



Notes

Збирка

2.1 $P = 8i + 2j + 5k$

a) $w = 2$ б) единичен вектор на правцу

a) $P = \begin{bmatrix} 16 \\ 4 \\ 10 \\ 2 \end{bmatrix}$

б) $P = \begin{bmatrix} 0.83 \\ 0.21 \\ 0.52 \\ 0 \end{bmatrix}$?

⇒ за да претставиме дескоординатен

script as r+

$r+.print3 \Rightarrow$ дода вектор во матрицата алик со w
 \Rightarrow ~~укажува~~ w
\$ знак \$ \Rightarrow мајсмајински знак

$r+.print3 \Rightarrow$ матрица без w

б) функција `unit-vector` \Rightarrow нормализира вектор

$\begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix} \Rightarrow$ само бидејќи не се знае кој к.с. е
 \Rightarrow мора да се кажејќи координати

shift + tab

2.4. $P(3, 5, 7)$ $2 \Rightarrow x$ $3 \Rightarrow y$ $4 \Rightarrow z$

RGB
x y z

CMYK
n o a

$$P_{nova} = T(2, 3, 4) P_{stara}$$

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \\ 7 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 11 \\ 1 \end{bmatrix}$$

r1. translation 3

r4. Animation Playground \Rightarrow интерактивная app

2.5. $B = \begin{bmatrix} 0 & 1 & 0 & 2 \\ 1 & 0 & 0 & 4 \\ 0 & 0 & -1 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$B_{nova} = T \cdot B_{stara}$$

2.6 $5 \rightarrow x$ $2 \rightarrow y$ $6 \rightarrow z$

$$F_{nova} = \begin{bmatrix} 0 & 1 & 0 & 2 \\ 1 & 0 & 0 & 4 \\ 0 & 0 & -1 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$F = T \cdot F_s \quad / \quad T^{-1} \text{ не бн}$$

$$T^{-1} F_n = F_s$$

2.8 $F = \begin{bmatrix} 0 & 1 & 0 & -2 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$x \times y = z$$

$$n \cdot a = 0 \quad n \cdot o = 0 \quad a \cdot o = 0$$

$$|n| = |a| = |o| = 1$$

$$n \perp a \\ n \perp o$$

$$o \perp a$$

$$\det F = 1$$

$$RR^T = I$$

$$\det R = +1$$

\Rightarrow модерна оријентација

\Rightarrow услови за десет координатен систем

$\det R \Rightarrow$ конуба како ќе се формира волумен

ротација слична од сирењето на големини

$\text{Mat}(f(n-1))$ - дабо од јурен \rightarrow фори

rotations - аџа б ротијатки

2.30

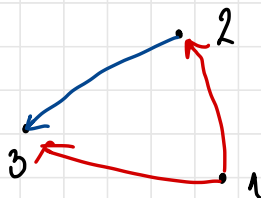
$$R R^T = I$$

$$R R^{-1} = I$$

}

$$R^T = R^{-1}$$

2.31.

 2F_3 

$${}^2F_1 {}^1F_3 = {}^2F_3$$

$$\circlearrowleft {}^1F_2^{-1}$$

2.33.

$$\frac{r \sin \beta \cos \gamma}{r \sin \beta \sin \gamma} \Rightarrow \sin \beta \neq 0$$

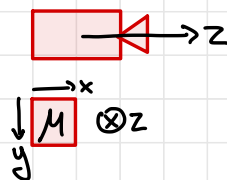
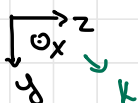
\downarrow
 $\tan \gamma$

$$\Rightarrow \sin \beta \neq 0$$

$$\frac{r \sin \beta \cos \gamma}{r \cos \gamma} = \tan \beta$$

2.34 \Rightarrow гипотенуза

$$\pi = \pi/2$$



$$R = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

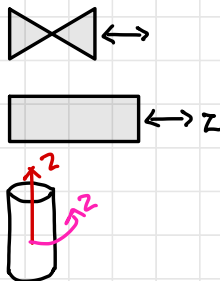
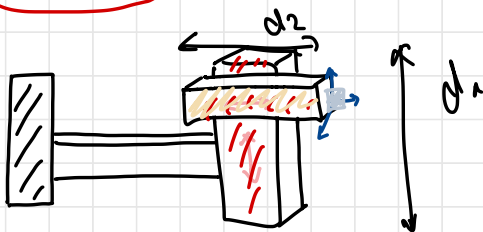
2.46 ${}^0R_1, {}^1R_2, {}^2R_3 \rightarrow {}^0R_3$

$$R_1 = \begin{bmatrix} x & y & z \\ 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

