Lecture 16:
Recurrent Neural Networks

Subhransu Maji, Chuang Gan and TAs
Some slides kindly provided by Fei-Fei Li, Jiajun Wu, Erik Learned-Miller

Visual Question Answering

Q: What endangered animal is featured on the truck?

A: **A bald eagle.**
A: A sparrow.
A: A humming bird.
A: A raven.

Q: Where will the driver go if turning right?

A: **Onto 24 ¾ Rd.**
A: Onto 25 ¾ Rd.
A: Onto 23 ¾ Rd.
A: Onto Main Street.

Q: When was the picture taken?

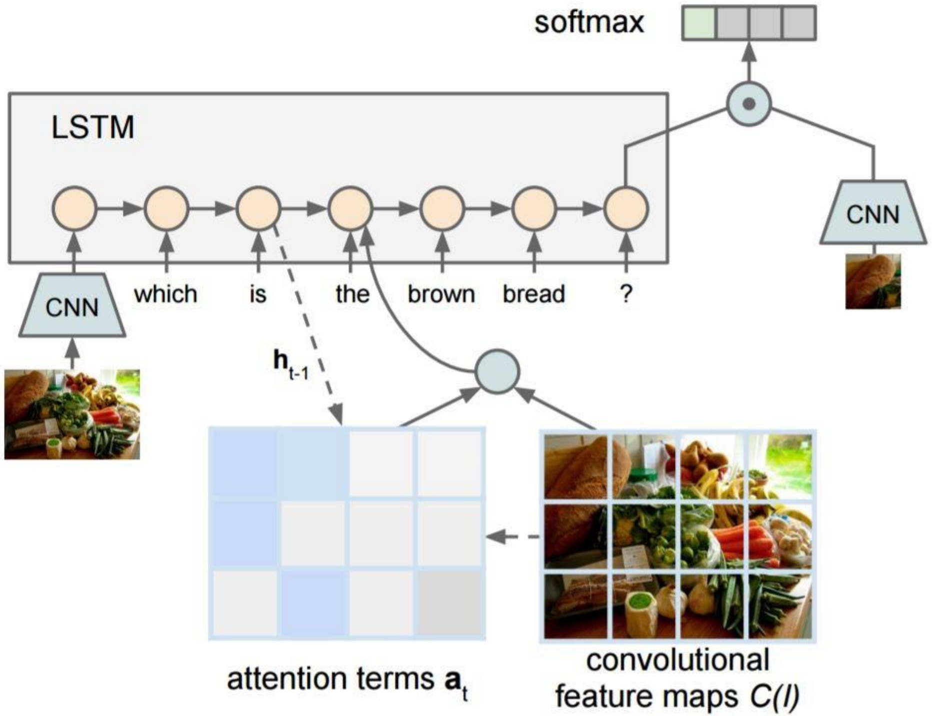
A: **During a wedding.**
A: During a bar mitzvah.
A: During a funeral.
A: During a Sunday church service

Q: Who is under the umbrella?

A: **Two women.**
A: A child.
A: An old man.
A: A husband and a wife.

Agrawal et al, "VQA: Visual Question Answering", ICCV 2015
Zhu et al, "Visual 7W: Grounded Question Answering in Images", CVPR 2016

Visual Question Answering: RNNs with Attention



What kind of animal is in the photo?
A **cat**.

Why is the person holding a knife?
To cut the **cake** with.

Zhu et al, "Visual 7W: Grounded Question Answering in Images", CVPR 2016

Multilayer RNNs

$$\boxed{h_t^l = \tanh W^l \begin{pmatrix} h_t^{l-1} \\ h_{t-1}^l \end{pmatrix}}$$

$h \in \mathbb{R}^n \qquad W^l \ [n \times 2n]$

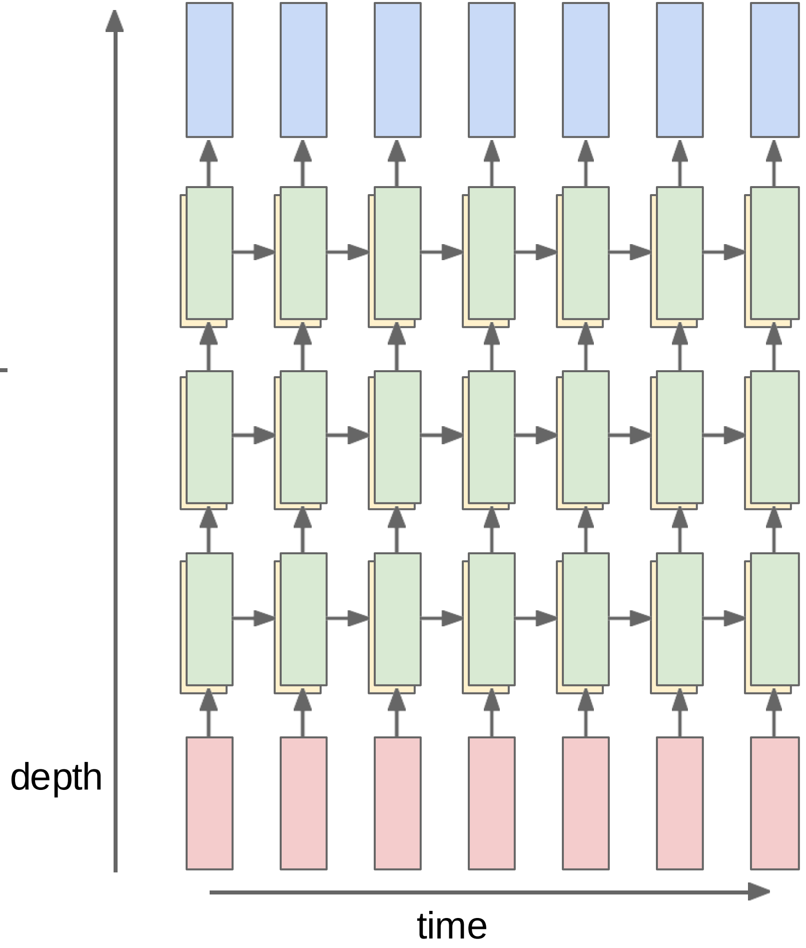
LSTM:

$W^l \ [4n \times 2n]$

$$\begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \text{sigm} \\ \text{sigm} \\ \text{sigm} \\ \tanh \end{pmatrix} W^l \begin{pmatrix} h_t^{l-1} \\ h_{t-1}^l \end{pmatrix}$$

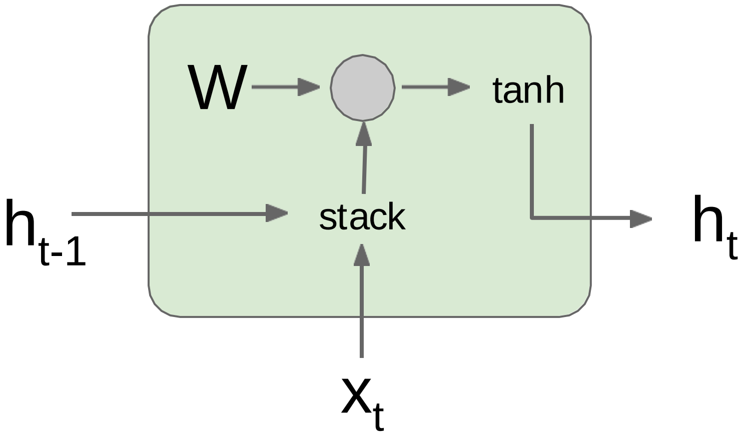
$$c_t^l = f \odot c_{t-1}^l + i \odot g$$

$$h_t^l = o \odot \tanh(c_t^l)$$



Vanilla RNN Gradient Flow

Bengio et al, "Learning long-term dependencies with gradient descent is difficult", IEEE Transactions on Neural Networks, 1994
Pascanu et al, "On the difficulty of training recurrent neural networks", ICML 2013



$$
\begin{aligned}
h_t &= \tanh(W_{hh}h_{t-1} + W_{xh}x_t) \\
&= \tanh\left(\begin{pmatrix} W_{hh} & W_{hx} \end{pmatrix} \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}\right) \\
&= \tanh\left(W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}\right)
\end{aligned}
$$

Vanilla RNN Gradient Flow

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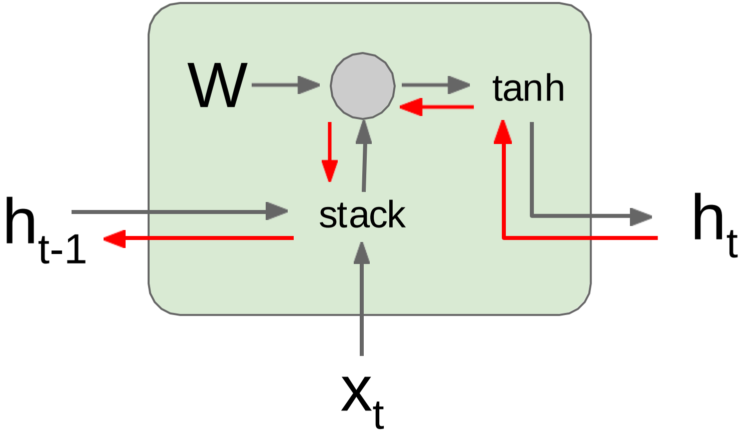
Backpropagation from h_t to h_{t-1} multiplies by W (actually W_{hh}^T)



$$h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t)$$
$$= \tanh\left(\begin{pmatrix}W_{hh} & W_{hx}\end{pmatrix}\begin{pmatrix}h_{t-1}\\ x_t\end{pmatrix}\right)$$
$$= \tanh\left(W\begin{pmatrix}h_{t-1}\\ x_t\end{pmatrix}\right)$$

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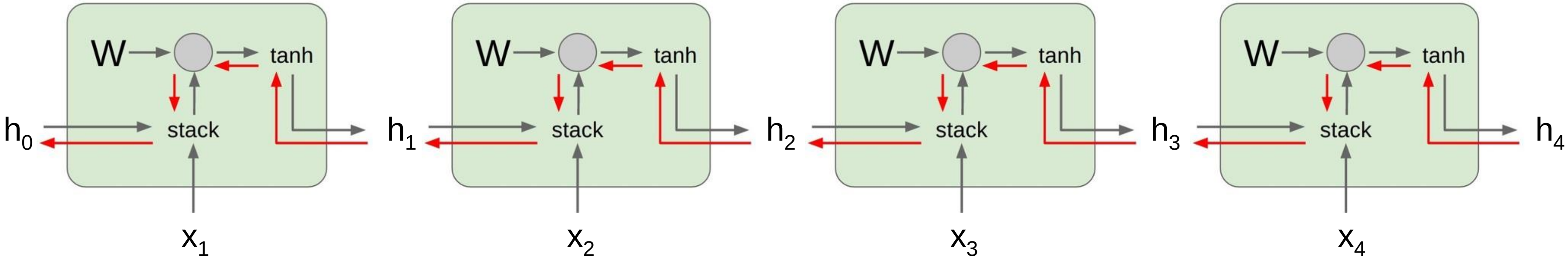

Computing gradient of h_0 involves many factors of W
(and repeated tanh)

