College Algebra - Stretch

Foundational and Challenge Workshops for
College Algebra at the University of Oklahoma

College Algebra - Stretch

Foundational and Challenge Workshops for College Algebra at the University of Oklahoma

Casey Violette Haskins
The University of Oklahoma

June 28, 2022

Website: Casey Haskin's Website

[1] creativecommons.org/licenses/by-nc-sa/4.0

Acknowledgements

I would like to thank the OU Libraries' Alternative Textbook Grant for helping me create an open educational resource for students in College Algebra.

I would also like to thank Carye Chapman and Dr. Nick Brown from The Department of Mathematics at OU for taking looks at the first drafts of this work.

Preface

This workbook is intended to serve as a supplement to the College Algebra -- Stretch materials at The University of Oklahoma. Introductions and Examples for each chapter are intended to be discussed during labs and discussions, while exercises serve as further examples for students to complete in their own time.

Students in this class are required to give some presentations or lead activities based on the content, so may use the exercises as inspiration for these assignments.

Contents

Chapter 1

Review for Variability and Functions

At the beginning of the semester, there are a lot of topics you might like to review alongside variability and an introduction to functions. This chapter serves to address the following:

- Using the order of operations to expand and condense linear expressions
- Using the order of operations to expand and condense polynomial and rational expressions
- Converting between algebraic and verbal representations
- Solving linear equations

1.1 Expanding and Condensing Expressions

For the purposes of this class, a **linear expression** involves up to a single variable with its highest exponent at one. The variable might be represented by any number of non-numeric symbols, typically letters (Latin or Greek) like x or α, but may be represented by any symbol you wish, like $\bigstar$ or $\heartsuit$.

Examples of linear expressions include

$$3x - 5$$

and

$$9 - 4x$$

and

$$\frac{3^2 - 4x + 6x - \frac{3}{2}}{10}$$

1.1.1 Condensing Linear Expressions

You may be wondering why the last example of a linear expression is considered as such if it involves both exponents and fractions. The definition for a linear expression did not include either. There are a few reasons why this expression is linear:

- the highest exponent of the *variable* is one
- the numbers in the linear expression may be any constant we can think of, including fractions
- the division is by a number, not a variable

We can see these things in action if we use a reasonable order of operations to condense the expression into a format with a single constant and a single variable term.

Example 1.1.1 Condensing a linear expression. Condense $\dfrac{3^2-4x+6x-\frac{3}{2}}{10}$ into a format with a single constant term and a single variable term.

Answer. $\dfrac{3}{4}+\dfrac{1}{5}x$

Solution.

$$\frac{3^2-4x+6x-\frac{3}{2}}{10}$$
$$\frac{1}{10}\left(9-4x+6x-\frac{3}{2}\right)$$
$$\frac{9}{10}-\frac{4}{10}x+\frac{6}{10}x-\frac{3}{20}$$
$$\frac{18}{20}-\frac{3}{20}+\frac{2}{10}x$$
$$\frac{15}{20}+\frac{2}{10}x$$
$$\frac{3}{4}+\frac{1}{5}x$$

□

Example 1.1.2 Analyzing Example 1.1.1. What is the order of operations used in the example above?

Hint. The typical operations to choose from are addition, multiplication, exponents, division, parenthetical statements, and subtraction

Answer. exponents, parenthetical statements, multiply, add, subtract, reduce

Solution.

1. applied *exponents*
2. reimagine division as multiplying by a fraction, and distributing it into a *parenthetical statement*
3. *add* like terms, begin to *subtract* constants by finding common denominators
4. *subtract* the constants
5. *divide* by reducing fractions (always allowed) or writing answers in decimal form (sometimes allowed)

□

Puzzle 1.1.1 Critiquing and Evaluating Example 1.1.2. Note that the previous examples used a reasonable order of operations to condense the linear expression. Did you use the same order of operations when you attempted the problem? Was the order you used reasonable as well? Why or why not?

Answer. There are many reasonable order of operations you can use. We typically use the operations in pairs from left to right: parentheses and exponents, multiplication and division, addition and subtraction. Consider why we might deviate from this order on occasion, for example in the previous example and checkpoint, it would have been reasonable to condense the entire numerator before dividing each term by 10. Why is that reasonable?

Exercises

Exercise Group. Use a reasonable order of operations to expand and condense the following linear expressions to a format which has one constant term and one variable term. When you are done, use a different, but still reasonable, order of operations to expand and condense the expression. The answers are provided.

1. $2(4w + 8) + 7(2w - 4) + 16$

Answer. $22w + 4$

2. $3(2z - 4) + 8(z - 9) + 84$

Answer. $14z$

3. $7 - \{8\bigstar + 4[2 - (5\bigstar - \sqrt{64})^2]\}$

Answer. $100\bigstar^2 - 328\bigstar + 255$

4. $\dfrac{2x-1}{5} + \dfrac{x}{4}$

Answer. $0.65x - 0.20$

1.1.2 Expanding and Condensing Other Expressions

Some expressions may not be linear, using exponents other than one on numerous variables. Note that using multiple variables does not necessarily make the expression non-linear, but involving elements like x^2, $\sqrt{\alpha}$, and $\dfrac{1}{\heartsuit}$ are *certainly* non-linear when x, α, $\heartsuit$ are variable. This does not change how we address using a reasonable order of operations for expanding or condensing a complicated expression. The only alteration might be additional considerations for what it means to combine "like terms".

Exercises

Exercise Group. Use a reasonable order of operations to expand and condense the following expressions into what might be considered simplest form.

1. $(ab)^2a^3b$

Answer. a^5b^3

2. $(x - 3)(3x^2 - 2x + 1)$

Answer. $3x^3 - 11x^2 + 7x - 3$

3. $\dfrac{\left(\dfrac{2x}{3} + \dfrac{1}{6}\right)^2}{\dfrac{2}{3} - \dfrac{5x}{9}}$

Answer. $\dfrac{16x^2 + 8x + 1}{24 - 20x}$ or $\dfrac{(4x+1)^2}{4(6-5x)}$

In a relatively simple (keep in mind that *simple* means that it is not overly complicated with a large number of steps, not that it is easy) expression, a reasonable order of operations may look much the same for everyone. In *complex* expressions, involving a large number of steps to expand and condense to what might be considered its "simplified" form, you might be able to find a reasonable order of operations that is perfectly correct, but different than how someone else approached it. Like with many things in mathematics, there is not always a *single correct solution*, though there may be *best solution* or what you consider to be the *easiest solution*. The terms "best" and "easiest" are subjective, and oftentimes up to the person doing the mathematics.

1.2 Converting Between Algebraic and and Verbal Representations of Information

If I told you that my dog eats 24 ounces of dog food per day, we'd be able to determine how much dog food she's consumed by the end of a typical week by creating an algebraic version of this relationship. It is best to define my input variable and units as well as my output variable and units. In this case, let's use c for *consumed* as the output variable to represent the number of ounces she consumes each day. We can use d to represent *days* since we've started counting her food consumption as the input variable. This means that the relationship looks like

$c = 24d$ ounces of dog food consumed,

where d is days since counting her food consumption.

Notice that saying $c = 24d$ isn't quite enough, since c and d don't hold meaning or context on their own. Even in the algebraic version, we need to use some words to describe what we are doing. The algebriac version above is what's called a **model** of the real-world information in algebraic form.

1.2.1 Turning Verbal Information into Algebraic Models

Example 1.2.1 Write an algebraic model for the following: The stock of a certain NASDAQ company began the day trading at 75 dollars per share, but decreased by 2 dollars per hour.

