is QED finite?

Predrag Cvitanović

Summer school on structures in local quantum field theory
Les Houches

June 5, 2018

overview

1. what this is about
2. QED finiteness conjecture
3. worldline approach
4. bye bye, Feynman diagrams

4- and 5-loop contributions to the anomaly

in 2017 Laporta completed
the twenty-year project of computing analytically
the 891 electron 4-loop magnetic moment diagrams[1]

here we shall discuss only the quenched set, i.e., the contributions of diagrams with no lepton loops

the 4- and 5-loop contributions are to the anomaly $a = \frac{1}{2}(g-2)$:

$$
\begin{aligned}
a^{(8)} &= -2.176866027739540077443259355895893938670 \\
a^{(10)} &= 8.726(336) \quad \text{Aoyama } et\ al.^{2}\ .
\end{aligned}
$$

Q : why are these numbers so small?

---

[1] S. Laporta, Phys. Lett. B **772**, 232–238 (2017).

[2] T. Aoyama et al., Phys. Rev. D **91**, 033006 (2015).

gauge cancellations?

as a prelude, you might enjoy the Dunne and Schubert[3] historical review of ideas about the QED perturbation series

they note:

a point which remains poorly understood

“is the influence of gauge cancellations on the divergence structure of a gauge theory.”

[3] G. V. Dunne and C. Schubert, J. Phys. Conf. Ser. **37**, 59–72 (2006).

a fun fact

the idea of how to avoid Feynman diagrams can be traced to 1950 Feynman paper[4], though it took a long time for it to gain traction

by the time I explained the gauge set conjecture to him in 1975, Feynman had forgotten all about it

[4] R. P. Feynman, Phys. Rev. **80**, 440–457 (1950).

compact space, infinite time cylinder

so far : Navier-Stokes on compact spatial domains, all times

yes, but

is space time?

notation

electron-photon vertex Γ_μ
with $p_i = p - q/2$ and $p_o = p + q/2$ the momenta of incoming and outgoing electron lines, evaluated on the electron mass shell $p_i^2 = p_o^2 = m^2$

Gordon decomposition into the Dirac and Pauli form factors $F_1(q^2)$ and $F_2(q^2)$:

$$\overline{u}(p_o)\Gamma_\mu(p,q)u(p_i) = \overline{u}(p_o)\left\{F_1(q^2)\gamma_\mu - \frac{F_2(q^2)}{2m}\,\sigma_{\mu\nu}q^\nu\right\}u(p_i)\,,$$

spinors $\overline{u}(p_i)$ and $u(p_i)$ satisfy the Dirac equation

$$\overline{u}(p_o)\ \not{p}_o = m\,\overline{u}(p_o)\,, \qquad \not{p}_i\,u(p_i) = m\,u(p_i)\,.$$

compact space, infinite time cylinder

we follow the notation of Bjorken and Drell [2] and Cvitanović and Kinoshita [4]. Z_1, Z_2, and Z_3, are the respectively the vertex, the electron wave function, and the photon wave function renormalization constants, and the electron mass will be set to $m = 1$ throughout. In what follows it is convenient to define $Z_1 = 1 + L$. For QED the charge conservation requires that the renormalized charge form factor satisfies $\tilde{F}_1(0) = 1$, which is guaranteed by the Ward identity [8] $Z_1 = Z_2$. The vertex renormalization constant L is given by the on-shell value of the unrenormalized charge form factor [3]

$$1 + L = F_1(0) = \frac{1}{4}\mathrm{tr}\left[(\not{p} + 1)p^\nu \Gamma^\nu\right]_{q=0} ,$$

and $a = (g-2)/2$, the anomalous magnetic moment of an electron is given by the static limit of the magnetic form factor $a = \tilde{F}_2(0) = M/(1+L)$, where [3]

$$M = \lim_{q \to 0} \frac{1}{4q^2}\mathrm{tr}\left\{\left[\gamma^\nu p^2 - (1 + q^2/2)p^\nu\right](\not{p}_o + 1)\Gamma_\nu(\not{p}_i + 1)\right\} .$$

The perturbative expansions for the magnetic moment anomaly

spatiotemporally compact solutions

a crude lattization of turbulence

(a)   (b)

model
(a) global turbulent field

by a lattice of nearest-neighbor coupled
(b) “minimal” turbulent cells

next :   pick the simplest such model

compute locally, adjust globally

it's rocket science[5]

[5] **WGBGQ13**.

summary

1. small computational domains reduce “turbulence” to “single particle” chaos
2. consider instead turbulence in infinite spatiatemporal domains
3. theory : classify all spatiotemporal tilings
4. numerics : future is spatiotemporal

there is no more time

there is only enumeration of spacetime solutions

summary

a proof of the QED finiteness conjecture might be within reach

so might be methods for computing gauge invariant QFT sets without recourse to traditional Feynman diagrams

Please download the current version of these notes:
ChaosBook.org/∼predrag/papers/finiteQED.pdf.

The source code: GitHub.com/cvitanov/reducesymm/QFT