Closed-Form Barrier Functions for Multi-Agent Ellipsoidal Systems with Uncertain Lagrangian Dynamics

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Abstract—In this paper, we design a decentralized control protocol for the collision avoidance of a multi-agent system, which is comprised of 3D ellipsoidal agents that obey 2nd-order uncertain Lagrangian dynamics. More specifically, we derive a novel closed-form smooth barrier function that resembles a distance metric between 3D ellipsoids and can be used by feedback-based control laws to guarantee inter-agent collision avoidance. Discontinuities and adaptation laws are incorporated in the control protocol to deal with the uncertainties of the dynamic model. The control laws are decentralized, in the sense that each agent uses only local sensing information. Simulation results verify the theoretical findings.

Index Terms—Cooperative control, Decentralized control, Agents-based systems, Robust adaptive control

I. INTRODUCTION

COLLISION avoidance in systems comprised of multiple robotic agents is a crucial safety property that needs to be always achieved. Except for the single-agent case [1], [2], multi-agent collision avoidance is tackled in a variety of works (e.g., [3]–[8]), where the multi-agent system aims for a primary objective (navigation, formation). The majority of the related works considers spherical agents, which provide a straightforward metric for the inter-agent or the agent-to-obstacle distances. However, since the shapes of real robotic vehicles can be far from spherical (e.g., robotic manipulators), that approach can be too conservative and may prevent the agents from fulfilling their primary objectives. Ellipsoids, on the other hand, can approximate more accurately the volume of autonomous agents (see Fig. 1).

The authors in [1], [9], [10] employ diffeomorphisms to transform arbitrarily-shaped obstacles, including ellipsoids, to points. This methodology, however, is not straightforwardly extendable to the case of moving obstacles (i.e., multiple autonomous agents). A point-world transformation of multi-agent systems was taken into account in [11], [12]. As described in [11] though, each agent's transformation deforms the other agents into shapes whose implicit closed-form equation (and hence a suitable distance metric) is not trivial to obtain. The methodology of [9] provides useful insight, where

the volume of each agent is "absorbed" to the other agents via Minkowski sums. The closed-form implicit equation of the resulting shapes, however, although possible to obtain [13], cannot be used to derive an appropriate distance metric in a straightforward way; [14] derives a conservative inter-ellipsoid distance by employing ellipsoid-to-sphere transformations and eigenvalue computations. An arithmetic algorithm that produces velocities for inter-agent elliptical agents is derived in [15], without, however, theoretical guarantees. Optimization-based techniques (e.g., Model Predictive Control), which can be employed for collision avoidance of convex-shaped agents [16], can be too complex to solve, especially in cases where the control must be decentralized and/or complex dynamics are considered. The latter property constitutes another important issue regarding the related literature. In particular, most related works consider simplified single- or double-integrator models, which deviate from the actual dynamics and can lead to performance decline and safety jeopardy.

Barrier functions constitute a suitable tool for expressing objectives like collision avoidance. Originated in optimization, they are continuous functions that diverge to infinity as their argument approaches the boundary of a desired/feasibly region. Barrier Lyapunov-like functions for general control systems can be found in [17], [18], and in [3], [19], [20] for multi-agent systems, for obstacle avoidance with spherical obstacles/agents and time-dependent tasks.

According to the authors's best knowledge, there are no existing works addressing collision avoidance between 3D ellipsoidal agents subject to dynamic uncertainties and external disturbances under closed-form control protocols, which is the focus of this work. In particular, we design smooth closed-form barrier functions for the collision avoidance of ellipsoidal agents. By employing results from the computer graphics field, we derive a novel closed-form expression that represents a distance metric[1] of two ellipsoids in 3D space. Moreover, we use the latter to design a control protocol that guarantees the collision avoidance of a multi-agent system that aims to achieve a primary objective, subject to uncertain 2nd-order Lagrangian dynamics. The derived control law is (i) decentralized, in the sense that each agent calculates its control signal based on local information, (ii) discontinuous and adaptive, in order to compensate for the uncertainties and external disturbances. We note that the derived barrier functions have

[1]By distance metric we mean that it is zero in a collision between the ellipsoids and positive otherwise.

appeared in our preliminary results [21], incorporated however with simpler dynamics. This work provides significant improvements, also from a practical viewpoint, by considering external disturbances and more general uncertainties in the agents' dynamics. Moreover, in contrast to [21], we present here an important symmetric property of the derived barrier functions (Proposition 3) that plays a key role in the multi-agent control design procedure.

The rest of the paper is organized as follows. Section II provides preliminary background and the used notation. Section III formulates the treated problem and Section IV illustrates the main results. Section V is devoted to a simulation example and Section VI concludes the paper.
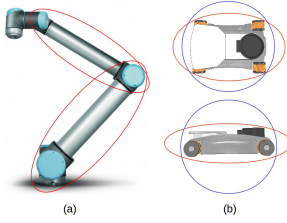

Fig. 1. Ellipsoid approximation of (a) the rigid links of a robotic manipulator, (b) a mobile robot (top and front view).

II. NOTATION AND PRELIMINARIES

A. Notation

The sets of natural and real numbers are denoted by $\mathbb{N}$, and $\mathbb{R}$, respectively, and $\mathbb{R}_{\geq 0}$, $\mathbb{R}_{>0}$ are the sets of nonnegative and positive real numbers, respectively; $\|x\|_1$ and $\|x\|$ denote the 1- and 2-norm, respectively, of a vector $x \in \mathbb{R}^n$; $\mathsf{SE}(3)$ is the special Euclidean group and S^{n-1} is the n-dimensional sphere. Given a set A, its interior is denoted by $\mathring{A}$. The identity matrix is $I_n \in \mathbb{R}^{n \times n}$. The open and closed balls with radius δ, centered at $x \in \mathbb{R}^n$, are denoted by $\mathcal{B}(x, \delta)$ and $\bar{\mathcal{B}}(x, \delta)$, respectively. The sign function is defined as $\text{sgn}(x) = -1$, if $x < 0$, $\text{sgn}(x) = 0$, if $x = 0$, and $\text{sgn}(x) = 1$, if $x > 0$; its vector counterpart is defined as $\text{sgn}(x) = [\text{sgn}(x_1), \ldots, \text{sgn}(x_n)]^\top \in \mathbb{R}^n$, for $x = [x_1, \ldots, x_n]^\top \in \mathbb{R}^n$. Given a discontinuous function $f : \mathbb{R}^n \to \mathbb{R}^k$, its Filippov regularization [22] is defined as $\mathsf{K}[f](x) := \bigcap_{\delta > 0} \bigcap_{\mu(\bar{N})=0} \overline{\text{co}}(f(\mathcal{B}(x, \delta) \backslash \bar{N}), t)$, where $\bigcap_{\mu(\bar{N})=0}$ is the intersection over all sets $\bar{N}$ of Lebesgue measure zero, and $\overline{\text{co}}(E)$ is the convex closure of the set E. The Filippov regularization of $\text{sgn}(x) \in \mathbb{R}$ is denoted by $\mathsf{K}[\text{sgn}](x) = \text{SGN}(x)$ where $\text{SGN}(x) := -1$, if $x < 0$, $\text{SGN}(x) := 1$, if $x > 0$, and $\text{SGN}(x) \in [-1, 1]$, if $x = 0$.

