymbol{B}} = \begin{bmatrix} 0 \\ 0 \\ -\mathrm{g} \end{bmatrix} + F\boldsymbol{Z_B}$$

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -\mathrm{g} \end{bmatrix} + F \begin{bmatrix} \beta \\ -\alpha \\ 1 \end{bmatrix}$$

At the hovering equilibrium, the gravity should be exactly counteracted, so by the third row we have: $F = \mathrm{g}$, so:

$$\ddot{x} = g \cdot \beta$$
$$\ddot{y} = -g \cdot \alpha$$

Which explains the nonzero numbers in rows 4 and 5 of Ac.

As we said before, ***U*** is the difference between the actual input and the input at the equilibrium, so supposed that ***u*** is the actual input:

$$\ddot{z} = F - g = k_{\mathrm{F}}(u_1 + u_2 + u_3 + u_4) - g = k_F(U_1 + U_2 + U_3 + U_4)$$

Since k_F = 3.5 from the model, which explains the fifth row of the Bc. Since the length of the arms L = 0.16m, we have $k_F \cdot L = 0.56$ which explains the sixth and seventh row of the Bc. The last row of the Bc shows that $\mathrm{k_M} = 0.7333$.

3. First MPC controller

3.1 Choice of tuning parameters and motivation for Q, R matrices and prediction horizon N

We set prediction horizon N = 30 since we don't use terminal constraints, the N should be large enough. The parameters Q and R can be designed to penalize the state variables and the control signals. The larger these values are, the more we penalize these signals. Basically, choosing large values for R means we try to stabilize the system with less (weighted) energy. While small values for R means, we don't want to penalize the control signal. Similarly, choose large values for Q means we try to stabilize the system with the least possible changes in the states and small Q implies less concern about the changes in the states. Since there is a trade-off between the two, we want to keep R as identity matrix and only alter Q since the 4 rotors work equivalently on this aircraft. Finally, the larger values of Q (2) and Q (3) (penalize weights on α and β), the faster α and β converge, hence we set Q as Q = diag(50, 1e4, 1e4, 2, 0.1, 0.1, 0.1) such that our system's settling time around 2s.

3.2 Plots of the response to an appropriate initial condition

We set the initial condition as x0 = (−1, 10, −10, 120, 0, 0, 0)). For the constraints of u, since the system is linearized around the hovering equilibrium point, where the input us = (0.7007, 0.7007, 0.7007, 0.7007). We aim to converge the system to the equilibrium, here the 'input' ***U*** we actually regulate is the difference between the real thrust input ***u*** and ***us***.
We have:

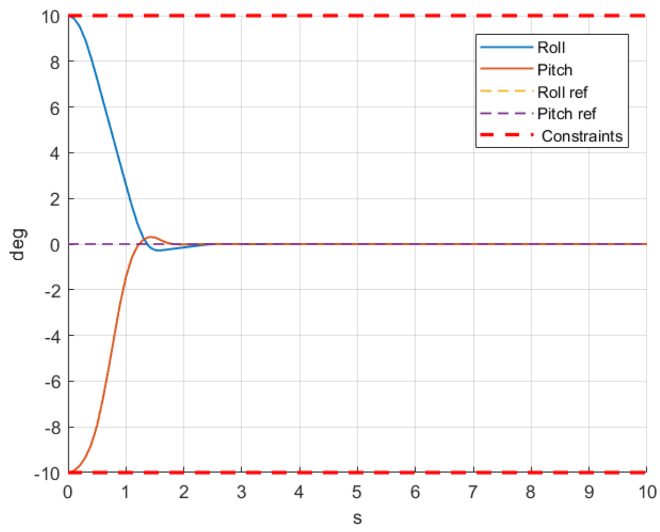
$$0 \leq \boldsymbol{u} \leq 1$$

So:

$$-\boldsymbol{u_s} \leq \boldsymbol{u} - \boldsymbol{u_s} \leq 1 - \boldsymbol{u_s}$$

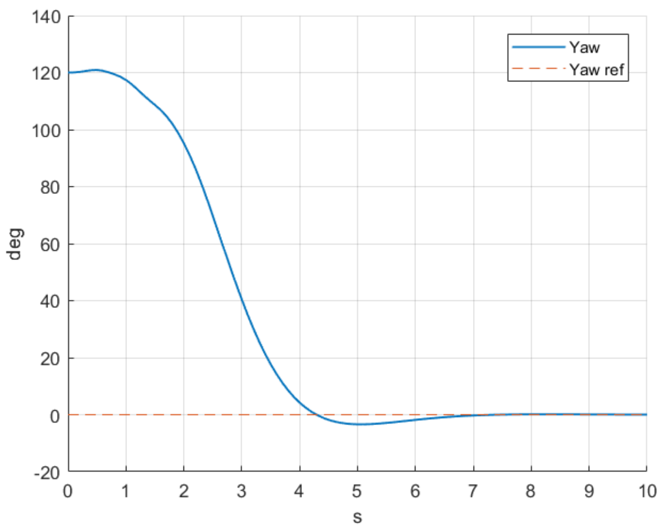
The input constraints we have is therefore: [-0.7007, 0.2993]

Simulation results are presented in following Fig.1. We can see that the pitch and rolls angles (a) and the vertical velocity (d) converge in 2s.



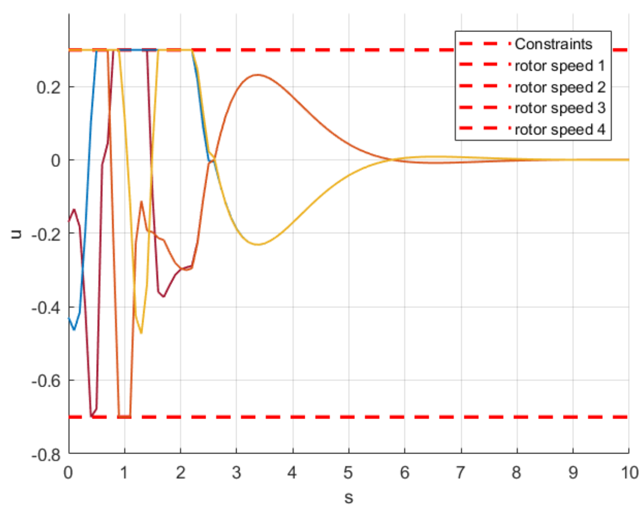
Roll
Pitch
Roll ref
Pitch ref
Constraints
deg
s

(a)



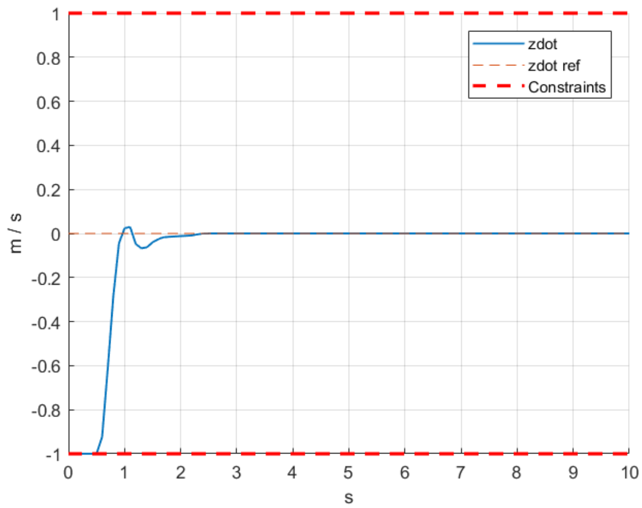
Yaw
Yaw ref
deg
s

(b)

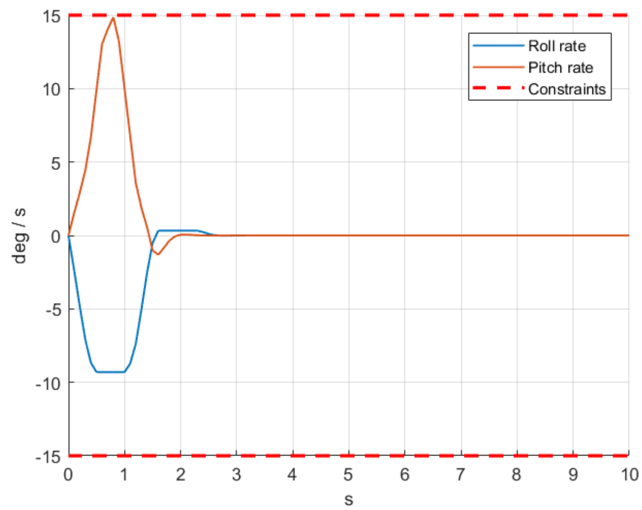


Constraints
rotor speed 1
rotor speed 2
rotor speed 3
rotor speed 4
u
s

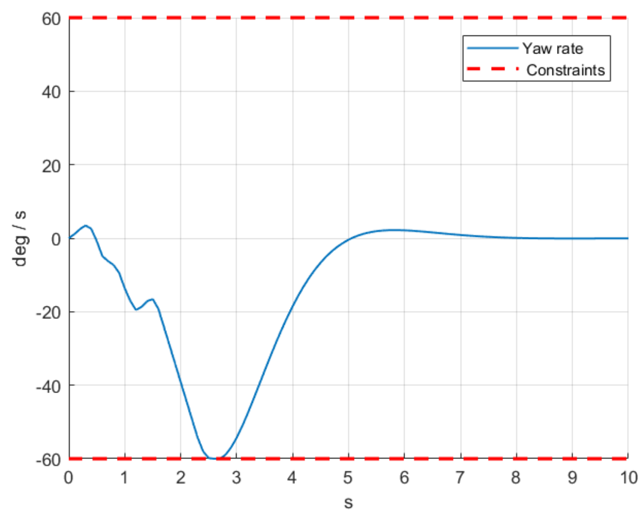
(c)



(d)



(e)



(f)

Figure 1. The simulated states, input signals and constraints for first MPC controller

4. Reference tracking

We consider the reference tracing problem, the set of equilibrium states and inputs **(x_r, u_r)** corresponding to the reference signal:

$$r_1 = [\dot{z}, \alpha, \beta, \gamma]$$

You can do so by solving the system of linear equations:

$$x_r = Ax_r + Bu_r$$
$$r_1 = Cx_r$$

4.1 Interpretation of the solution (x_r, u_r) to system above for arbitrary r_1

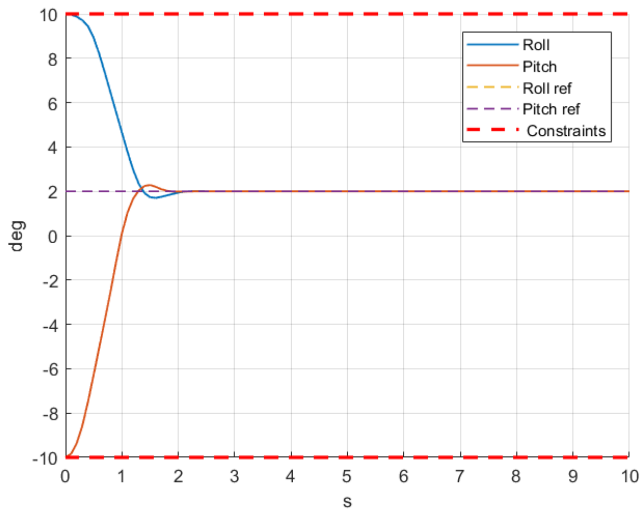
We solved the equations above, the equilibrium states and inputs (x_r, u_r) is:

$$x_r = (\dot{z}_1, \alpha_1, \beta_1, \gamma_1, 0, 0, 0) = (r_1, 0, 0, 0)$$
$$u_r = (0, 0, 0, 0)$$

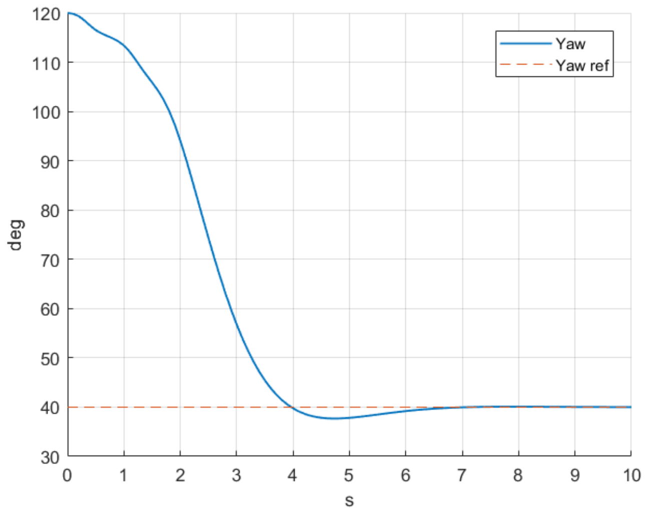
It means that the system can converge to the equilibrium point which is defined by given arbitrary reference signals. At the hovering state, 3 angle rates are 0 which means it's stay without turning around. And u is 0 which means input signal reaches $\boldsymbol{u_s}$.

4.2 Plots of the response to a constant reference signal

In this case, we set the reference signal r=(0.5,2pi/180,2pi/180,40)[rad] and obtained following results.