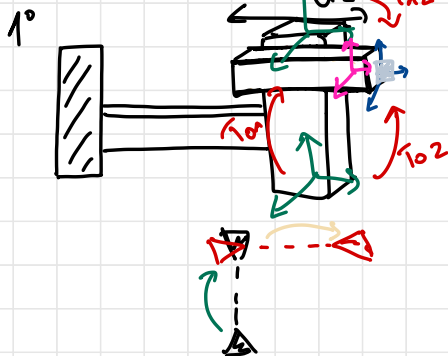
→ ckeжe нeт

$$R_2 = \begin{bmatrix} x & y & z \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_3 = \begin{bmatrix} x & y & z \\ 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



da a) $\sharp \uparrow$ δ $\mu \uparrow$

$$d_2 \quad \text{---} \rightarrow$$


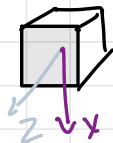
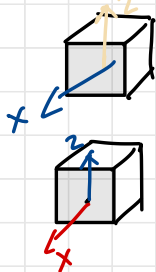
- работник проситр \Rightarrow рамнина

0	d	a	2
0	d_1^*	0	0
0	d_2^*	0	0

→ за оборотный к.с.

⇒ само транскрира

2.48



θ	d	a	α
0	d_1^*	0	$\pi/2$
$\pi/2$	d_2^*	0	$\pi/2$

2.49

θ	d	a	α
θ_1^*	0	e_1	0
θ_2^*	0	e_2	0
θ_3^*	0	e_3	-90°

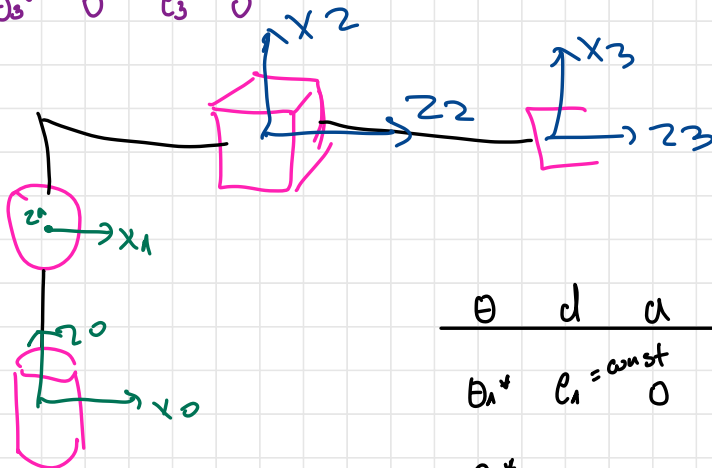
2.53

θ	d	a	α
θ_1^*	e_1	0	$-30^\circ/90^\circ$
θ_2^*	0	e_2	0
θ_3^*	0	e_3	0

2.52

θ	d	a	α
θ_1^*	0	0	0
0	d_2^*	0	90°
θ_3^*	0	e_3	0

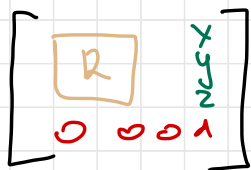
\Rightarrow coordinate system $(0, 0, 0)$



