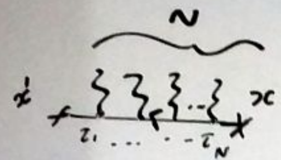
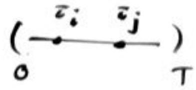
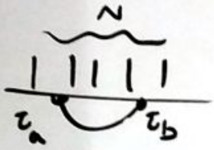


$$= \int_0^\infty dT\, e^{-m^2T}\int_{x(0)=x}^{x(T)=x'} \mathcal{D}x\, e^{-\underset{\sim}{S}[x]} = (ie)^N\int dT\,(4\pi T)^{-D/2}e^{-m^2T}\int_0^T d\tau_1\dots d\tau_N\; e^{\sum_{i,j=1}^N k_i\cdot k_j\,\Delta_{ij}}\; e^{-(x-x')^2/4T + i\sum_i k_i\cdot x'}$$

$$x(\tau) = \underbrace{x'}\, + (x-x')\tau/T + \underbrace{q(\tau)}_{\text{DBC.}}$$



$$\Delta_{ij} = \tfrac{1}{2}|\tau_i-\tau_j| - \tfrac{1}{2}(\tau_i+\tau_j) + \frac{\tau_i\tau_j}{T}$$



$$= \int_0^\infty dT\, e^{-m^2T}\int_{x(0)=x}^{x(T)=x}\mathcal{D}x\, e^{-S_N[x]}\int_0^T d\tau_a\int_0^T d\tau_b\,\underbrace{\frac{\#}{[(x(\tau_a)-x(\tau_b))^2]^{D/2-1}}}_{\int_0^\infty d\bar T\,(4\pi\bar T)^{-D/2}\,e^{-\frac{(x_a-x_b)^2}{4\bar T}}}$$

$$S_{\text{Free}}[x] \to \int \frac{\dot x^2}{4T}d\tau + (x_a-x_b)^2/4\bar T$$

$$B_{ab}(\tau_1,\tau_2) = [\delta(\tau_a-\tau_1)-\delta(\tau_b-\tau_1)][\delta(\tau_a-\tau_2)-\delta(\tau_b-\tau_2)]$$

$$S_{\text{Free}} \to \int d\tau\, d\tau'\, x(\tau)\left[-\frac{1}{4T}\frac{d^2}{d\tau'^2}\delta(\tau-\tau') + \frac{B_{ab}}{4\bar T}(\tau,\tau')\right]x(\tau')$$

- Ingredient 1: $\mathrm{Det}\left[-\frac{1}{4T}\frac{d^2}{d\tau^2} + \frac{B_{ab}}{4\bar T}\right] = [4\pi T]^{-D/2}\left[1 + \frac{G_{Bab}}{\bar T}\right]^{-D/2}$ loop G.f.
- Ingredient 2: $\Delta^{(1)}(\tau_1,\tau_2|\tau_a,\tau_b) := \left\langle \tau_1\right| \frac{-1}{4T}\frac{d^2}{d\tau^2} + \frac{B_{ab}}{4\bar T}\left|\tau_2\right\rangle$

$$= \Delta(\tau_1,\tau_2) + \frac{[\Delta_{1a}-\Delta_{1b}][\Delta_{a2}-\Delta_{b2}]}{\bar T + G_{Bab}}$$

Final Result

$$(ie)^N\int_0^\infty dT\,(4\pi T)^{-D/2}e^{-m^2T}e^{-\frac{(x-x')^2}{4T}}\int_0^\infty d\bar T\left[4\pi\bar T\left(1+\frac{G_{ab}}{\bar T}\right)\right]^{-D/2}e^{-m_\varphi^2\bar T}$$

$$e^{-\frac{(x-x')^2}{T}\frac{(\tau_a-\tau_b)^2}{T}\Big/\left[4\bar T\left(1+\frac{G_{ab}}{\bar T}\right)\right] + \sum_{i,j=1}^N k_i\cdot k_j\,\Delta^{(1)}_{ij} + i\sum_i k_i\cdot x'}$$

Take $N=1$: $\sum$