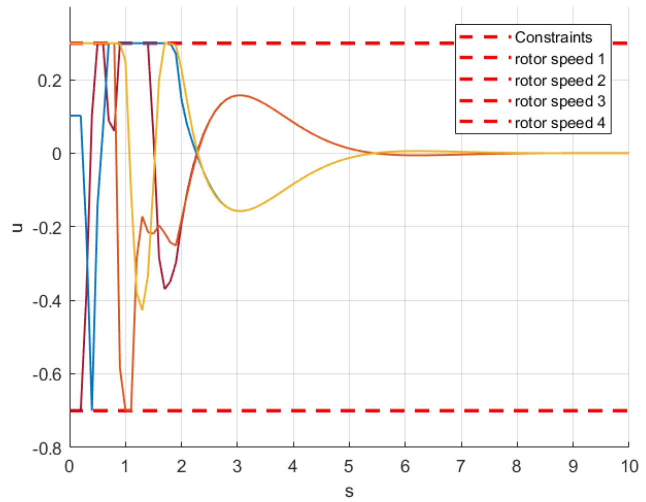


(a)



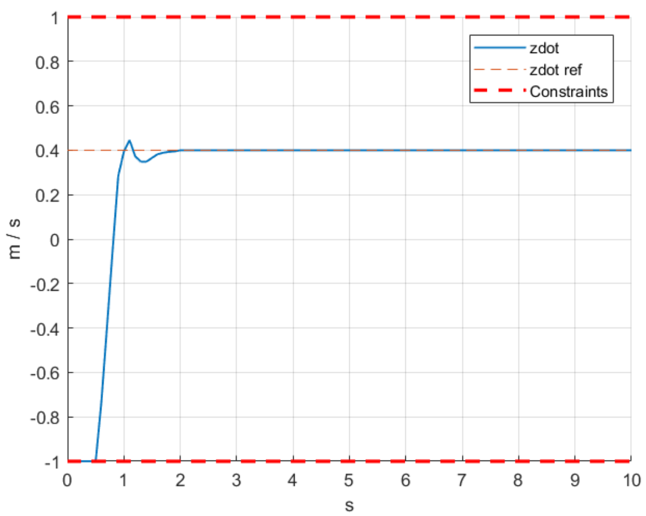
Yaw
Yaw ref
deg
s

(b)



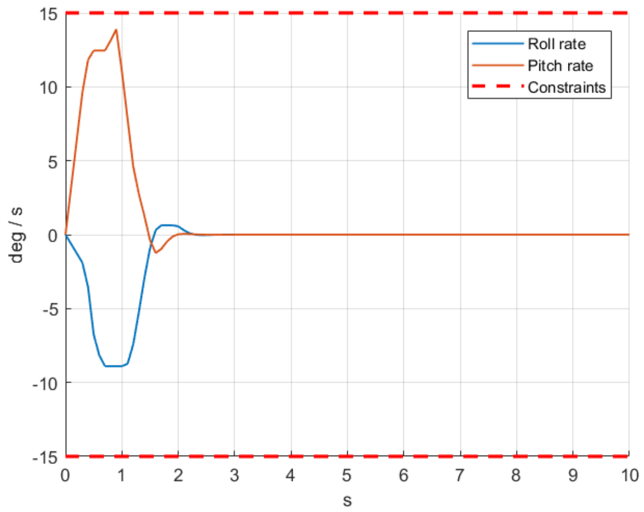
Constraints
rotor speed 1
rotor speed 2
rotor speed 3
rotor speed 4
u
s

(c)

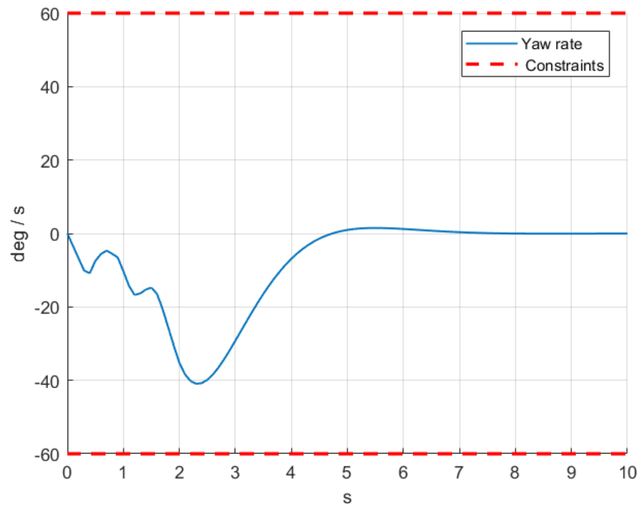


zdot
zdot ref
Constraints
m / s
s

(d)



(e)



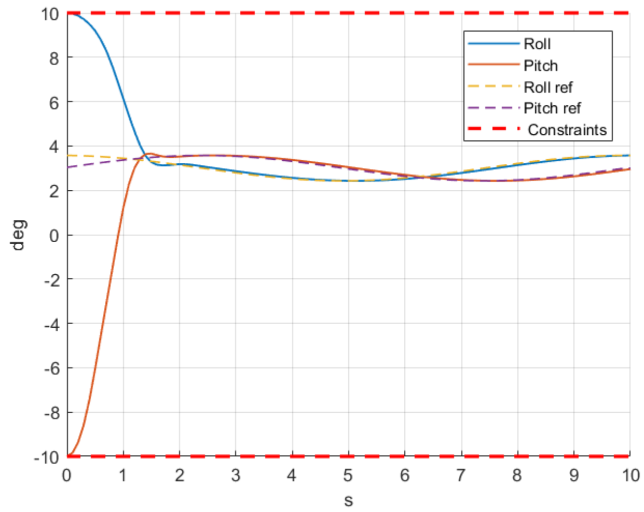
(f)

Figure 2. The simulated states, input signals and constraints for constant reference tracking

We can verify that all the states converge to the reference.

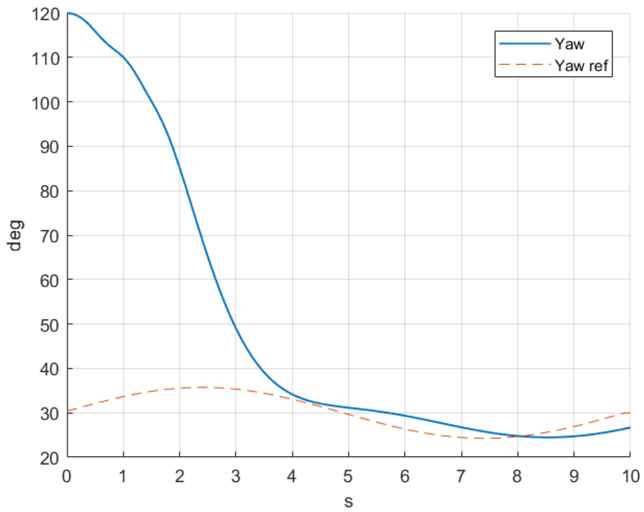
4.3 Plots of the response to a slowly varying reference signal

In this case, we use the same reference signal r=(0.5,2pi/180,2pi/180,40) and we add a small sin/cos signal to each element of reference signal. And we can obtain following results.



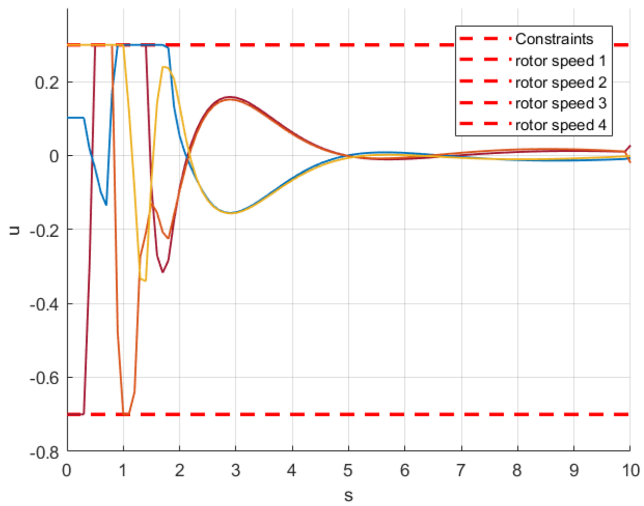
Roll
Pitch
Roll ref
Pitch ref
Constraints
deg
s

(a)



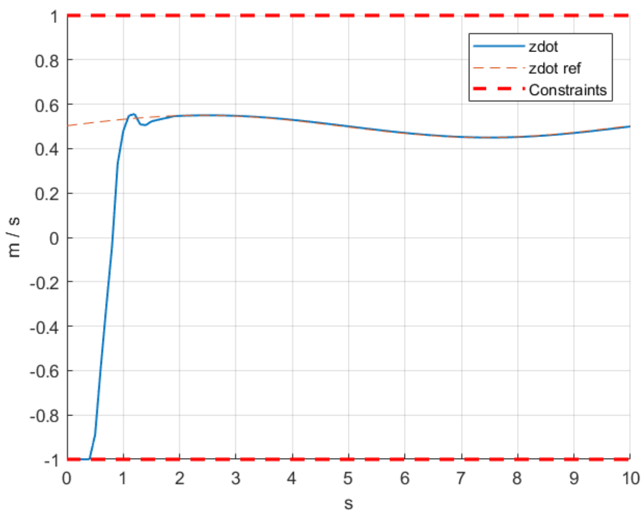
Yaw
Yaw ref
deg
s

(b)



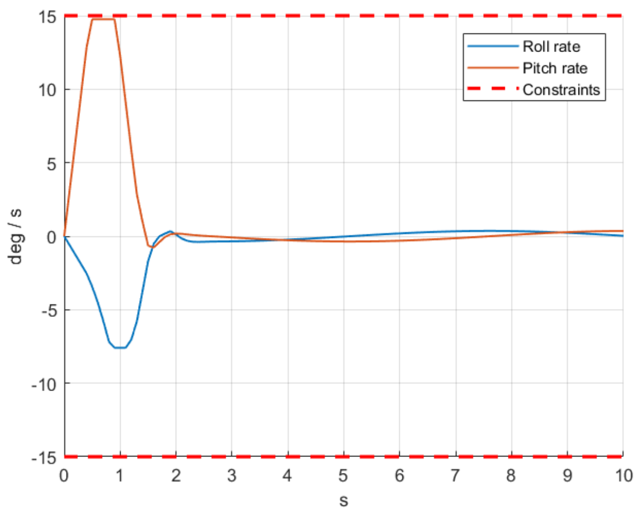
Constraints
rotor speed 1
rotor speed 2
rotor speed 3
rotor speed 4
u
s

(c)



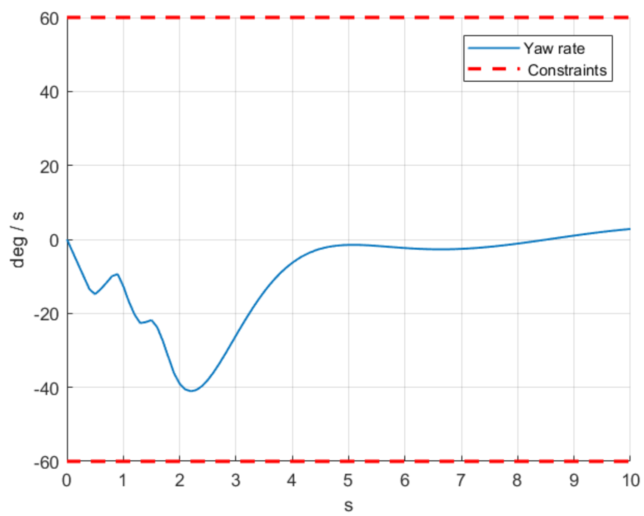
zdot
zdot ref
Constraints
m / s
s

(d)



Roll rate
Pitch rate
Constraints
deg / s
s

(e)



Yaw rate
Constraints
deg / s
s

(f)

Figure 3. The simulated states, input signals and constraints for varying reference tracking

In this case, we can verify that all the states except yaw angle are perfectly track the time varying reference signal since the settling time of yaw angle is a little bigger than others'.

5. First simulation of the nonlinear model

Now we try out your controller on the nonlinear model with simulink. And we obtained following result.

