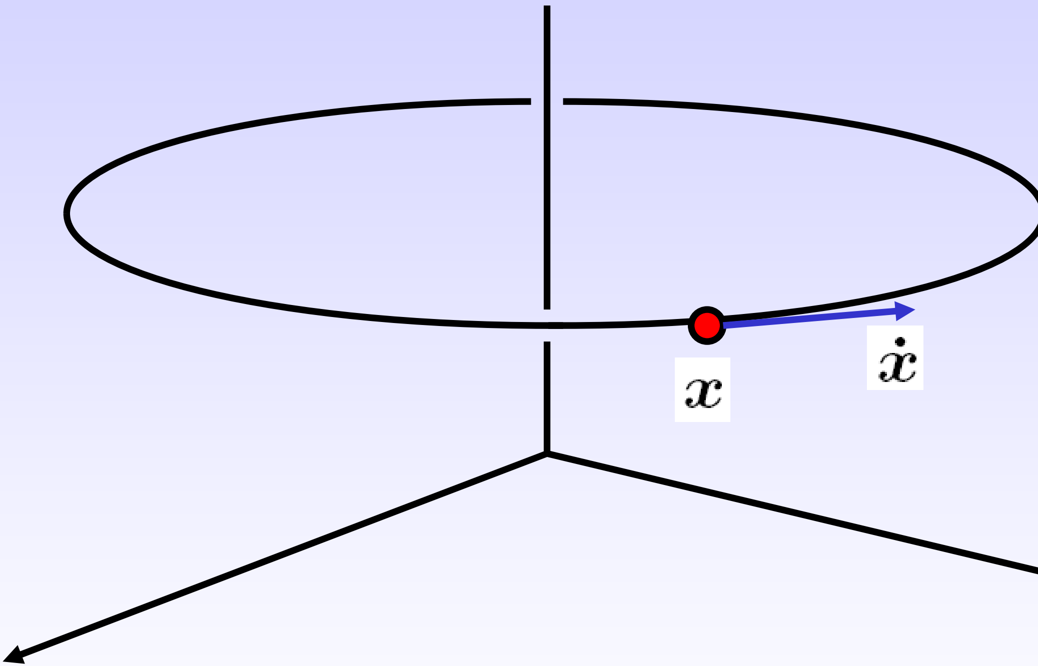
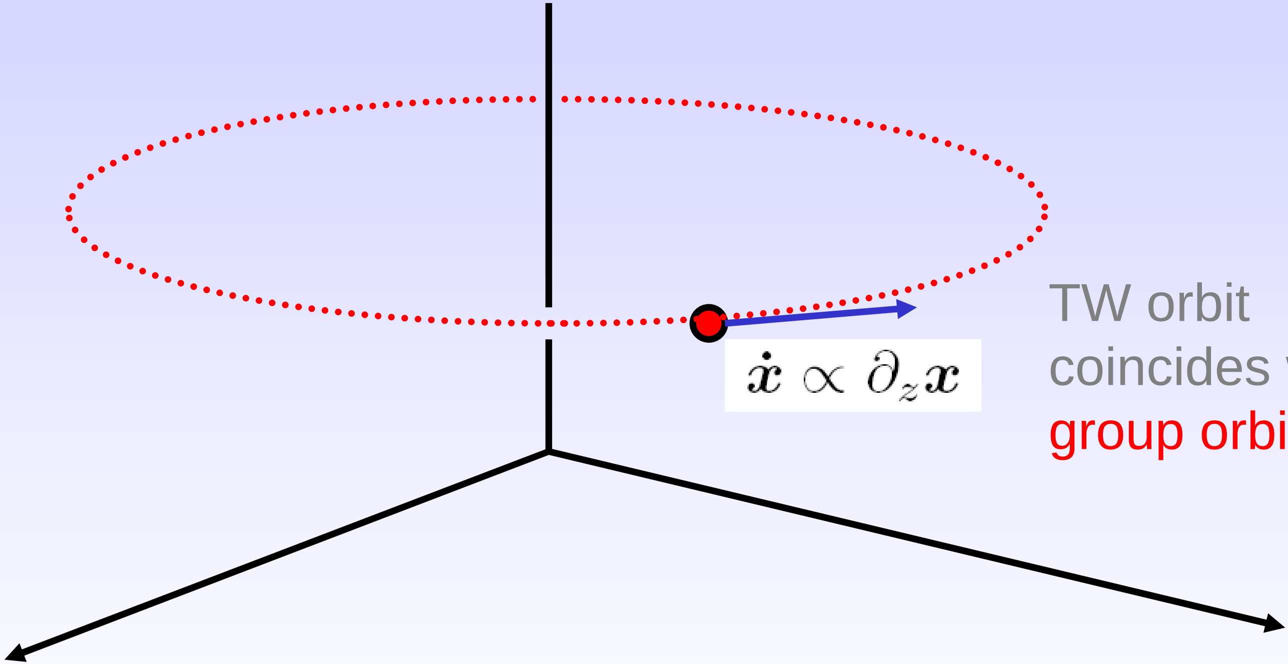
TW orbit in phase space