θ	d	a	α
θ_1^*	$e_1 = \text{const}$	0	90°
θ_2^*			90°

Инверзная кинематика

2.60. p.o. - рамка

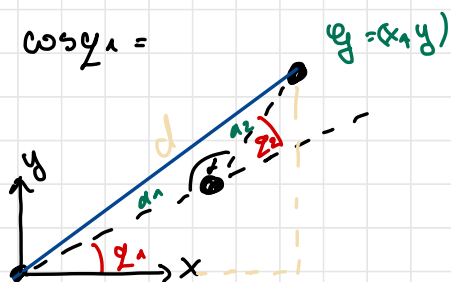


$$x = C_1 \cos \alpha_1 + C_2 C_2$$

$$y = C_1 \sin \alpha_1 + C_2 \sin \alpha_2$$

$$z = 0$$

$$\cos \varphi_2 = \frac{x^2 + y^2 - a_1^2 - a_2^2}{2a_1 a_2}$$



$$\cos \varphi_1 =$$

$$y = (x, y)$$

$$\alpha + \varphi_2 = 180^\circ$$

$$= d^2$$

$$a_2^2 + a_1^2 - 2 \cos \alpha \cdot a_1 \cdot a_2 = d^2$$

$$\frac{\sin \beta}{a_2} = \frac{\sin \alpha}{d}$$

$$\beta + \varphi_1$$

2.61

θ_i	d_i	a_i	α_i
θ_1^*	d_1	0	-90°
θ_2^*	0	0	90°
0	d_3^*	0	0

$$\theta_1 \quad \theta_2 \quad d_3 = ?$$

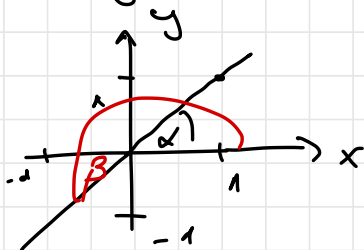
$$x = d_3 s_2 c_1$$

$$y = d_3 s_1 c_2$$

$$z = d_1 + d_3 c_2$$

$$\frac{y}{x} = \frac{\sin \theta_1}{\cos \theta_1} = \tan \theta_1 \Rightarrow \theta_1 = \arctan \frac{y}{x}$$

$$\text{sp. atan}(y/x) \Rightarrow 0,78$$



$$\text{sp. atan2}(y, x) \Rightarrow \text{оглущува кој кбуграње е за да даде точно агол}$$

$$\text{ако } \sin \theta_2 = 0 \Rightarrow \text{сингуларност}$$

2.60

$$\cos(\theta_1 + \theta_2) = -0,29$$

$$1^\circ \quad \cos \theta_1 \cos \theta_2 - \sin \theta_1 \sin \theta_2 = -0,29$$

$$\sin(\theta_1 + \theta_2) = 0,96$$

$$2^\circ \quad \sin \theta_1 \cos \theta_2 + \sin \theta_2 \cos \theta_1 = 0,96$$

$$3^\circ \quad \cos \theta_1 + \cos(\theta_1 + \theta_2) = 0,7$$

$$\cos \theta_1 - 0,29 = 0,7$$

$$\cos \theta_1 = 0,99$$

$$\theta_1' = 0,14 \text{ rad} = \theta_1'' \approx 0^\circ$$

$$\Rightarrow \theta_1 \text{ } 60^\circ \quad 1^\circ \Rightarrow$$

$$\Rightarrow \theta_1 \text{ } 60^\circ \quad 2^\circ \Rightarrow$$

$$1 \cos \theta_2 = -0,29$$

$$\sin \theta_2 = 0,96$$

$$\theta_2' \approx 107^\circ$$

$$\theta_2'' \approx -107^\circ$$



$\sin \theta$, а $\cos \theta \Rightarrow$ II клограну



2.61.

θ_i	d_i	a_i	α_i
θ_1^*	d_1	0	-90
θ_2^*	0	0	90
0	d_3^*	0	0

$$\begin{aligned} X &= d_3 \sin \theta_2 \cos \theta_1 \\ Y &= d_3 \sin \theta_1 \sin \theta_2 \\ Z &= d_1 + d_3 \cos \theta_2 \end{aligned}$$

ако се дадени само x, y, z ,
а не y, z трансформационна
матрица

$$\theta_1 \Rightarrow \frac{y}{x} = \frac{d_3 \sin \theta_1 \sin \theta_2}{d_3 \sin \theta_2 \cos \theta_1} = \tan \theta_1$$

$$\Rightarrow \theta_1 = \arctan\left(\frac{y}{x}\right)$$

в jupyter за точно решение в
кбодражи `atan2(y, x)`

$$\theta_2 \Rightarrow d_3 \sin \theta_2 \cos \theta_1 = X \Rightarrow d_3 \sin \theta_2 = \frac{X}{\cos \theta_1}$$

$$d_3 \cos \theta_2 = Z - d_1$$

$$\Rightarrow \frac{\sin \theta_2}{\cos \theta_2} = \tan \theta_2 = \frac{X}{(Z - d_1) \cos \theta_1}$$

$$\Rightarrow \theta_2 = \arctan\left(\frac{X}{(Z - d_1) \cos \theta_1}\right)$$

$$d_3 \Rightarrow Z = d_1 + d_3 \cos \theta_2 \Rightarrow d_3 = \frac{Z - d_1}{\cos \theta_2}$$

2.62.

