College Algebra - Stretch

Foundational and Challenge Workshops for College Algebra at the University of Oklahoma

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My school

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For ...

Acknowledgements

I would like to thank...

Preface

About this book:

Not just that..

I still have more to say!

Contents

Chapter 1

Review for Variability and Functions

At the beginning of the semester, there are a lot of topics you might like to review alongside variability and an introduction to functions. This worksheet serves to address the following:

- Using the order of operations to expand and condense linear expressions
- Using the order of operations to expand and condense polynomial and rational expressions
- Converting between algebraic and verbal representations
- Solving linear equations

1.1 Expanding and Condensing Expressions

For the purposes of this class, a **linear expression** involves up to a single variable with its highest exponent at one. The variable might be represented by any number of non-numeric symbols, typically letters (Latin or Greek) like x or α, but may be represented by any symbol you wish, like $\bigstar$ or $\heartsuit$.

Examples of linear expressions include

$$3x - 5$$

and

$$9 - 4x$$

and

$$\frac{3^2 - 4x + 6x - \frac{3}{2}}{10}$$

1.1.1 Condensing Linear Expressions

You may be wondering why the last example of a linear expression is considered as such if it involves both exponents and fractions. The definition for a linear expression did not include either. There are a few reasons why this expression is linear:

- the highest exponent of the *variable* is one
- the numbers in the linear expression may be any constant we can think of, including fractions
- the division is by a number, not a variable

We can see these things in action if we use a reasonable order of operations to condense the expression into a format with a single constant and a single variable term.

Example 1.1.1 Condensing a linear expression. Condense $\dfrac{3^2 - 4x + 6x - \frac{3}{2}}{10}$ into a format with a single constant term and a single variable term.

Answer. $\dfrac{3}{4} + \dfrac{1}{5}x$

Solution.

$$\frac{3^2 - 4x + 6x - \frac{3}{2}}{10}$$

$$\frac{1}{10}\left(9 - 4x + 6x - \frac{3}{2}\right)$$

$$\frac{9}{10} - \frac{4/10}{x} + \frac{6/10}{x} - \frac{3}{20}$$

$$\frac{18}{20} - \frac{3}{20} + \frac{2}{10}x$$

$$\frac{15}{20} + \frac{2}{10}x$$

$$\frac{3}{4} + \frac{1}{5}x$$

□

Example 1.1.2 Analyzing Example 1.1.1. What is the order of operations used in the example above?

Hint. The typical operations to choose from are addition, multiplication, exponents, division, parenthetical statements, and subtraction

Answer. exponents, parenthetical statements, multiply, add, subtract, reduce

Solution.

1. applied *exponents*
2. reimagine division as multiplying by a fraction, and distributing it into a *parenthetical statement*
3. *add* like terms, begin to *subtract* constants by finding common denominators
4. *subtract* the constants
5. *divide* by reducing fractions (always allowed) or writing answers in decimal form (sometimes allowed)

□

Puzzle 1.1.1 Critiquing and Evaluating Example 1.1.2. Note that the previous examples used a reasonable order of operations to condense the linear expression. Did you use the same order of operations when you attempted the problem? Was the order you used reasonable as well? Why or why not?

Answer. There are many reasonable order of operations you can use. We typically use the operations in pairs from left to right: parentheses and exponentns, multiplication and division, addition and subtraction. Consider why we might deviate from this order on occasion, for example in the previous example and checkpoint, it would have been reasonable to condense the entire numerator before dividing each term by 10. Why is that reasonable?

Exercises

Exercise Group. Use a reasonable order of operations to expand and condense the following linear expressions to a format which has one constant term and one variable term. When you are done, use a different, but still reasonable, order of operations to expand and condense the expression. The answers are provided.

1.1.2 Expanding and Condensing Other Expressions

Some expressions may not be linear, using exponents other than one on numerous variables. Note that using multiple variables does not necessarily make the expression non-linear, but involving elements like x^2, $\sqrt{\alpha}$, and $\frac{1}{\heartsuit}$ are *certainly* non-linear when x, α, $\heartsuit$ are variable. This does not change how we address using a reasonable order of operations for expanding or condensing a complicated expression. The only alteration might be additional considerations for what it means to combine ``like terms''.

Exercises

Exercise Group. Use a reasonable order of operations to expand and condense the following expressions into what might be considered simplest form.

1. $(ab)^2a^3b$

Answer. a^5b^3

2. $(x-3)(3x^2-2x+1)$

Answer. $3x^3-11x^2+7x-3$

3. $$\frac{\left(\frac{2x}{3}+\frac{1}{6}\right)^2}{\frac{2}{3}-\frac{5x}{9}}$$

Answer. $\dfrac{16x^2+8x+1}{24-20x}$ or $\dfrac{(4x+1)^2}{4(6-5x)}$

In a relatively simple (keep in mind that *simple* means that it is not overly complicated with a large number of steps, not that it is easy) expression, a reasonable order of operations may look much the same for everyone. In *complex* expressions, involving a large number of steps to expand and condense to what might be considered its ``simplified'' form, you might be able to find a reasonable order of operations that is perfectly correct, but different than how someone else approached it. Like with many things in mathematics, there is not always a *single correct solution*, though there may be *best solution* or what you consider to be the *easiest solution*. The terms ``best'' and ``easiest'' are subjective, and oftentimes up to the person doing the mathematics.