Returns at
T = L / c ,
wavelength L

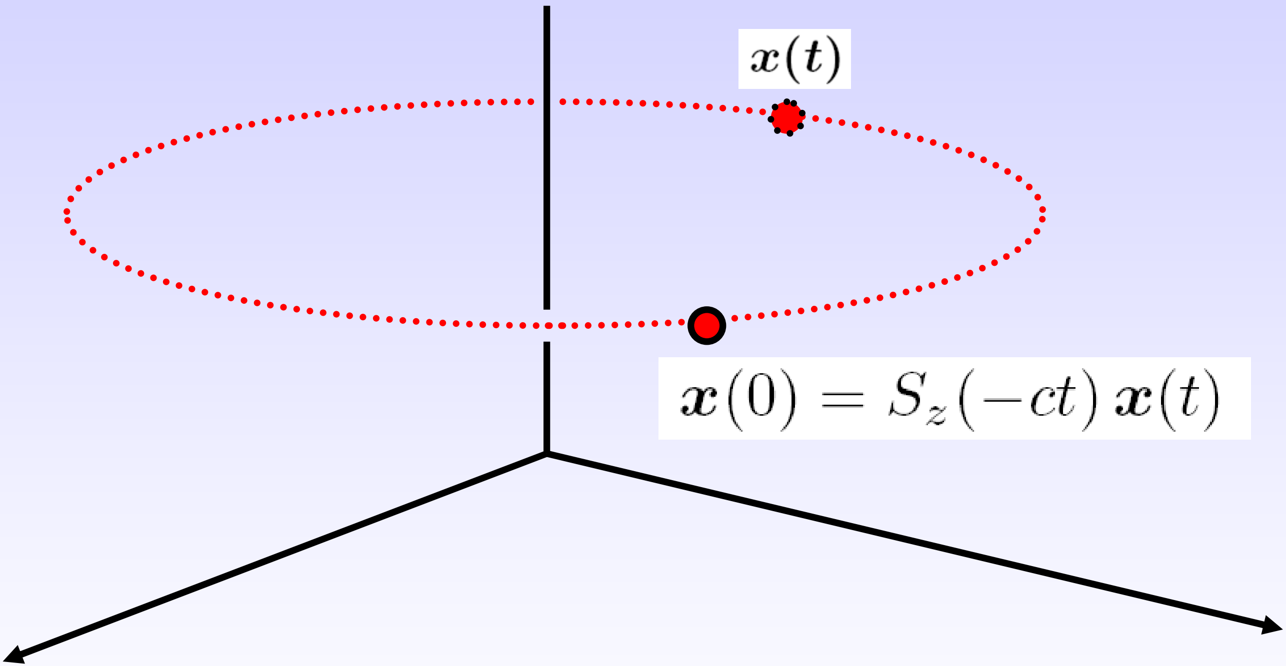
Axial shifts of TW state



$$\dot{\boldsymbol{x}} \propto \partial_z \boldsymbol{x}$$

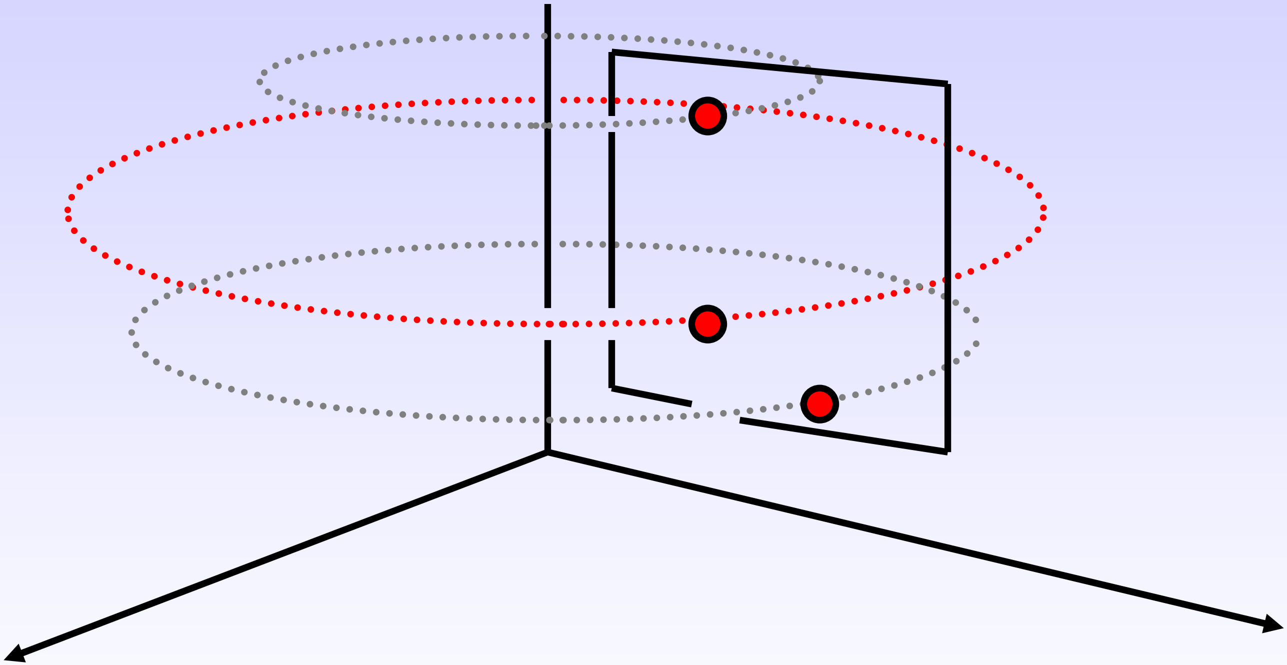