∂_i	d_i	a_i	α_i
∂_1^*	d_1	0	-90°
∂_2^*	d_2	0	90°
0	d_3	0	0

• Неизвестны: θ_1, θ_2, d_3

$$\begin{aligned} x &= -d_2 \sin \theta_1 + d_3 \sin \theta_2 \cos \theta_1 \\ y &= d_2 \cos \theta_1 + d_3 \sin \theta_1 \sin \theta_2 \\ z &= d_1 + d_3 \cos \theta_2 \end{aligned}$$

$$X = X_e \quad x_e - x$$

$$\frac{x + d_2 \sin \theta_1}{y - d_2 \cos \theta_1} = \frac{\cos \theta_1}{\sin \theta_1} \Rightarrow \frac{y - d_2 \cos \theta_1}{x + d_2 \sin \theta_1} = \frac{\sin \theta_1}{\cos \theta_1}$$

$$y \cos \theta_1 - d_2 \cos^2 \theta_1 = x \sin \theta_1 + d_2 \sin^2 \theta_1$$

$$y \cos \theta_1 - x \sin \theta_1 = d_2 \quad /: \cos \theta_1$$

$$y - x \frac{\sin \theta_1}{\cos \theta_1} = \frac{d_2}{\cos \theta_1}$$

$$y - x \frac{\sqrt{1 - t^2}}{t} = \frac{d_2}{t}$$

$$yt - x \sqrt{1 - t^2} = d_2$$

$$yt - d_2 = x \sqrt{1 - t^2}$$

$$y^2 t^2 - 2y d_2 t + d_2^2 = x^2 (1 - t^2)$$

$$(y^2 - x^2) t^2 - 2y d_2 t + d_2^2 - x^2 = 0$$

$$x^2 + y^2 = d_3^2 \sin^2 \theta_2$$

$$(z - d_1)^2 = d_3^2 \cos^2 \theta_2$$

$$\tan \theta_2 = \sqrt{\frac{x^2 + y^2}{(z - d_1)^2}}$$

$$\theta_2 = \arctan \sqrt{\frac{x^2 + y^2}{(z - d_1)^2}}$$

$$d_3 = \frac{z - d_1}{\cos \theta_2}$$

$$\underbrace{x + d_2 \sin \theta_1}_c = \underbrace{d_3 \sin \theta_2 \cos \theta_1}_c \quad \sin \theta_1 = t$$

$$x + d_2 t = d_3 \sin \theta_2 \sqrt{1 - t^2}$$

$$x^2 + 2x d_2 t + t^2 = d_3^2 \sin^2 \theta_2 (1 - t^2)$$

$$\Rightarrow \text{выбираем } t \Rightarrow \text{2 pers. } \sin \theta_1 = 1 \cos \theta_1$$

sp.nonlin solve (equations, [theta1, theta2, d3])