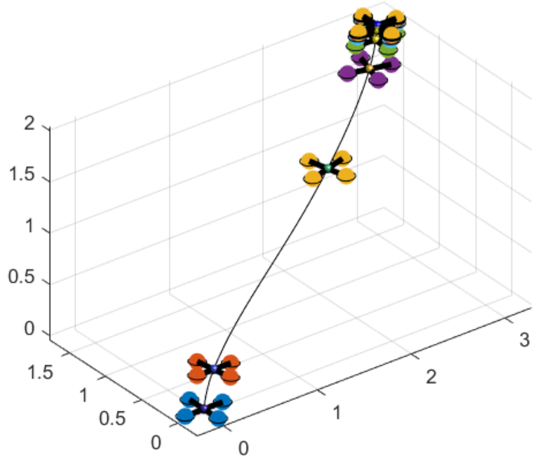
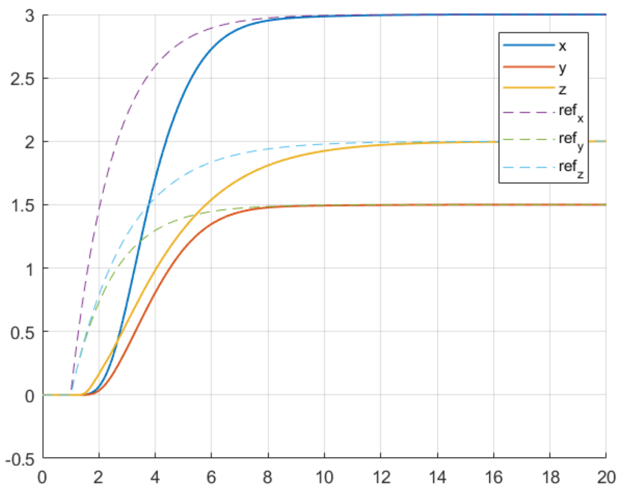
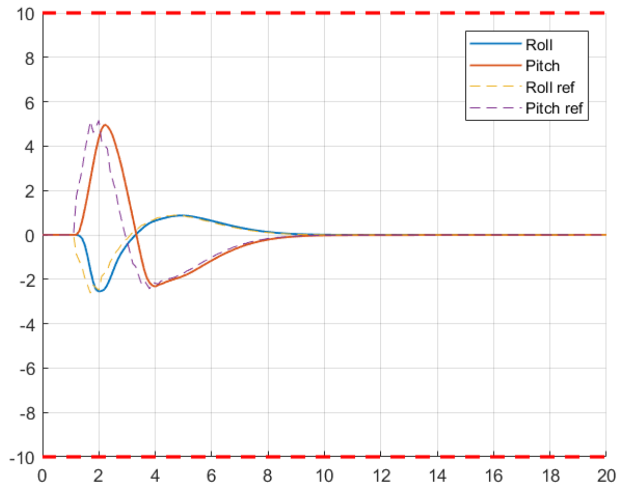


Figure 4. The trajectory of nonlinear model

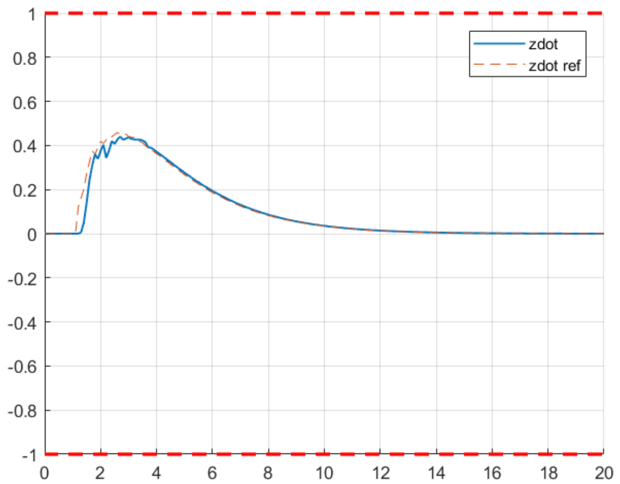


(a)



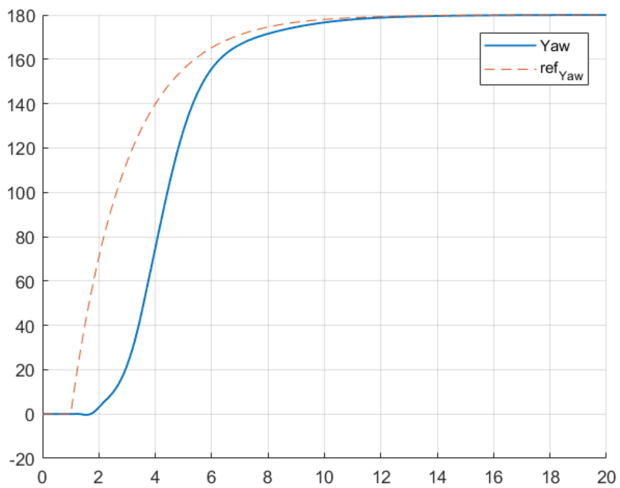
Roll
Pitch
Roll ref
Pitch ref

(b)



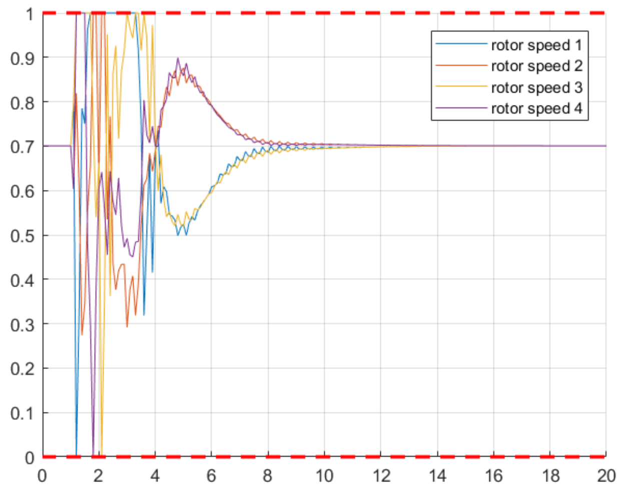
zdot
zdot ref

(c)



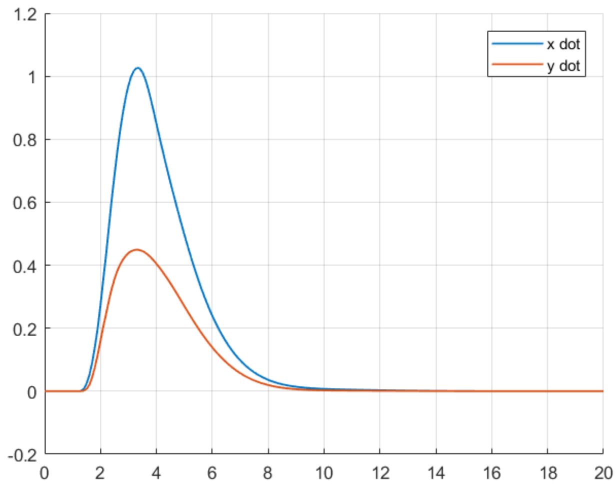
Yaw
ref_{Yaw}

(d)



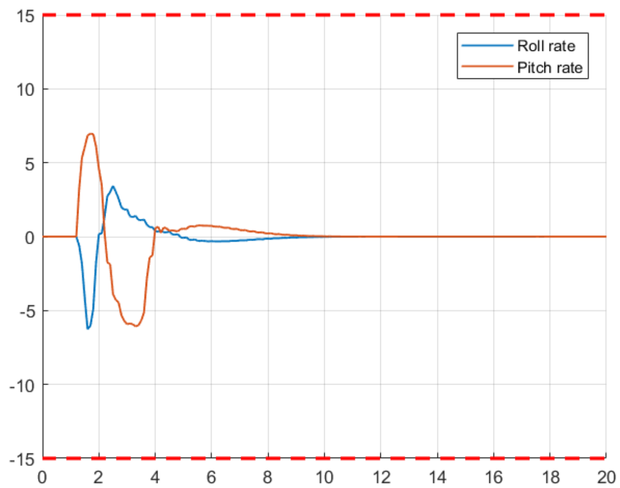
rotor speed 1
rotor speed 2
rotor speed 3
rotor speed 4

(e)



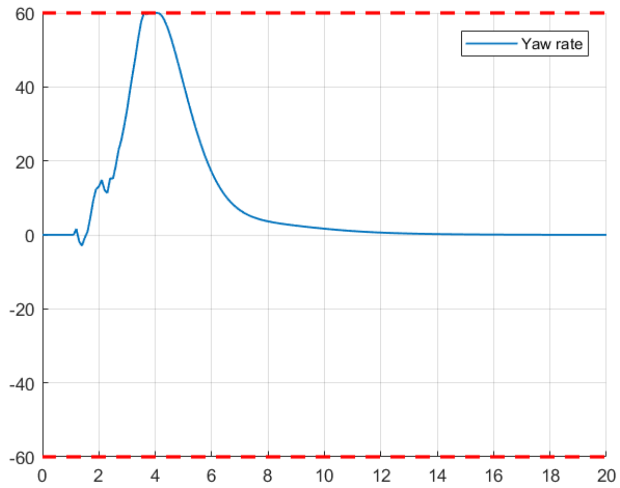
x dot
y dot

(f)



Roll rate
Pitch rate

(g)



(h)

Figure 5. The related tracking states, input signals of nonlinear model

6. Offset free MPC

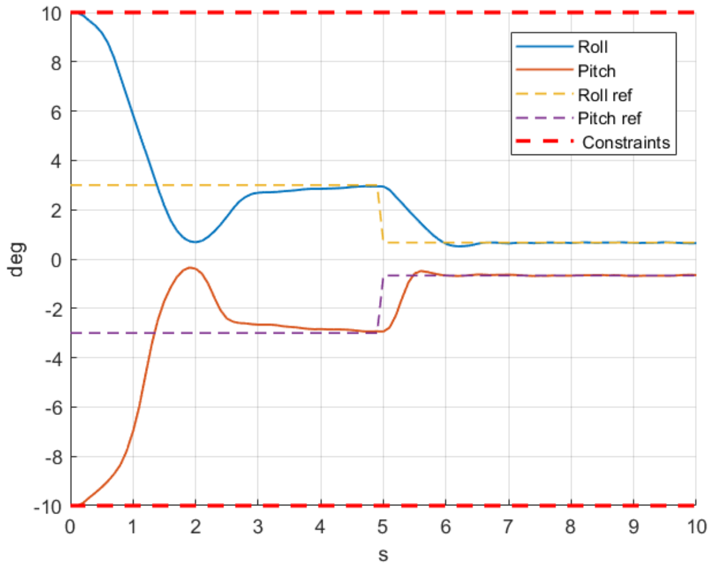
The nonlinear model is subject various disturbances. We consider all these disturbances can be packed into a single disturbance (random) variable ***d***. In this section, we designed an observer to estimate the mean value of disturbance and make the error converge to zero. If you get an infeasible problem, just re-run this section several times.

6.1 Motivation for the choice of the estimation error dynamics

Here we use poles place method to design the observer. In order to estimate error dynamics, we want to converge the error to its mean value. We should set the poles locate in the unit circle (Discrete system), and we make the 14 poles linearly located in [0.88 0.91] such that the error can converge faster. In order to avoid infeasible problem, we tune slightly the parameters of Q matrix.

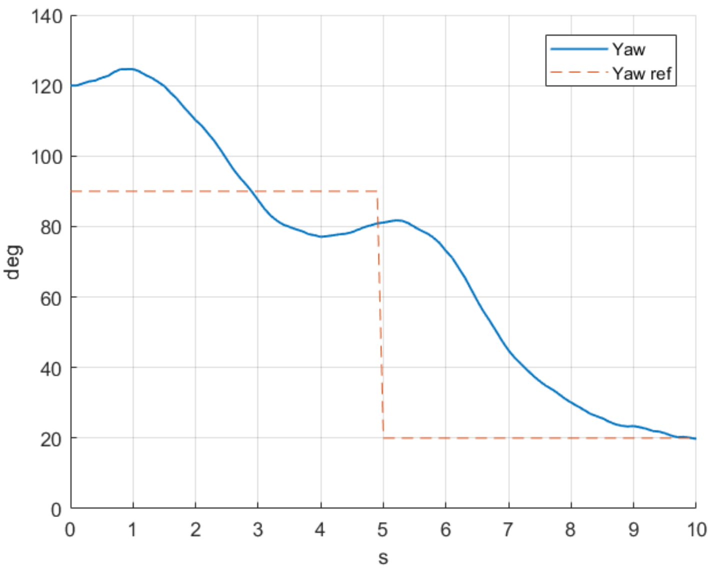
6.2 Step reference tracking plots in the presence of disturbance

We use two reference signals to remap a new step-like signal. Then we obtain the following step response.