TW orbit coincides with its group orbit

Reduction of TW orbit to point by shifts



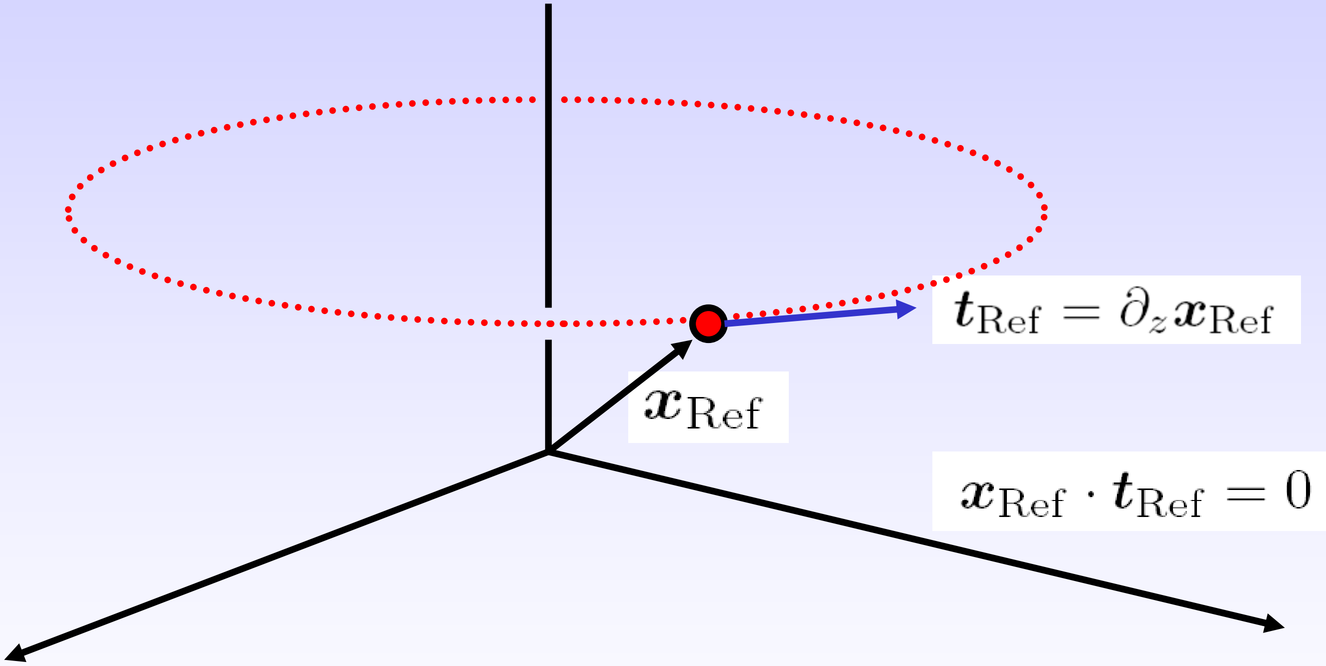
$$\boldsymbol{x}(t) \to \bar{\boldsymbol{x}}$$