B. Cubic Equations and Ellipsoid Collision

Proposition 1: Consider the cubic equation $f(\lambda) = c_3 \lambda^3 + c_2 \lambda^2 + c_1 \lambda + c_0 = 0$ with $c_\ell \in \mathbb{R}, \forall \ell \in \{0, \ldots, 3\}$ and roots $\lambda_1, \lambda_2, \lambda_3$, with $f(\lambda_1) = f(\lambda_2) = f(\lambda_3) = 0$. Then, given its discriminant $\Delta := (c_3)^4 \prod_{\substack{i \in \{1,2\} \\ j \in \{i+1, \ldots, 3\}}} (\lambda_i - \lambda_j)^2$, the following hold:

(i) $\Delta = 0 \Leftrightarrow \exists i, j \in \{1, 2, 3\}$, with $i \neq j$, such that $\lambda_i = \lambda_j$, i.e., at least two roots are equal,

(ii) $\Delta > 0 \Leftrightarrow \lambda_i \in \mathbb{R}, \forall i \in \{1, 2, 3\}$, and $\lambda_i \neq \lambda_j, \forall i, j \in \{1, 2, 3\}$, with $i \neq j$, i.e., all roots are real and distinct.

Proposition 2: [23] Consider two planar ellipsoids $\mathcal{A} = \{z \in \mathbb{R}^3 \text{ s.t. } z^\top A(t) z \leq 0\}$, $\mathcal{B} = \{z \in \mathbb{R}^3 \text{ s.t. } z^\top B(t) z \leq 0\}$, with $z = [p^\top 1]^\top$, $p \in \mathbb{R}^2$, and $A, B : \mathbb{R}_{\geq 0} \to \mathbb{R}^{3 \times 3}$ terms that describe their motion in 2D space. Given their characteristic polynomial $f(\lambda) = \det(\lambda A - B)$, which has degree 3, the following hold:

(i) $\exists \lambda^* > 0$ s.t. $f(\lambda^*) = 0$, i.e, the polynomial $f(\lambda)$ has always one positive real root,

(ii) $\mathcal{A} \cap \mathcal{B} = \emptyset$ if and only if the characteristic equation $f(\lambda) = 0$ has two distinct negative roots, i.e., $\exists \lambda_1^*, \lambda_2^* < 0$, with $\lambda_1^* \neq \lambda_2^*$, and $f(\lambda_1^*) = f(\lambda_2^*) = 0$.

(iii) $\mathcal{A} \cap \mathcal{B} \neq \emptyset$ and $\mathring{\mathcal{A}} \cap \mathring{\mathcal{B}} = \emptyset$, i.e., $\mathcal{A}$ and $\mathcal{B}$ touch externally, if and only if and only if $f(\lambda) = 0$ has a negative root with multiplicity 2.

C. Nonsmooth Analysis

Consider the following differential equation with a discontinuous right-hand side:

$$\dot{x} = f(x, t), \tag{1}$$

where $f : \mathcal{D} \times [t_0, \infty) \to \mathbb{R}^n$, $\mathcal{D} \subset \mathbb{R}^n$, is Lebesgue measurable and locally essentially bounded.

Definition 1 (Def. 1 of [24]): A function $x : [t_0, t_1) \to \mathbb{R}^n$, with $t_1 > t_0$, is called a Filippov solution of (1) on $[t_0, t_1)$ if $x(t)$ is absolutely continuous and if, for almost all $t \in [t_0, t_1)$, it satisfies $\dot{x} \in \mathsf{K}[f](x, t)$.

Lemma 1 (Lemma 1 of [24]): Let $x(t)$ be a Filippov solution of (1) and $V : \mathcal{D} \times [t_0, t_1) \to \mathbb{R}$ be a locally Lipschitz, regular function[2]. Then $V(x(t), t)$ is absolutely continuous, $\frac{\partial}{\partial t} V(x(t), t)$ exists almost everywhere (a.e.), i.e., for almost all $t \in [t_0, t_1)$, and $\dot{V}(x(t), t) \overset{\text{a.e}}{\in} \tilde{\dot{V}}(x(t), t) := \bigcap_{\xi \in \partial V(x, t)} \xi^\top [\mathsf{K}[f](x, t)^\top, 1]^\top$, where $\partial V(x, t)$ is the Clarke's generalized gradient [24].

Theorem 1 (Corollary 2 of [24]): For the system given in (1), let $\mathcal{D} \subset \mathbb{R}^n$ be an open and connected set containing $x = 0$ and suppose that f is Lebesgue measurable and $x \mapsto f(x, t)$ is essentially locally bounded, uniformly in t. Let $V : \mathcal{D} \times [t_0, t_1) \to \mathbb{R}$ be locally Lipschitz and regular such that $W_1(x) \leq V(x, t) \leq W_2(x)$, $\forall t \in [t_0, t_1)$, $x \in \mathcal{D}$, and $z \leq -W(x(t))$, $\forall z \in \tilde{\dot{V}}(x(t), t)$, $t \in [t_0, t_1)$, $x \in \mathcal{D}$, where W_1 and W_2 are continuous positive definite functions and W is a continuous positive semi-definite on $\mathcal{D}$. Choose $r > 0$ and $c > 0$ such that $\bar{\mathcal{B}}(0, r) \subset \mathcal{D}$ and $c < \min_{\|x\|=r} W_1(x)$. Then for all Filippov solutions $x : [t_0, t_1) \to \mathbb{R}^n$ of (1), with $x(t_0) \in \bar{\mathcal{D}} := \{x \in \bar{\mathcal{B}}(0, r) : W_2(x) \leq c\}$, it holds that $t_1 = \infty$, $x(t) \in \bar{\mathcal{D}}$, $\forall t \in [t_0, \infty)$, and $\lim_{t \to \infty} W(x(t)) = 0$.

III. PROBLEM FORMULATION

Consider $N > 1$ ellipsoidal autonomous agents, with $\mathcal{N} := \{1, \ldots, N\}$, operating in $\mathsf{SE}(3)$, and described by the ellipsoids $\mathcal{A}_i(x_i) := \{y \in \mathbb{R}^4 : y^\top A_i(x_i) y \leq 0\}$; $x_i := [p_i^\top, \eta_i^\top]^\top \in \mathbb{M} := \mathbb{R}^3 \times \mathsf{S}^3$ is the ith agent's center of mass pose, where $p_i \in \mathbb{R}^3$ is its inertial position and

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[2]See [24] for a definition of regular functions.