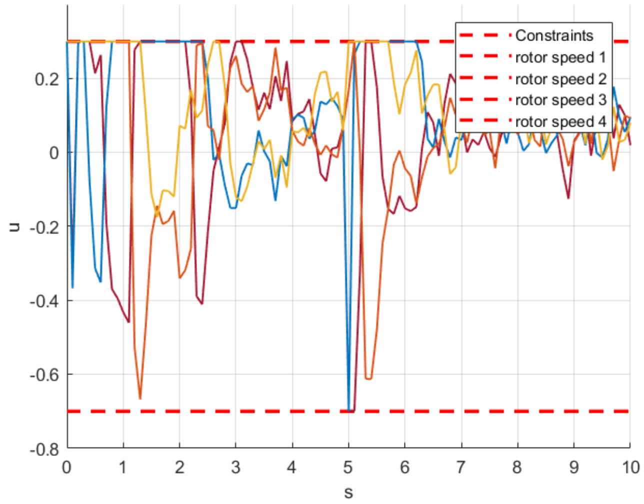
10
8
6
4
2
0
-2
-4
-6
-8
-10
deg
0
1
2
3
4
5
6
7
8
9
10
s
Roll
Pitch
Roll ref
Pitch ref
Constraints

(a)

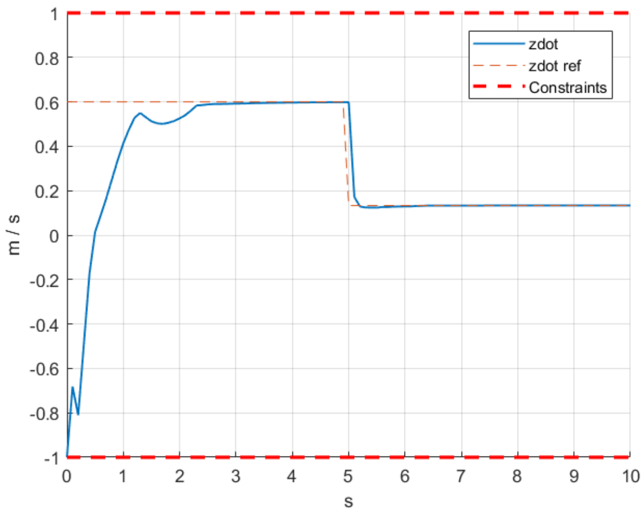


140
120
100
80
60
40
20
0
deg
0
1
2
3
4
5
6
7
8
9
10
s
Yaw
Yaw ref

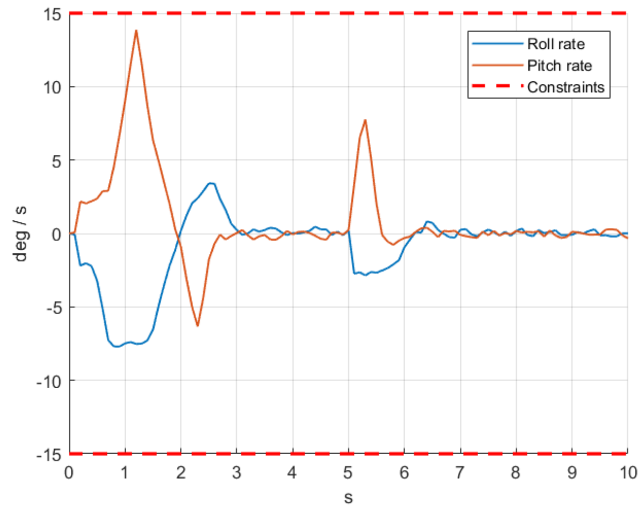
(b)


Constraints
rotor speed 1
rotor speed 2
rotor speed 3
rotor speed 4
u
s

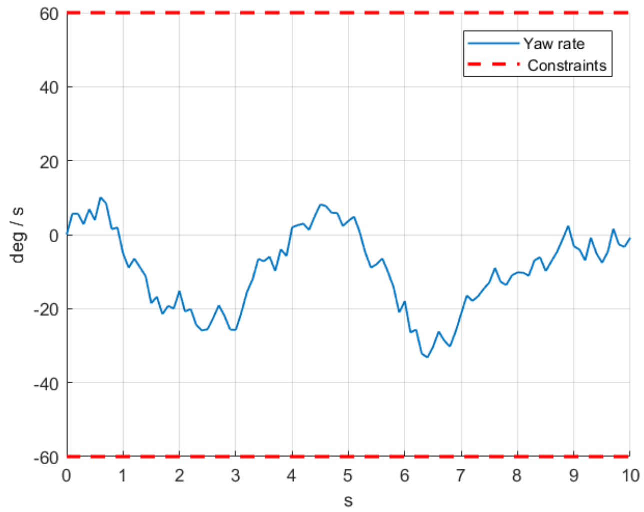
(c)


zdot
zdot ref
Constraints
m / s
s

(d)

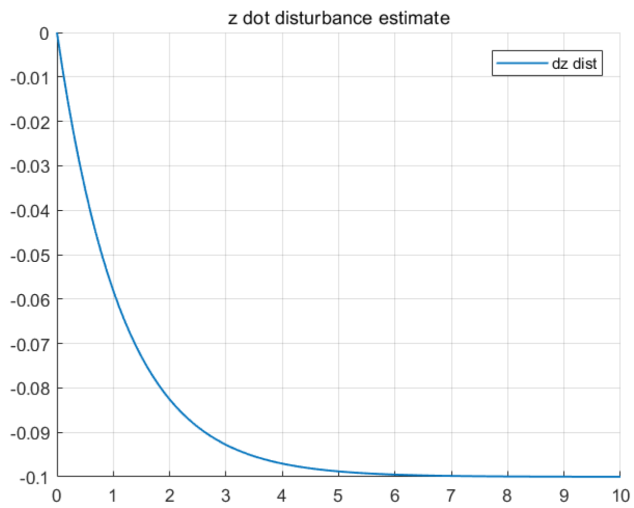

Roll rate
Pitch rate
Constraints
deg / s
s

(e)



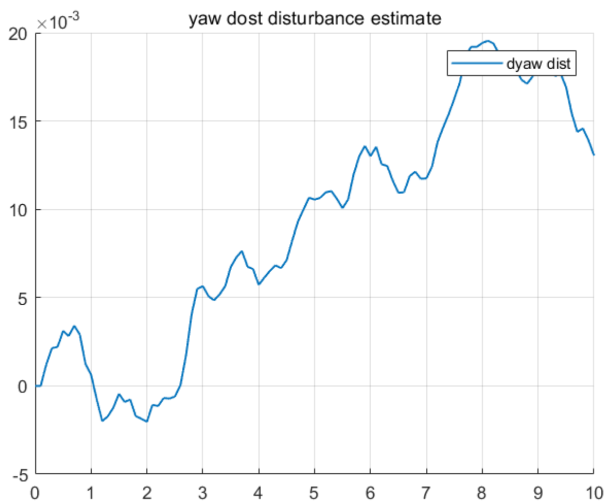
60
40
20
0
-20
-40
-60
deg / s
Yaw rate
Constraints
0
1
2
3
4
5
6
7
8
9
10
s

(f)



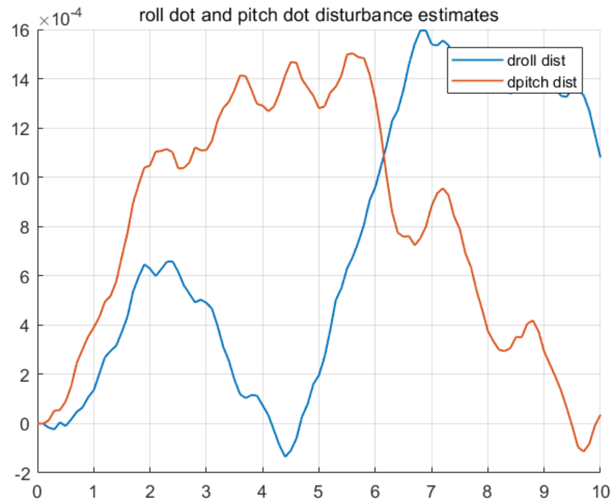
z dot disturbance estimate
dz dist
0
-0.01
-0.02
-0.03
-0.04
-0.05
-0.06
-0.07
-0.08
-0.09
-0.1
0
1
2
3
4
5
6
7
8
9
10

(g)



$\times 10^{-3}$
yaw dost disturbance estimate
dyaw dist
20
15
10
5
0
-5
0
1
2
3
4
5
6
7
8
9
10

(h)



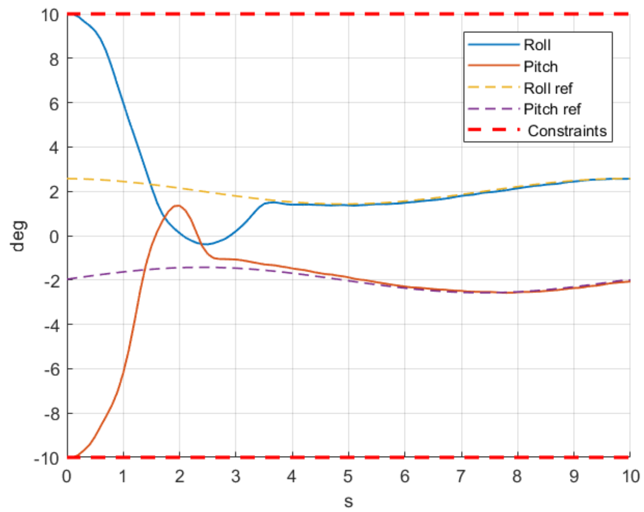
(i)

Figure 6. Step reference tracking of states with estimated disturbance

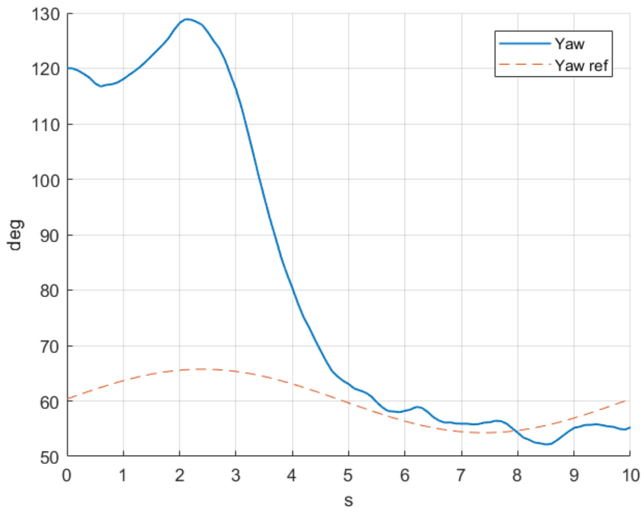
We can verify that our controller can track the step-like reference with small settling time. At the same time, we can estimate the mean value of disturbance of Z dot is -0.1. While for other states, we cannot obtain a clear value of disturbance since the mean value is much smaller than the absolute value.

6.3 Slowly-varying reference tracking plots in the presence of disturbance

We again injected the same slowly varying reference to our system with a noise disturbance.

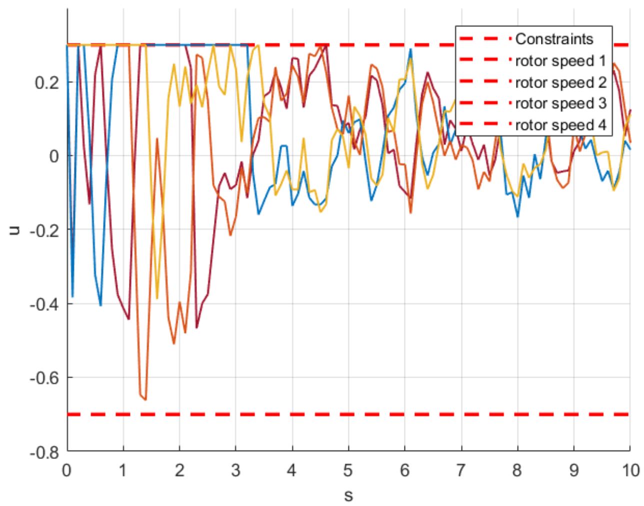


(a)



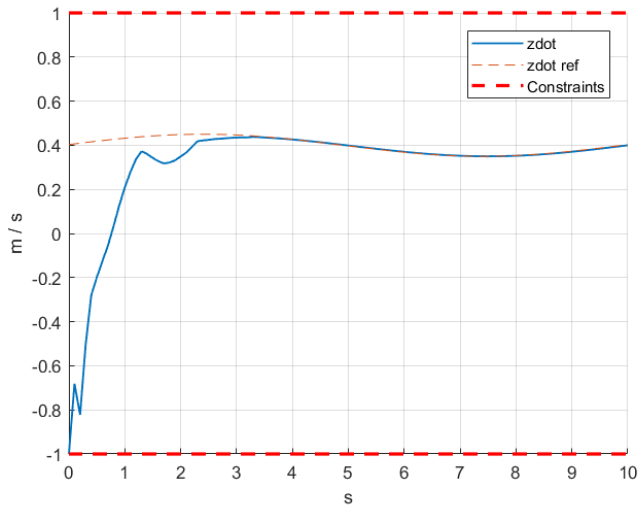
Yaw
Yaw ref
deg
s

(b)



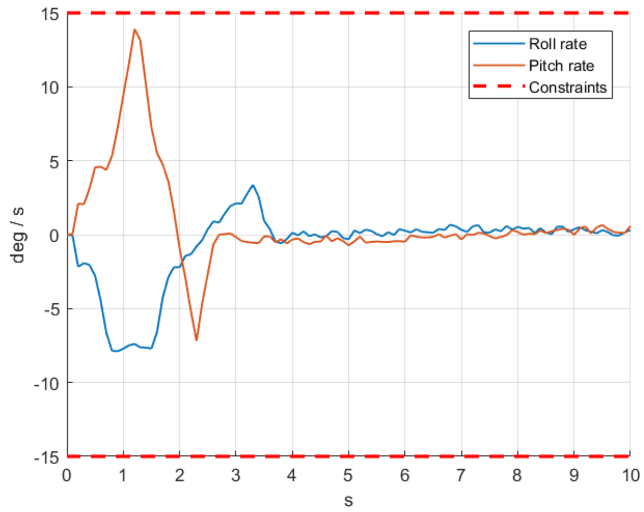
Constraints
rotor speed 1
rotor speed 2
rotor speed 3
rotor speed 4
u
s