Answer. $p = 75 - 2h$ □

Example 1.2.2 We can use these skills to solve other problems without creating a model. For example, we want to find two numbers so that one of them is three more than twice the other number, and the sum of the two numbers is 36. Create a setup for this situation.

Answer.

$$\begin{aligned} 2x + 3 &= y \\ x + y &= 36 \end{aligned}$$

□

Example 1.2.3 Suppose you want to find the dimensions of a small rectangular performance stage which has perimeter of 110 ft (feet) and has length that is 1 foot more than twice the width. Create a setup for this situation.

Answer.

$$\begin{aligned} 2\ell + 2w &= 110 \\ \ell &= 1 + 2w \end{aligned}$$

□

1.2.2 Turning Algebraic Information into Verbal Information

Exercises

Exercise Group. Take each of the algebraic scenarios below and convert them into a verbal format. Note that a verbal format does not involve algebraic expressions.

1. The cumulative amount of dog food eaten by my dog is given by $C = 0.75f + 1$ pounds of food, after f number of feedings.

2.

$$\begin{aligned} V &= LWH \\ L &= 2W \\ H &= 8 \end{aligned}$$

Note that some information is missing. Make up a scenario which this would reasonably represent, including the units for each variable.

3. Use your own words to describe this situation where n is a number: $3(n+5) = 5n$.

1.3 Solving Linear Equations

A **linear equation** is an equation which sets two linear expressions equal to each other. The linear equation may have a single variable, like

$$3(x-5) + 2x = \frac{1}{2}x - 5$$

or have multiple variables like

$$y = 5x + 6$$

and

$$2x = 3y - 7x + 5$$

Solving linear equations might include expanding or condensing the expressions on either side, but will also include what is often referred to as "algebraic manipulations", or performaning a certain operation on both sides of the equation simultaneously. This means each subsequent step of the solution is equivalent to the last.

Example 1.3.1 Solving a Linear Equation. Solve $3(x-5) + 2x = \frac{1}{2}x - 5$ for the variable x

Answer. $x = \dfrac{20}{9}$

Solution.

$$\begin{aligned}3(x-5)+2x &= \frac{1}{2}x-5\\ 3x-15+2x &= \frac{1}{2}x-5\\ 5x-15 &= \frac{1}{2}x-5\\ 10x-30 &= x-10\\ 9x-30 &= -10\\ 9x &= 20\\ x &= \frac{20}{9}\end{aligned}$$

□

Example 1.3.2 Analyzing Example 1.3.1. Describe the steps for solving the equation in the previous example.

Solution.

- In the left-hand expression, multiply 3 and $x-5$ by distribution.
- In the left-hand expression, combine the like terms of $3x$ and $2x$ through addition
- Multiply both sides of the equation by 2.
- Subtract x from both sides of the equation, and combine like terms.
- Add 30 to both sides of the equation, and combine like terms.
- Divide both sides by 9, the answer is fully reduced.

□

Puzzle 1.3.1 Critiquing and Evaluating Example 1.3.2. Note that the previous examples used a reasonable order of operations and manipulations to solve for x. Did you use the same order of operations and manipulations when you attempted the problem? Was the order you used reasonable as well? Why or why not?

Answer. There are many reasonable orders of operations and manipulations you can use. The typical order of operations is the same as before, while the typical order of manipulations might go in the opposite order. Consider why we might deviate from this order on occasion, for example in the previous examples, it would have been reasonable to never distribute the 3 to $x-5$, and never multiply both sides by 2. Why is that reasonable? What might you do instead?

Exercises

Exercise Group. Solve the following linear equations for the variable b, describing each step in words. The answers are provided.

1. $-6b-4=20$

Answer. $b=-4$

2. $2.3=4.5b+30.2$

Answer. $b=-6.2$

3. $-3(b-4)+5=10-(b+1)$

Answer. $b=4$

4. $\dfrac{b-1}{4}=\dfrac{2b+1}{5}$

Answer. $b=-3$

5. $\dfrac{b-6}{3}=\dfrac{b+3}{2}+1$

Answer. $b=-27$

Exercise Group.

6. If my dog consumes 15 pounds of food per week, how many pounds of food will she consumer over the next year?

780 pounds of food

7. A number is four more than twice another number. If one of the numbers is 6, what could the other number be? There are two answers.

Answer. The other number could be either 1 or 16.

8. You are selling t-shirts at a music festival, and start the festival with 470 t-shirts. You sell about 23 per hour during festival hours. How many hours must the festival run in order for you to sell out of shirts?

Answer. The festival needs to run at least 21 hours to sell out of the 470 available t-shirts.

Chapter 2

Review for Factoring and Functions

Throughout the semester, we will be solving a variety of equations in many contexts. This chapter serves to address the following:

- Using factoring methods on polynomial expressions
  - Greatest Common Factor (GCF)
  - Factoring by grouping
  - AC Method
- Converting between numerical and graphical representations

2.1 Methods of Factoring

A **polynomial expression** is an expression with at least one term, exponents on variables that are non-negative integers (like counting numbers), and coefficients that are any real number. The following are examples of polynomial expressions with variables represented by symbols:

- $k - 5$
- $m^2 - 5m + 6$
- $-7\heartsuit + 7\heartsuit^6 - 9\heartsuit^2$
- $(x-4)(3x-4)^2(x+2)^3$

2.1.1 Factoring the GCF

The **Greatest Common Factor** is the largest factor which can be "factored out" of each term. A **factor** is something which can multiply with other factors to create the term. For example:

- 3 has factors: -1, 1, -3, 3
- 4 has factors: -1, 1, -2, 2, -4, 4
- 12 has factors: -1, 1, -2, 2, -3, 3, -4, 4, -6, 6, -12, 12

- $9\heartsuit^2$ has factors: -1, 1, -3, 3, -9, 9, $-\heartsuit$, $\heartsuit$, $-3\heartsuit$, $3\heartsuit$, $-9\heartsuit$, $9\heartsuit$, $-\heartsuit^2$, $\heartsuit^2$, $-3\heartsuit^2$, $3\heartsuit^2$, $-9\heartsuit^2$, $9\heartsuit^2$

Example 2.1.1 How to Factor the GCF. In a polynomial expression like $-7\heartsuit + 7\heartsuit^6 - 9\heartsuit^2$, factoring out the GCF means knowing the factors of each term *and* what the greatest combination of factors from each term could be. Find the GCF and factor it out.

Hint. Factoring the GCF is like dividing it from each term, but it doesn't "go away" like it might in a division problem. Factoring is the opposite of distribution.

Answer. GCF is $\heartsuit$ or $-\heartsuit$, so the factored version of the expression could then be $\heartsuit(-7 + 7\heartsuit^5 - 9\heartsuit)$ or $-\heartsuit(7 - 7\heartsuit^5 + 9\heartsuit)$ □

Exercises

Exercise Group. Factor the GCF from each of the following polynomial expressions

1. $30x^3 - 45x^2 + 135x$

Answer. $15x(2x^2 - 15x + 9)$

2. $200p^3m^3 - 30p^2m^3 + 40m^3$

Answer. $10m^3(20p^3 - 3p^2 + 4)$

3. $36j^4k^2 - 18j^3k^3 + 54j^2k^4$

Answer. $18j^2k^2(2j^2 - jk + 3k^2)$

4. $3x(x + 2) + 4(x + 2)$

Answer. $(x + 2)(3x + 4)$

2.1.2 Factoring by Grouping

Factoring by grouping is a way to factor out "groups of terms" at a time, instead of a single factor. This method of factoring utilizes the GCF.

