herding cats
a chaotic field theory

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ChaosBook.org/overheads/spatiotemporal
→ Chaotic field theory slides

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overview

1. what is this about
2. chaos - a short course
3. temporal cat
4. spatiotemporal cat
5. chaotic field theory
6. space is time
7. bye bye, dynamics

Q. what is a chaotic field theory?

A. it is a field theory

field configuration X probability

$$p(\mathrm{X}) = \frac{1}{Z} e^{-S[\mathrm{X}]}, \qquad Z = Z[0]$$

partition function = sum over configurations

$$Z[\mathrm{M}] = \int [d\phi]\, e^{-S[\mathrm{X}]+\mathrm{X}\cdot\mathrm{M}}, \qquad [d\phi] = \prod_{z}^{\mathcal{L}} \frac{d\phi_z}{\sqrt{2\pi}}$$

example : Euclidean ϕ^4 theory action

$$S[\mathrm{X}] = \int dx^d \left\{ \frac{1}{2} \sum_{i=1}^{d} (\partial_\mu \phi(x))^2 + \frac{\mu^2}{2} \phi(x)^2 + \frac{g}{4!} \phi(x)^4 \right\}$$

Q. why a "chaotic" field theory?

turbulence !

a motivation : need a theory of large turbulent domains

pipe flow close to onset of turbulence [1]

we have a detailed theory of small turbulent fluid cells

can we can we construct the infinite pipe by coupling small turbulent cells ?

what would that theory look like ?

[1] M. Avila and B. Hof, Phys. Rev. **E 87** (2013)

the goal

build
a chaotic field theory

from
the simplest chaotic blocks

using

- time invariance
- space invariance

of the defining partial differential equations

Dreams of Grand Schemes : solve

Navier-Stokes

$$\rho \frac{\partial u_i}{\partial t} + \rho u_j \frac{\partial u_i}{\partial x_j} = \rho X_i - \frac{\partial p}{\partial x_i} + \mu \nabla^2 u_i$$

Einstein

$$R_{ik} - \frac{1}{2} g_{ik} R = \frac{8\pi k}{c^4} T_{ik}$$

$$R^i_{klm} = \frac{\partial \Gamma^i_{km}}{\partial x^l} - \frac{\partial \Gamma^i_{kl}}{\partial x^m} + \Gamma^i_{ne} \Gamma^n_{km} - \Gamma^i_{nm} \Gamma^n_{kl}$$

Yang-Mills

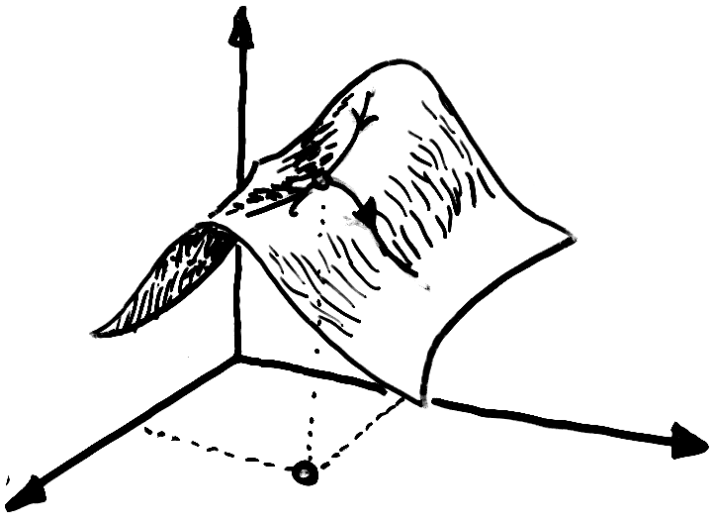
$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g C_{abc} A^b_\mu A^c_\nu$$

Quantum ?

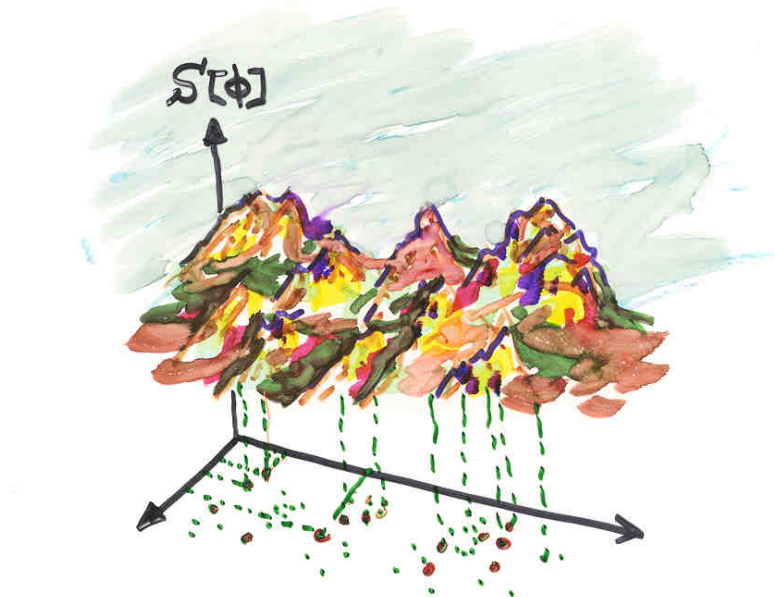
QFT path integrals : semi-classical WKB quantization