$\eta_i := [\varphi_i, \epsilon_i^\top]^\top \in S^3$ its unit quaternion-based orientation, with $\varphi_i \in \mathbb{R}$, $\epsilon_i \in \mathbb{R}^3$ its scalar and vector parts, respectively, subject to $\|\eta_i\| = 1$; $A_i(x_i) := T_i^{-\top}(x_i)\hat{A}_i T_i^{-1}(x_i)$, with $\hat{A}_i := \mathrm{diag}\{l_{x,i}^{-2}, l_{y,i}^{-2}, l_{z,i}^{-2}, -1\}$, corresponding to the principal axis lengths $l_{x,i}, l_{y,i}, l_{z,i} \in \mathbb{R}_{>0}$ of agent i's ellipsoid, and $T_i \in \mathsf{SE}(3)$ is the transformation matrix describing the translation and orientation of agent i's center of mass, $\forall i \in \mathcal{N}$. The agents' motion follows the 2nd-order dynamics:

$$\dot{x}_i = \bar{E}_\eta(\eta_i)v_i \tag{2a}$$

$$M_i(x_i)\dot{v}_i + C_i(x_i, v_i)v_i + g_i(x_i) + f_i(v_i) + d_i(t) = u_i, \tag{2b}$$

where $v_i := [\dot{p}_i^\top, \omega_i^\top]^\top$ is agent i's velocity, with $\omega_i \in \mathbb{R}^3$ being its angular velocity, $\bar{E}_\eta : S^3 \to \mathbb{R}^{7\times 6}$ is the matrix mapping the quaternion rates to velocities [25], $M_i : \mathbb{M} \to \mathbb{R}^{6\times 6}$ are positive definite inertia matrices, satisfying the property $\underline{m} \leq M_i(x) \leq \bar{m}$, $\forall x \in \mathbb{M}, i \in \mathcal{N}$, for positive constants $\underline{m}$, $\bar{m}$, $C_i : \mathbb{M} \times \mathbb{R}^6 \to \mathbb{R}^{n\times n}$ are the Coriolis terms, $g_i : \mathbb{M} \to \mathbb{R}^6$ are the gravity vectors, $f_i : \mathbb{R}^6 \to \mathbb{R}^6$ are unknown vector fields that represent static friction-like terms, $d_i : \mathbb{R}_{\geq 0} \to \mathbb{R}^6$ are unknown external disturbances, and $u_i \in \mathbb{R}^6$ are the robots' control inputs, $\forall i \in \mathcal{N}$. The terms M_i, C_i and g_i are continuous everywhere, the terms f_i are locally bounded and continuous almost everywhere, and d_i are measurable and uniformly bounded. We also consider that u_i is decomposed as $u_i = u_{f,i} + u_{s,i}$, where u_{f_i} is a bounded term that is responsible for some (potentially cooperative) task, and $u_{s,i}$ is a control term to be designed in order to achieve multi-agent decentralized collision avoidance, $\forall i \in \mathcal{N}$. More specifically, we consider that $\phi(x) \in \mathbb{R}_{\geq 0}$ is a term that corresponds to the cooperative task dictated by $u_{f,i}$, with $u_{f,i} = \bar{E}_\eta(\eta_i)^\top \frac{\partial \phi(x)}{\partial x_i}$, $\forall i \in \mathcal{N}$, $c_1(x) \leq \phi(x) \leq c_2(x)$, for continuous positive definite functions c_1, c_2, and nonempty sets $\{x \in \mathcal{X} : x = \phi^{-1}(y)\}$, $\forall y \in \mathbb{R}_{\geq 0}$, where $x := [x_1^\top, \ldots, x_N^\top]^\top$, and $\mathcal{X} := \{x \in \mathbb{M}^N : \mathcal{A}_i(x_i) \cap \mathcal{A}_j(x_j) = \emptyset, \forall i, j \in \mathcal{N}, i \neq j\}$; ϕ can be also a function of $\widetilde{x} := [p_1^\top - p_2^\top, \ldots, p_N^\top - p_{N-1}^\top, \eta_1^\top, \ldots, \eta_N^\top]^\top$ that concerns potential formation control objectives. Then $\mathcal{X}$ becomes $\{\widetilde{x} \in \mathbb{R}^{\frac{3N(N-1)}{2}} \times S^{3N} : \mathcal{A}_i(x_i) \cap \mathcal{A}_j(x_j) = \emptyset, \forall i, j \in \mathcal{N}, i \neq j\}$. The conditions for ϕ are satisfied by standard quadratic functions, e.g., $\phi(x) = \sum_{i \in \mathcal{N}}\{\|p_i - \alpha_i\|^2 + e_{\eta_i}^2\}$ (for multi-agent navigation) or $\phi(\widetilde{x}) = \sum_{(i,j) \in \mathcal{F}}\{\|p_i - p_j - \alpha_{i,j}\|^2 + e_{\eta_{i,j}}^2\}$ (for formation) for sufficiently distant $\alpha_i, \alpha_{i,j}$, where $\mathcal{F}$ is a potential formation set and $e_{\eta_i}, e_{\eta_{i,j}}$ represent appropriate quaternion errors [25]. Note that ϕ and $u_{f,i}$ are *not* responsible for collision avoidance or compensating model uncertainties.

The dynamics (2) have the following properties [26]:

Property 1: The terms C_i can be chosen such that $\dot{M}_i(x) - 2C_i(x, \dot{x})$ are skew-symmetric, i.e., $y^\top(\dot{M}_i(x) - 2C_i(x, \dot{x}))y = 0$, $\forall x \in \mathbb{M}, \dot{x}, y \in \mathbb{R}^6, i \in \mathcal{N}$.

Property 2: The gravity terms of (2) can be written as $g_i(x) = Y_i(x)\theta_i$, $\forall x \in \mathbb{M}, i \in \mathcal{N}$, where $Y_i : \mathbb{M} \to \mathbb{R}^{6\times\ell}$ are known continuous matrices, and $\theta_i \in \mathbb{R}^\ell$, $\ell \in \mathbb{N}$, are constant but unknown dynamic parameters of the agents, $\forall i \in \mathcal{N}$.

Property 3: [26] The friction terms are dissipative, i.e., $v_i^\top f_i(v_i) > 0$, $\forall v_i \neq 0, i \in \mathcal{N}$.

Moreover, the following assumption is needed:

Assumption 1: It holds that $\|d_i(t)\|_1 \leq d_{b_i}$, $\forall t \in \mathbb{R}_{\geq 0}$, where d_{b_i} are *unknown* positive constants, $i \in \mathcal{N}$.