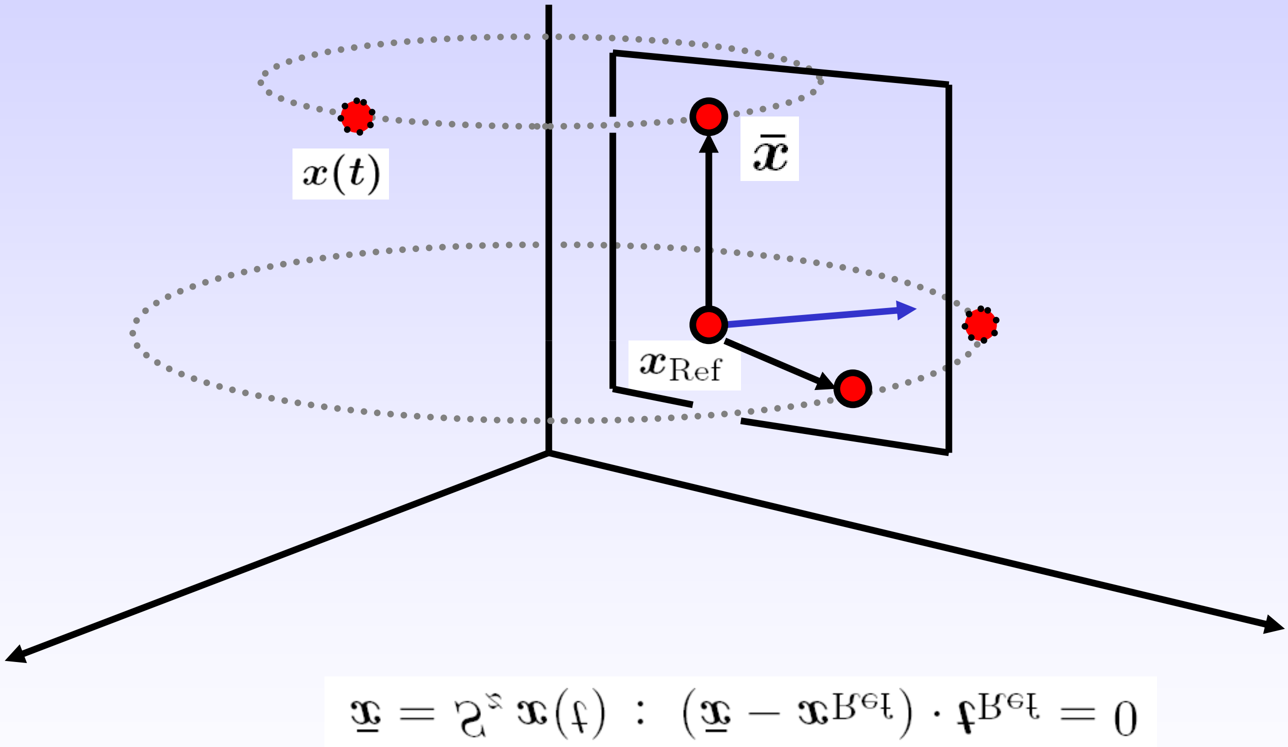
Reduce all TWs into a single slice



$$\boldsymbol{x}_i(0) = S_z(-c_i\, t)\, \boldsymbol{x}_i(t)$$

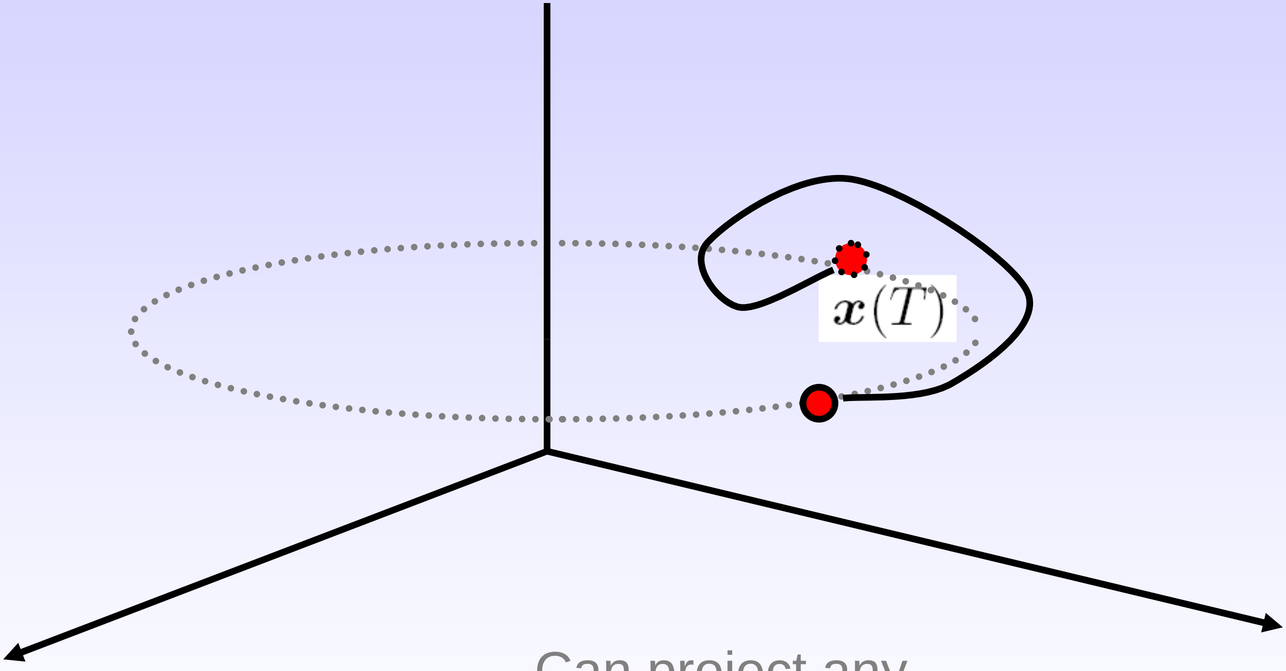
How? - several speeds *c*, possibly unknown


$\boldsymbol{t}_{\mathrm{Ref}} = \partial_z \boldsymbol{x}_{\mathrm{Ref}}$
$\boldsymbol{x}_{\mathrm{Ref}}$
$\boldsymbol{x}_{\mathrm{Ref}} \cdot \boldsymbol{t}_{\mathrm{Ref}} = 0$