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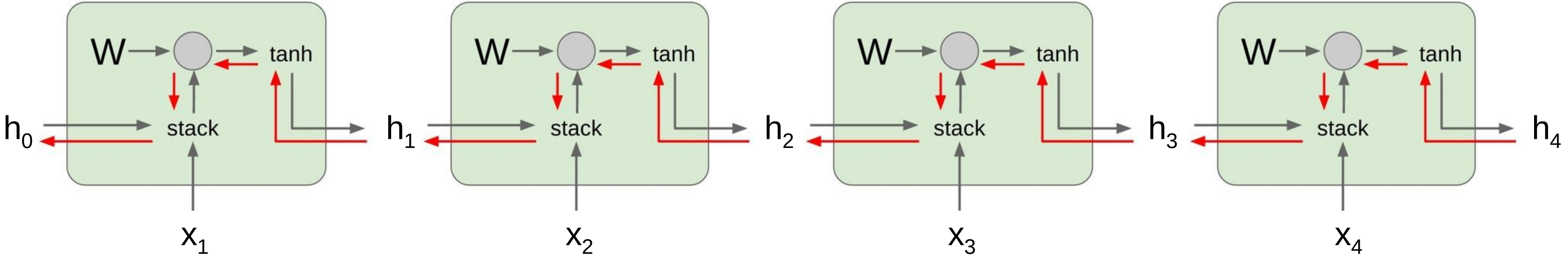

Computing gradient of h_0 involves many factors of W
(and repeated tanh)

Largest singular value > 1:
Exploding gradients

Largest singular value < 1:
Vanishing gradients

Vanilla RNN Gradient Flow

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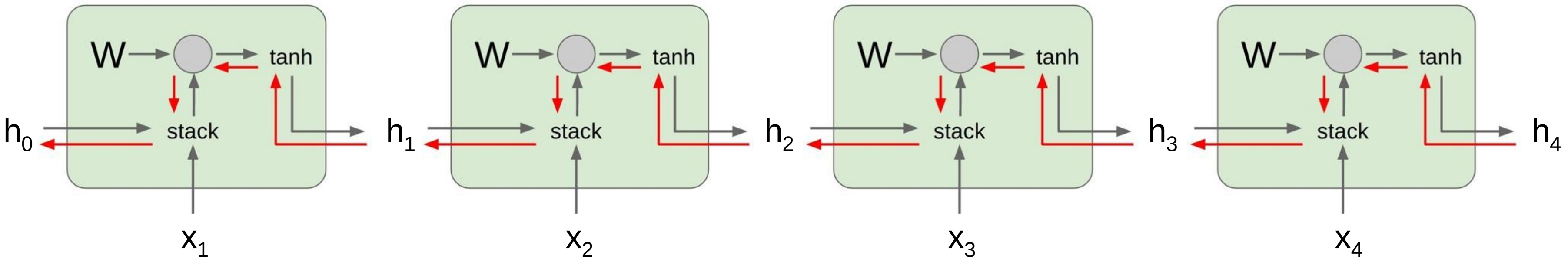

Computing gradient of h_0 involves many factors of W (and repeated tanh)

Largest singular value > 1:
Exploding gradients

→ **Gradient clipping**: Scale gradient if its norm is too big

```
grad_norm = np.sum(grad * grad)
if grad_norm > threshold:
  grad *= (threshold / grad_norm)
```

Largest singular value < 1:
Vanishing gradients

Vanilla RNN Gradient Flow

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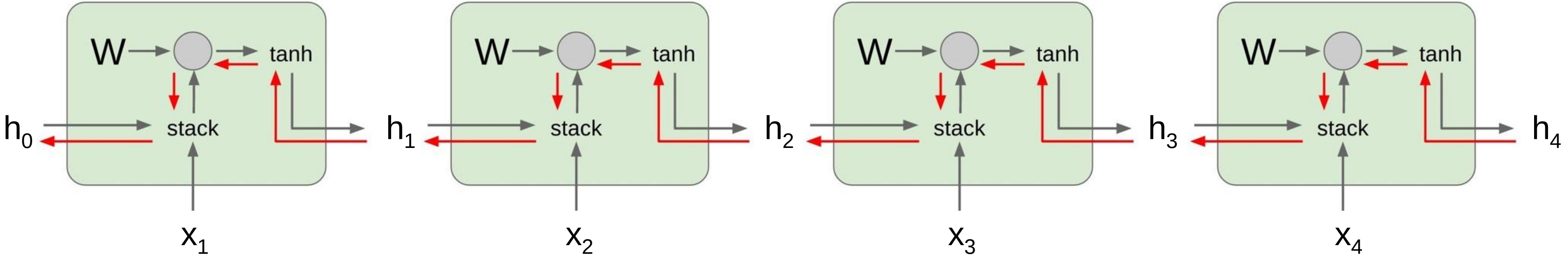


Computing gradient of h_0 involves many factors of W
(and repeated tanh)

Largest singular value > 1:
Exploding gradients

Largest singular value < 1:
Vanishing gradients

→ Change RNN architecture

Long Short Term Memory (LSTM)

Vanilla RNN

$$\boxed{h_t = \tanh\left(W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}\right)}$$

LSTM

$$\begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ \tanh \end{pmatrix} W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}$$