Note that in our previous work [21] we imposed a growth condition on the terms $f_i(\cdot)$ and we did not consider any form of external disturbances. In addition, we consider that each robot has a limited sensing radius $d_{\mathrm{con},i} \in \mathbb{R}_{>0}$, with $d_{\mathrm{con},i} > \max\{l_{x,i}, l_{y,i}, l_{z,i}\} + \max_{j\in\mathcal{N}}\{\max\{l_{x,j}, l_{y,j}, l_{z,j}\}\} + \varepsilon$ for an arbitrarily small positive constant ε, which implies that the agents can sense each other without colliding. Based on this, we can model the topology of the multi-robot network through the undirected time-varying graph $\mathcal{G}(p) := (\mathcal{N}, \mathcal{E}(p))$, with $\mathcal{E}(p) := \{(i,j) \in \mathcal{N}^2 : \|p_i - p_j\| \leq \min\{d_{\mathrm{con},i}, d_{\mathrm{con},j}\}\}$, $p := [p_1^\top, \ldots, p_N^\top]^\top$, and we further define the agent time-varying neighborhood $\mathcal{N}_i(p) := \{j \in \mathcal{N} : \|p_i - p_j\| < d_{\mathrm{con},i}\}$, $\forall i \in \mathcal{N}$. Moreover, we consider the complete graph $\bar{\mathcal{G}} := (\mathcal{N}, \bar{\mathcal{E}})$, with $\bar{\mathcal{E}} := \{(i,j), \forall i, j \in \mathcal{N}, i < j\}$, $\bar{M} := |\bar{\mathcal{E}}| = \frac{N(N-1)}{2}$ and an edge numbering set $\bar{\mathcal{M}} := \{1, \ldots, \bar{M}\}$. Finally, given an edge $m \in \bar{\mathcal{M}}$, we use the notation $(m_1, m_2) \in \mathcal{N}^2$ for the robot indices of edge $m \in \bar{\mathcal{M}}$. As discussed in Section I, the agents need to avoid collisions with each other, while executing their task, dictated by $u_{f,i}$. To that end, we aim to design *closed-form* barrier functions and decentralized feedback control laws $u_{s,i}$ that guarantee collision avoidance among the ellipsoidal agents, while compensating appropriately for the model uncertainties and the external disturbances. By "decentralized", we mean here that agents can use only local information, in view of their limited sensing radius (i.e., only p_j, η_j, $j \in \mathcal{N}_i(p)$). Formally, the treated problem is the following:

Problem 1: Given N 3D ellipsoidal autonomous agents with the uncertain Lagrangian dynamics (2) executing tasks dictated by $u_{f,i}$, design

1) closed-form barrier functions that encode collision avoidance of the agents,
2) decentralized control laws in $u_{s,i}$ that guarantee inter-agent collision avoidance, i.e., $\mathcal{A}_i(x_i(t)) \cap \mathcal{A}_j(x_j(t)) = \emptyset$, $\forall i, j \in \mathcal{N}$, $i \neq j$, as well as boundedness of all closed loop signals.

IV. MAIN RESULTS

This section describes the proposed solution to Problem 1. In order to deal with the ellipsoidal collision avoidance, we employ results from computer graphics that are related to detection of ellipsoid collision and we build appropriate barrier functions whose boundedness implies the collision-free trajectories. Moreover, we use adaptive and discontinuous control laws to appropriately compensate for the uncertainties and external disturbances of (2).

We employ first the results described in Proposition 2 to build an appropriate ellipsoidal barrier function. Note, however, that these results concern planar ellipsoids and cannot be straightforwardly extended to the 3D case, which is the case of the considered multi-agent system. For that reason, we consider the respective planar projections. For an ellipsoid $\mathcal{A}_i, i \in \mathcal{N}$, we denote as $\mathcal{A}_i^{xy}, \mathcal{A}_i^{xz}, \mathcal{A}_i^{yz}$ its projections on the planes x-y, x-z and y-z, respectively, with corresponding matrix terms $A_i^{xy}, A_i^{xz}, A_i^{yz}$ (i.e., $\mathcal{A}_i^s(x_i) := \{y \in \mathbb{R}^3 : y^\top A_i^s(x_i)y \leq 0\}$, $\forall s \in \{xy, xz, yz\}$). Note that in order for $\mathcal{A}_i, \mathcal{A}_j$ to collide (touch externally), all their projections on the three planes must also collide, i.e.,

$\mathcal{A}_i(x_i) \cap \mathcal{A}_j(x_j) \neq \emptyset \wedge \mathring{\mathcal{A}}_i(x_i) \cap \mathring{\mathcal{A}}_j(x_j) = \emptyset \Leftrightarrow \mathcal{A}_i^s(x_i) \cap \mathcal{A}_j^s(x_j) \neq \emptyset \wedge \mathring{\mathcal{A}}_i^s(x_i) \cap \mathring{\mathcal{A}}_j^s(x_j) = \emptyset, \forall s \in \{xy, xz, yz\}$, i.e., Therefore, $\mathcal{A}_i$ and $\mathcal{A}_j$ do not collide if and only if $\mathcal{A}_i^s(x_i) \cap \mathcal{A}_j^s(x_j) = \emptyset$ for some $s \in \{xy, xz, yz\}$. In view of Proposition 2, that means that the characteristic equations $f_{i,j}^s(\lambda) := \det(\lambda A_i^s(x_i) - A_j^s(x_j)) = 0$ must always have one positive real root and two negative distinct roots for at least one $s \in \{xy, xz, yz\}$. Hence, by denoting the discriminant of $f_{i,j}^s(\lambda) = 0$ as $\Delta_{i,j}^s(x_i, x_j)$, Proposition 1 suggests that $\Delta_{i,j}^s(x_i, x_j)$ must remain always positive for at least one $s \in \{xy, xz, yz\}$, since a collision would imply $\Delta_{i,j}^s(x_i, x_j) = 0, \forall s \in \{xy, xz, yz\}$. Therefore, by defining the smooth function [9] $\sigma(x) := \exp(-\frac{1}{x})$, if $x > 0$ and $\sigma(x) := 0$, if $x \leq 0$, we conclude that $\mathcal{A}_i$ and $\mathcal{A}_j$ do not collide if and only if $\sigma(\Delta_{i,j}^{xy}(x_i, x_j)) + \sigma(\Delta_{i,j}^{xz}(x_i, x_j)) + \sigma(\Delta_{i,j}^{yz}(x_i, x_j)) > 0$, since a collision would result in $\Delta_{i,j}^s(x_i, x_j) = 0 \Leftrightarrow \sigma(\Delta_{i,j}^s(x_i, x_j)) = 0, \forall s \in \{xy, xz, yz\}$. We aim now at defining a decentralized continuously differentiable function for each edge $m \in \bar{\mathcal{M}}$ that incorporates the collision avoidance property of agents m_1, m_2. We need first the following result regarding the discriminant of $f_{i,j}^s(\lambda) = 0$:

Proposition 3: Let Δ_1, Δ_2 be the discriminants of $f_1(\lambda) := \det(\lambda A - B) = 0$, $f_2(\lambda) := \det(\lambda B - A) = 0$, respectively, where $A, B \in \mathbb{R}^{3 \times 3}$. Then $\Delta_1 = \Delta_2$.