Example 2.1.2 An Example of Grouping. For example, we want to factor the polynomial expression $8r^3 - 64r^2 + r - 8$, we can see that it does not have a GCF besides 1 or -1. We might be able to factor by grouping.

(a) We'll start with "grouping" the terms. Rewrite the expression, but with some indication that $8r^3$ and $-64r^2$ are in the same group and that r and -8 are in the same group.

(b) Factor the GCF out of the first group, then factor the GCF out of the second group.

(c) If we have chosen our groups well *and* our polynomial expression is factorable, the two terms (our factored groups) will have the same multiplier. What is it?

(d) If the polynomial expression is factorable, we should now be able to factor that multiplier out of both terms, like we did in the last exercise from the last exercise in Example 2.1.1.

□

Exercises

Exercise Group. Factor each of the following polynomial expressions, if possible.

1. $12x^3 + 9x^2 - 20x - 15$

Answer. $(3x^2 - 5)(4x + 3)$

2. $9mz - 4nc + 3mc - 12nz$

Hint. Try rearranging the terms before setting your groups.

Answer. $(3m - 4n)(3z + c)$

3. $8m^2 - 12m + 6m - 9$

Answer. $(2m - 3)(4m + 3)$

2.1.3 Factoring with the AC Method

Some polynomials (usually a trinomial, or a binomial with highest exponent of 2 on its variable) may need a different factoring method. This method of factoring utilizes the method of grouping within it.

Example 2.1.3 Example of the AC Method. For example, if we want to factor the trinomial $5x^2 + 7x + 2$, we can see that there is not a GCF aside from 1 or -1, and there are only three terms, so we cannot get groups of two terms at a time. To get to the point where we can use the method of grouping, we need to have an even number of terms.

(a) This method is called "the AC Method" because it begins by thinking of the polynomial as $Ax^2 + Bx + C$ and multiplying the A and C together. What is AC?

(b) Find all possible combinations of numbers which multiply to get AC

(c) Choose from your list the pair which also adds to get B (or in this case, 7).

(d) Split the middle term into two represented by these numbers. The expression should be equivalent to what you were given, but should now have four terms.

(e) If everything was done well and the polynomial is factorable, you may now factor by grouping.

□

Note that the AC Method is also used when $A = 1$, but is streamlined by the idea of finding a pair of numbers which multiply to get C and add to get B.

2.1.4 Exercises

Exercise Group. Factor each of the following polynomial expressions, if possible.

1. $x^2 - 10x + 25$

Answer. $(x - 5)(x - 5)$ or $(x - 5)^2$

2. $x^2 + 9x + 20$

Answer. $(x + 4)(x + 5)$

3. $4a^2 + 16a + 16$

Answer. $4(a + 2)(a + 2)$ or $4(a + 2)^2$

4. $-2y^2 + 20y - 32$

Answer. $-2(y - 2)(y - 8)$

5. $3x^2 - 8x + 4$

Answer. $(3x - 2)(x - 2)$

6. $3x^2 + 4x - 4$

Answer. $(3x - 2)(x + 2)$

7. $6x^3 + x^2 - 2x$

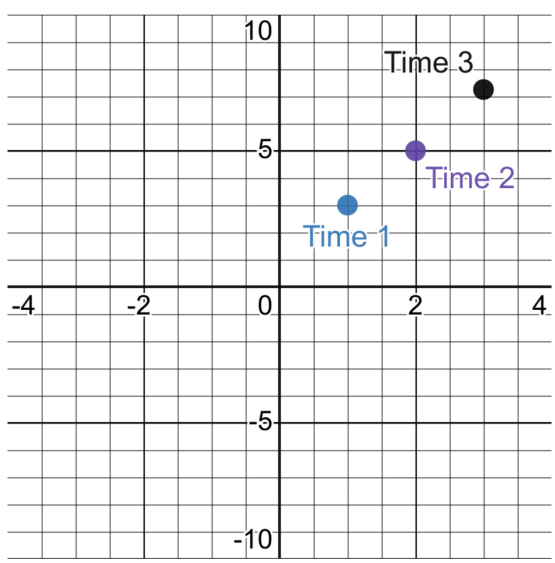
Answer. $x(3x + 2)(2x - 1)$

8. $18x^2y - 12xy - 30y$

Answer. $6y(x + 1)(3x - 5)$

2.2 Converting Between Graphical and Numerical Information

In addition to verbal and algebraic information, you may need to take in or produce graphic or numerical representations of mathematical information. For example, consider the coordinate plane below with three labeled points.



You are asked to describe this graphical information in numerical form. Typically, this means that you will create a table to summarize the information.

Because this graph is three discrete (or separate) points, this summary will be easier to create than if the graph were a full curve. We can either represent the table vertically:

Table 2.2.1

x	y
1	3
2	5

or horizontally:

Table 2.2.2

x	1	2	3
y	3	5	7.25

Note that choosing x to represent the horizontal component and y to represent the vertical component isn't necessary. You might instead see function notation, like x and $f(x)$, or any combination of symbols as relevant to the context of the graph.

Exercises

Exercise Group. Graph each of the following numeric representations.

1.

Table 2.2.3

α	-3	-2	-1	0	1	2
$g(\alpha)$	9	4	1	0	1	4

2.

Table 2.2.4

A	B
-2	$-\frac{1}{2}$
-1	-1
$-\frac{1}{2}$	-2
$-\frac{1}{3}$	-3
$\frac{1}{3}$	3
$\frac{1}{2}$	2
1	1
2	$\frac{1}{2}$

Chapter 3

Communication

Doing mathematics involves solving problems. There is a distinction between solving problems and practicing exercises. An **exercise** is routine, while a **problem** is not. Some problems are non-routine in that you might need to use a tool (mathematics concept) in a new way or with other tools you haven't previously thought about combining. These problems might still produce a single correct answer or solution set like exercises. Other problems are both non-routine and not entirely well-stated. These problems may have a number of solutions, depending on whether you take a certain path or another. Here's one example of a problem being non-routine and the mathematician's suggested solutions before finding the correct answer:

YouTube: `https://www.youtube.com/watch?v=m2eyq9qTOQY`

Figure 3.0.1

Here is another example of a problem which is multiple choice in a higher stakes environment.

YouTube: `https://www.youtube.com/watch?v=BbX44YSsQ2I`

Figure 3.0.2

No matter whether you are working on an exercise or a problem, communication of what you understand in the solution process is incredibly important. Your understanding isn't directly measurable (or grade-able), but your communication is.

3.1 Interpreting Problem Statements

In Figure 3.0.1, the mathematician in the video was able to solve the problem only after the question was restated. At first, they were asked "If you are going 80 miles per hour, how long does it take you to go 80 miles?" After awhile, they are asked "If you are going 80 miles per hour, so an hour at 80 miles, so how long will it take you to go 80 miles?" This reframing is often an important step to solve problems, but we do not always have another person there to do it for us. Instead, we should practice reframing for ourselves.

Example 3.1.1 Consider the question from Figure 3.0.2: "Which of these square numbers also happens to be the sum of two smaller square numbers?" The answer choices were 16, 25, 36, and 49. How would you reframe the question in order to answer it if you were in this mathematician's shoes? □

Exercises

Exercise Group. In each of the following problem statements, figure out what you are being asked, what tools you need, and what tools you have. Tools might be considered to be definitions, theorems, equations, formulas, etc.