1.2 Converting Between Algebraic and and Verbal Representations of Information

If I told you that my dog eats 24 ounces of dog food per day, we'd be able to determine how much dog food she's consumed by the end of a typical week by creating an algebraic version of this relationship. It is best to define my input variable and units as well as my output variable and units. In this case, let's use c for *consumed* as the output variable to represent the number of ounces she consumes each day. We can use d to represent *days* since we've started counting her food consumption as the input variable. This means that the relationship looks like

$c = 24d$ ounces of dog food consumed, where d is days since counting her food consumption.

Notice that saying $c = 24d$ isn't quite enough, since c and d don't hold meaning or context on their own. Even in the algebraic version, we need to use some words to describe what we are doing. The algebriac version above is what's called a **model** of the real-world information in algebraic form.

1.2.1 Turning Verbal Information into Algebraic Models

Example 1.2.1 Write an algebraic model for the following: The stock of a certain NASDAQ company began the day trading at 75 dollars per share, but decreased by 2 dollars per hour. □

Example 1.2.2 We can use these skills to solve other problems without creating a model. For example, we want to find two numbers so that one of them is three more than twice the other number, and the sum of the two numbers is 36. Create a setup for this situation.

Answer.

$$2x + 3 = y$$
$$x + y = 36$$

□

Example 1.2.3 Suppose you want to find the dimensions of a small rectangular performance stage which has perimeter of 110 ft (feet) and has length that is 1 foot more than twice the width. Create a setup for this situation.

Answer.

$$\ell + w = 110$$
$$\ell = 1 + 2w$$

□

1.2.2 Turning Algebraic Information into Verbal Information

Exercises

Exercise Group. Take each of the algebraic scenarios below and convert them into a verbal format. Note that a verbal format does not involve algebraic expressions.

1. The cumulative amount of dog food eaten by my dog is given by $C = 0.75f + 1$ pounds of food, after f number of feedings.

2.

$$V = LWH$$
$$L = 2W$$
$$H = 8$$

Note that some information is missing. Make up a scenario which this would reasonably represent, including the units for each variable.

3. Use your own words to describe this situation where n is a number: $3(n + 5) = 5n$.

1.3 Solving Linear Equations

A **linear equation** is an equation which sets two linear expressions equal to each other. The linear equation may have a single variable, like

$$3(x - 5) + 2x = \frac{1}{2}x - 5$$

or have multiple variables like

$$y = 5x + 6$$

and

$$2x = 3y - 7x + 5$$

Solving linear equations might include expanding or condensing the expressions on either side, but will also include what is often referred to as ``algebraic manipulations'', or performaning a certain operation on both sides of the equation simultaneously. This means each subsequent step of the solution is equivalent to the last.

Example 1.3.1 Solving a Linear Equation. Solve $3(x - 5) + 2x = \frac{1}{2}x - 5$ for the variable x

Answer. $x = \frac{20}{9}$

Solution.

$$3(x - 5) + 2x = \frac{1}{2}x - 5$$
$$3x - 15 + 2x = \frac{1}{2}x - 5$$
$$5x - 15 = \frac{1}{2}x - 5$$

$$\begin{aligned} 10x - 30 &= x - 10 \\ 9x - 30 &= -10 \\ 9x &= 20 \\ x &= \frac{20}{9} \end{aligned}$$

□

Example 1.3.2 Analyzing Example 1.3.1. Describe the steps for solving the equation in the previous example.

Solution.

- In the left-hand expression, multiply 3 and $x - 5$ by distribution.
- In the left-hand expression, combine the like terms of $3x$ and $2x$ through addition
- Multiply both sides of the equation by 2.
- Subtract x from both sides of the equation, and combine like terms.
- Add 30 to both sides of the equation, and combine like terms.
- Divide both sides by 9, the answer is fully reduced.

□

Puzzle 1.3.1 Critiquing and Evaluating `[cross-reference to target(s) "analyze-solve-liner" missing or not unique]`. Note that the previous examples used a reasonable order of operations and manipulations to solve for x. Did you use the same order of operations and manipulations when you attempted the problem? Was the order you used reasonable as well? Why or why not?

Answer. There are many reasonable orders of operations and manipulations you can use. The typical order of operations is the same as before, while the typical order of manipulations might go in the opposite order. Consider why we might deviate from this order on occasion, for example in the previous examples, it would have been reasonable to never distribute the 3 to $x - 5$, and never multiply both sides by 2. Why is that reasonable? What might you do instead?

Exercises

Exercise Group. Solve the following linear equations for the variable b, describing each step in words. The answers are provided.

1. $-6b - 4 = 20$

Answer. $b = -4$

2. $2.3 = 4.5b + 30.2$

Answer. $b = -6.2$

3. $-3(b - 4) + 5 = 10 - (b + 1)$

Answer. $b = 4$

4. $\dfrac{b-1}{4} = \dfrac{2b+1}{5}$

Answer. $b = -3$

5. $\dfrac{b-6}{3} = \dfrac{b+3}{2} + 1$

Answer. $b = -27$

Exercise Group.

6. If my dog consumes 15 pounds of food per week, how many pounds of food will she consumer over the next year?

780 pounds of food

7. A number is four more than twice another number. If one of the numbers is 6, what could the other number be? There are two answers.

Answer. The other number could be either 1 or 16.

8. You are selling t-shirts at a music festival, and start the festival with 470 t-shirts. You sell about 23 per hour during festival hours. How many hours must the festival run in order for you to sell out of shirts?

Answer. The festival needs to run at least 21 hours to sell out of the 470 available t-shirts.

Chapter 2

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Chapter 3

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Selected Hints

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Selected Solutions

Appendix C

List of Symbols

Symbol	Description	Page

Colophon

This book was authored in PreTeXt.