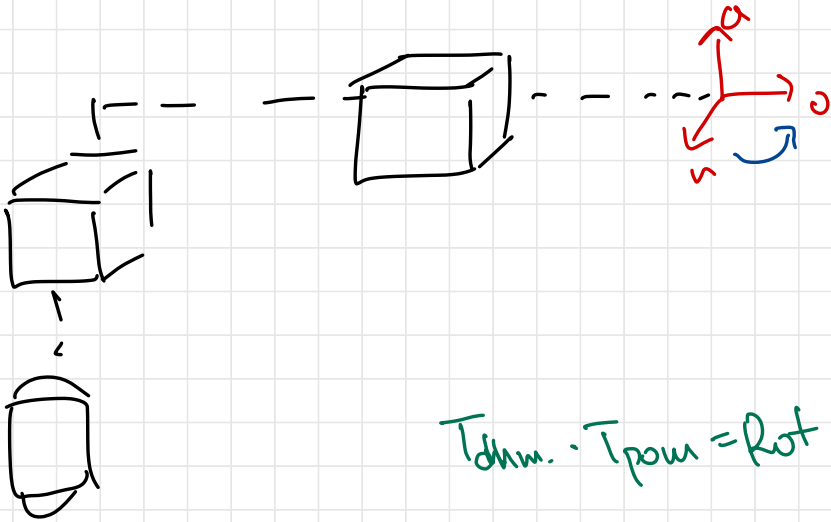
↓

$x, y, z = \text{sp.symbols}('x, y, z')$
 $\text{equations} = \begin{bmatrix} x_e - x, \\ y_e - y, \\ z_e - z \end{bmatrix}$

$s1, c1, s2, c2, d3$

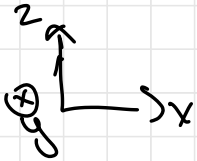
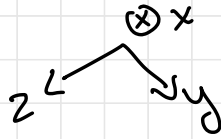


2.66

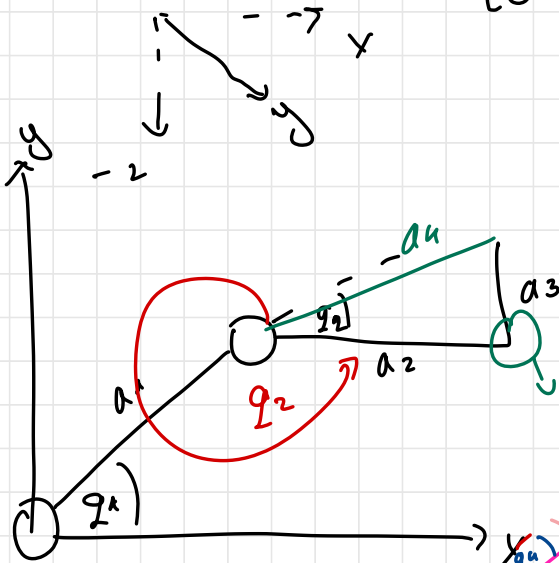


$T_{dhm} \cdot T_{pwm} = \text{Rot around } a$

$T_{dh_matrix} \cdot R(a, 90^\circ)$



$$R = \begin{bmatrix} 0 & +\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & x \\ 1 & 0 & 0 & y \\ 0 & -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & z \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ перенос}$$



$$a_4^2 = a_2^2 + a_3^2$$

много точек
- 0°

$$q_2 = 25^\circ$$

$$q_3 = \text{фиксир}$$

$$q_4 = 37^\circ$$

$$q_4 + q_3 = q_2$$

$$\text{rad} = \frac{180}{\text{deg}}$$

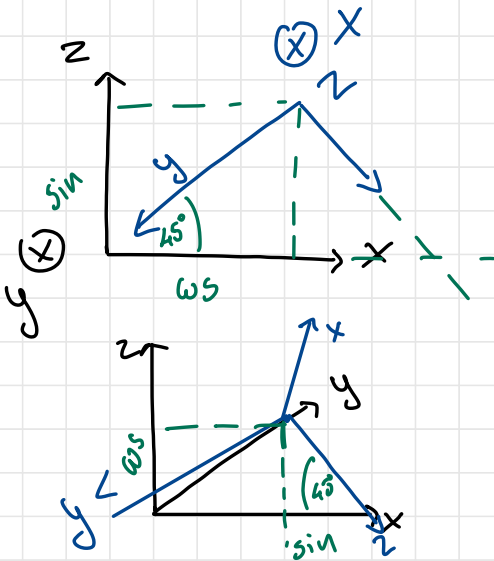
$$\text{deg} = \frac{180}{\text{rad}}$$

$\odot, 74$
 $\odot, 12$
 $\odot, 57$

$\odot \rightarrow$ көп таңпа

$\odot \rightarrow$ әлқой мере

$$\sin 45 = \frac{\sqrt{2}}{2} \quad \cos 45 = \frac{\sqrt{2}}{2}$$



$$R = \begin{bmatrix} x & y & z \\ 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ 1 & 0 & 0 \\ 0 & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$$

Ниспекция
 на y е әрпайыт
 ана cos тәңіш \odot

$$P = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$