1. If $f(x) = 11 - 3x - 2x^2$, then find the average rate of change between x and $x + h$, where h is a constant.
2. According to Ohm's law, the electric current l, in amperes, in a circuit varies directly with its voltage V. When 14 volts are apllied, the current is 2 amperes. What is the current when 29 volts are applied.

3.2 "Show Your Work"

In Figure 3.0.2, the mathematician was given a multiple choice question, so did not need to explain their work or show how they determined the correct answer. Let's say that you needed to answer a similar question "Find a perfect square which also happens to be the sum of two smaller perfect squares."

We can answer this question by first thinking of some perfect sqaures $1^2 = 1$, $2^2 = 4$, $3^2 = 9$, $4^2 = 16$, $5^2 = 25$, $6^2 = 36$, $7^2 = 49$, etc. This means that we can use the numbers 1, 4, 9, 16, 25, 36, 49, etc. in order to make another perfect square *OR* we can think about those perfect squares and what kind of sums we'd need to make them.

Even within these two options, there are numerous mental organizations you can take to finding a solution (note there is not a single correct solution here). This means that because you will likely solve it in a different way than I'm thinking of it, you should show enough of your reasoning to explain why your solution is equivalently correct to someone who took a different route.

Exercises

Exercise Group. Answer each of the following. Show your work and explanation.

1. Newton's Law of Gravitation says that two objects with masses m_1 and m_2 attract each other with a force F that is jointly proportional to their masses and inversly proportional to the square of the distance

r between the objects. Newton discovered the constant of variation is $6.67 \cdot 10^{-11}$. In a small laboratory experiment, two 400 kg masses are separated by 0.8 meters. What would the gravitational force between the objects be?

2. The velocity V (in centimeters per second) of blood in an artery at a distance x cm from the center of the artery can be modeled by the function $V = f(x) = 500(0.04 - x^2)$ for $0 \leq x \leq 0.2$. What is the lowest velocity in centimeters per second the blood might be traveling? What is the highest velocity it might travel?

Chapter 4

Challenge Workshop #1

This chapter is not about seeing new things, but doing new things with topics you've seen before. You will be asked to take on two challenging tasks; one involving algebraic and numeric representations of mathematics, the other involving verbal and graphical representations of mathematics.

4.1 Converting Between Algebraic and Numeric Representations

Example 4.1.1 You are given the following algebraic representation for a function:

$$f(x) = \frac{5x-3}{x^2-4}$$

Your challenge is to give me a numeric representation (a table) that has enough points that I'd be able to make a graph of it that is relatively accurate. You will not be allowed to graph it, only choose appropriate points to include in your table. You may choose the number of points to include, what the inputs are, then find the outputs using the algebraic representation. □

Example 4.1.2 Using your numeric representation from Example 4.1.1, what would you expect $f(38)$ to be? What about $f(-2)$? Can you estimate whether the average rate of change between $f(3)$ and $f(48)$ is positive or negative? □

Example 4.1.3 Use the table of points to estimate an algebraic representation for a function

Table 4.1.4

t	$h(t)$
4	DNE
5	DNE
6	8
7	3
10	-2
15	-7
22	-12
31	-17

□

Exercises

Exercise Group. Create a table with enough information about each of the following functions that someone else could create a relatively accurate graph or answer some follow up questions about the behavior of the function.

1. $g(t) = -6\sqrt{t^2 - 7}$

2. $m(b) = |b^3 - 6b + 7|$

Exercise Group. Use the table to estimate an algebraic representation for the function.

3.

Table 4.1.5

x	-2.5	-2.4	-2.3	-2.2	-2.1
$g(x)$	3.1	3	3.1	3.2	3.3

4.

Table 4.1.6

$\clubsuit$	-2	-1	-0.5	-0.25	0.25	0.5	1	2
$d(\clubsuit)$	-5.5	-6	-7	-9	-1	-3	-4	-4.5

4.2 Converting Between Verbal and Graphical Representations

Example 4.2.1 You are given the following list of facts about a situation which can be modeled by a function.

- We are measuring the number of bacteria in a culture, measuring time in minutes as the input.
- At 5 minutes, we measure there to be 40 bacteria in the culture.
- The average rate of change from 5 minutes to 55 minutes is 16 bacteria per minute.
- Growth of the bacteria slows to almost constant between 60 minutes and 90 minutes.
- After 2 hours, the bacteria can no longer be supported by their environment and begin to die off.
- At 3 hours, we measure there to be 40 live bacteria in the culture.
- At 4 hours, we measure there to be 0 live bacteria in the culture.

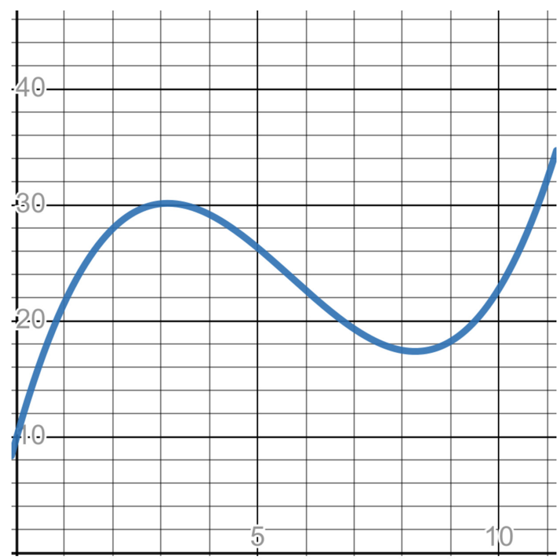
Create a graph for a function which satisfies all of these facts. □

Exercises

Exercise Group. For each of the descriptions below, create a graph for a function which satisfies each component of the description.

1. The amount of time it takes me to write an essay is two hours total for the introduction and conclusion and 30 minutes per page of the middle sections.
2. We are keeping track of the height of the ball after being thrown. It reaches a maximum height of 12 feet, then lands on the ground at 6 seconds, 9 seconds, 11 seconds, and 12 seconds.
3. You want to keep track of good deeds, starting with yourself. You do good deeds for two people you know and ask them to do the same. Assuming all people do their two good deeds, the input is measured in how far down the line the favors are "paid forward", while the output is measured in number of good deeds done.

Example 4.2.2 In this example, you will use the following coordinate plane:

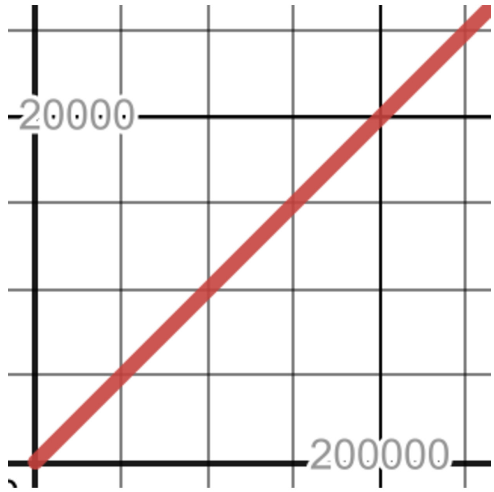


Suppose that the graph above represents the amount of revenue, in billions of dollars, of iPads where the input is years since 2010. Your challenge is to give a description, in words, of this graph with enough detail that someone else could either redraw the graph accurately from your description or answer follow-up questions about iPad revenue. □

Exercises

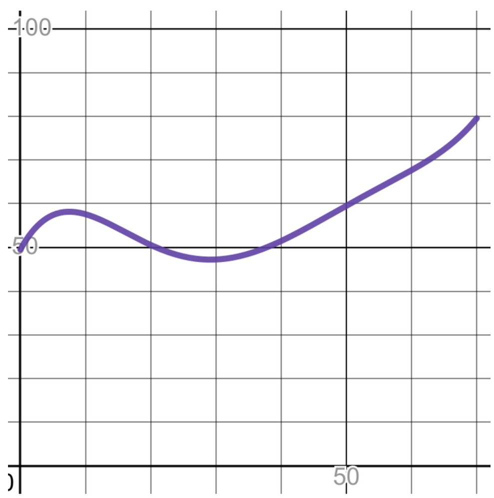
Exercise Group. Give a description of each of these graphs with enough detail that someone else could either redraw the graph accurately from your description or answer follow-up questions about the scenario. You are also provided with inputs and outputs.

