Using Exact Coherent Structures to tile the infinite spacetime Kuramoto-Sivashinsky equation

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Abstract

Finding Exact coherent structures [1], (time invariant solutions) has become a very popular area of investigation in fluid dynamics. The reason for this is that the solutions can be used to forward a theory that hopes to explain spatiotemporally recurrences as well as dynamical averages for difficult computational problems such as turbulent pipe flow. In this study we will formulate a truly spatiotemporal foundation which serves to provide a new perspective on one of the most important problems in classical Physics.

Kuramoto-Sivashinsky Equation

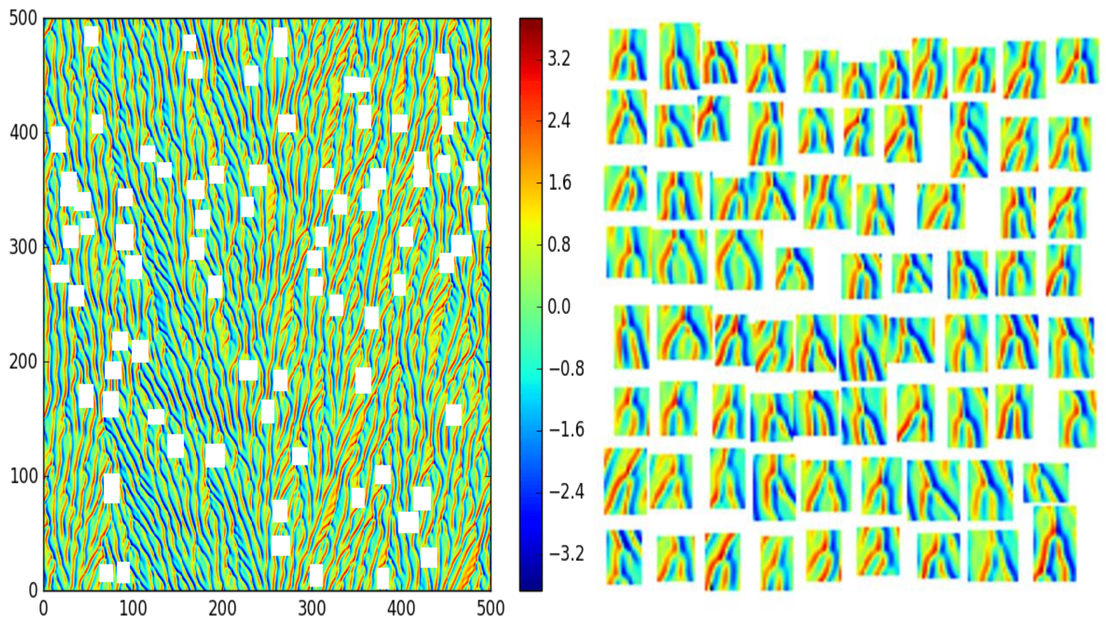
$$u_t(x,t) = -u_{xx}(x,t) - u_{xxxx}(x,t) - u(x,t)u_x(x,t)$$

Diffusion (wrong sign) "Hyper"-diffusion Advection

$$u(x,t) = u(x+L,t) = u(x,t+T) = u(x+L,t+T)$$

Motivation via spatiotemporal recurrences

Frequent repetition of finite number of similar patterns across **space *and* time.** Initial value problems in chaotic systems are ***ill-posed*** problems [2].



Spatiotemporal Kuramoto-Sivashinsky Eqn.

$$(iw_n - {q_m}^2 + {q_m}^4)\tilde{u} + \frac{iq_m}{2}\mathrm{F}(\mathrm{F}^{-1}(\tilde{u})^2) = \mathrm{G}(\tilde{u}, \mathrm{L}, \mathrm{T}) = 0$$