(c)



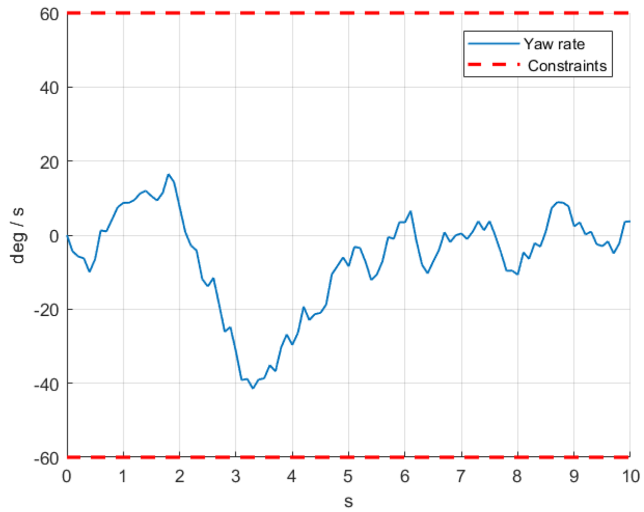
zdot
zdot ref
Constraints
m / s
s

(d)



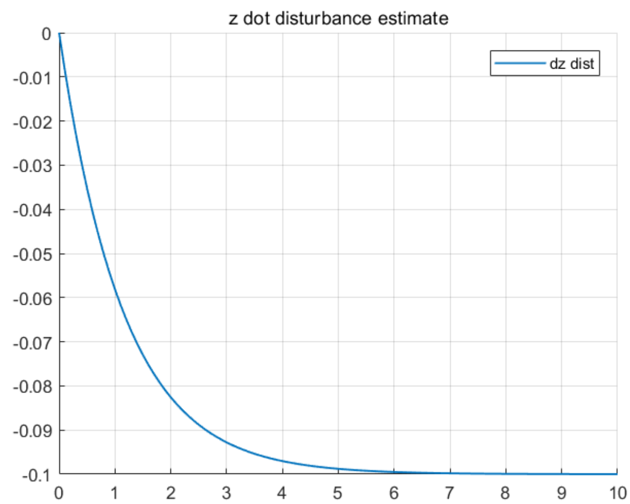
Roll rate
Pitch rate
Constraints
deg / s
s

(e)



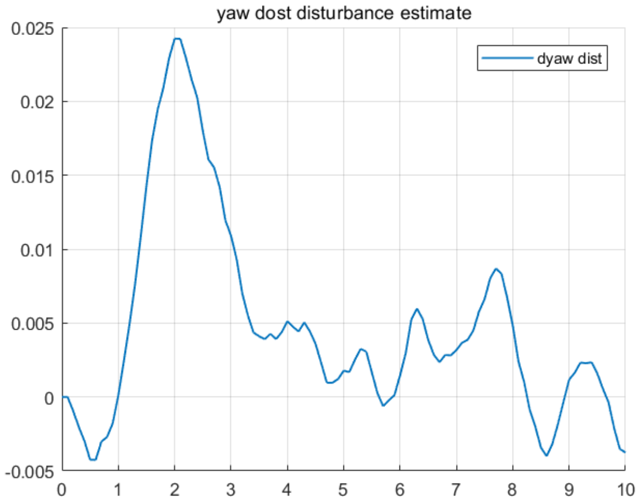
Yaw rate
Constraints
deg / s
s

(f)

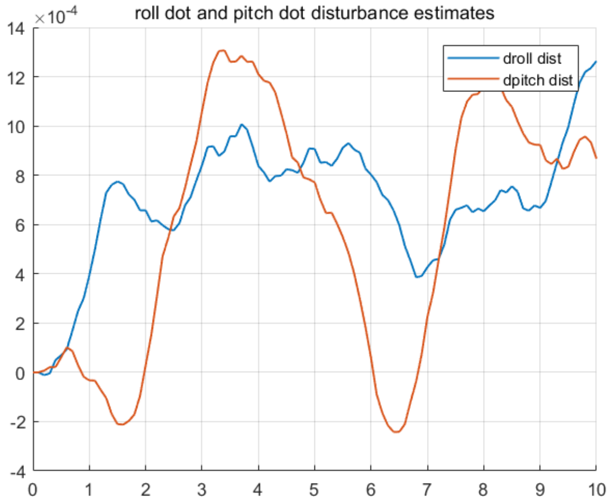


z dot disturbance estimate
dz dist

(g)



(h)



(i)

Figure 7. Slowly-varying reference tracking of states with estimated disturbance

We verify again that our controller can track the slowly varying reference. And we can estimate the mean value of disturbance of Z dot is -0.1.

7. Simulations on the nonlinear model

We turn to the nonlinear model again with the same configuration. There are two reference signals (step and hexagon) we need to track. We can verify the tracking by the following figures.

7.1 Plots of the reference tracking of a step signal

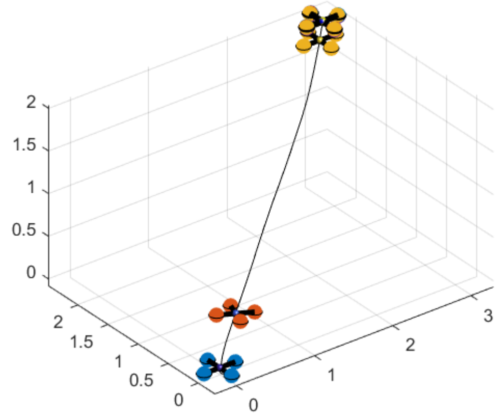
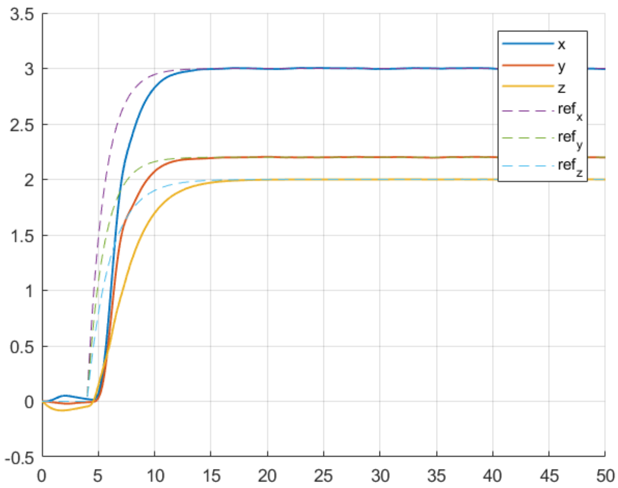
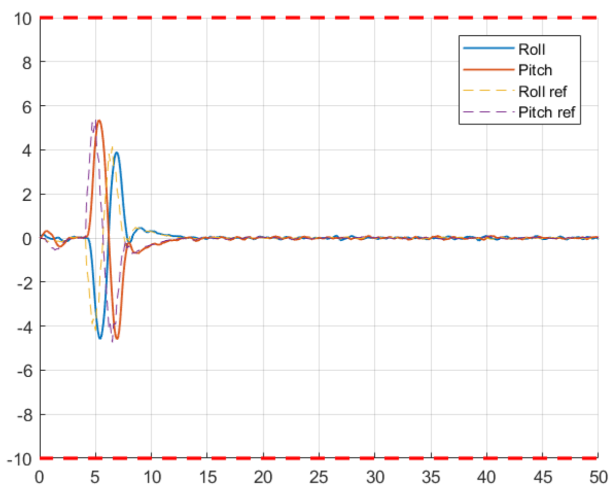


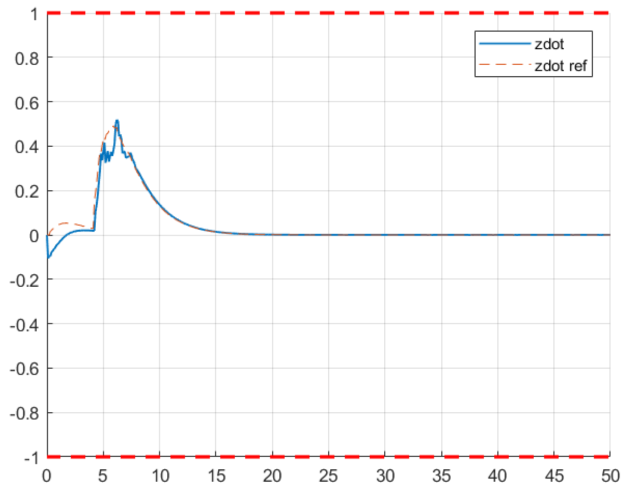
Figure 8. Trajectory of a step tracking with disturbance



(a)

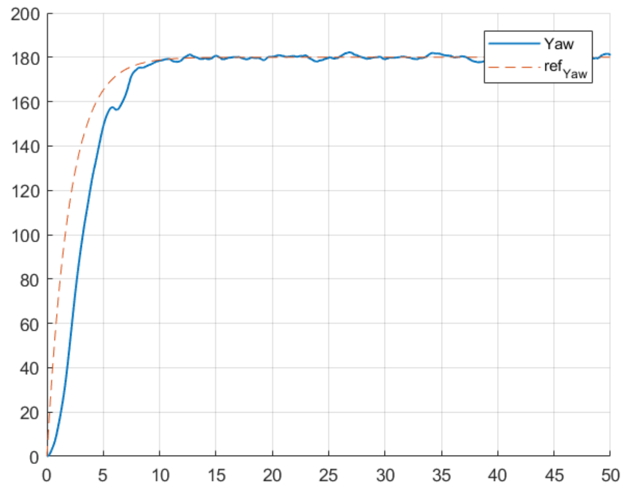


(b)



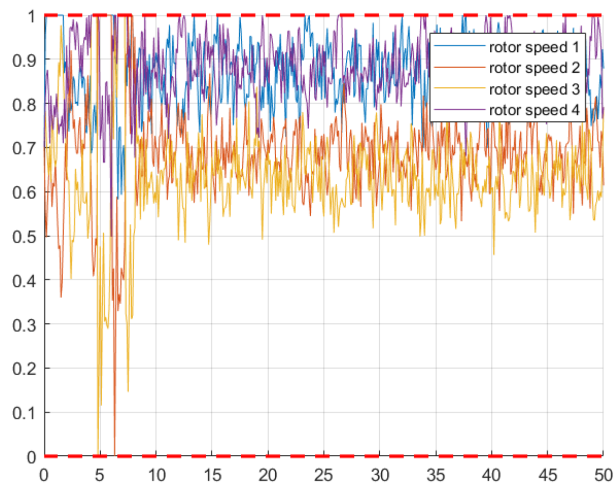
zdot
zdot ref
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
5
10
15
20
25
30
35
40
45
50

(c)



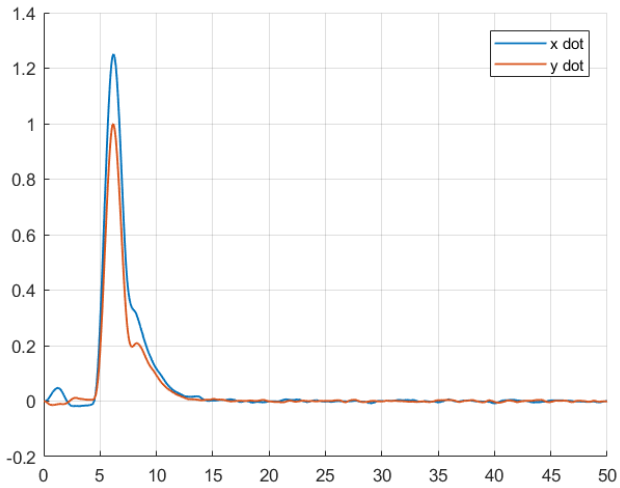
Yaw
ref_{Yaw}
200
180
160
140
120
100
80
60
40
20
0
0
5
10
15
20
25
30
35
40
45
50

(d)

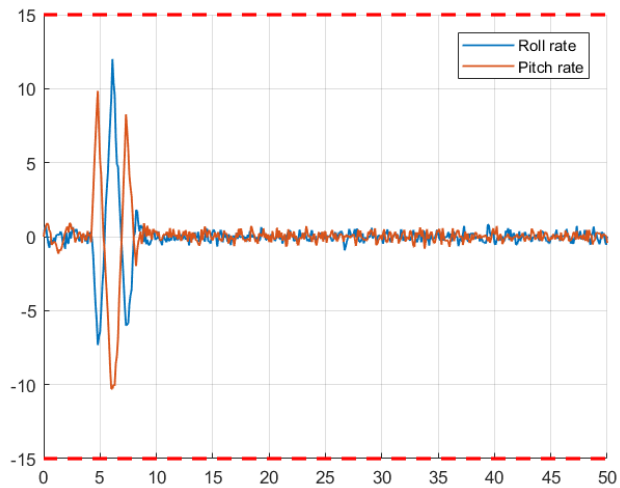


rotor speed 1
rotor speed 2
rotor speed 3
rotor speed 4
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
5
10
15
20
25
30
35
40
45
50

