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All mixed up

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(Dated: Draft Mar 2 2009)

Abstract

We undertake an exploration of Lagrangian mixing in recurrent patterns in the ... For a small, but already rather "turbulent" system, the long-time dynamics takes place on a low-dimensional invariant manifold. A set of equilibria offers a coarse geometrical partition of this manifold. ... The mixing dynamics appears decomposable into chaotic dynamics within such local repellers, interspersed by rapid jumps between them.

PACS numbers: recheck! 95.10.Fh, 02.60.Lj, 47.52.+j, 05.45.-a

Keywords: mixing, Lagrangian chaos, turbulence, equilibria, periodic orbits

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Statistical approaches to the study of turbulence [**?**], rely on assumptions which break down in presence of large-scale coherent structures typical of fluid motions [**?**]. Description of such coherent structures requires detailed understanding of the dynamics of underlying equations of motion. In E. Hopf's dynamical systems vision [**?**] turbulence explores a repertoire of distinguishable patterns; as we watch a turbulent system evolve, every so often we catch a glimpse of a familiar whorl. At any instant and a given finite spatial resolution the system approximately tracks for a finite time a pattern belonging to a finite alphabet of admissible patterns, and the dynamics can be thought of as a walk through the space of such patterns, just as chaotic dynamics with a low dimensional attractor can be thought of as a succession of nearly periodic (but unstable) motions.

Exploration of Hopf's program close to the onset of spatiotemporal chaos was initiated in ref. [**?**] which was the first to extend the periodic orbit theory to a PDE, the 1-spatial dimension Kuramoto-Sivashinsky [**?** **?**] system, a flow embedded in an infinite-dimensional state space. Many recurrent patterns were determined numerically, and the recurrent-patterns theory predictions tested for several parameter values. Continuous symmetries of the full periodic domain problem lead to new important features of dynamics - such as relative periodic orbits - that merit study on their own [**?**]. For that reason both in ref. [**?**] and in this paper we found it advantageous to focus on the dynamics confined to the antisymmetric subspace, space for which periodic orbits characterize "turbulent" dynamics. In what follows we shall often refer to such periodic orbit solutions of truncated Kuramoto-Sivashinsky equation as "recurrent patterns" in order to emphasize their spatio-temporal periodicity. In this paper (and, in a much greater detail, in ref. [**?**]) we venture into a Kuramoto-Sivashinsky system bigger than the one studied in ref. [**?**], just large enough to exhibit "turbulent" dynamics arising through competition of several unstable coherent structures.

Basic properties of the Kuramoto-Sivashinsky equation are reviewed in sect. ??. Determining equilibria and periodic orbits in high-dimensional state spaces opens new challenges, and in sect. ?? we sketch the Newton descent method that we have developed and deployed in our searches for recurrent patterns. Informed by the topology of the flow, the method can determine even very long periodic orbits, such as the orbit of figure ?? (c). Equilibria, which play a key role in organizing the global topology of state space dynamics, are investigated in sect. ??. We then fix the size of Kuramoto-Sivashinsky system in order to illustrate our methodology on a concrete example. Not all equilibria influence the dynamics equally, and in sect. I A we show how to gauge the relative importance of an equilibrium by its proximity to the most recurrent state space regions. For this small Kuramoto-Sivashinsky system the dynamics is shaped by the competition between "center" and "side" equilibria. In sect. ?? we turn this observation into a dynamical description of the flow by constructing local, equilibrium-centered Poincaré sections. In sect. ?? we show that with intrinsic curvilinear coordinates built along unstable manifolds of equilibria and short periodic orbits (the key observation of ref. [?]), the dynamics can be reduced to iteration of low-dimensional Poincaré return maps. The long road from an infinite-dimensional PDE to essentially 1-dimensional iteration is now completed, its crowning achievement the bimodal return map of figure ?? (e). Such return maps enable us construct a symbolic dynamics, and initiate a systematic search for periodic orbits that build up local Smale horseshoe repellers, as many as desired. The periodic points so determined are overlayed over the return map in figure ?? (f). Interestingly, this systematic parsing of state space leads to a discovery of a non-trivial attracting periodic orbit of short period, an orbit highly unlikely to show up in random initial condition simulations of Kuramoto-Sivashinsky dynamics. The hierarchy of periodic orbits so determined can then be used to predict long-time dynamical averages via periodic orbit theory. Our results are summarized

(a) (b)

(c) (d)

FIG. 1: Example of a figure (plots omitted): Equilibria in $[u, u_x]$ representation, and as $u(x)$ spatially periodic profiles: (a), (b) C_1. (c), (d)

in sect. II.

I. PLANE COUETTE FLOW

A. Search for dynamically important equilibria

For small system sizes L the number of equilibria is small and concentrated on the low wavenumber end of the Fourier spectrum.

II. SUMMARY

The recurrent patterns program was first implemented in detail [**?**] on the 1-d Kuramoto-Sivashinsky system at the onset of chaotic dynamics. For these specific parameter values many recurrent patterns were determined numerically, and the periodic-orbit theory predictions tested. In this paper we venture into a large Kuramoto-Sivashinsky system, just large enough to exhibit "turbulent" dynamics of topologically richer structure, arising through competition of several unstable coherent structures. Both papers explore dynamics confined to the antisymmetric subspace, space for which periodic orbits characterize "turbulent" dynamics. Ref. [**?**] studies Kuramoto-Sivashinsky in the full periodic domain, where relative periodic orbits due to the continuous translational symmetry play a key role, and ref. [**?**] applies the lessons learned to a full $3D$ Navier-Stokes flow. In this context Kawahara and Kida [**?**] have demonstrated that the recurrent patterns can be determined in turbulent hydrodynamic flows by explicitly computing several important unstable spatio-temporally periodic solutions in the 3-dimensional plane Couette turbulence.

We have applied here the "recurrent pattern program" to the Kuramoto-Sivashinsky system in a periodic domain, antisymmetric subspace, in a larger domain size than explored previously [**?**]. The state space non–wandering set for the system of this particular size appears to consist of three repelling Smale horseshoes and orbits communicating between them. Each subregion is characterized by qualitatively different spatial u-profiles in the 1-dimensional physical space. The "recurrent patterns," identified in this investigation by nearby equilibria and periodic orbits, capture well the state space geometry and dynamics of the system. Both the equilibria and periodic orbits are efficiently determined by the Newton descent method. The equilibria so determined, together with their unstable manifolds, provide the global frame for the non–wandering set. We utilize these unstable manifolds to build 1-dimensional curvilinear coordinates along which the infinite-dimensional PDE dynamics is well approximated by 1-dimensional return maps and the associated symbolic dynamics. In principle, these simple models of dynamics enable us to systematically classify and search for recurrent patterns of arbitrary periods. For the particular examples studied, the approach works well for the "central" repeller but not so well for the "side" repeller.

Above advances are a proof of principle, first steps in the direction of implementing the recurrent patterns program. But there is a large conceptual gap to bridge between what has been achieved, and what needs to be done: Even the flame flutter has been probed only in its weakest turbulence regime, and it is an open question to what extent Hopf's vision remains viable as such spatio-temporal systems grow larger and more turbulent.

III. FLOTSAM

Turbulence is a ubiquitous phenomena. It was first observed in the fluid motion [?] and more recently in the chemical reactions, optics, coupled oscillators and so on [? ?].

The claim explored in this article is that periodic orbit theory is a candidate for a dynamical theory of turbulence. It has been applied to many low-dimensional chaotic systems with remarkable success.