"Recent progress on geometric wave equations"
Sung-Jin Oh (University of California, Berkeley)

Notebook: First Notebook
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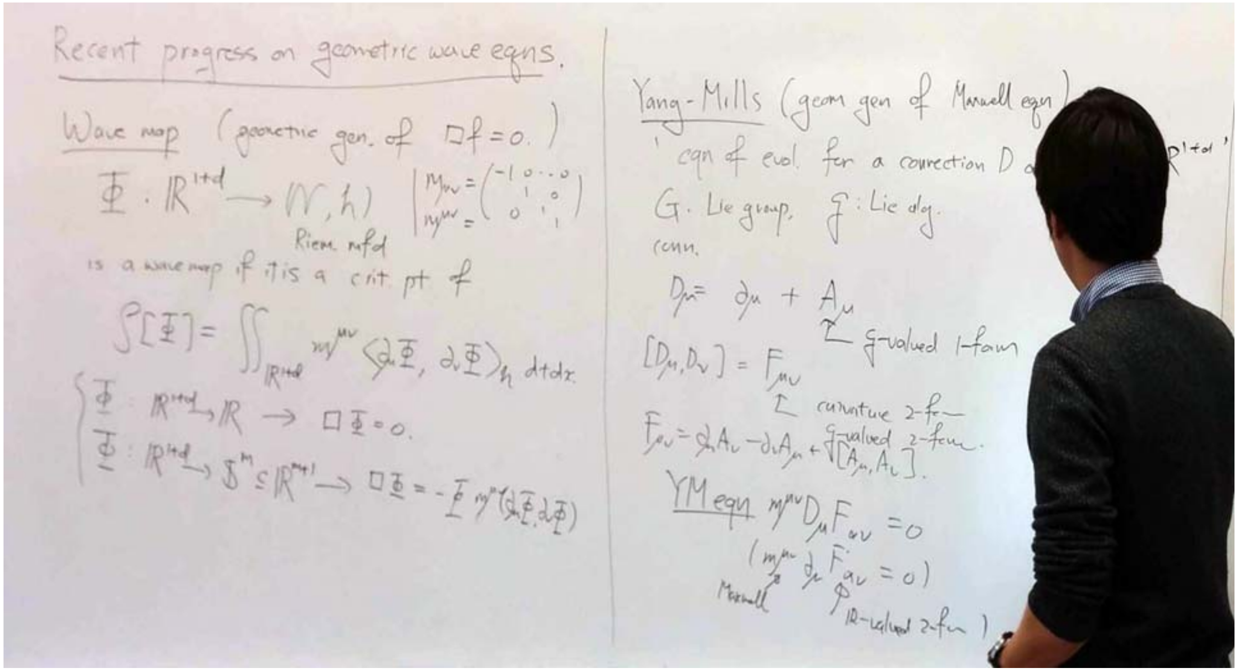
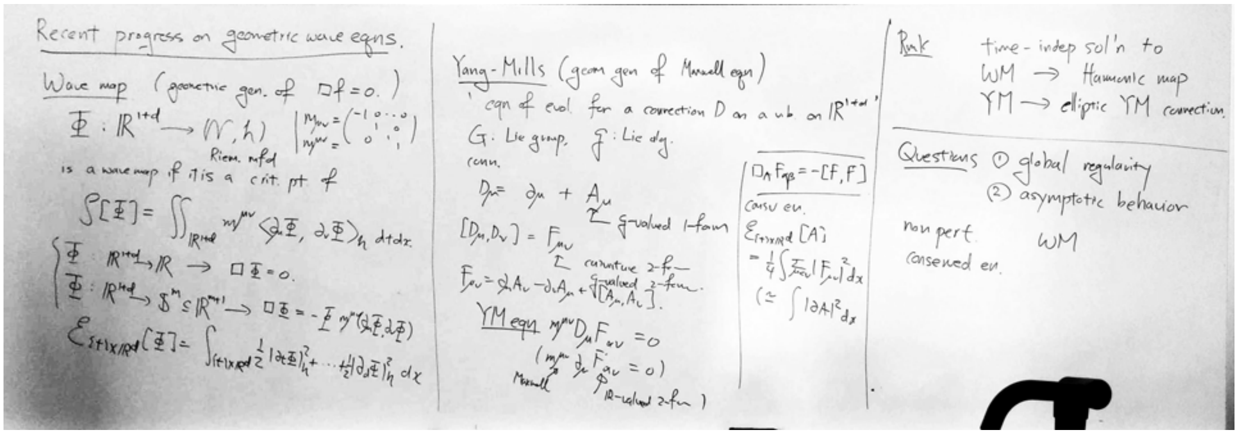