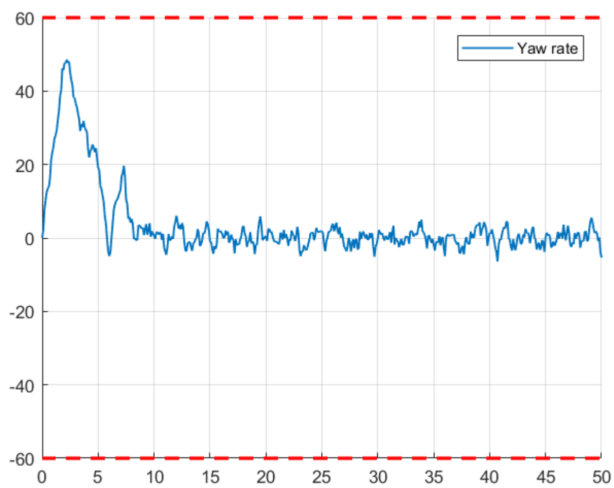
(e)


x dot
y dot
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0
5
10
15
20
25
30
35
40
45
50

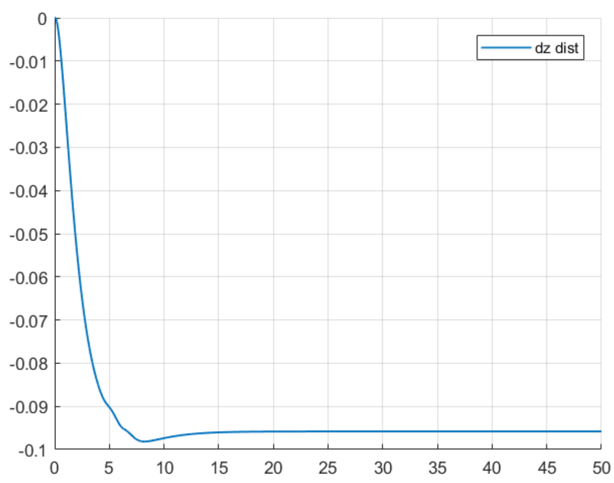
(f)


Roll rate
Pitch rate
15
10
5
0
-5
-10
-15
0
5
10
15
20
25
30
35
40
45
50

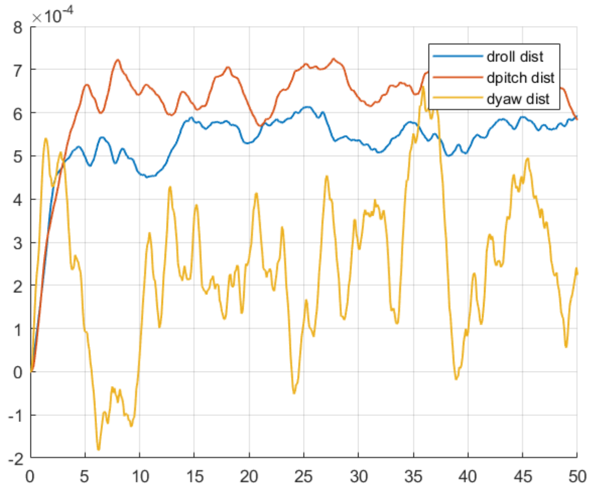
(g)


Yaw rate
60
40
20
0
-20
-40
-60
0
5
10
15
20
25
30
35
40
45
50

(h)



(i)



(j)

Figure 9. Step signal reference tracking of states with estimated disturbance

7.2 Plots of the reference tracking of the hexagon signal

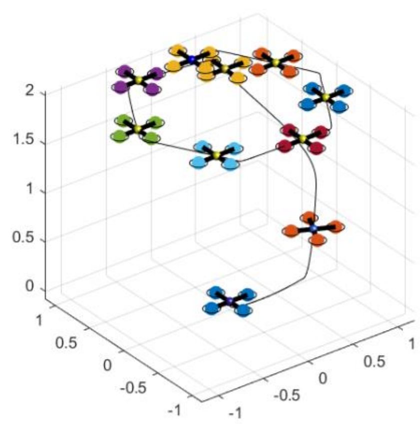
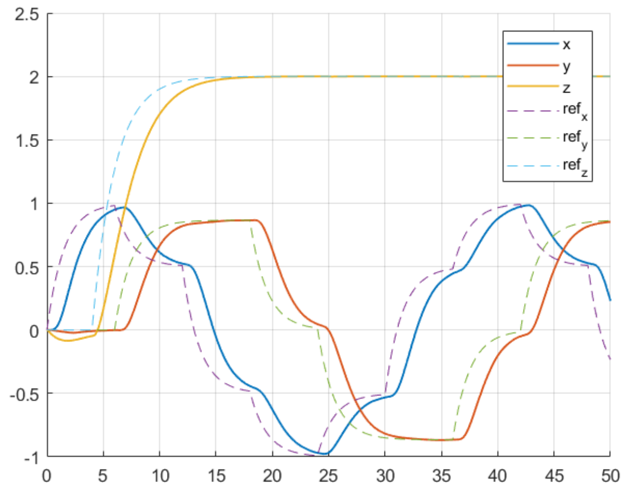
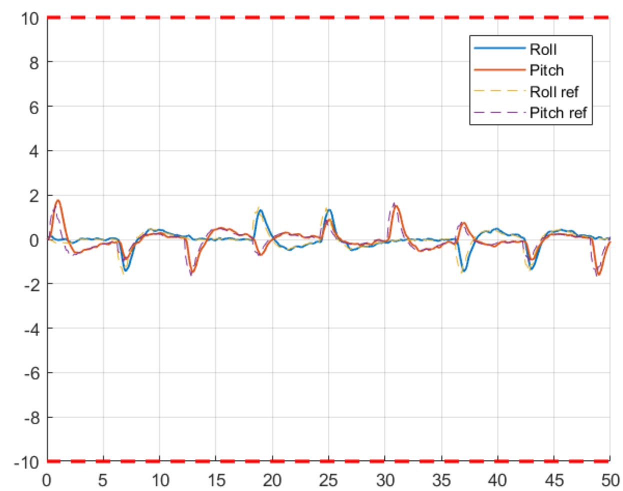


Figure 10. Trajectory of a hexagon tracking with disturbance



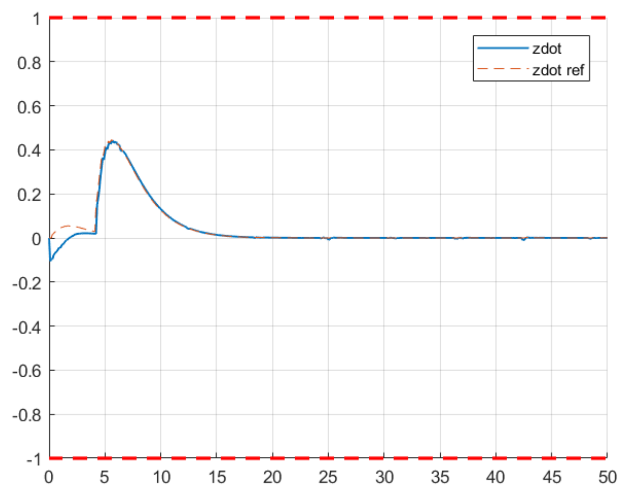
x
y
z
ref
x
ref
y
ref
z

(a)



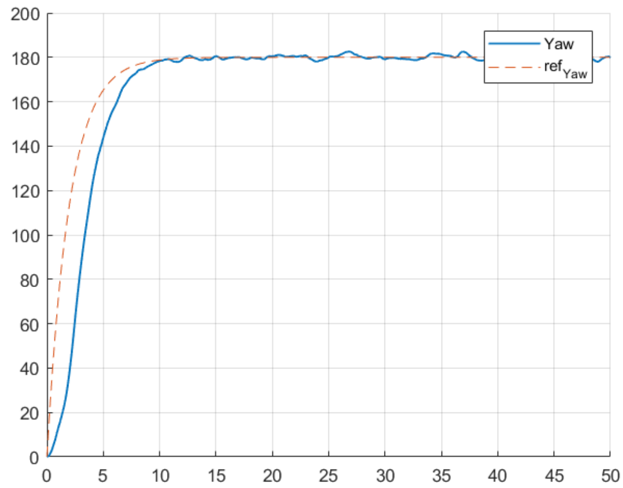
Roll
Pitch
Roll ref
Pitch ref

(b)

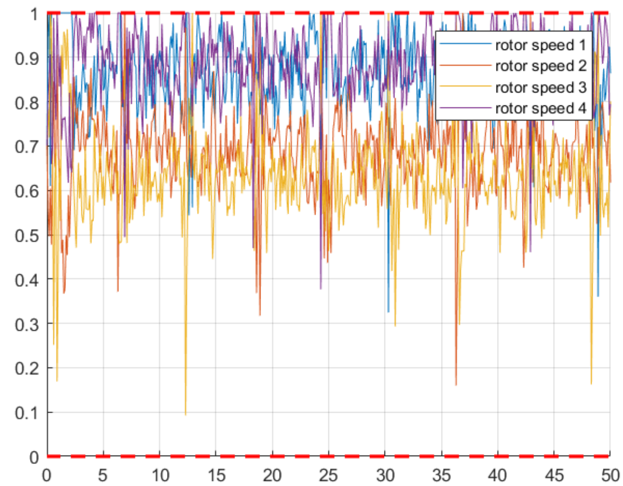


zdot
zdot ref

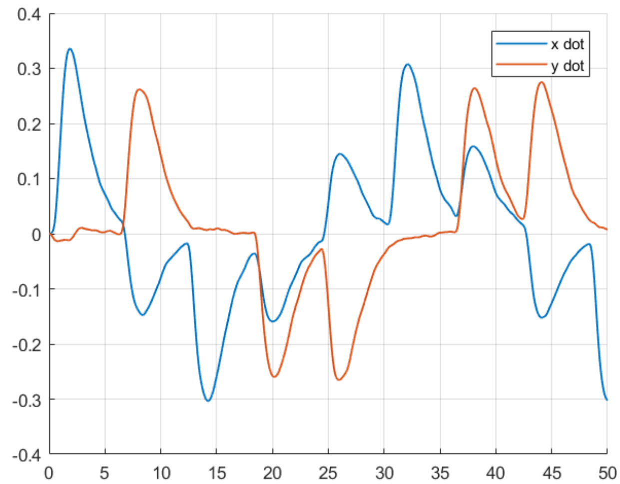
(c)


Yaw
ref$_{Yaw}$
200
180
160
140
120
100
80
60
40
20
0
0
5
10
15
20
25
30
35
40
45
50

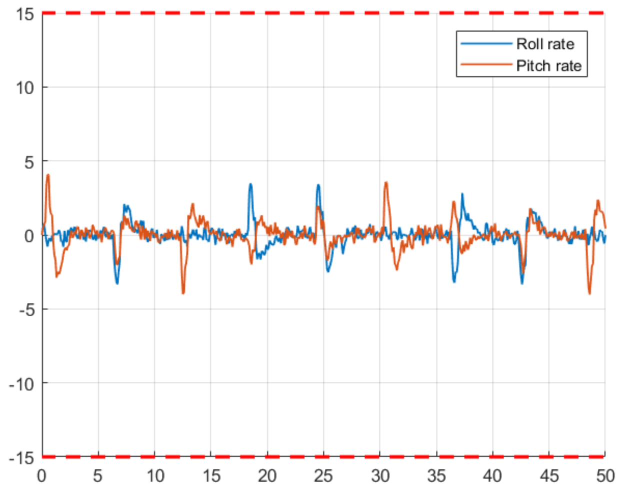
(d)


rotor speed 1
rotor speed 2
rotor speed 3
rotor speed 4
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
5
10
15
20
25
30
35
40
45
50

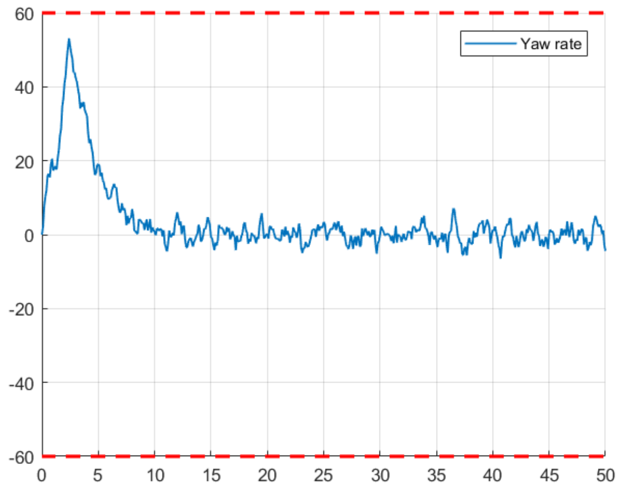
(e)


x dot
y dot
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
0
5
10
15
20
25
30
35
40
45
50