$$c_t = f \odot c_{t-1} + i \odot g$$

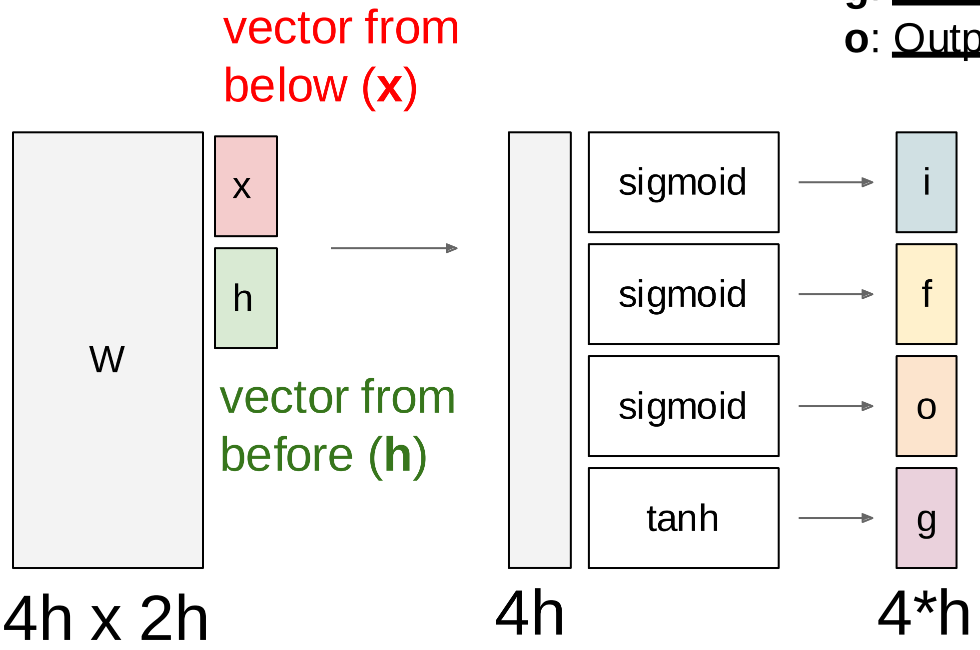
$$h_t = o \odot \tanh(c_t)$$

Hochreiter and Schmidhuber, “Long Short Term Memory”, Neural Computation 1997

Long Short Term Memory (LSTM)

[Hochreiter et al., 1997]

f: Forget gate, Whether to erase cell
i: Input gate, whether to write to cell
g: Gate gate (?), How much to write to cell
o: Output gate, How much to reveal cell



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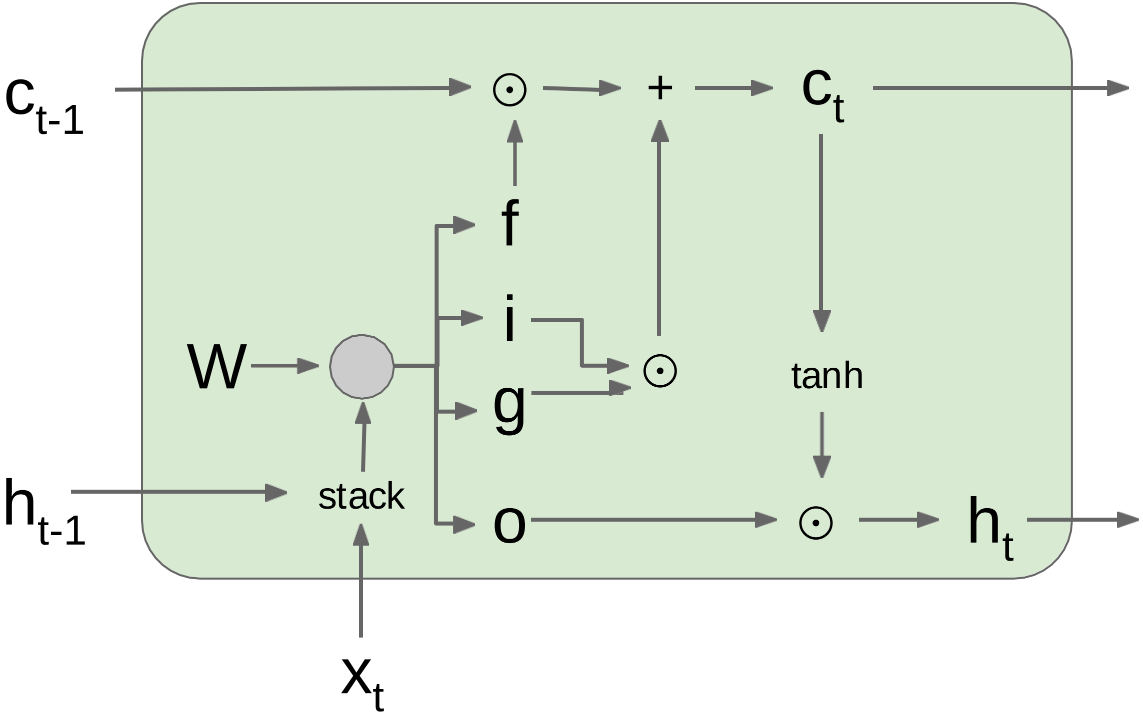
Long Short Term Memory (LSTM)

[Hochreiter et al., 1997]



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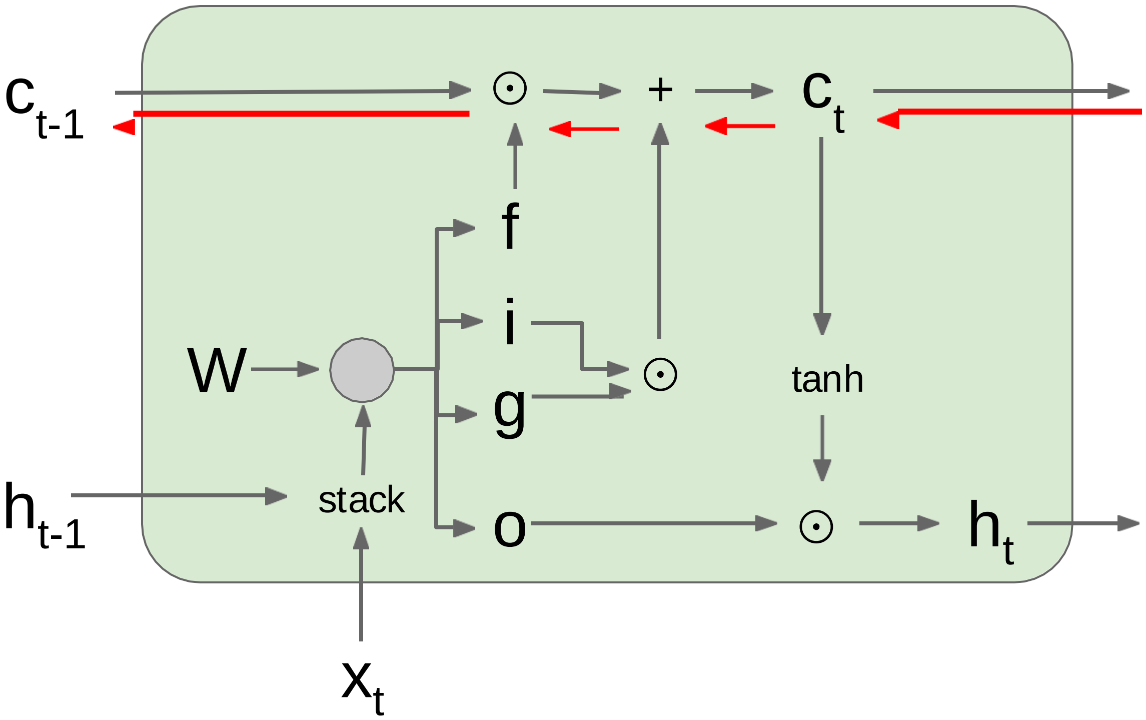
Long Short Term Memory (LSTM): Gradient Flow