https://math.berkeley.edu/people/faculty/sung-jin-oh
Wednesday, April 6, 2016

Sung-Jin Oh has PDE focus: are there finite time singularities?

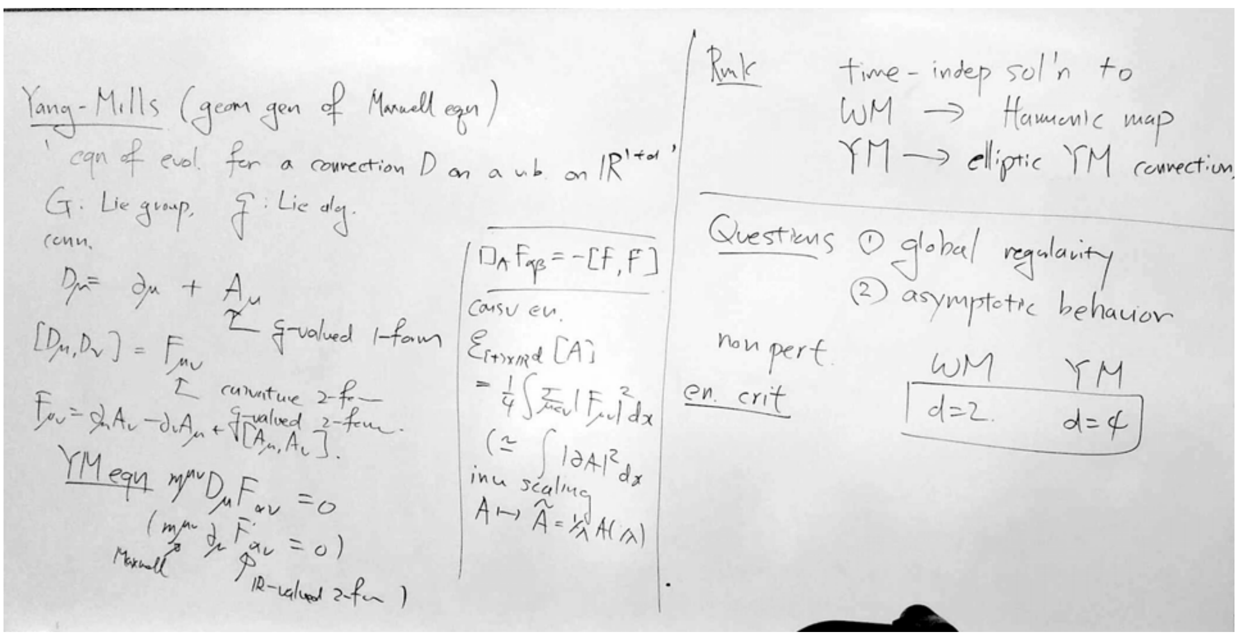
The subject of this talk is wave equations that arise from geometric considerations. Prime examples include the wave map equation and the Yang-Mills equation on the Minkowski space. On one hand, these are fundamental field theories arising in physics; on the other hand, they may be thought of as the hyperbolic analogues of the harmonic map and the elliptic Yang-Mills equations, which are interesting geometric PDEs on their own. I will discuss the recent progress on the problem of global regularity and asymptotic behavior of solutions to these PDEs.

Conserved energy enables one to define a "ground state" (the lowest energy state). "Threshold" theorems: for energies below the ground state, scattering amplitudes are smooth. Above: one has many counter examples.