1.



The graph represents the amount of money you should spend on a car based on the amount of annual income you have.

2.



The graph represents a model of the number of tornados in Oklahoma during each year for inputs representing years since 1950.

Chapter 5

Converting Algebraic Forms

So far, we've practiced converting algebraic forms of mathematical information into and out of numerical and verbal forms. We'll now focus on graphical forms. You may choose to use graphical transformations of library functions or first converting to a numerical form, as needed.

Example 5.0.1 Consider the algebraic form:

$$f(x) = \frac{\sqrt{3x-4}}{x+5}$$

Sketch a graph of $y_1 = \sqrt{3x-4}$ and another graph of $y_2 = x+5$. How might each output of y_1 change by getting divided by each output of y_2?

Now convert $f(x)$ into numerical form and plot the points on a graph. Does the image make sense based on what you see of y_1 and y_2? □

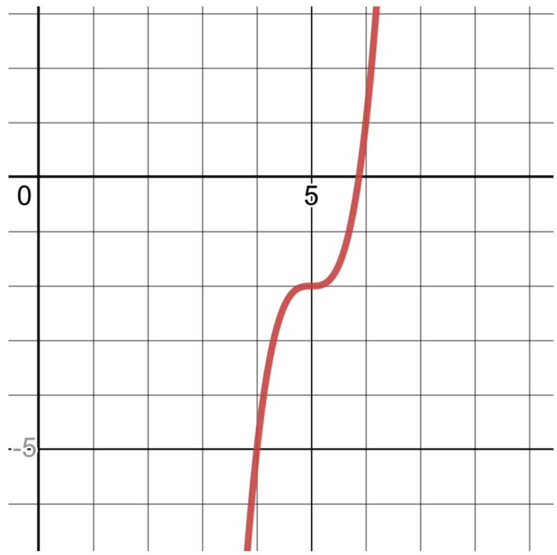
Exercises

Exercise Group. For each of the following algebraic forms for functions below, sketch a graph of the function.

1. $g(t) = -3(t-8)^2 + 6$

2. $h(r) = \sqrt{r^2 - 8} + 5$

Example 5.0.2 Consider the graph below:

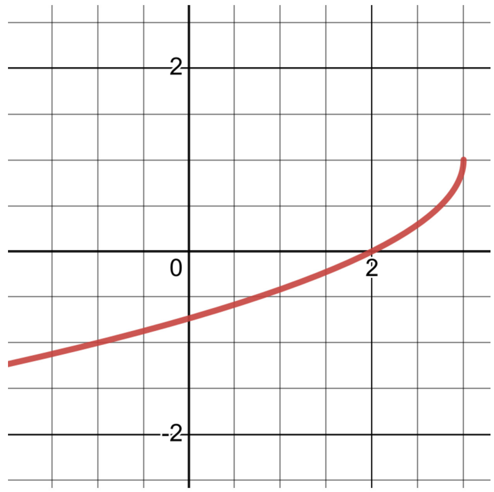


This looks like a function that is not in our Library of Functions, but is very common. It looks almost like $y = x^3$, but graphically transformed. How can you use this to create an algebraic form for the graphed function? Do so. □

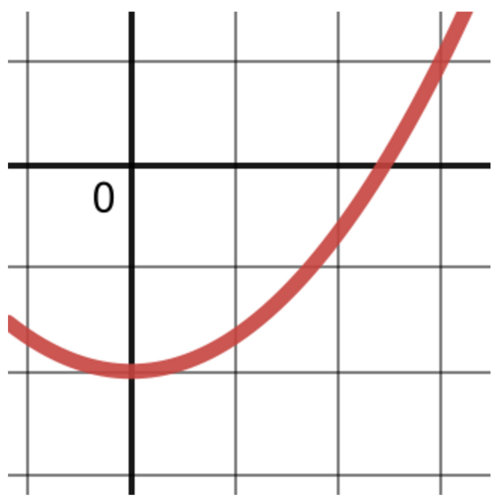
Exercises

Exercise Group. For each of the graphed functions below, convert them to algebraic form.

1.



2.



Chapter 6

Perseverance

An important aspect of problem solving is being able to persevere through challenging problems. Perseverance does not look like pushing through to the end while ignoring any red flags that come up along the way, rather it looks like stopping to consider red flags and considering a new path, only moving forward when it makes sense to do so. Perseverance sometimes looks like turning back to a previous step. Perseverance oftentimes looks like starting over.

Example 6.0.1 In Calculus, one must do a variety of complex things. Sometimes, the "aftermath" is using basic and intermediate algebra in order to simplify something rather ugly. Given the function $f(x) = (2x-3)^2(x^2+x+1)^2$, using the Chain Rule for Derivatives in a Calculus class would give us this derivative:

$$f'(x) = (2x-3)^2 3(x^2+x+1)^2(2x+1) + (x^2+x+1)^3 2(2x-3)2$$

This is not the way we'd want to look at this answer, though, in using it moving forward. Instead, we might want it to be a list of individual terms (a.k.a. expanded and "simplified"). Expand and simplify $f'(x)$.

Hint. Try considering small chunks at a time. You could expand the first term

$$(2x-3)^2 3(x^2+x+1)^2(2x+1)$$

separately from the second term

$$(x^2+x+1)^3 2(2x-3)2$$

and within each term, do small steps at a time. For example, in the first term, try expanding $(2x-3)^2$, then multplying the result by 3, then multiplying that result by $2x+1$, etc.

Answer. $f'(x) = 32x^7 - 18x^5 - 85x^4 - 24x^3 + 9x^2 + 44x + 15$

Solution. There are many different ways to go about expanding and simplifying $f'(x)$. It is up to you to find a solution which makes sense to you without breaking any rules. Keep working. □

Example 6.0.2 There are many applications in which we are told information using one unit, say days, but in order to use this information we need it be in a different unit, say hours. We can use relationships between equivalent numbers in order change the given information into a usable format. If I take a road trip where I average 1500 miles per day, and I'd like to see my average speed in mph, I can use a method called dimensional analysis and the relationship between days and hours:

$$\frac{1500 \text{ miles}}{1 \text{ day}} \cdot \frac{1 \text{ day}}{24 \text{ hours}}$$

To find my speed in mph, I cancel out both units and numbers where possible, and simplify this to 62.5 mph.

Let's say that a worker is able to build a short 4-ft tall garden fence on their own at a rate of about 6 inches (the width of a panel) per minute. Determine the rate the fence is built in yards per hour, then determine how many hours it will take to build a fence that is 45 yards long.

Hint. Take things one step at a time. You don't need to convert inches straight to yards, you could convert to feet first. You don't need to convert both length and time in the same step, you can do one and then the other.

Solution. You got this. Look for things that stick out. Work with someone else. Turn back to a previous step. Ask for help. Take a break, but don't give up. □

Example 6.0.3 Solve the equation:

$$0 = x^{2/3} + 5x^{1/3} + 6$$

Hint. This equation looks almost quadratic, but the $x^{1/3}$ is in the way. Try saying that $w = x^{1/3}$ and solve for w first.