[Hochreiter et al., 1997]



Backpropagation from c_t to c_{t-1} only elementwise multiplication by f, no matrix multiply by W

$$\begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ \tanh \end{pmatrix} W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}$$

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Long Short Term Memory (LSTM): Gradient Flow

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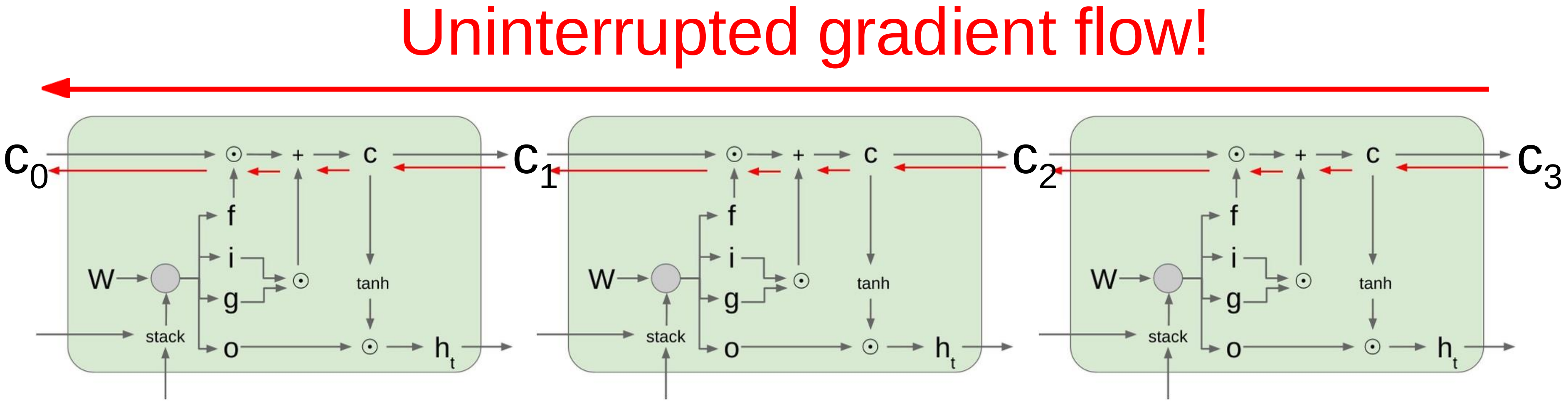
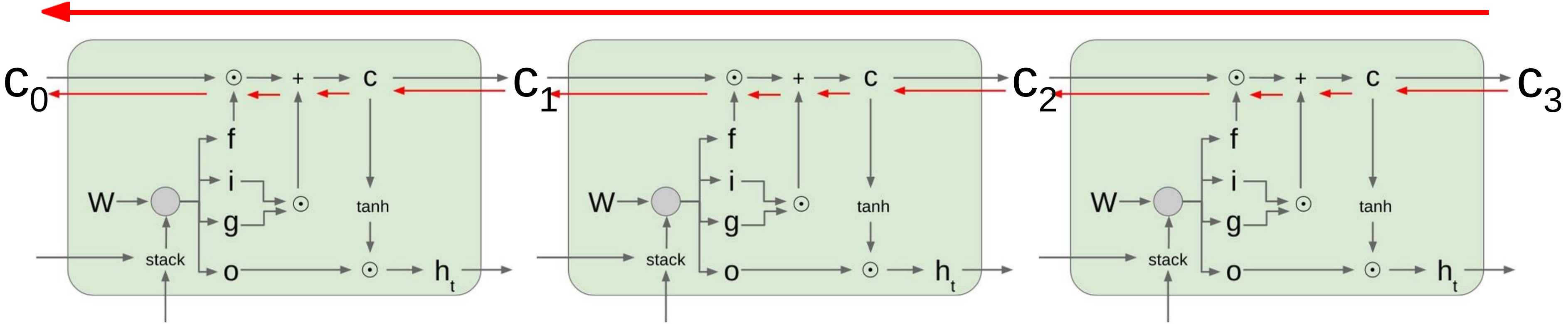


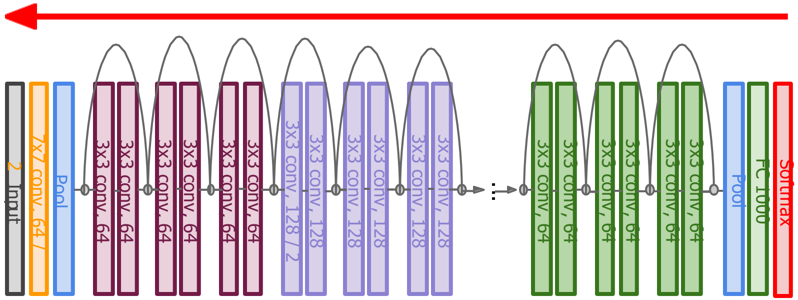
Long Short Term Memory (LSTM): Gradient Flow

[Hochreiter et al., 1997]

Uninterrupted gradient flow!