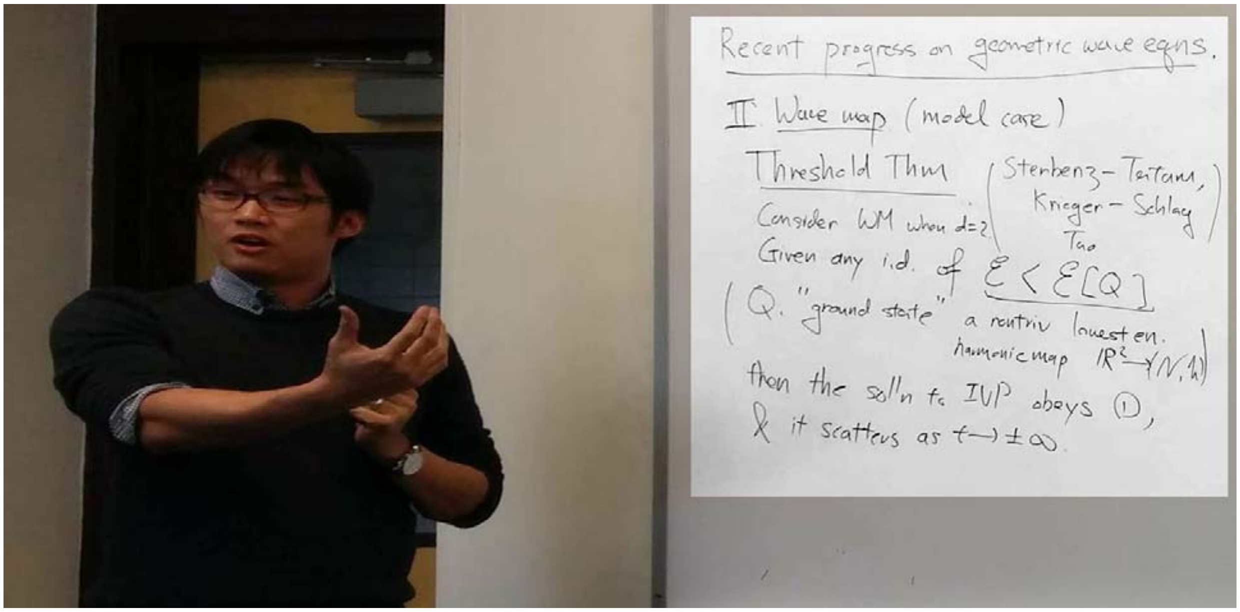


Critical dimension is 4+1 for YM and QED, 2+1 for the wave map equation

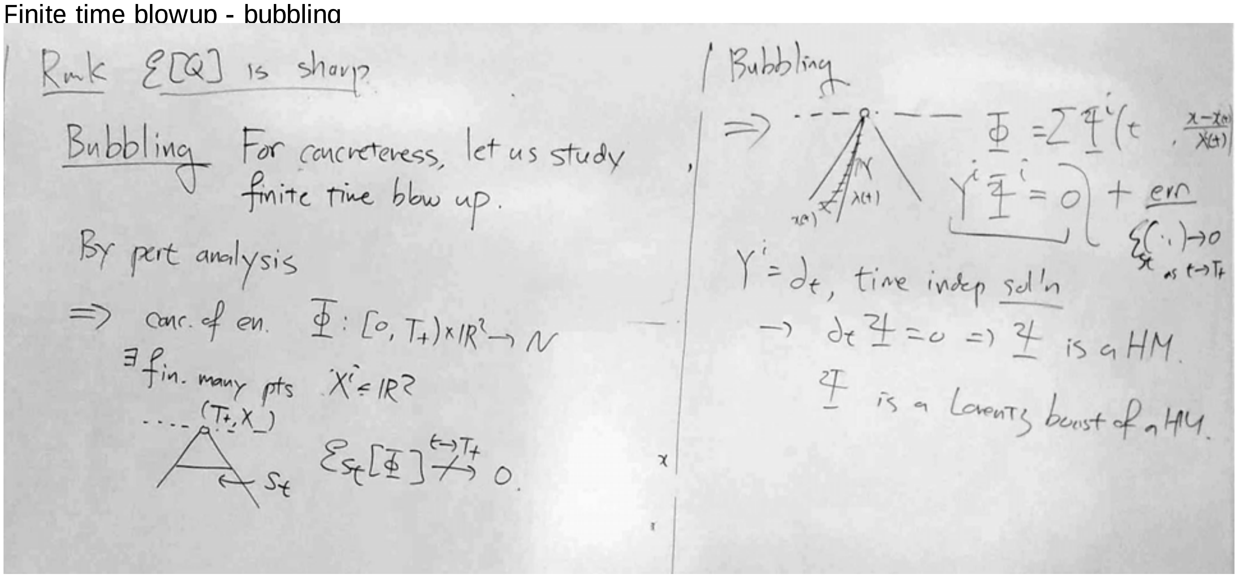
10 *A refined threshold theorem for (1+2)-dimensional wave maps into surfaces*, with A. Lawrie, to appear in **Comm. Math. Phys.** arXiv:1502.03435 [math.AP]