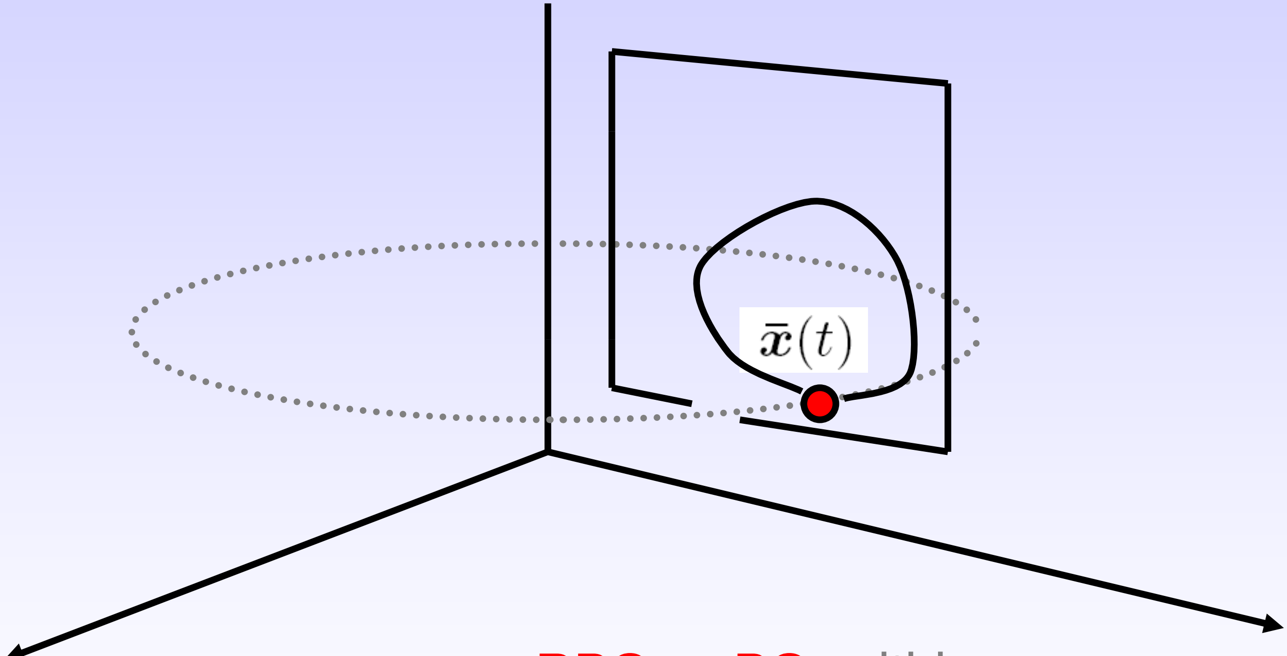
All TWs $\rightarrow$ points in the slice

Application to a relative periodic orbit (RPO)

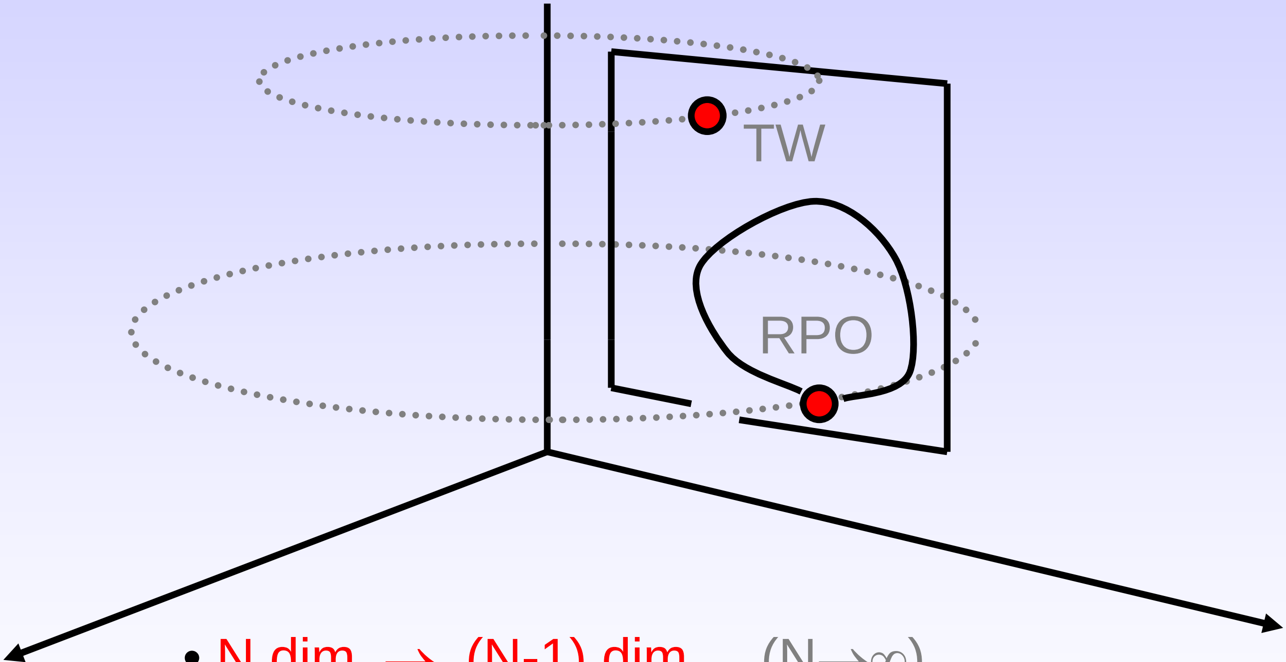


Can project any
trajectory into the slice

Application to a relative periodic orbit (RPO)