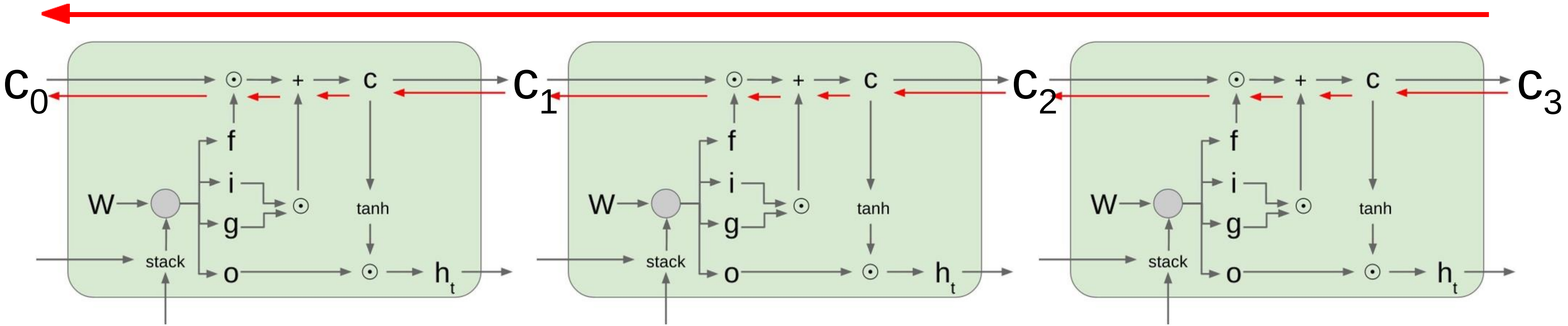
Similar to ResNet!



Long Short Term Memory (LSTM): Gradient Flow

[Hochreiter et al., 1997]

Uninterrupted gradient flow!



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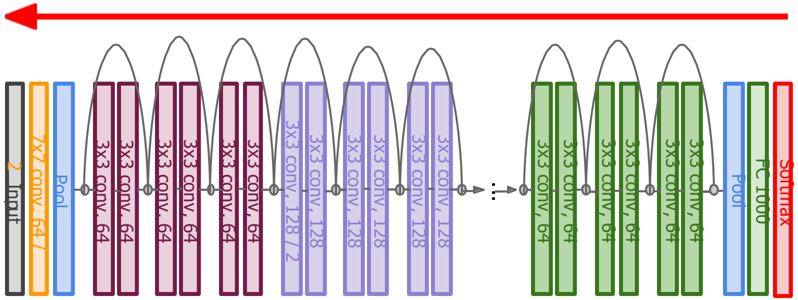


In between:

Highway Networks

$$g = T(x, W_T)$$

$$y = g \odot H(x, W_H) + (1 - g) \odot x$$

Srivastava et al, “Highway Networks”, ICML DL Workshop 2015

Other RNN Variants

[*An Empirical Exploration of Recurrent Network Architectures*, Jozefowicz et al., 2015]

GRU [*Learning phrase representations using rnn encoder-decoder for statistical machine translation*, Cho et al. 2014]

$$r_t = \sigma(W_{xr}x_t + W_{hr}h_{t-1} + b_r)$$
$$z_t = \sigma(W_{xz}x_t + W_{hz}h_{t-1} + b_z)$$
$$\tilde{h}_t = \tanh(W_{xh}x_t + W_{hh}(r_t \odot h_{t-1}) + b_h)$$
$$h_t = z_t \odot h_{t-1} + (1 - z_t) \odot \tilde{h}_t$$

[*LSTM: A Search Space Odyssey*, Greff et al., 2015]

MUT1:

$$\begin{aligned} z &= \mathrm{sigm}(W_{\mathrm{xz}}x_t + b_{\mathrm{z}}) \\ r &= \mathrm{sigm}(W_{\mathrm{xr}}x_t + W_{\mathrm{hr}}h_t + b_{\mathrm{r}}) \\ h_{t+1} &= \tanh(W_{\mathrm{hh}}(r \odot h_t) + \tanh(x_t) + b_{\mathrm{h}}) \odot z \\ &+ h_t \odot (1 - z) \end{aligned}$$

MUT2:

$$\begin{aligned} z &= \mathrm{sigm}(W_{\mathrm{xz}}x_t + W_{\mathrm{hz}}h_t + b_{\mathrm{z}}) \\ r &= \mathrm{sigm}(x_t + W_{\mathrm{hr}}h_t + b_{\mathrm{r}}) \\ h_{t+1} &= \tanh(W_{\mathrm{hh}}(r \odot h_t) + W_{xh}x_t + b_{\mathrm{h}}) \odot z \\ &+ h_t \odot (1 - z) \end{aligned}$$

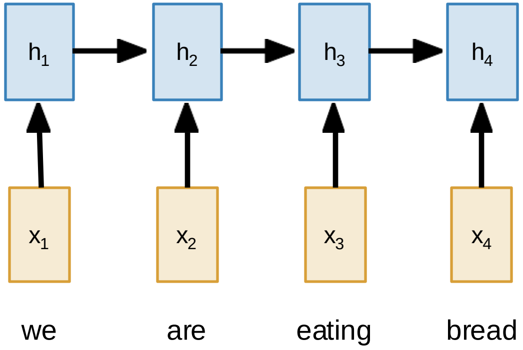
MUT3:

$$\begin{aligned} z &= \mathrm{sigm}(W_{\mathrm{xz}}x_t + W_{\mathrm{hz}}\tanh(h_t) + b_{\mathrm{z}}) \\ r &= \mathrm{sigm}(W_{\mathrm{xr}}x_t + W_{\mathrm{hr}}h_t + b_{\mathrm{r}}) \\ h_{t+1} &= \tanh(W_{\mathrm{hh}}(r \odot h_t) + W_{xh}x_t + b_{\mathrm{h}}) \odot z \\ &+ h_t \odot (1 - z) \end{aligned}$$