Threshold theorem (Tao, Krigier, ...)



Finite time blowup - bubbling

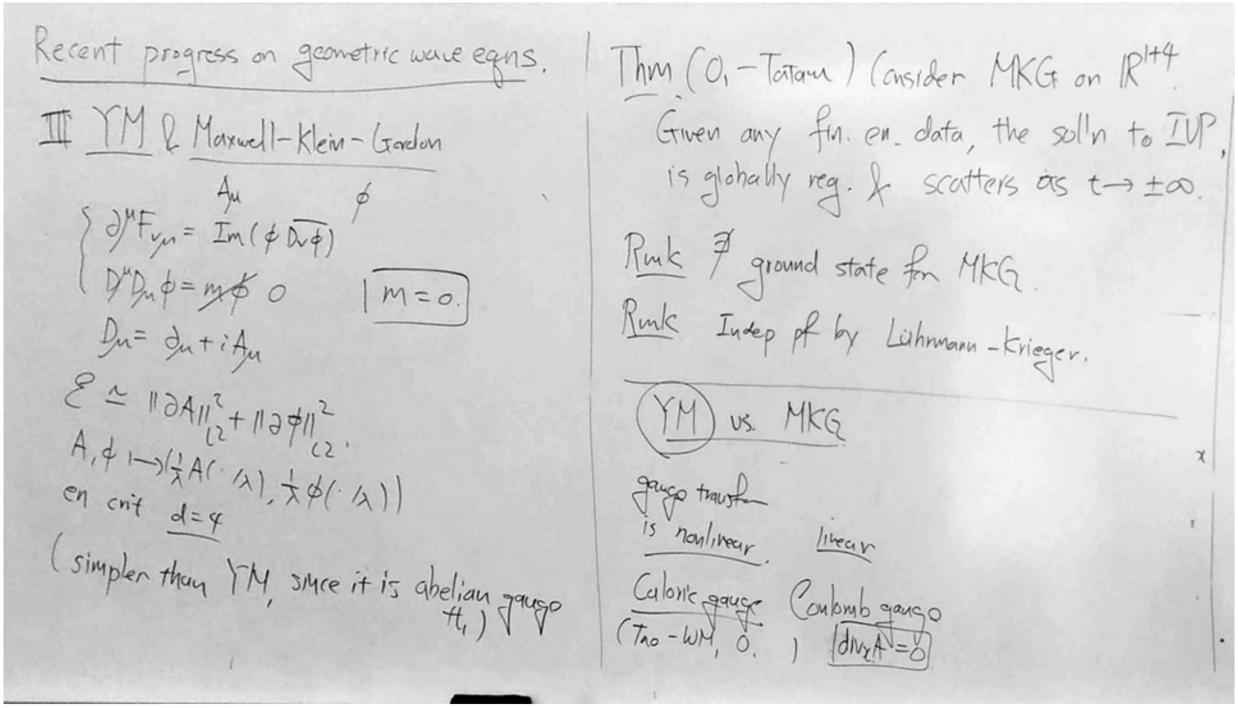


Maxwell-Klein-Gordon (ie, scalar QED, but with massless scalar particle) in 4+1 OK, but YM not.
Perhaps in these references:

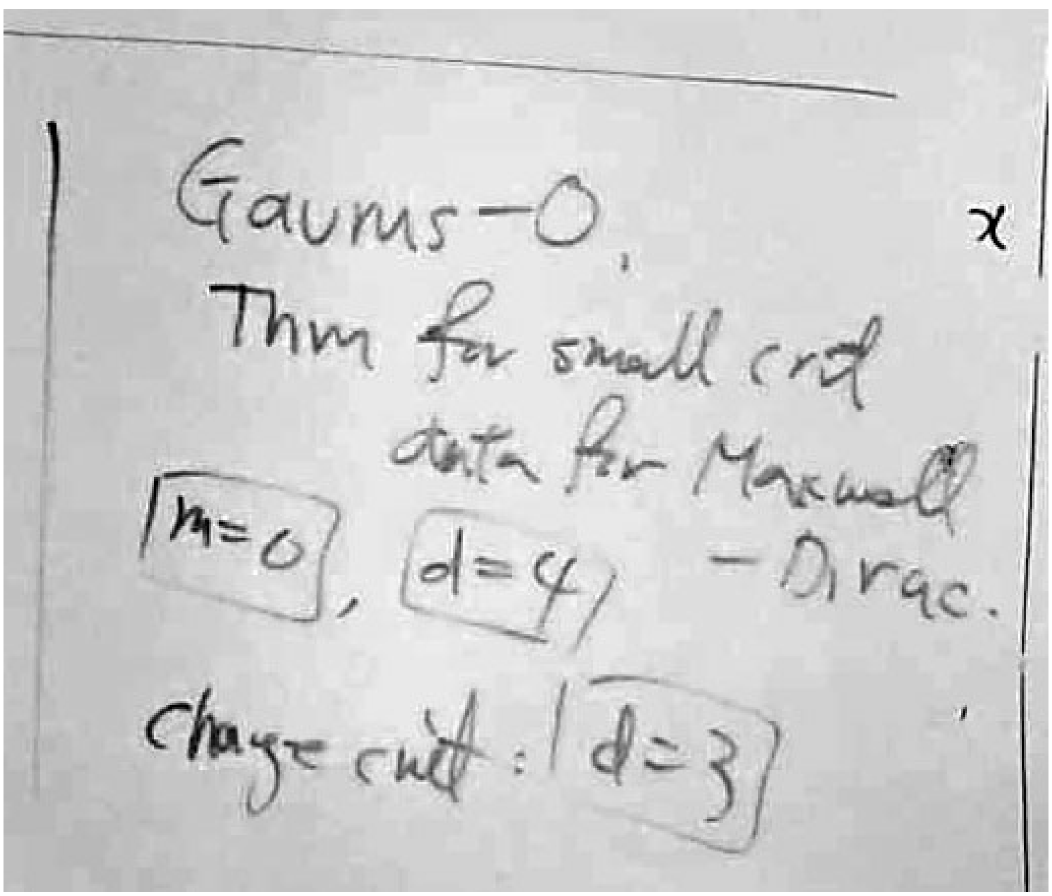
7 *Local well-posedness of the (4+1)-dimensional Maxwell-Klein-Gordon equation*, with D. Tataru, to appear in **Annals of PDE**. arXiv:1503.01560 [math.AP]

8 *Energy dispersed large energy solutions to the (4+1) dimensional Maxwell-Klein-Gordon equation*, with D. Tataru. arXiv:1503.01561 [math.AP]

9 *Finite energy global well-posedness and scattering of the (4+1) dimensional Maxwell-Klein-Gordon equation*, with D. Tataru, to appear in **Invent. Math.** arXiv:1503.01562 [math.AP]



Maxwell-Klein-Dirac (ie, spin 1/2 QED, massless only), with energy `ground states' critical dimension is 4+1. With conserved charge critical dim is 3, but results are only perturbative.



From talking to Sung-Jin Oh: there is advance in finding non-singular surfaces within Kerr black holes. The results are on weak solutions (not smooth), not clear to Predrag where this work is going.

Predrag's (undigested) impressions: This might be a path to computing non-perturbative classical solutions of Yang-Mills. Talking to Sung-Jin Oh I kept confusing the quantum and the classical problem - it is for a reason that PDE people do not know what expressions like "on-mass-shell amplitudes" and "Ward identities" mean.