Answer. $x = -8, -27$

Solution. There are a number of ways to solve this problem. Find one that makes sense to you. □

Example 6.0.4 We don't know things about trigonometry, but we can use the idea of substituting symbols in for scary looking things to reduce a trigonometric expression. Here are some rules:

$$\sec\theta = \frac{1}{\cos\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Use these to reduce:

$$\frac{\csc\theta\cos\theta + \cot\theta}{\sec\theta\cot\theta}$$

Write your answer in terms of θ

Hint. Try saying that $\sin\theta = A$ and $\cos\theta = B$.

Answer. $2\cos\theta$

Solution. Nah, you got this. □

Chapter 7

Challenge Workshop #2

This chapter is not about seeing new things, but doing new things with topics you've seen before. You will be asked to take on two challenging tasks, both involving setting up an equation based on a problem statement, expanding and simplifying polynomial expressions, and solving polynomial equations.

7.1 Building a Box

In the following example, you might need the fact that the area of a rectangle is $A = \ell w$ where A ia the area, ℓ is the length, and w is the width. You might also want to use the fact that the volume of a rectangular prism (or a box) is $V = \ell wh$, where V is the volume, ℓ is the length, w is the width, and h is the height.

Example 7.1.1 Suppose a large rectangular piece of cardboard with length of 24 feet and width of 36 feet can have squares with length and width of x cut out from each corner in order to fold up into the shape of a box (with no lid). Draw and label both a picture of the flat piece of cardboard and a picture of when it is folded up into the shape of a box. Answer each of the following questions:

- What values are allowed for x? Give reasoning in terms of the equation for the volume of the box. (In other words, when is the volume positive)?
- What values are allowed for x? Give reasoning in terms of the real-world context of this problem.
- When will the volume of the box be equal to $24,920$ cubic feet?
- When will the area of the base of the box be equal to 160 square feet?

Hint. When you are thinking through problems like this, you should focus on what's possible. What is possible, mathematically? What is possible in the real world? When things go wrong, is it because you are trying to do something that's not possible?

Answer. The values allowed (overall) for x are in the interval $(0, 12)$. The value of x for a volume of $24,920$ cubic feet is 30 feet, which isn't possible given the real-world context. The area of the base of the box will be equal to 160 square feet when x is 8 feet. □

7.2 An Exploration of Profit

For the following example, you might need the fact that revenue for selling a product is $r = su$ where r is the revenue in dollars, s is the selling price, and u is the number of units sold. You might also need the fact that the profit from selling a product is $P = r - c$ where P is the profit in dollars, r is revenue, and c is cost of creating and selling the product.

Example 7.2.1 The following table represents the number of units sold for an item, given the selling price of the item.

Table 7.2.2

Selling Price	1	5	10	50
Units Sold	1180	1100	1000	200

Answer the following questions:

- What is the revenue, given the selling price of the item?
- Assuming the cost of producing items is $c = 10u$, find the equation for profit, given the selling price of the item.
- What is the smallest selling price of the item in order to turn a profit?
- What is the largest selling price of the item before no longer being able to turn a profit?

Hint. All of your equations should have s as an input.

Answer. The selling price s can be in the interval $(10, 60)$ in order to turn a profit. □

Chapter 8

Expanding and Condensing Complicated Expressions

Now that we have a lot of experience with polynomials (linear, quadratic, and beyond), we might want to work on other types of expressions in terms of expanding them (like distribution) and condensing (like combining like-terms). In this chapter we will work on ways we are allowed to expand and condense expressions that are:

- rational
- radical

8.1 Rational Expressions

A rational expression is a polynomial divided by a polynomial. Because of this, you still need the skills used in expanding and condensing polynomial expressions, but will also need to take into account that condensing may happen between the numerator and denominator.

Example 8.1.1 Consider the rational expression

$$\frac{(x+2)^2(x-6)(x-8)}{(x+2)(x+4)(x-8)}$$

Write it in the most simple way imaginable.

Hint. Terms attached through addition and subtraction are "besties" and can't be canceled away from each other. Instead, you may cancel common *factors* between the numerator and denominator

Answer. $\dfrac{(x+2)(x-6)}{(x+4)}$ or $\dfrac{x^2-4x-12}{x+4}$ □

Example 8.1.2 Write the following rational expression in the most simple way imaginable.

$$\frac{x^2-3x-4}{x+1}$$

Hint. You will need to factor the numerator

Answer. $x-4$ □

Example 8.1.3 Write the following rational expression in the most simple way imaginable.

$$\frac{(x-3)(x+2)}{14x^2-14x-84}$$

Answer. $\frac{1}{14}$ □

Exercises

Exercise Group. Fully simplify each of the following expressions.

1. $\frac{(x^2-4)(x-4)^2}{(x+2)^2(x+4)}$

Answer. $\frac{(x-2)(x-4)^2}{(x+2)(x+4)}$

2. $\frac{(x+2)^{1/2}}{(x+2)^{3/2}}$

Answer. $\frac{1}{(x+2)^{1/2}}$

3. $\frac{(x-7)}{(x-7)^{-1}}$

Answer. $(x-7)^2$

8.2 Radical Expressions

Radical expressions may have exponents which are fractions. An example of a radical expression is one which involves a square root, or an exponent of one-half.

Example 8.2.1 Find a different way to write the following radical expression, which has as few "things" under the square root symbol as possible:

$$\sqrt{8x^3(x^2-4x+4)}$$

Hint. Because it is a square root, you can pull out "two things" at a time. For example, if you had some multiple of 9 as a factor inside the square root, then you'd be able to pull it out of the square root as a 3.

Answer. $2|x(x-2)|\sqrt{2x}$ □

Example 8.2.2 Sometimes your radical expression won't be written with a square root symbol, but you will still want to reduce it, if possible. Try the following:

$$(8x^3(x^2-4x+4))^{2/3}$$

The numerator of the exponent still represents the term being multiplied by itself that many times. The denominator of the exponent represents the "root" you are taking. For example, a square root would be an exponent of one-half.

$4x^2(x-2)(x-2)^{1/3}$ □

Exercises

Exercise Group. Fully simplify the following expressions.

1. $\dfrac{\sqrt{(x-3)^3}}{\sqrt{x-3}}$

Hint. It might help to use fractional exponents instead of square root symbols when the symbols get in the way of your thinking.

Answer. $|x-3|$

2. $\dfrac{(x^2-6x+8)^{4/5}}{(x-2)^{-1/5}(x-4)^{-1/5}}$

Hint. Think about all the rules of exponents you know. This will help for future lessons as well.

Chapter 9

Modeling

One of your course goals is to model mathematical information. We've been doing this throughout the semester, usually in the context of a "word problem". In this section we will focus on a few aspects of modeling:

- Using data to estimate a model.
- Use an estimated model to answer questions about the real-world context.

9.1 Estimates for Models

In this section, you will see some tables of data representing real-world data. Your job will be to determine what type of function the data might represent and estimate an algebraic representation for that data. The algebraic functions you create will not be the function "of best fit", but should be a reasonable representation of the information.

Example 9.1.1 Let's say you frequently go to restaurants with your friends. The following figure gives how much money is spent based on the number of people dining. Each dot represents one meal out with friends.

Specify a static image with the `@preview` attribute;
Or create and provide an automatic screenshot as
`generated/preview/interactive-1-preview.png`
via the `PreTeXt-CLI` application or `pretext/pretext` script.