Sequence to Sequence with RNNs

Input: Sequence $x_1, \ldots x_T$
Output: Sequence $y_1, \ldots, y_{T'}$

Encoder: $h_t = f_W(x_t, h_{t-1})$



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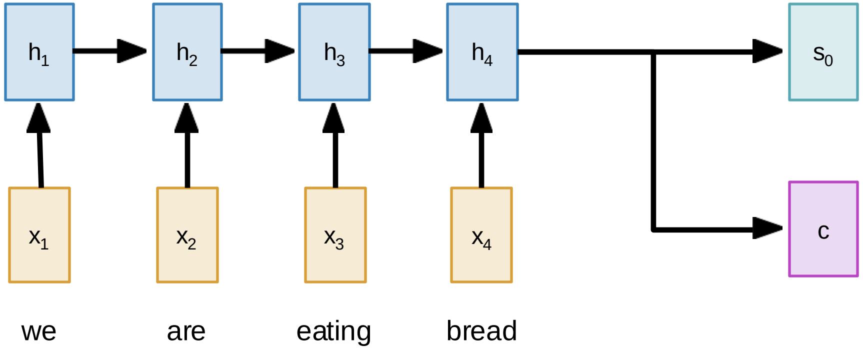
From final hidden state predict:
Initial decoder state s_0
Context vector c (often $c=h_T$)



Sequence to Sequence with RNNs

Input: Sequence $x_1, \ldots x_T$
Output: Sequence $y_1, \ldots, y_{T'}$

Decoder: $s_t = g_U(y_{t-1}, s_{t-1}, c)$

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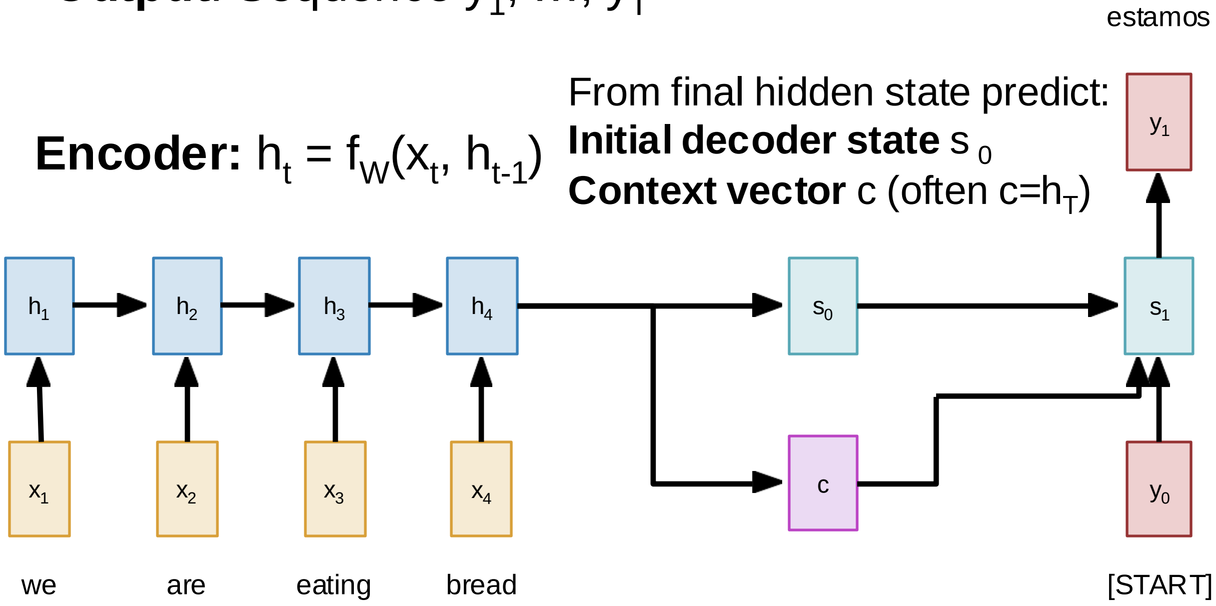

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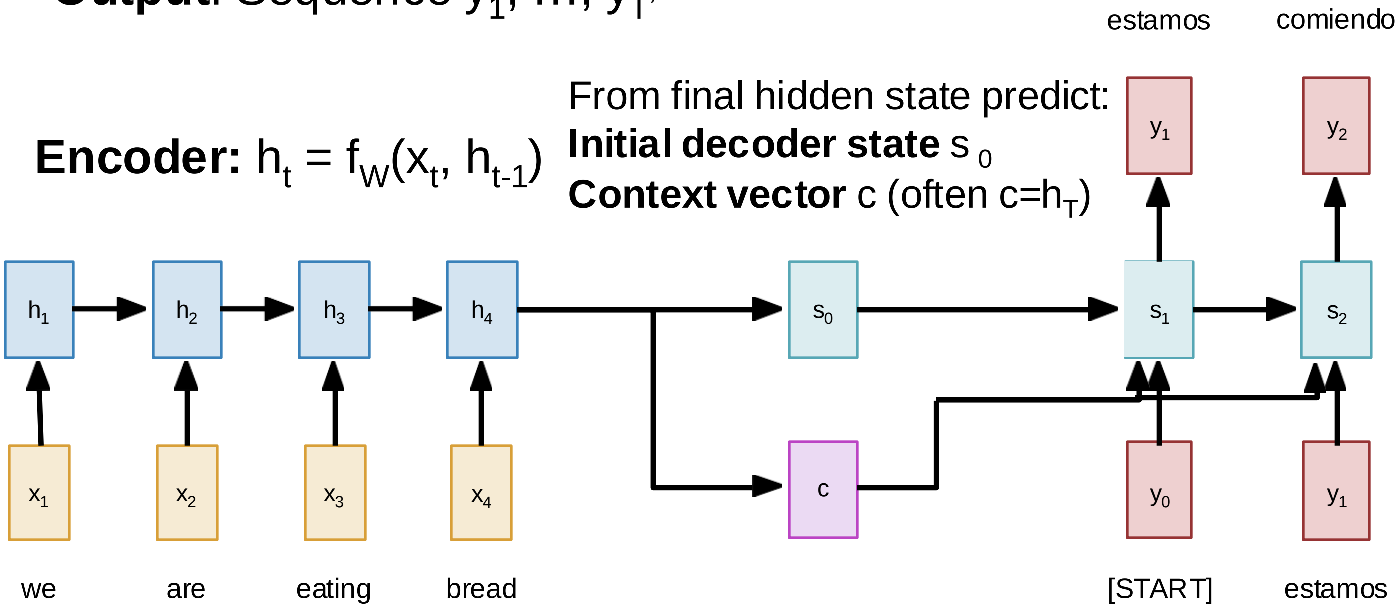

