The quadratic Dynkin index is given by the ratio of $\mathrm{tr}\, X^2$ and $\mathrm{tr}_A\, X^2$ for the adjoint rep (7.30):

$$\ell_2 = \frac{\mathrm{tr}\, X^2}{\mathrm{tr}_A\, X^2} = \frac{d_s p(p+n)}{2n^2(n+1)} \,. \tag{9.116}$$

To take a random example from the Patera-Sankoff tables [273], the $SU(6)$ rep dimension and Dynkin index

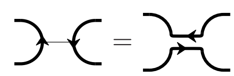
| rep | dim | $\ell_2$ |
|---|---|---|
| (0,0,0,0,0,14) | 11628 | 6460 |

(9.117)

check with the above expressions.

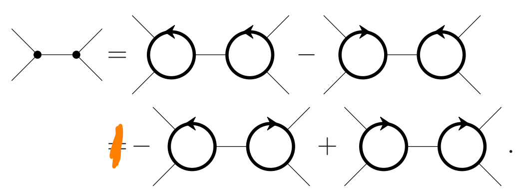
## 9.14 $SU(n), U(n)$ EQUIVALENCE IN ADJOINT REP

The following simple observation speeds up evaluation of pure adjoint rep group-theoretic weights $(3n\text{-}j)$'s for $SU(n)$: The adjoint rep weights for $U(n)$ and $SU(n)$ are identical. This means that we can use the $U(n)$ adjoint projection operator

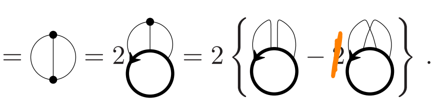
$U(n)$ :  (9.118)

instead of the traceless $SU(n)$ projection operator (9.54), and halve the number of terms in the expansion of each adjoint line.

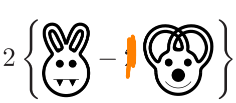
**Proof**: Any internal adjoint line connects two $C_{ijk}$'s:



The trace part of (9.54) cancels on each line; hence, it does not contribute to the pure adjoint rep diagrams. As an example, we reevaluate the adjoint quadratic casimir for $SU(n)$:

$C_A N =$ 

Now substitute the $U(n)$ adjoint projection operator (9.118):

$C_A N = 2$  $= 2n(n^2-1)$ ,

in agreement with the first exercise of section 2.2.