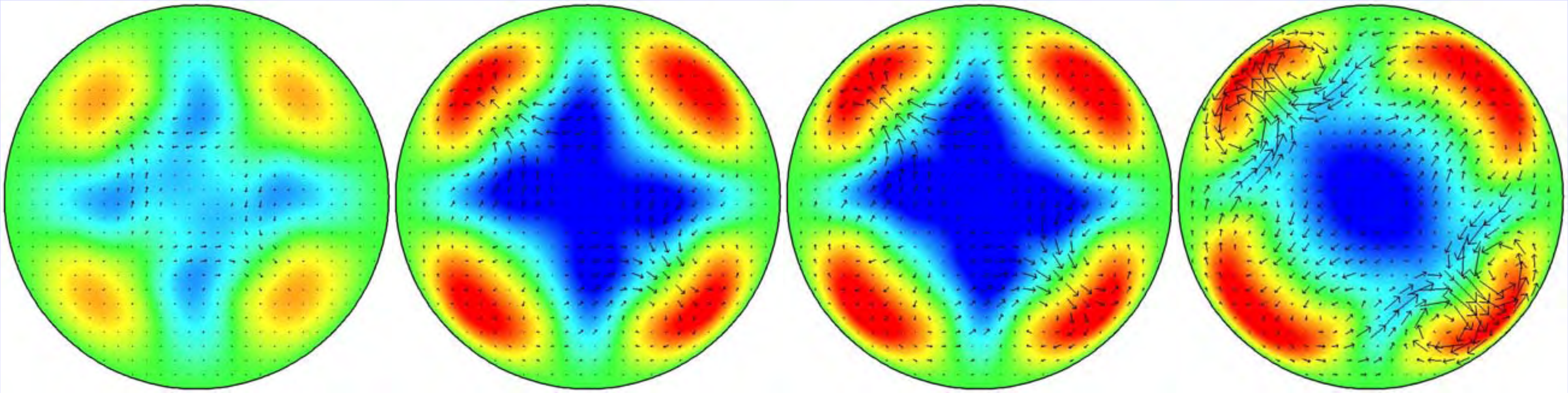


RPO $\rightarrow$ PO within the slice



- N dim $\rightarrow$ (N-1) dim (N$\rightarrow\infty$)
- Automatic removal of strong shift (gives c for TW)
- TW $\rightarrow$ point
- RPO $\rightarrow$ PO

