Relative periodic orbits in Kuramoto-Sivashinsky Equation

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The Kuramoto-Sivashinsky partial differential equation (KSe):

$$u_t = 2uu_x - u_{xxxx} - u_{xx}\,,$$

$x \in [0, L]$ with periodic boundary conditions $u(0, t) = u(L, t)$.

Appears in the description of

- hydrodynamic instabilities in laminar flame fronts,
- reaction-diffusion systems,
- the interface between two viscous fluids,
- the evolution of liquid falling films.

Our motivation for KSe study

It is one of the simplest PDE's that exhibit spatiotemporal chaos.

Our approach

Use dynamical system language and tools.

- Phase space dynamics,
- Organization of dynamics by invariant objects (equilibria, periodic orbits, their stable/unstable manifolds)
- Role of symmetries:

Translational invariance $\longrightarrow$ relative equilibria and relative periodic orbits.

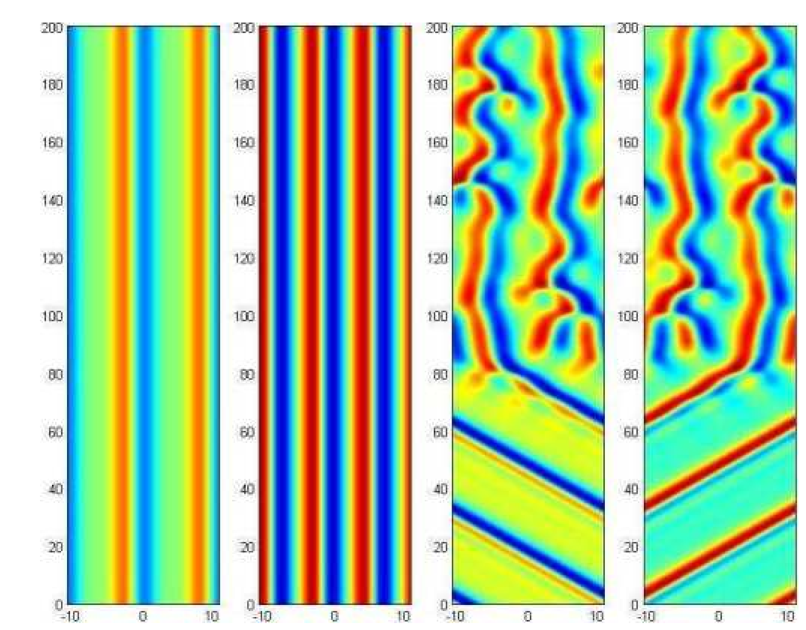
Goal

Apply Periodic Orbit Theory to predict statistical quantities for specific system size.

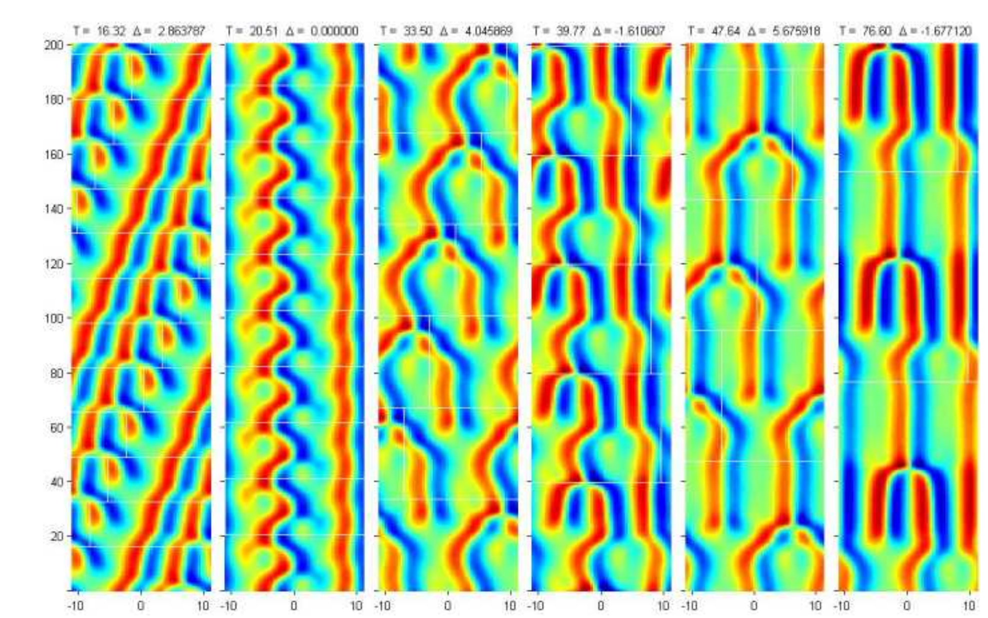
Phase space v.s. physical space

PDE:	∞-dimensional dynamical system
Transform to Fourier space:	system of ∞-many, nonlinearly coupled ODE's
Dissipation:	only a finite number of modes active
Truncate system:	d-dimensional system of nonlinearly coupled ODE's

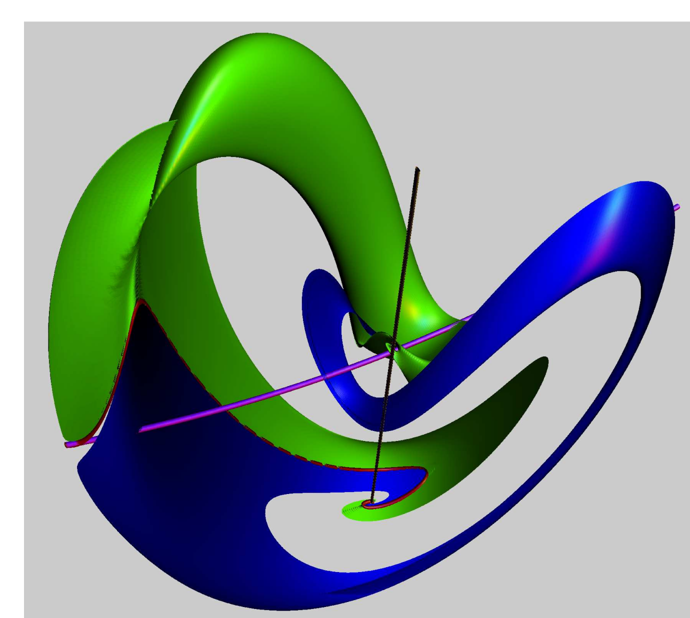
KSe for L=22.0



Unstable Equilibria and Relative Equilibria (traveling waves) for $L = 22.0$. From left to right: 2-cell equilibrium, 3-cell equilibrium, 1-cell traveling wave (to the left) and it's $u(x, t) \to -u(-x, t)$ counterpart. The traveling waves are shown decaying to a typical chaotic state.



Relative Periodic Orbits satisfying the condition $u(x + \Delta, t + T) = u(x, t)$ for time period T and spatial displacement Δ.



Unstable manifold of 2-cell equilibrium of KSe for $L = 22.0$. The black line represents the family of 2-cell equilibria obtained by application of the translation operator, while the purple line represents the family of 3-cell equilibria. The red trajectory represents the heteroclinic connection from a 2-cell equilibrium to a 3-cell equilibrium. The connection splits the manifold into two parts, colored blue and green here.

Future

- Construct symbolic dynamics
- Find all periodic and relative periodic orbits up to a given period
- Use trace formulas that incorporate continous symmetries

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