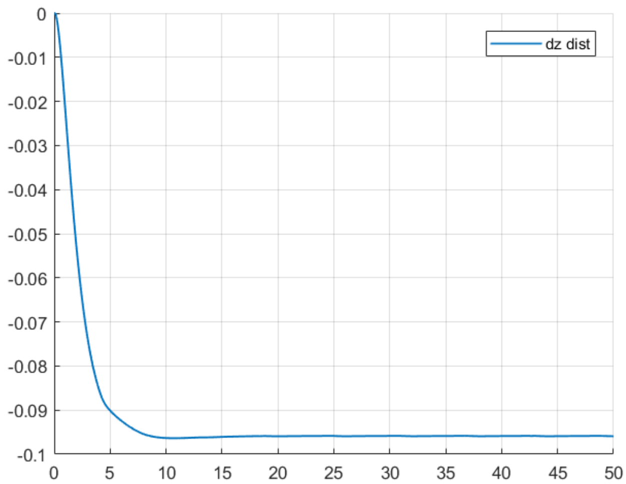
(f)


15
10
5
0
-5
-10
-15
0
5
10
15
20
25
30
35
40
45
50
Roll rate
Pitch rate

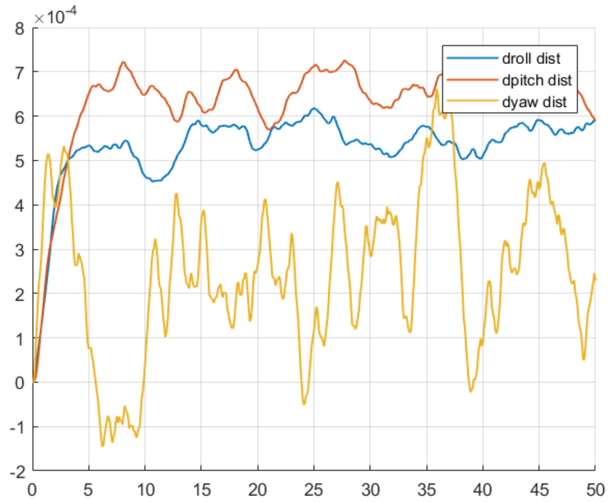
(g)


60
40
20
0
-20
-40
-60
0
5
10
15
20
25
30
35
40
45
50
Yaw rate

(h)


0
-0.01
-0.02
-0.03
-0.04
-0.05
-0.06
-0.07
-0.08
-0.09
-0.1
0
5
10
15
20
25
30
35
40
45
50
dz dist

(i)



(j)

Figure 11. Slowly-varying reference tracking of states with estimated disturbance

7.3 Bonus: plots of the reference tracking of a lemniscate

In this bonus section, we should inject the mathematical expression of a lemniscate as position X, Y reference signals. Then we set the position Z and yaw angle as step signals. The parametric equation of lemniscate of Bernoulli is following:

$$x = \frac{a\sqrt{2}\cos(t)}{\sin^2(t)+1}$$

$$y = \frac{a\sqrt{2}\cos(t)\sin(t)}{\sin^2(t)+1}$$

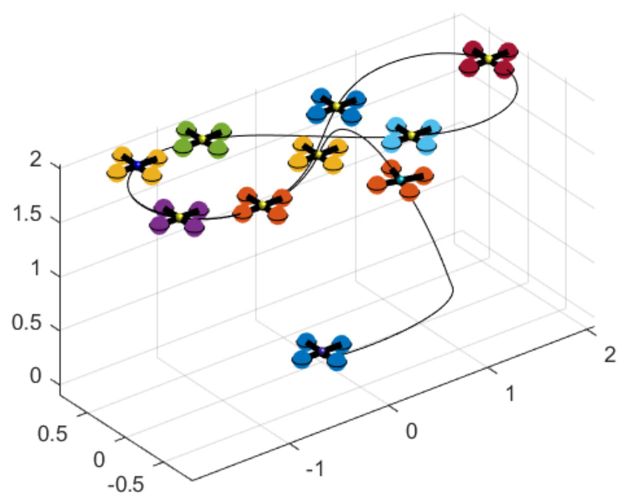
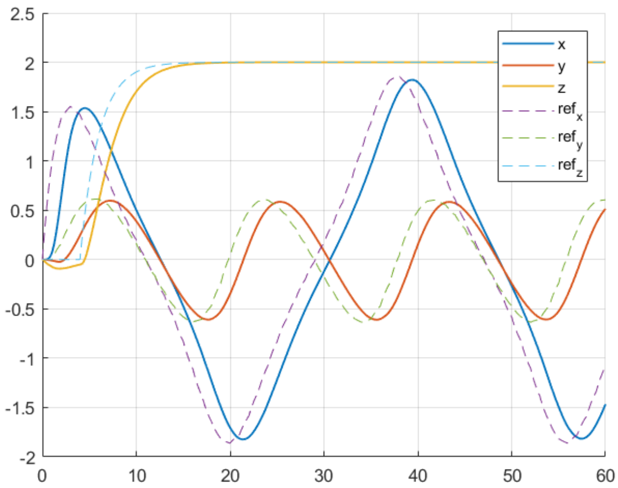
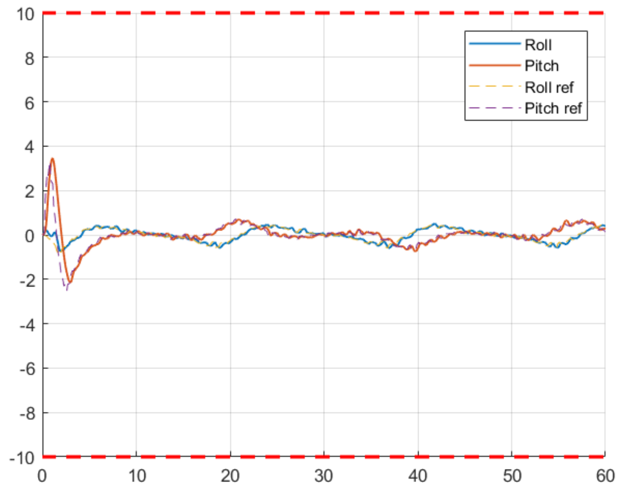


Figure 12. Trajectory of a lemniscate tracking with disturbance



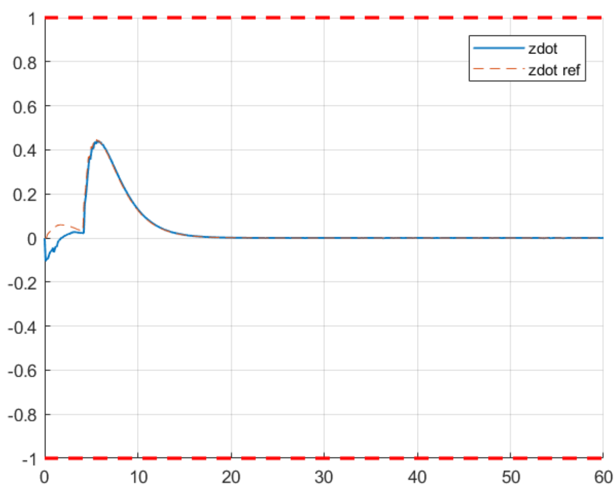
x
y
z
ref$_x$
ref$_y$
ref$_z$

(a)



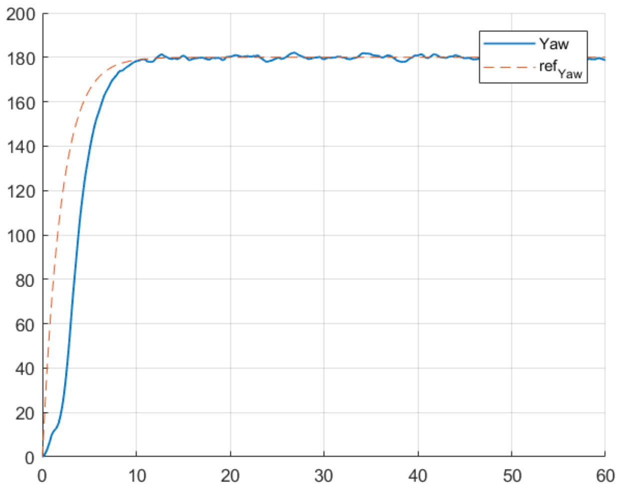
Roll
Pitch
Roll ref
Pitch ref

(b)



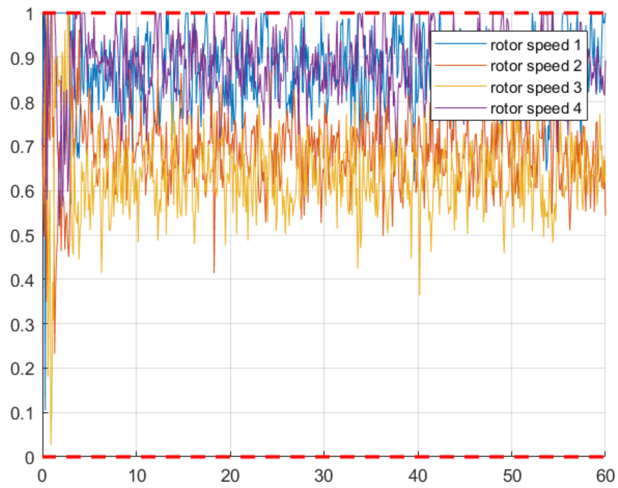
zdot
zdot ref

(c)



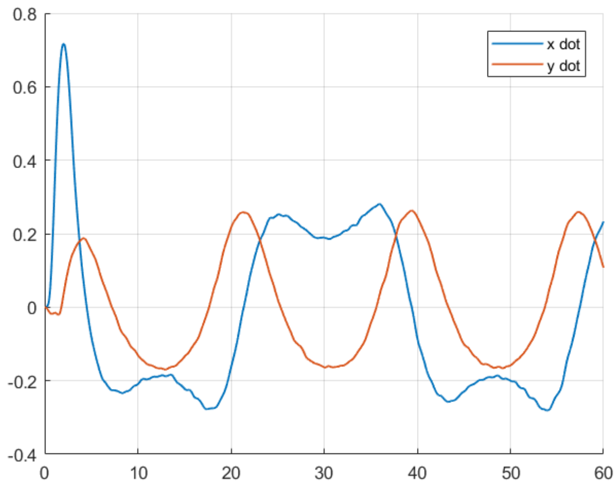
Yaw
ref$_{Yaw}$
200
180
160
140
120
100
80
60
40
20
0
0
10
20
30
40
50
60

(d)



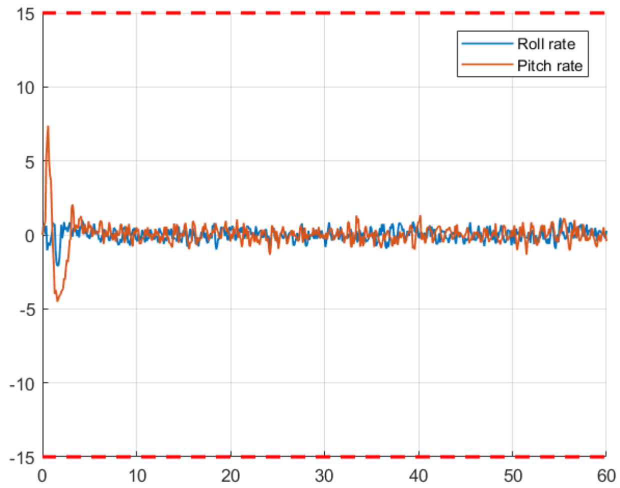
rotor speed 1
rotor speed 2
rotor speed 3
rotor speed 4
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
10
20
30
40
50
60

(e)

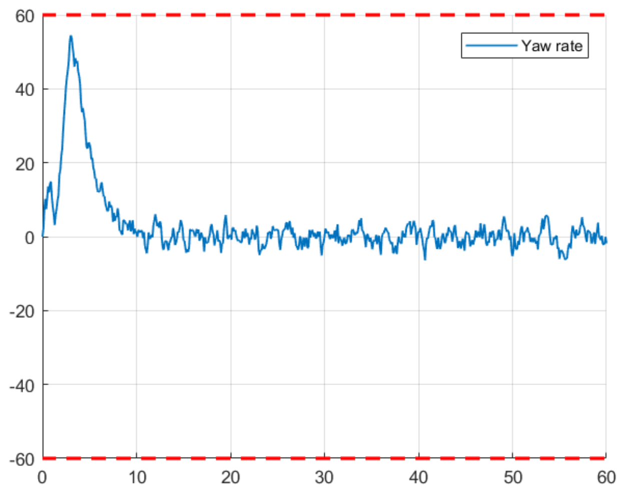


x dot
y dot
0.8
0.6
0.4
0.2
0
-0.2
-0.4
0
10
20
30
40
50
60

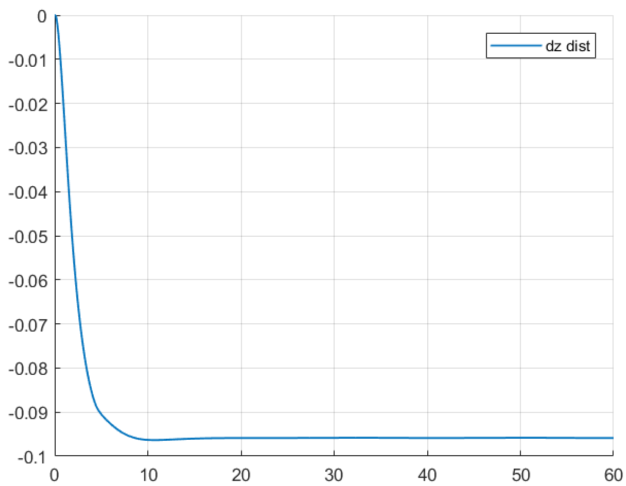
(f)