Application to Pipe Flow, N2 L=2.5 D states



LB M1 M2 UB

1 real | 1 complex / 1 real | 2 real | 4 complex

LB	M1	M2	UB
1 real	1 complex	2 real	4 complex
	1 real		

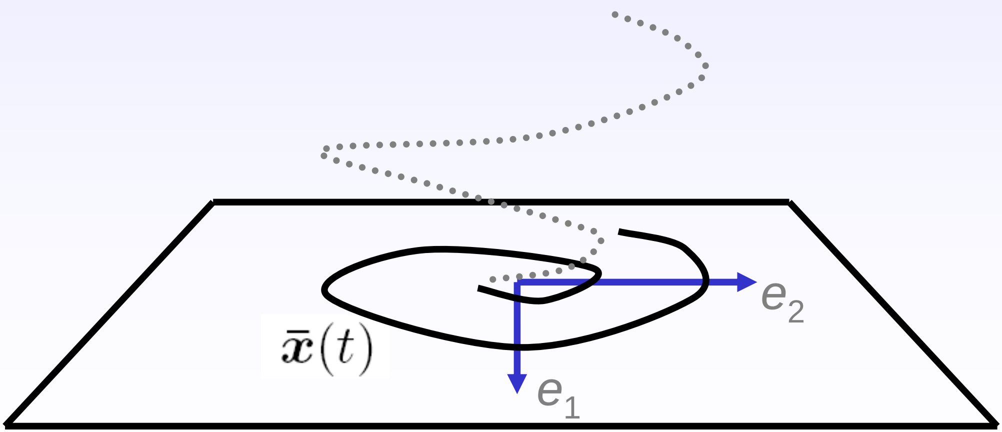
unstable eigenvalues

Projection within slice, (N-1) dim $\rightarrow$ 2 dim

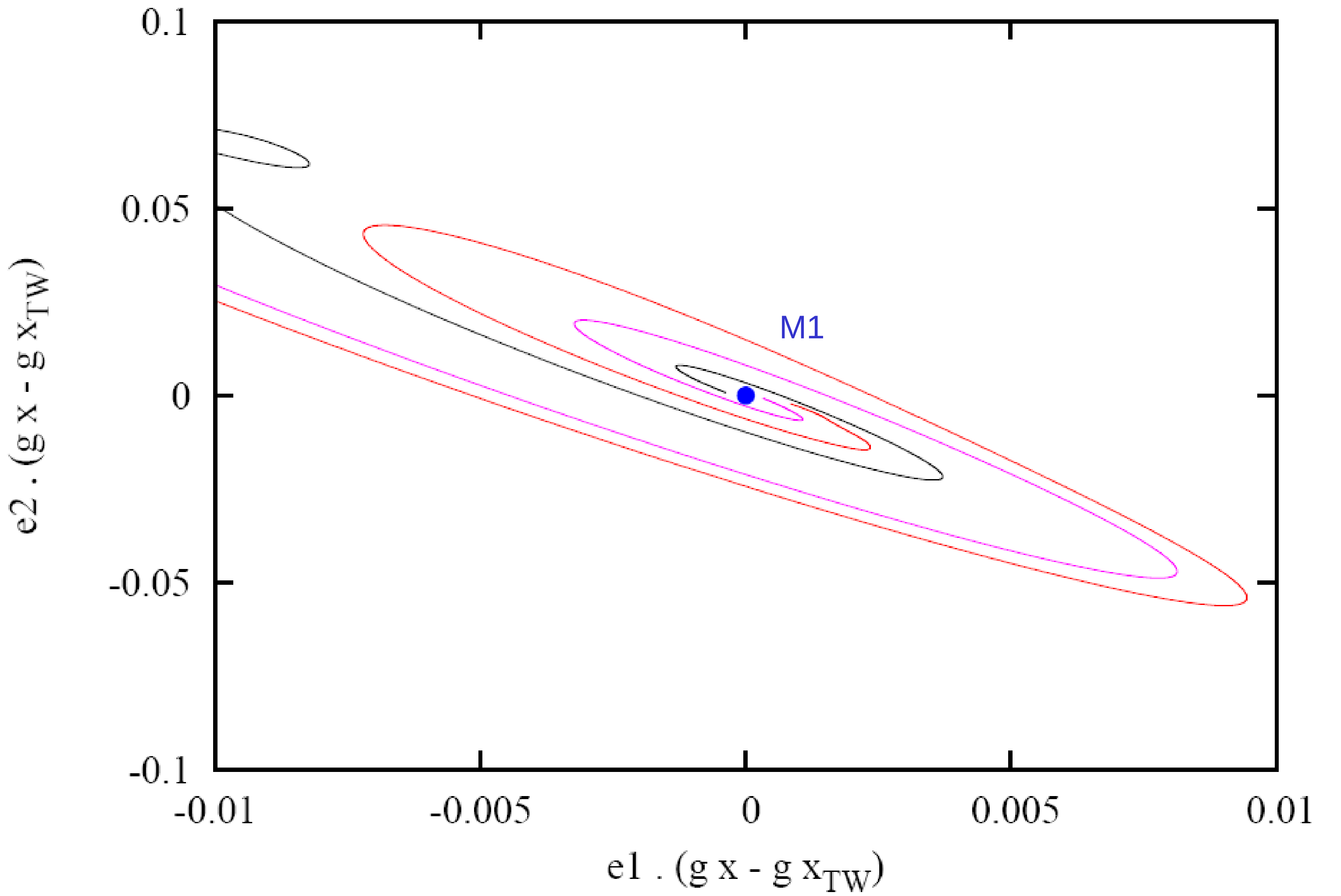
$$\boldsymbol{e}_1 \cdot (\bar{\boldsymbol{x}}(t) - \boldsymbol{x}_{\mathrm{Ref}})$$

$$\underline{\boldsymbol{e}_2} \cdot (\bar{\boldsymbol{x}}(t) - \boldsymbol{x}_{\mathrm{Ref}})$$

unstable directions of $\boldsymbol{x}_{\mathrm{Ref}}$ (M1 state)



Local dynamics, projection within slice



Embedded RPO