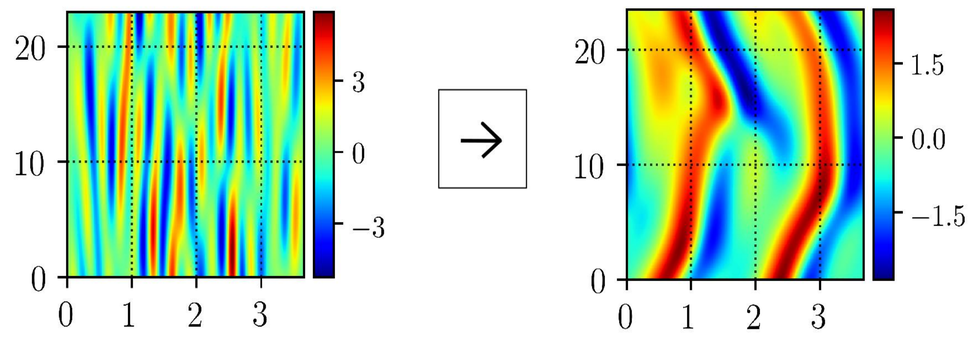
Numerical Algorithms

Solve nonlinear optimization problem,

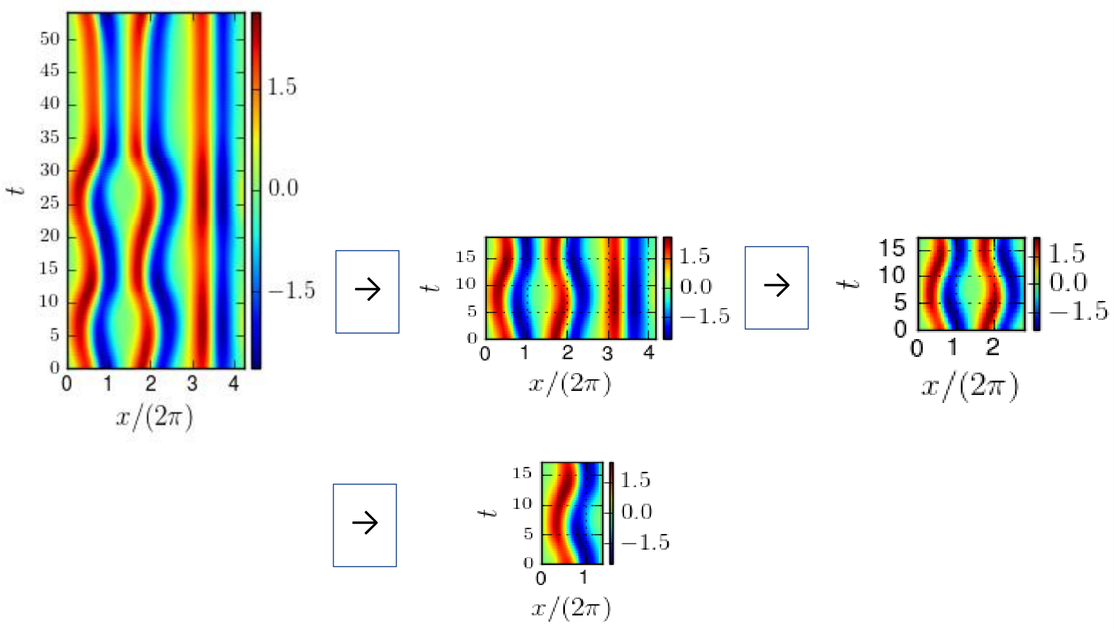
$$I = \frac{1}{2}|\mathrm{G}|^2 = 0$$

We solve this by using the *Adjoint descent method* [3] and a custom version of *Newton's method*.

Library of new solutions

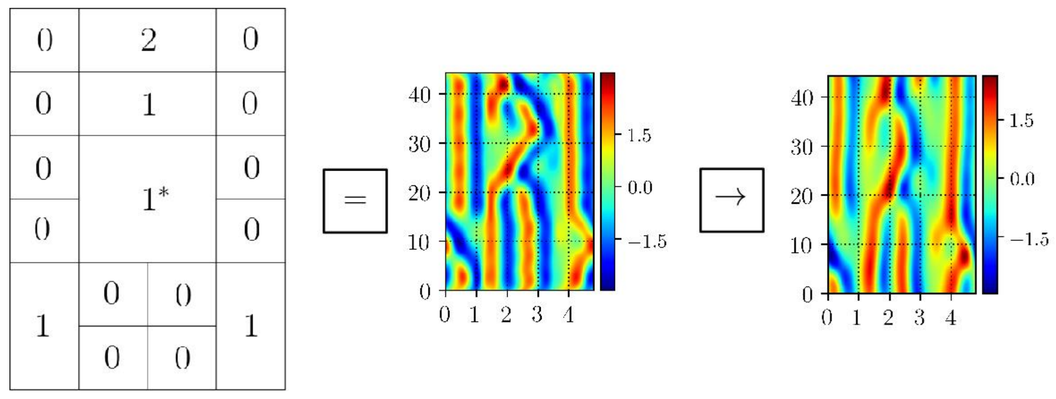


Extracting "tile" solutions



Spatiotemporal symbolic dynamics

By using two dimensional symbolic dynamics for spatiotemporal ("tile") solutions, we generate initial conditions that converge to both new and known solutions.



Gluing ECS together

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References

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[2] Wang et al. (2013) Towards scalable parallel-in-time turbulent flow simulations, Phys. Fluids 25, 110818.
[3] M. Farazmand, (2016) An adjoint-based approach for finding invariant solutions of Navier-Stokes equations, J. Fluid M. 795. 278—312.