Proof: Let $\det(\lambda A - B) = 0 \Leftrightarrow f_1(\lambda) := c_3\lambda^3 + c_2\lambda^2 + c_1\lambda + c_0 = 0$, with $c_\ell \in \mathbb{R}, \forall \ell \in \{0, \dots, 3\}$. It can be verified that $\det(\lambda B - A) = 0 \Leftrightarrow f_2(\lambda) = -c_0\lambda^3 - c_1\lambda^2 - c_2\lambda - c_3 = 0$. Let $\lambda_1, \lambda_2, \lambda_3$ be the solutions of $f_1(\lambda) = 0$, i.e. $f_1(\lambda_1) = f_1(\lambda_2) = f_1(\lambda_3) = 0$, and $\lambda_1\lambda_2\lambda_3 = -\frac{c_0}{c_3}$. By substituting $\frac{1}{\lambda_\ell}$ in $f_2(\lambda)$, $\ell \in \{1, 2, 3\}$, we obtain $-c_0\lambda_\ell^{-3} - c_1\lambda_\ell^{-2} - c_2\lambda_\ell^{-1} - c_3 = -(c_3\lambda_\ell^3 + c_2\lambda_\ell^2 + c_1\lambda_\ell + c_0) = -f_1(\lambda_\ell) = 0$. Hence, $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \frac{1}{\lambda_3}$ are the solutions of $f_2(\lambda) = 0$. The discriminants of $f_1(\lambda) = 0$ and $f_2(\lambda) = 0$ are $\Delta_1 = c_3^4(\lambda_1 - \lambda_2)^2(\lambda_1 - \lambda_3)^2(\lambda_2 - \lambda_3)^2$ and $\Delta_2 = (-c_0)^4\left(\lambda_1^{-1} - \lambda_2^{-1}\right)^2\left(\lambda_1^{-1} - \lambda_3^{-1}\right)^2\left(\lambda_2^{-1} - \lambda_3^{-1}\right)^2 = c_0^4(\lambda_1\lambda_2\lambda_3)^{-4}(\lambda_2 - \lambda_1)^2(\lambda_3 - \lambda_1)^2(\lambda_3 - \lambda_2)^2$, which, by substituting $c_0 = -c_3\lambda_1\lambda_2\lambda_3$, becomes $\Delta_2 = \Delta_1$. ∎

Therefore, we conclude that the discriminants $\Delta_{i,j}^s(x_i, x_j)$ and $\Delta_{j,i}^s(x_j, x_i)$ of $\det(\lambda A_i^s(x_i) - A_j^s(x_j)) = 0$ and $\det(\lambda A_j^s(x_j) - A_i^s(x_i)) = 0$, respectively, are the same, for all $s \in \{xy, xz, yz\}$. Hence, we can define uniquely for each edge $m \in \bar{\mathcal{M}}$ the continuously differentiable function $\Delta_m : \mathbb{M}^2 \to \mathbb{R}_{\geq 0}$, with

$$\Delta_m(x_{m_1}, x_{m_2}) := \sigma(\Delta_{m_1,m_2}^{xy}(x_{m_1}, x_{m_2})) + \sigma(\Delta_{m_1,m_2}^{xz}(x_{m_1}, x_{m_2})) + \sigma(\Delta_{m_1,m_2}^{yz}(x_{m_1}, x_{m_2})), \quad (3)$$

which needs to remain positive for all times in order to achieve the collision avoidance property, i.e., $\Delta_m(x_{m_1}(t), x_{m_2}(t)) > 0, \forall t \in \mathbb{R}_{\geq 0}, m \in \bar{\mathcal{M}}$. Note that, in view of Proposition 3, the agents m_1 and m_2 can calculate (3) based on $\Delta_{m_1,m_2}^s(x_{m_1}, x_{m_2})$ and $\Delta_{m_2,m_1}^s(x_{m_2}, x_{m_1})$, respectively, $\forall s \in \{xy, xz, yz\}, m \in \bar{\mathcal{M}}$.

We still need to incorporate the fact the that agents have a limited sensing radius, and that agent i does not have access to the functions $\Delta_{i,j}^s(x_i, x_j)$, when $j \notin \mathcal{N}_i(p)$. To that end, we define first the greatest lower bound of Δ_m when both agents m_1, m_2 are in each other's sensing radius,

i.e., $\widetilde{\Delta}_m := \inf_{(x_{m_1}, x_{m_2}) \in \mathbb{M}^2}\{\Delta_m(x_{m_1}, x_{m_2}) \text{ s.t. } \|p_{m_1} - p_{m_2}\| \leq \min\{d_{\text{con}_{m_1}}, d_{\text{con}_{m_2}}\}\}, \forall m \in \bar{\mathcal{M}}$. Since $d_{\text{con},i} > \max\{l_{x,i}, l_{y,i}, l_{z,i}\} + \max_{j \in \mathcal{N}}\{\max\{l_{x,j}, l_{y,j}, l_{z,j}\}\} + \varepsilon, \forall i \in \mathcal{N}$, it follows that there exists a positive constant ε_Δ such that $\widetilde{\Delta}_m \geq \varepsilon_\Delta > 0, \forall m \in \bar{\mathcal{M}}$. Next, we define the smooth switching functions $\beta_m : \mathbb{R}_{\geq 0} \to [0, \bar{\beta}_m]$, with [9]

$$\beta_m(x) = \bar{\beta}_m \frac{\sigma(x)}{\sigma(x) + \sigma(\bar{\Delta}_m - x)}, \quad (4)$$

where $\bar{\Delta}_m$ is a positive constant satisfying $\bar{\Delta}_m < \widetilde{\Delta}_m, \forall m \in \bar{\mathcal{M}}$. Then, by choosing $\beta_m := \beta_m(\gamma_\sigma \Delta_m(x_{m_1}, x_{m_2}))$, where γ_σ is a positive scaling constant, we incorporate the limited sensing radius of the agents in the collision avoidance scheme, since $\frac{\partial \beta_m(x)}{\partial x}$ vanishes when $m_1 \notin \mathcal{N}_{m_2}(p)$ or $m_2 \notin \mathcal{N}_{m_1}(p)$, i.e., when at least one of the agents that form edge m lies outside the sensing range of the other agent. The terms $\bar{\beta}_m$ can be any positive constants, $\forall m \in \bar{\mathcal{M}}$. All the necessary information for the construction of the functions β_m, Δ_m, i.e., the constants $\bar{\Delta}_m, \bar{\beta}_m$ and the lengths $l_{x,i}, l_{y,i}, l_{z,i}, i \in \mathcal{N}$, can be transmitted off-line to the agents.