a local unstable extremum



a fractal set of saddles

TURBULENT Q.F.T. ?



$$\text{(observable)} = \sum_{\text{set}}^{\text{fractal}} \frac{e^{i S_n[\phi_c]/\hbar}}{\sqrt{\frac{\partial^2 S}{\partial\phi_i \partial\phi_j}}}$$

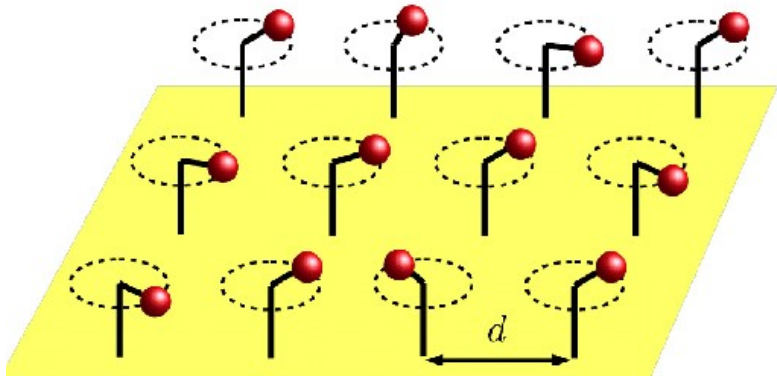
learn to count + weigh unstable saddles

take-home :

harmonic field theory

tight-binding model
(Helmholtz)

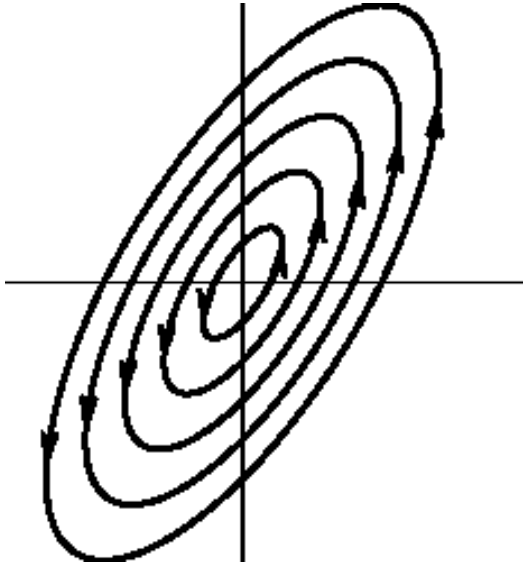
chaotic field theory



Euclidean Klein-Gordon
(damped Poisson)

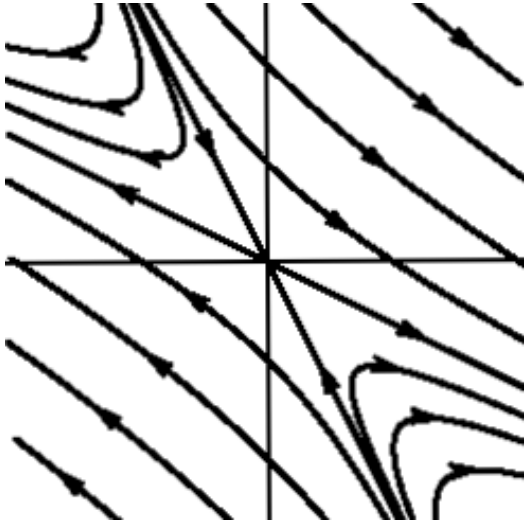
take-home :

harmonic field theory



oscillatory eigenmodes

chaotic field theory



hyperbolic instabilities

the very short answer : POT

if you win : I teach you how

(for details, see ChaosBook.org)

Mephistopheles knocks at Faust's door and says, "Du mußt es dreimal sagen!"

"You have to say it three times"

— Johann Wolfgang von Goethe
Faust I - Studierzimmer 2. Teil