Geometry of Inertial Manifolds in Nonlinear Dissipative Dynamical Systems

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As an exponentially attracting subset of the state space, an inertial manifold serves as a tool to reduce the study of an infinite-dimensional system to the study of a finite set of determining modes. We determine the dimension of the inertial manifold for the one-dimensional Kuramoto-Sivashinsky equation using the information about the linear stability of system’s unstable periodic orbits. In order to attain the numerical precision required to study the exponentially unstable periodic orbits, we formulate and implement “periodic eigendecomposition”, a new algorithm that enables us to calculate all Floquet multipliers and vectors of a given periodic orbit, for a given discretization of system’s partial differential equations (PDEs). It turns out that the O(2) symmetry of Kuramoto-Sivashinsky equation significantly complicates the geometrical structure of the global attractor, so a symmetry reduction is required in order that the geometry of the flow can be clearly visualized. We reduce the continuous symmetry using so-called slicing technique. The main result of the thesis is that for one-dimensional Kuramoto-Sivashinsky equation defined on a periodic domain of size L = 22, the dimension of the inertial manifold is 8, a number considerably smaller that the number of Fourier modes, 62, used in our simulations. Based on our results, we believe that inertial manifolds can, in general, be approximately constructed by using sufficiently dense sets of periodic orbits and their linearized neighborhoods. we hope that the method of using periodic orbits to determine the dimension of an inertial manifold can be ported to higher-dimensional physical nonlinear dissipative systems, such as Navier-Stokes equations.