We can now define a suitable barrier function for each edge $m \in \bar{\mathcal{M}}$ as any continuously differentiable function $b_m : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ with the property $\lim_{x \to 0} b_m(x) = \infty$, e.g., $b_m(x) = \frac{1}{x}, m \in \bar{\mathcal{M}}$. The barrier function for edge m is then $b_m := b_m(\beta_m), \forall m \in \bar{\mathcal{M}}$.

We propose now a decentralized feedback control law for the solution of Problem 1. Firstly, we define the estimations of the unknown terms $\theta_i \in \mathbb{R}^\ell$ and $d_{b_i} \in \mathbb{R}$ of Property 2 and Assumption 1, respectively, as $\hat{\theta}_i \in \mathbb{R}^\ell$ and $\hat{d}_{b_i} \in \mathbb{R}$, with the respective errors $\widetilde{\theta}_i := \hat{\theta}_i - \theta_i$ and $\widetilde{d}_{b_i} := \hat{d}_{b_i} - d_{b_i}, \forall i \in \mathcal{N}$. By using adaptive and discontinuous control techniques, we prove in the following that these estimations compensate appropriately for the unknown terms, without necessarily converging to them. In particular, we design the feedback control laws for $u_{s,i}$ as

$$u_{s,i} = \sum_{m \in \bar{\mathcal{M}}} \alpha(i, m)\kappa_m \bar{E}_\eta(\eta_i)^\top \frac{\partial \Delta_m}{\partial x_i} + Y_i(x_i)\hat{\theta}_i - k_{v_i}v_i - \hat{d}_{b_i}\text{sgn}(v_i), \quad (5)$$

where $\alpha(i, m) = -1$ if agent i is part of edge m, and $\alpha(i, m) = 0$ otherwise, $\forall i \in \mathcal{N}, m \in \bar{\mathcal{M}}$, and $\kappa_m := \frac{\partial b_m(\beta_m)}{\partial \beta_m} \frac{\partial \beta_m(\Delta_m)}{\partial \Delta_m}, \forall m \in \bar{\mathcal{M}}$. Moreover, we design the associated adaptation laws

$$\left.\begin{aligned} \dot{\hat{\theta}}_i &:= -\gamma_{i,\theta}Y_i(x_i)^\top v_i \\ \dot{\hat{d}}_{b_i} &:= \gamma_{i,d}\|v_i\|_1 \end{aligned}\right\} \forall i \in \mathcal{N}, \quad (6)$$

with arbitrary bounded initial conditions, where $\gamma_{\theta,i}$ and $\gamma_{d,i}$ are positive gains, $\forall i \in \mathcal{N}$. The correctness of (5)-(6) is shown in the following theorem:

Theorem 2: Consider a multi-agent system comprised of 3D ellipsoidal agents and subject to the dynamics (2) at a collision-free initial configuration, i.e., $\mathcal{A}_i(x_i(0)) \cap \mathcal{A}_j(x_j(0)) = \emptyset, \forall i, j \in \mathcal{N}$ with $i \neq j$. Then, application of the control and adaptation laws (5), (6) guarantees that the agents avoid collisions for all times, i.e., $\mathcal{A}_i(x_i(t)) \cap \mathcal{A}_j(x_j(t)) = \emptyset$,

$\forall i, j \in \mathcal{N}$ with $i \neq j$, $t \in \mathbb{R}_{\geq 0}$, with all closed loop signals being bounded. Moreover, $\lim_{t\to\infty} v_i(t) = 0, \forall i \in \mathcal{N}$.

Proof: Consider the vector $\zeta_x := [x^\top, v^\top, \widetilde{\theta}^\top, \widetilde{d}_b^\top]^\top$ $\in \mathcal{Z}_x := \mathcal{X} \times \mathbb{R}^{7N+\ell N}$, where $\mathcal{X} := \{x \in \mathbb{M}^N : \mathcal{A}_i(x_i) \cap \mathcal{A}_j(x_j) = \emptyset, \forall i, j \in \mathcal{N}, i \neq j\}$ as defined in Section III, $v := [v_1^\top, \ldots, v_N^\top]^\top \in \mathbb{R}^{6N}$, $\widetilde{d}_b := [\widetilde{d}_{b_1}, \ldots, \widetilde{d}_{b_N}]^\top \in \mathbb{R}^N$, $\widetilde{\theta} := [\widetilde{\theta}_1^\top, \ldots, \widetilde{\theta}_N^\top] \in \mathbb{R}^{\ell N}$. Since the initial configuration is collision-free, it holds that $\zeta_x(0) \in \mathcal{Z}_x$. By combining (2), (5), and (6), we obtain the closed-loop system dynamics $\dot{\zeta}_x = F_\zeta(\zeta_x, t)$. It can be verified that F_ζ is measurable in t over $\mathbb{R}_{\geq 0}$ and measurable and locally bounded in ζ_x over $\mathcal{Z}_x$. Hence, by invoking Prop. 3 of [27], we conclude that at least one Filippov solution exists and any such solution satisfies $\zeta_x : [0, t_1) \to \mathcal{Z}_x$ for a positive t_1. Define $\zeta := [\phi, b_1, \ldots, b_{\bar{M}}, v^\top, \widetilde{\theta}^\top, \widetilde{d}_b^\top]^\top$ $\in \mathcal{Z} := \mathbb{R}^{\bar{M}+7N+\ell N+1}$, where ϕ is the cooperative term defined in Section III. Note that $\zeta(0) \in \mathcal{Z}$ and, for any finite r, $\zeta \in \bar{\mathcal{B}}(0, r) \subset \mathcal{Z} \Leftrightarrow \zeta_x \in \mathcal{Z}_x$, which we prove in the following. Define the function $V(\zeta) := \phi(x) + \sum_{m \in \bar{\mathcal{M}}} b_m + \sum_{i \in \mathcal{N}} \{ \frac{1}{2} v_i^\top M_i(x_i) v_i + \frac{1}{2\gamma_{i,d}} \widetilde{d}_{b_i}^2 + \frac{1}{2\gamma_{i,\theta}} \|\widetilde{\theta}_i\|^2 \}$, for which it holds that $W_1(\zeta) \leq V(\zeta) \leq W_2(\zeta)$ for positive definite functions W_1, W_2 on $\mathcal{Z}$. Since $\zeta(0) \in \mathcal{Z}$, we conclude that $V(\zeta(0))$ is well defined, and hence there exists a finite constant $\bar{V}$ such $V(\zeta(0)) \leq \bar{V}$ and $b_m(0) \leq \bar{V}$, $\forall m \in \bar{\mathcal{M}}$. We aim to show that V, given its initial boundedness, remains bounded $\forall t \in \mathbb{R}_{\geq 0}$, and so do the terms b_m, $\forall m \in \bar{\mathcal{M}}$. By differentiating $\bar{V}$ along the solutions of the closed loop system and in view of Lemma 1 we obtain $\dot{V} \in \dot{\tilde{V}} := \cap_{\xi \in \partial V(\zeta)} \xi^\top \mathsf{K}[\dot{\zeta}]$. Since V is continuously differentiable, the generalized gradient reduces to the standard gradient and therefore, after using Properties 1, 2, and grouping terms, we obtain