`www.desmos.com/calculator/qo7d4naisp`

Use this graphing utility feature to answer the questions

- Scrolling down below the table should show you an equation, which is also plotted onto the calculator. Change the numbers for the slope and y-intercept to find a linear equation, which would reasonably fit the data.
- Consider, though, that the data might not be best estimated by a linear function. Try finding a quadratic equation which also reasonably fits the data.
- Using both equations (the linear and quadratic), estimate the amount of the tab when there are 3 people dining.

- You and three of your friends go out to eat and order the typical number and types of dishes and drinks. When the bill comes, your friends asks them to bring the itemized version (listing all items ordered alongside their prices) so they can compute the total from scratch. How large might the bill have been to cause this concern?

□

Example 9.1.2 You are gathering data about summer enrollment at The University of Oklahoma. The following table and plotted points represent the change in enrollment from year to year, starting where $x = 0$ represents the year 2012. Each dot represents the difference between the current year and the previous. A positive number indicates that enrollment went up that summer, while a negative number indicates that enrollment went down.

Specify a static image with the `@preview` attribute;
Or create and provide an automatic screenshot as
`generated/preview/interactive-2-preview.png`
via the `PreTeXt-CLI` application or `pretext/pretext` script.

`www.desmos.com/calculator/b9ov86yz8x`

Use this graphing utility feature to answer the questions:

- Let's assume that the data can be best represented as a polynomial function of degree 4. There are four sliders available in the graphing utility, one for each zero (or root) that is not at the origin and one for the leading coefficient. Find values for a, b, c, and d that you think are reasonable (there will not be a perfect fit, so find something you can live with).

- *Interpolation* is when you estimate based on things happening during the original domain of the data. We are using interpolation when answering questions about anything from $x = 0$ to $x = 9$. What is the maximum amount of change in enrollment possible, according to your model?

- *Extrapolation* is when you estimate what might happen outside of the original domain of the data. We would be using extrapolation to estimate what happened before 2012 or after 2021. Extrapolation is always much more risky, depending on the behavior of the data. What do you think the enrollment change will be going into Summer 2022?

□

Chapter 10

Rules of Exponents

There are a number of rules we have come to know and love about exponents and their bases. To prepare for the equivalent logarithmic rules, we'll do some review of those rules in this chapter. We'll review the following:

- Product Rule for Exponents
- Negative Exponent Rule
- Quotient Rule for Exponents
- Zero Exponent Rule
- Power Rule for Exponents

10.1 Product Rule for Exponents

The **Product Rule for Expoennts** helps us to condense a statement like $a^m a^n$ into something more concise. Let's start with something more concrete.

Example 10.1.1 You may already know the product rule for exponents. Resist the urge in this example to use it immediately. Instead, try thinking through the problem "the long way" and give a full explanation. Condense the following:

$$a^2 a^3$$

Hint. Write out the full version of a^2 and a^3

Answer. a^5

Solution. "The long way" is to say that

$$a^2 a^3 = a * a * a * a * a = a^5$$

Once you have a hang of it, you can use the product rule which says that $a^m a^n = a^{m+n}$ □

Exercises

Exercise Group. Use the product rule for exponents to condense the following:

1. $2^3 2^6$

2. $3^6 3^{-6}$

Hint. You need not continue condensing this one until you've finished the other examples in the other sections.

10.2 Negative Exponent, Quotient, and Zero Exponent Rules for Exponents

We'll cover three of the rules in this section, because they build off of the product rule and each other. First, the negative exponent rule.

Example 10.2.1 Imagine that we have been using the product rule all day on things like $2^2 2^5$ and $3^4 3^7$, when we come upone one exercise that looks like $3^3 3^{-1}$. Use the produce rule to determine the value of this expression, then tell me another way of saying 3^{-1}, and why this is the case. □

Example 10.2.2 The previous example should tell us that the negative exponent rule is $a^{-1} = \frac{1}{a}$. Using this negative exponent rule, find the value of $\frac{2^8}{2^6}$ and explain all your reasoning. □

Example 10.2.3 It is not much of a stretch from the product rule, then, to say that the quotient rule is $\frac{a^m}{a^n} = a^{m-n}$. What if we need to know a^0? Any combination of rules so far to explain the value of a^0. □

Exercises

Exercise Group. Use any combination of rules so far, so condense the following expressions.

1. $\dfrac{a^2 b^{-4} c^6}{a^3 b^{-5} c^3}$

2. $\dfrac{2^2 3^5 x^{-1}}{3^4 x}$

10.3 Power Rule for Exponents

One other rule that's useful is **The Power Rule for Exponents**, but it operates similarly to the product rule, using a different operation.

Example 10.3.1 Using similar reasoning as the other sections, describe the condensing of the expression $(a^3)^2$ □

So we can use the power rule for exponents which states $(a^m)^n = a^{mn}$

Exercises

Exercise Group. Use any combination of rules for exponents to condense the following.

1. $(2^3)^2$

2. $(x^{0.5})^2$

3. $(45^7)^0$

Chapter 11

Challenge Workshop #3

In this section, you will do some problem solving. In particular, you will be working on examples which focus on verbal and numerical representations more than they do about explicit algebraic manipulations or graphs. These are examples of ill-structured problems, which don't necessary have a single correct solution.

11.1 Converting Between Verbal and Numerical Representations

Example 11.1.1 The following table represents a movie start time for a certain theatre in the first row, the number of children's tickets for that time in the second row, the number of adult tickets for that time in the third row, and the number of senior tickets in the fourth row. Note that there are 80 seats in the theatre.

Table 11.1.2

10:00 AM	1:35 PM	5:40 PM	9:15 PM
25	38	23	12
31	28	31	61
15	6	26	7

Answer the following questions:

- If a character from the film were to have their likeness stand outside the theatre for pictures, what time would be best to schedule this event? Why?
- What time of day should the movie theatre assign two bartenders instead of one? Why?
- The movie theatre would like to offer a special senior discount next week. Based on this information, what time of day should they offer it? Why?

□

Example 11.1.3 You are assigned to schedule tutors for the Math Center's yellow room (where College Algebra and PreCalculus Trig students go for studying help). The busiest days tend to be Tuesday and Wednesday at around 12pm to 4pm, where at least 3 to 4 tutors should be scheduled, while other times (for the sake of this example, let's say that they are open from 9AM to

5PM, Monday through Friday) are either empty or not so busy that one or two tutors can't cover the entire room. There are 10 tutors to schedule and each has their own course schedule and preferences. The maximum number of hours a tutor can work in the Math Center is 20 hours. Using the table of information below with each tutor's class schedule and preferences, create their weekly schedule for the semester.

