Theoretical Concepts of Computer Science

Answered Questions for Sperner's Theorem

Presentation is on Sperner's Lemma

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Q1. How did sperner's theory came into the mathematic world and what is sperner's theorem?

Answer: Sperner's theorem is a result in extremal set theory with relations to algebra & combinatorics. This theorem was discovered by Emanuel Sperner around 1928, which gives the size of the largest family of subsets of [n] not containing a 2-chain $F1 \subset F2$.

Q2. What are some applications of Sperner's theorems?

Sperner's Thereom can be applied to resolve the Littlewood–Offord problem; this describes the number of sub-sums of a set of vectors that fall in a given convex set. Hungarian mathematician Paul Erdős, discovered the first upper bound for this problem was proven (for d = 1 and d = 2) in 1938, and around 1945 he modified d = 1 to 2n (1/squareroot(n)) using Sperner's Theorem.

Q3. Why this theorem and why not other one?

Answer: Fate has led me to answer questions about Sperner's Theorem however I am presenting on Sperner's Lemma.

Q4. Given a set system F over the base set {1, . . . , n}, we call F semi-independent if it contains no three sets A,B,C such that $A \subset B \subset C$.

Prove that $|F| \leq 2 \lfloor n/2 \rfloor$

Answer: If F does not contain a 3-element chain $A \subset B \subset C$, then F is essentially the *union of two antichains*. This could mean that each permutation could contain twice of the sets of F.

Q5. What is chain and anti-chain in Sperner's theorem?

Answer: A chain in a Poset P is a subset $C \subseteq P$ such that any two elements in C are comparable where an antichain in a Poset P is a subset $A \subseteq P$ such that no two elements in A are comparable. In Sperner's theorem, the size of a largest antichain of an n-set is (n (n/2))

Q6. Sperner’s Lemma

Answer: Your curiosity on Sperner’s Lemma will be presented on the PowerPoint Presentation

Q7. How does Sperner Theorem work?

Answer:

Here’s how it works:

Let A be a set with n elements. Let S1,…,Sm be subsets of A, such that no two of them are related by containment, example for any distinct $i \neq j$, we have $Si \not\subset Sj$. What would the largest possible value of m be? How many mutually incomparable subsets can we fit inside A? Sperner's theorem gives the answer:

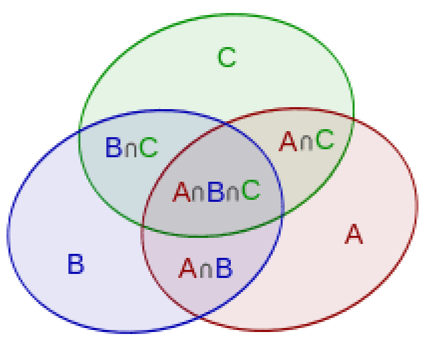
$m \leq (n\lfloor n2 \rfloor)$

Q8. When is a graded partially ordered set said to have the Sperner theory?

Answer: A partially ordered set “poset” is a set P together with a binary relation. A graded partially ordered set is said to have what is called a “Sperner property” when one of its largest antichains is formed by a set of elements that all have the same rank (which is the value of the rank function for an element of the poset).

Q9. How does Partial Order width relate to Sperner’s Theorem?

Answer: The family of all subsets of an n-element set can be partially ordered by set inclusion



Example of set inclusion

Q10. The proof of sperner's theorem

Answer

There are many different proofs, here is one claim:

The number of chains containing a given set A where $|A|=k$ is $k!(n-k)!$

<u>Proof:</u> If $|A|=k$, then it must happen that $A=Ak$. So, the chain containing A is obtained by joining together a chain for A and a chain for $X\setminus A$. Thus, A lies in $k!(n-k)!$ permutations.