$$
\begin{aligned}
\max_{z \in \dot{\tilde{V}}} \{z\} \leq \sum_{i \in \mathcal{N}} \Bigg\{ & \sum_{m \in \bar{\mathcal{M}}} \left[\alpha(i, m) \kappa_m \frac{\partial \Delta_m}{\partial x_i}^\top \bar{E}_\eta(\eta_i) \right] v_i + \\
& \|v_i\|_1 \|d_i(t)\|_1 + v_i^\top \left(u_i - Y_i(x_i)\theta_i + \bar{E}_\eta(\eta_i)^\top \frac{\partial \phi(x)}{\partial x_i} \right) \\
& - v_i f_i(v_i) + \frac{1}{\gamma_{i,d}} \widetilde{d}_{b_i} \dot{\hat{d}}_{b_i} + \frac{1}{\gamma_{i,\theta}} \widetilde{\theta}_i^\top \dot{\hat{\theta}}_i \Bigg\},
\end{aligned}
$$

By also using Property 3, Assumption 1, substituting $u_i = u_{f,i} + u_{s,i}$ with $u_{f,i} = \bar{E}_\eta(\eta_i)^\top \frac{\partial \phi(x)}{\partial x_i}$ and (5), the adaptation laws (6), and using $\widetilde{d}_{b_i} = \hat{d}_{b_i} - d_{b_i}$, $\widetilde{\theta}_i = \hat{\theta}_i - \theta_i$ and the property $x^\top \text{sign}(x) = \|x\|_1$, $\forall x \in \mathbb{R}^n$, we obtain $\max_{z \in \dot{\tilde{V}}} \{z\} \leq -\sum_{i \in \mathcal{N}} k_{v_i} \|v_i\|^2 =: W(\zeta)$. Therefore, $z \leq -W(\zeta(t))$, $\forall z \in \dot{\tilde{V}}(\zeta(t))$, $t \in [0, t_1)$, where $W : \mathcal{Z} \to \mathbb{R}_{\geq 0}$ is a positive semi-definite function defined on $\mathcal{Z}$. Hence, by applying Theorem 1, we conclude that $t_1 = \infty$, $\zeta(t)$ is bounded in the compact set $\{\zeta \in \bar{\mathcal{B}}(0, r) : W_2(\zeta) \leq c\}$, $\forall t \in \mathbb{R}_{\geq 0}$ for any r and c satisfying $\bar{\mathcal{B}}(0, r) \subset \mathcal{Z}$, $c < \min_{\|x\|=r} W_1(\bar{\zeta})$, and $\lim_{t\to\infty} W(\zeta(t)) = 0 \Rightarrow \lim_{t\to\infty} v(t) = 0$. Note that, since the sets $\{x \in \mathbb{M}^N : x = \phi^{-1}(y), x \in \mathcal{X}\}$ are nonempty, r can be chosen arbitrarily large, corresponding to all collision-free initial configurations. Therefore, inter-agent collisions are avoided, and the adaptation signals $\hat{\theta}_i$, $\hat{d}_{b_i}$, remain bounded, $\forall i \in \mathcal{N}$, $t \in \mathbb{R}_{\geq 0}$. The continuity of the terms $Y_i(\cdot)$ implies also their boundedness and hence the boundedness of the control signals (5), (6), $t \in \mathbb{R}_{\geq 0}$. Note that

convergence of $\widetilde{\theta}_i$, $\widetilde{d}_{b_i}$ to zero is *not* needed, and hence we do not require persistence of excitation or sufficient richness of the respective signals, as is usually assumed in adaptive control when parameter convergence is taken into account. ∎

Remark 1: It can be verified that $\det(\lambda A_{m_1}^s(x_{m_1}) - A_{m_2}^s(x_{m_2}))$, and hence b_m, are functions of $p_{m_1} - p_{m_2}$, η_{m_1}, η_{m_2}. Therefore, if ϕ is a function of $\widetilde{x}$, the aforementioned analysis still holds by setting $\mathcal{X} = \{\widetilde{x} \in \mathbb{R}^{\frac{3N(N-1)}{2}} \times \mathsf{S}^3 : \mathcal{A}_i(x_i) \cap \mathcal{A}_j(x_j) = \emptyset, \forall i, j \in \mathcal{N}, i \neq j\}$. Moreover, note that achievement of the objectives expressed by ϕ is not pursued in this paper and may not be necessarily guaranteed due to the potentially counteracting terms of u_i. The control scheme could be extended, however, by appropriately designing the terms $\phi(x)$ such that the resulting configuration of the agents implies $\phi(x) = 0$, like, e.g., [3], [4], [9].

Remark 2: Since $\Delta_{i,j}^s = \Delta_{j,i}^s$ (due to Proposition 3), $\forall i, j \in \mathcal{N}$, $i \neq j$, the control scheme can be extended to directed communication graphs, by setting for the ith agent $b_{i,j} = b_{i,j}(\beta_{i_j}(\Delta_{i,j}(x_i, x_j)))$, $\forall j \in \mathcal{N} \setminus \{i\}$, with $\Delta_{i,j}(x_i, x_j)$ as in (3) and β_{i_j} as in (4), $\widetilde{\Delta}_{i,j} := \inf_{(x_i, x_j) \in \mathbb{M}^2} \{\Delta_{i,j}(x_i, x_j) \text{ s.t. } \|p_i - p_j\| \leq d_{\text{con}_i}\}$, and appropriately modifying the control law. Similarly, collision avoidance with static environment obstacles could be incorporated in the overall scheme. An event-triggered extension is also possible by employing results from the related literature, e.g., [28]. Finally, for implementation purposes in real applications, the discontinuous part of (5) could be approximated by a continuous function via the boundary layer technique [29], e.g., $\frac{|v_{i_j}|}{|v_{i_j}| + \epsilon_{i_j}}$ instead of $\text{sgn}(v_{i_j})$, when $|v_{i_j}| < \epsilon_{i_j}$, where $v_{i_j} \in \mathbb{R}$ is the jth component of v_i, $\forall i \in \mathcal{N}$, and ϵ_{ij} is a small positive constant.