Table 11.1.4

Name	*Course Schedule*	*Other Preferences*
Antoine	MWF 9:30am-10:20am, MW 10:30am - 11:45am, MWF 12:30pm - 1:20pm, TR 9:00am - 10:15am, TR 12:00pm - 1:15pm	Antoine also works in a another department on campus, so will need to have a maximum of 8 hours in the Math Center.
Bella	MWF 8:30am - 9:20am, TR 9:00am - 10:15am, TR 1:30pm - 2:45pm	Bella is a graduate student with a teaching assignment, so will need to have a maximum of 6 hours in the Math Center.
Cadie	TR 9:00am-10:15am, TR 10:30am-11:45am, TR 12:00pm-1:15pm, TR 1:30pm-2:45pm	Cadie will only work on Mondays or Wednesdays.
Diego	MTWRF 12:30pm-1:20pm, TR 1:30pm-2:45pm, MWF 1:30pm-2:20pm, W 4:25pm-7:05pm	Diego does not want to work in the morning.
Emersyn	MTWRF 9:30am-10:30am, TR 1:30pm-2:45pm, MW 3:00pm-4:15pm, TR 3:00pm-4:15pm	Emersyn needs to work the maximum number of hours, if possible.
Fatma	MWF 10:30am-11:20am, TR 10:30am-11:45am	Fatma is a graduate student with two classes to teach, so will need to have a maximum of 2 hours in the Math Center.
Graceann	MWF 9:30am - 10:20am, MWF 11:30am - 12:20pm, MWF 1:30pm-2:20pm, MW 3pm-4:15pm, TR 12:00pm-1:15pm	Graceann has no preferences.
Harrison	MW 3:00pm-4:15pm, MW 4:30pm - 5:45pm, TR 3:00pm-4:15pm, TR 4:30pm-5:45pm	Harrison prefers to not work before noon on any day, and cannot be on campus on Fridays, but would like as many hours as possible.
Ignacius	MWF 10:30am-11:20am, MWF 11:30am-12:20am, TR 3:00pm-4:15pm, MWF 3:30pm-4:20pm, MW 4:30pm-5:45pm	Ignacius prefers to work in large chunks at a time, with only one shift per day.
Jamar	TR 9:00am-10:15am, TR 10:30am-11:45am, MWF 12:30pm-1:20pm, MW 1:30pm-2:45pm, R 12:00pm-12:50pm	Jamar wants as many hours as possible, but only wants to work shifts that are one hour in length, with breaks in between.

□

Chapter 12

Critiquing and Evaluating

An important aspect of problem solving is being able to critique and evaluate your work and the work of others. How do you know your answer is the correct or best answer? How do you know that what someone hands you is the correct or best solution to a problem?

12.1 Evaluating Last Chapter's Solutions

Example 12.1.1 Find someone else who did Example 11.1.3 and present your solutions to each other. Identify the parts that were done differently between the solutions and determine if one solution is a better fit than the other. Were more tutors able to have their preferences met in one solution over the other? Are all the busiest times appropriately covered by enough tutors? What else should be used as criteria for what would make the "best" solution? □

12.2

Evaluating Fresh Soltuions

Example 12.2.1 A quiz asks the following question:

"Consider the relation: $\{(0,1),(3,6),(4,-4),(2,3),(5,1),(0,-1)\}$. Discuss whether this relation is a function. Why or why not?"

A student gave the following answer and explanation:

The relation is not a function because there is a repeating input value of 0.

Using our standard scoring rubric for the course, determine what score this student should recieve and explain why. □

12.3 Exercises

Exercise Group. Each of the following exercises gives a question as it would be presented on a quiz or exam, followed by a partially or entirely incorrect solution. Determine which parts are incorrect, and what could be corrected in order to get the correct or best solution.

1. Q: Find the average rate of change of $f(x)$ between $x=-8$ and $x=4$, where $f(x)=x^2+2$.

A:

$$(-8)^2+2=66$$

$$(4)^2 + 2 = 18$$

$$\frac{-8-4}{66-18} = \frac{-12}{48} = -\frac{1}{4}$$

When you plug in the correct numbers and use the formula, $\frac{f(x_1)-f(x_1)}{f(x_2)-f(x_2)}$, you get $-\frac{1}{4}$.

2. Q: If $f(x) = \frac{3-5x}{3x}$ and $g(x) = \frac{10}{3x+5}$, then find and simplify $(f+g)(x)$.

A:

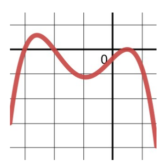
$$\frac{3-5x}{3x} + \frac{10}{3x+5} = \frac{-7+5x}{6x+5}$$

3. Q: Find the equation of the line which is parallel to the line $8x+2y+6 = 0$ and that passes through the point $(-3, 5)$

A:

$$y = 4x + 3$$
$$y - 5 = 4(x + 3)$$
$$y - 5 = 4x + 3$$
$$y = 4x + 8$$

4. Q: Estiamte an equation for the graph of $b(x)$ drawn below. Leave your answer in factored form:



A:

$$x = -3, -2, 0.5$$
$$1(x + 2)^2(x - 0.5)(x + 3)$$

End behavior = even, going down

$$-(x + 2)^2(x + 3)(x - 0.5)$$

5. Q: What is the domain of the function $f(x) = y + \log(2x + 10)$?

A:
input: left 10, compress 2
output: up 7
Parent domain: $(0, \infty)$
Parent range: $(-\infty, \infty)$
Parent VA: $x = 0$
D: $(7, \infty)$

6. Q: $f(x) = 16x^4 + kx^2 + 6x + 10$ has a zero at $x = \frac{1}{2}$. Find the value of k.

A:

$$16\left(\frac{1}{2}\right)^4 + k\left(\frac{1}{2}\right)^2 + 6\left(\frac{1}{2}\right) + 10$$

$$16\left(\frac{1}{16}\right) = 1$$

$$1 + k\frac{1}{4} + 3 + 10$$

$$1 + k\frac{1}{4} + 13$$

$$k\frac{1}{4} + 12$$

$$k = 48$$

Chapter 13

Challenge Workshop #4

The problems in this chapter will include condensing and expanding exponential expressions

Example 13.0.1 Show that

$$\frac{f(x+h)-f(x)}{h} = 2^x \frac{2^h - 1}{h}$$

when $f(x) = 2^x$. $\square$

Example 13.0.2 Solve the exponential equation:

$$\left(\frac{1}{6}\right)^{3x+6} \cdot 216^{3x} = \frac{1}{216}$$

$\square$

Example 13.0.3 Solve the exponential equation:

$$\frac{81^{3x+2}}{243^x} = 3$$

$\square$

Chapter 14

Expanding and Condensing Logarithmic Expressions

In this section you will continue working on what you know about the properties of logarithms to solve problems.

Example 14.0.1 Expand and simplify the logarithm:

$$\log\left(\frac{x^{15}y^{13}}{z^{19}}\right)$$

□

Exercises

Exercise Group. Expand and simplify the logarithmic expressions.

1. $\log(\sqrt{x^3y^{-4}})$
2. $\log\left(\frac{a^{-2}}{b^{-4}c^5}\right)$

Example 14.0.2 Condense and simplify the logarithmic expression:

$$\log(2x^4) + \log(3x^5)$$

□

Exercises

Exercise Group. Condense and simplify the logarithmic expressions.

1. $2\log(x) - 3\log(x+1)$
2. $4\log_7(c) + \frac{\log_7(a)}{3} + \frac{\log_7(b)}{2}$

Example 14.0.3 Suppose that $\log_5(6) = a$ and $\log_5(11) = b$. Use the change-of-base (using a base of 5) along with properties of logarithms to rewrite the logarithmic expression below in terms of a and b:

$$\log_{11}(5)$$

□

Exercises

Exercise Group. Using the same definition for a and b, rewrite each logarithmic expression below in terms of a and b.

1. $\log_6(11)$

2. $\log_{11}\left(\frac{6}{11}\right)$

Exercise Group. As an added challenge, work on these problems which use different properties of logarithms.

3. Use the product rule for logarithms to find all x-values such that $\log_{12}(2x+6)+\log_{12}(x+2)=2$.

4. Using change-of-base, show that $\log_b(n)=\dfrac{1}{\log_n(b)}$ for any positive integers $b>1$ and $n>1$.

5. Does $\log_{81}(2401)=\log_3(7)$? Verify this algebraically.