Roll rate
Pitch rate
15
10
5
0
-5
-10
-15
0
10
20
30
40
50
60

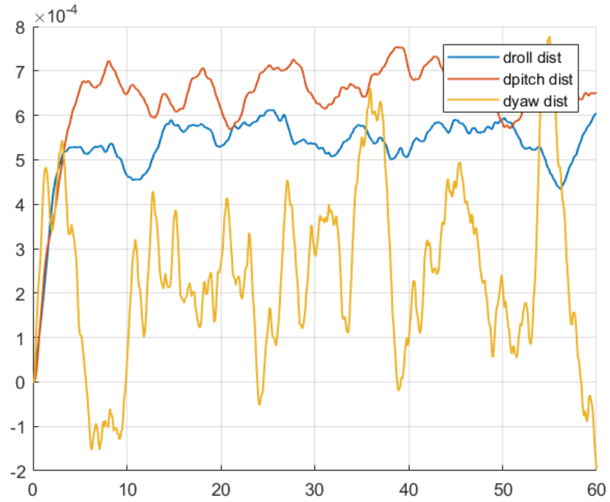
(g)


Yaw rate
60
40
20
0
-20
-40
-60
0
10
20
30
40
50
60

(h)


dz dist
0
-0.01
-0.02
-0.03
-0.04
-0.05
-0.06
-0.07
-0.08
-0.09
-0.1
0
10
20
30
40
50
60

(i)



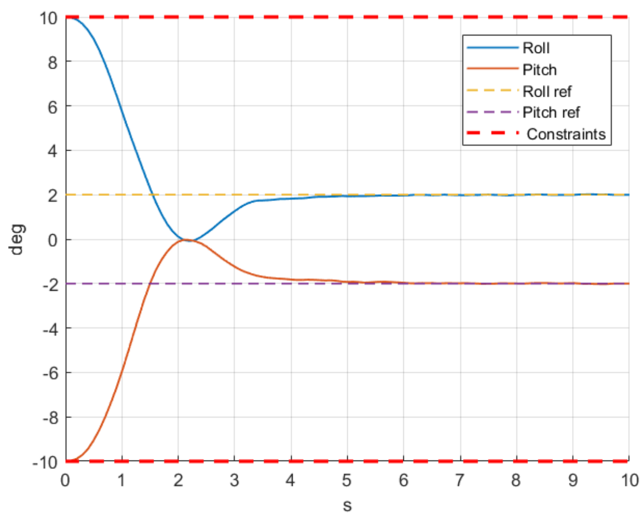
(j)

Figure 13. Lemniscate reference tracking of states with estimated disturbance

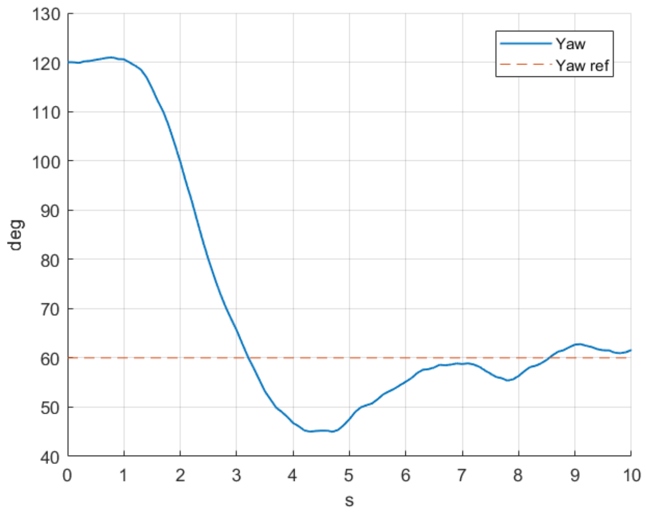
7.4 Bonus: slew rate constraints

In this section, we consider rate constraints on the control inputs which are of the form $|u_{k+1} - u_k| \leq \Delta(tolerance)$. These constraints will limit the change of input signals and deteriorate system's performance in some cases (depends on tolerance). In our controller, if the tolerance smaller than 0.02, the settling time would increase to 4s and the performance is deteriorated significantly.

In practice, we can use soft constraints to improve feasibility. In many cases, some constraints are not crucial, the corresponding state valuables can violate these constraints and then go back to feasible set. On the other hand, if we can guarantee the stability, we can set the predictive horizon N smaller to reduce the constraints.

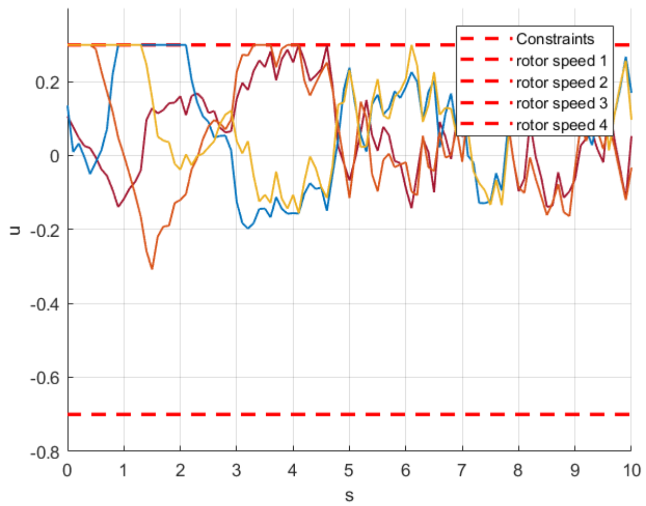


(a)



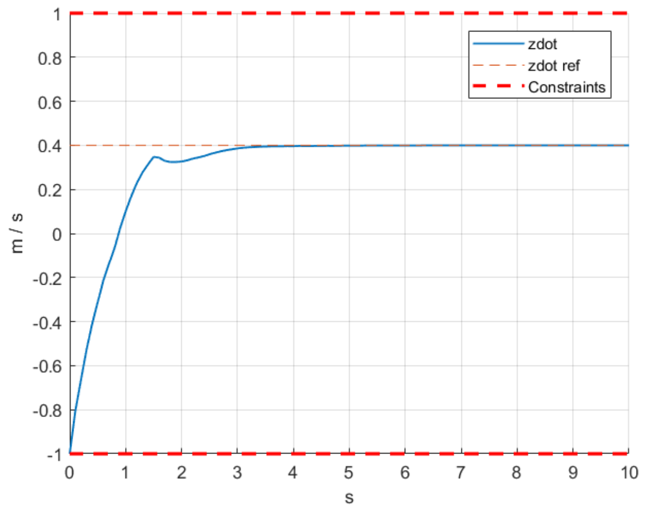
130
120
110
100
90
80
70
60
50
40
deg
0 1 2 3 4 5 6 7 8 9 10
s
Yaw
Yaw ref

(b)



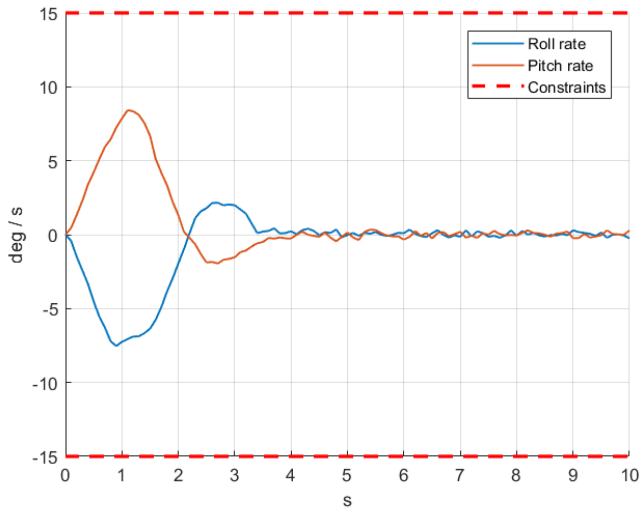
0.2
0
-0.2
-0.4
-0.6
-0.8
u
0 1 2 3 4 5 6 7 8 9 10
s
Constraints
rotor speed 1
rotor speed 2
rotor speed 3
rotor speed 4

(c)



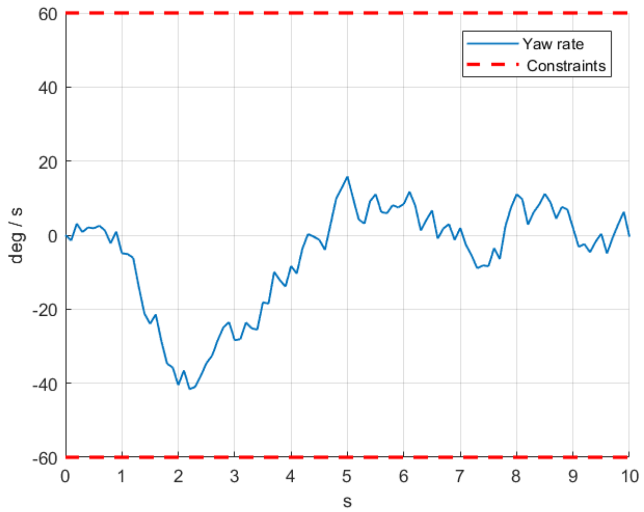
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
m / s
0 1 2 3 4 5 6 7 8 9 10
s
zdot
zdot ref
Constraints

(d)



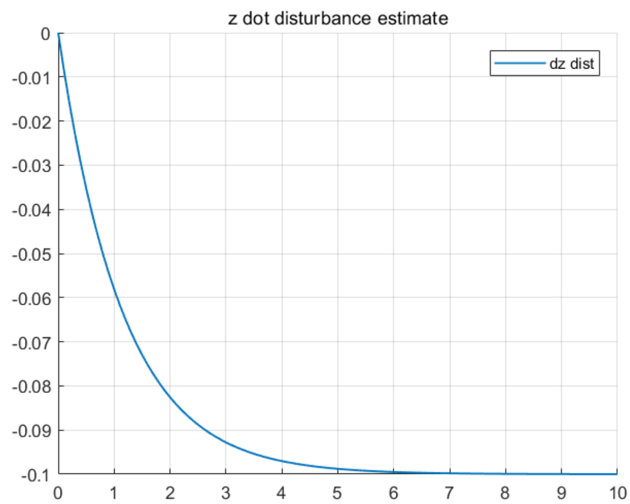
Roll rate
Pitch rate
Constraints
deg / s
s

(e)



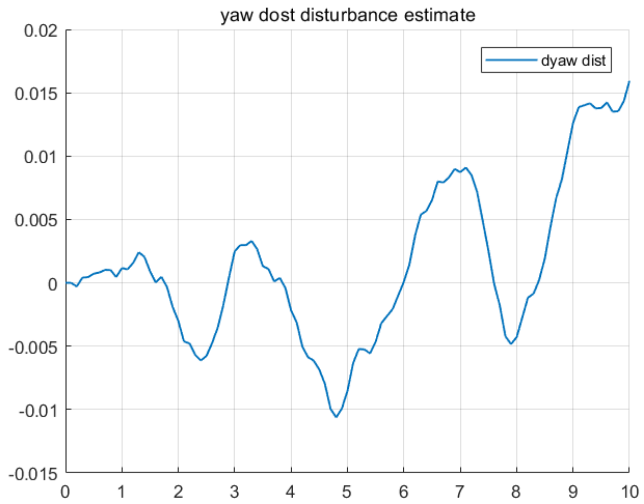
Yaw rate
Constraints
deg / s
s

(f)

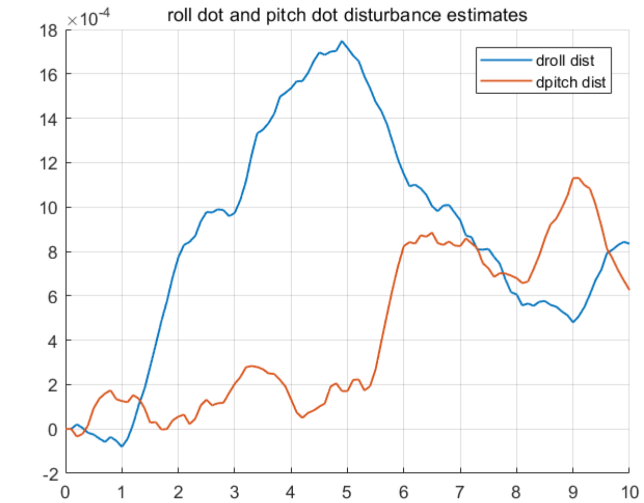


z dot disturbance estimate
dz dist

(g)



(h)



(i)

Figure 14. The related tracking valuables of linear model with disturbance and rate constraints