V. SIMULATION RESULTS

We consider a simulation example with $N = 8$ rigid bodies in $\mathsf{SE}(3)$, described by ellipsoids with axes lengths $l_{x,i} = 0.5$m, $l_{y,i} = 0.3$m, $l_{z,i} = 0.2$m, $\forall i \in \mathcal{N}$. The initial poses are $p_1 = [3,3,0]^\top$, $p_2 = -[3,3,0]^\top$, $p_3 = [3,-3,0]^\top$, $p_4 = [-3,3,0]^\top$, $p_5 = [3,3,3]^\top$, $p_6 = -[3,3,3]^\top$, $p_7 = [3,-3,3]^\top$, $p_8 = [-3,3,-3]^\top$, $\eta_1 = \eta_8 = [0.769, 0.1696, 0.6153, 0.0358]^\top$, $\eta_2 = \eta_6 = [0.8488, -0.3913, -0.0598, -0.3505]^\top$, $\eta_3 = \eta_5 = [0.7638, -0.5283, -0.3275, -0.1738]^\top$, $\eta_4 = \eta_7 = [0.7257, 0.3081, 0.3714, 0.4904]^\top$. We consider that $\phi(x)$ describes an independent multi-agent navigation objective, with desired configurations as $p_{1_d} = p_2$, $p_{2_d} = p_1$, $p_{3_d} = p_4$, $p_{4_d} = p_3$, $p_{5_d} = p_6$, $p_{6_d} = p_5$, $p_{7_d} = p_8$, $p_{8_d} = p_7$, $\eta_{i_d} = [1,0,0,0]^\top$, $\forall i \in \mathcal{N}$. We set the errors $e_{p_i} := p_i - p_{i_d}$ and $e_{\eta_i} := [e_{\varphi_i}, e_{\epsilon_i}^\top]^\top := \eta_{i_d} \otimes \bar{\eta}_i$, where $\bar{\eta}_i := [\varphi_i, -\epsilon_i^\top]^\top \in \mathsf{S}^3$ is the quaternion conjugate, $\otimes$ denotes the quaternion product, and $e_{\varphi_i}, e_{\epsilon_i}$ are the scalar and vector parts, respectively, of the quaternion error [25]. The desired quaternion configuration is achieved when $e_{\eta_i} = [\pm 1, 0, 0, 0]^\top$ and hence the function $\phi(x)$ is chosen as $\phi(x) = \sum_{i \in \mathcal{N}} \left(\frac{1}{2} \|p_i - p_{i_d}\|^2 + 1 - e_{\varphi_i}^2 \right)$, with $\dot{\phi} = \sum_{i \in \mathcal{N}} \left((p_i - p_{i_d})^\top \dot{p}_i - e_{\varphi_i} e_{\epsilon_i}^\top \omega_i \right)$ [25]. The control inputs $u_{f,i}$ are therefore chosen as $u_{f,i} = [p_{i_d}^\top - p_i^\top, e_{\varphi_i} e_{\epsilon_i}^\top]^\top$, $\forall i \in \mathcal{N}$. The agent masses are chosen as $(0.1, 0.2, 0.01, 0.1, 0.1, 0.2, 0.1, 0.2)$ and the principal moments of inertia as $\text{diag}\{0.05, 0.03, 0.01\}$, $\forall i \in \mathcal{N}$. We also set

$f_i(v_i) = m_{f_i} \sin(w_{f_i} t + \phi_{f_i}) v_i$, $d_i(t) = (1/m_{f_i}) \sin(w_{f_i} t + \phi_{f_i})$, $\forall i \in \mathcal{N}$, with $[m_{f_1}, \ldots, m_{f_8}] = 0.1 \cdot [1,2,0.1,1,1,2,1,2]$, $[\omega_{f_1}, \ldots, \omega_{f_8}] = 0.01 \cdot [1,2,0.1,1,1,2,1,2]$, and $[\phi_{f_1}, \ldots, \phi_{f_8}] = 0.01 \cdot [5,1,0.05,0.5,1,0.5,1]$. We choose $b_m = \frac{1}{\beta_m}$, with $\bar{\beta}_m = 1$, $\bar{\Delta}_m = 10^4$, $\gamma_\sigma = 10^{-40}$, $\forall m \in \mathcal{M}$, and $\hat{\theta}_i(0) = 0.1$, $\hat{d}_{b_i}(0) = 0.2$, $k_{v_i} = 1$, $\forall i \in \mathcal{N}$. The expressions for $\Delta_m(x_{m_1}, x_{m_2})$ were derived by using the symbolic toolbox of MATLAB. Fig. 2 shows a 3D plot of the agent trajectories, and Fig. 3 shows the minimum of the barrier functions $\min_{m \in \bar{\mathcal{M}}} \{b_m(t)\}$ (left), which is always positive, and the signals $\gamma_i(t) := \|p_i - p_{i_d}\|^2 + 1 - e_{\varphi_i}^2$ and $v_i(t)$ (right), $\forall i \in \mathcal{N}$, $t \in [0, 15]$. A short video that demonstrates the aforementioned simulation example can be found in https://youtu.be/IAni7zIMM7k.
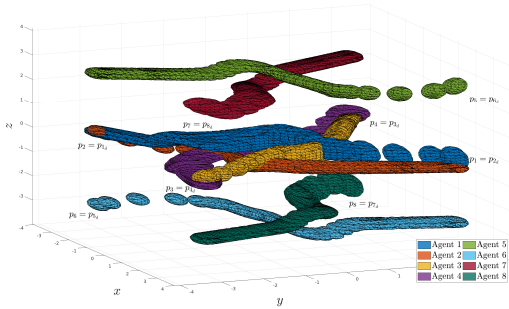

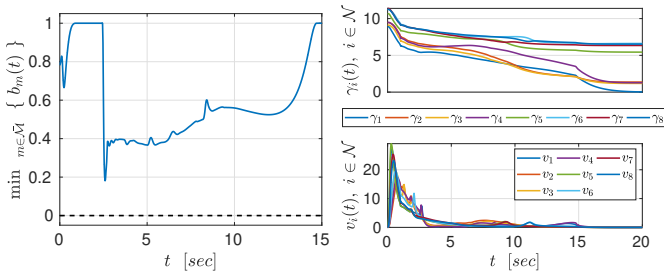Fig. 2. The evolution of agent trajectories $\forall t \in [0, 15]$.

Fig. 3. Left: The evolution of the minimum of the barrier functions $\min_{m \in \bar{\mathcal{M}}} \{b_m(t)\}$. Right: The evolution of the signals $\gamma_i(t)$ and $v_i(t)$, $\forall i \in \mathcal{N}$, $\forall t \in [0, 15]$.

VI. CONCLUSIONS AND FUTURE WORK

This paper proposes a closed-form barrier function as well as a robust decentralized control scheme for the multi-agent collision avoidance of 3D ellipsoids, using discontinuous and adaptive controllers. Future efforts will be devoted towards adding connectivity properties to the current framework and resolving issues of local minima.

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