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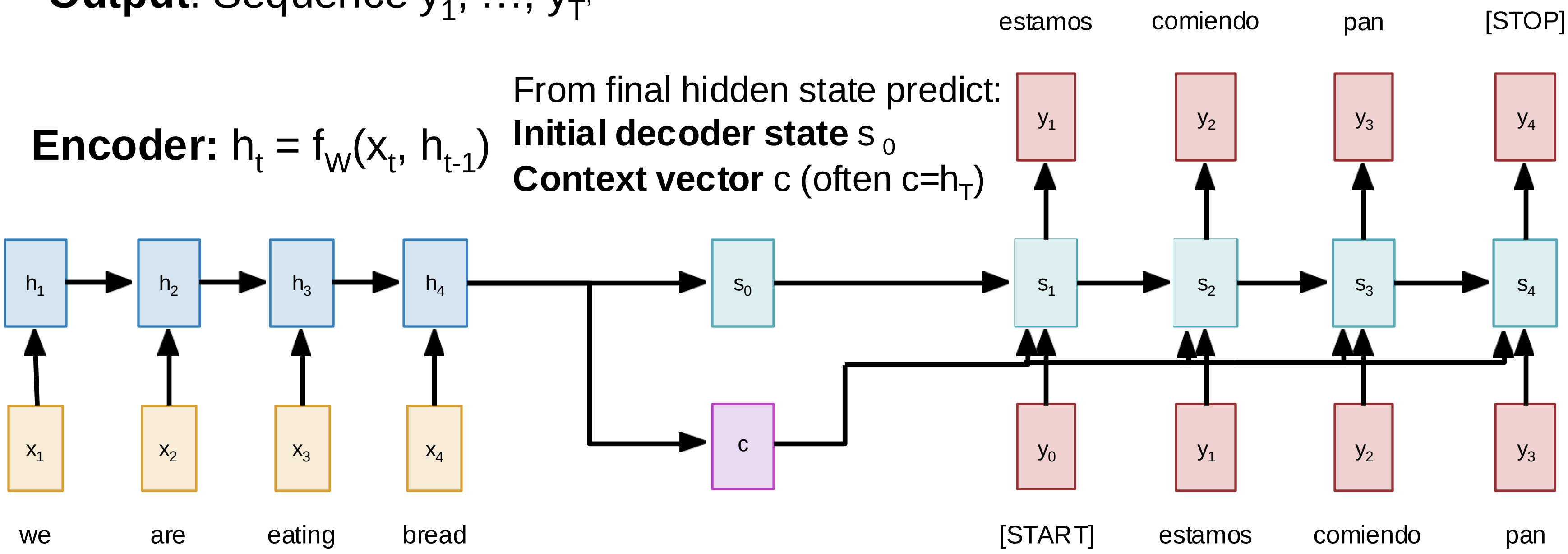

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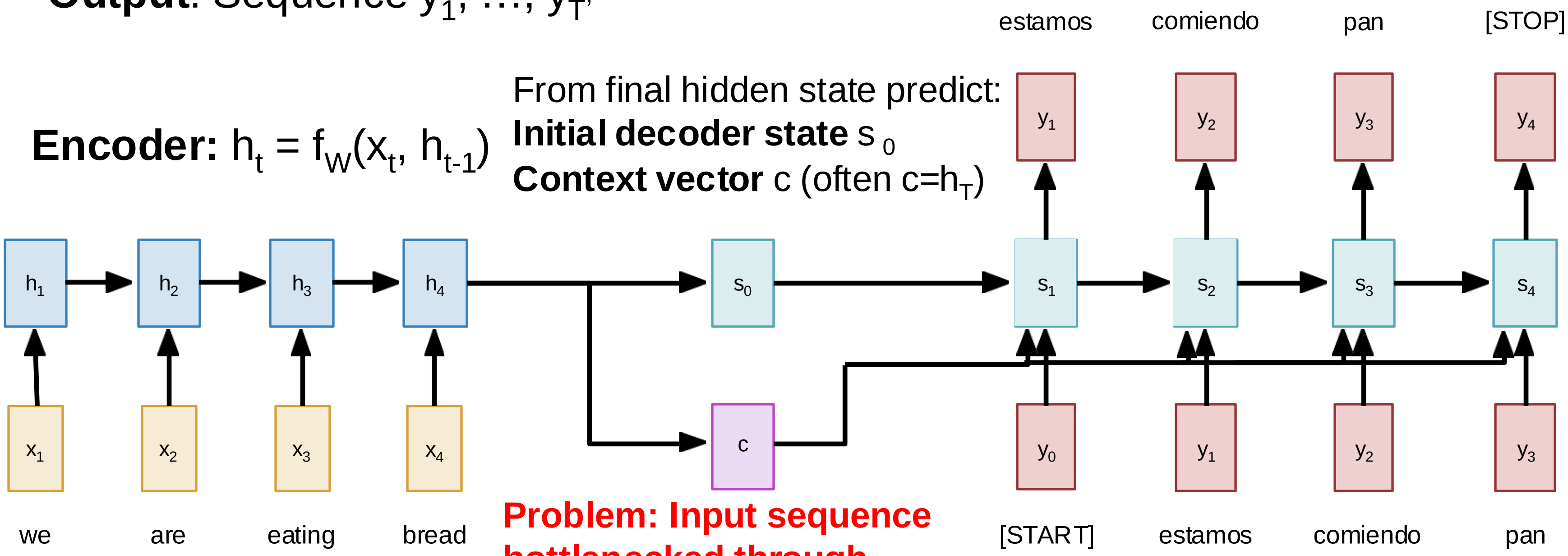

Problem: Input sequence bottlenecked through fixed-sized vector. What if

Sequence to Sequence with RNNs

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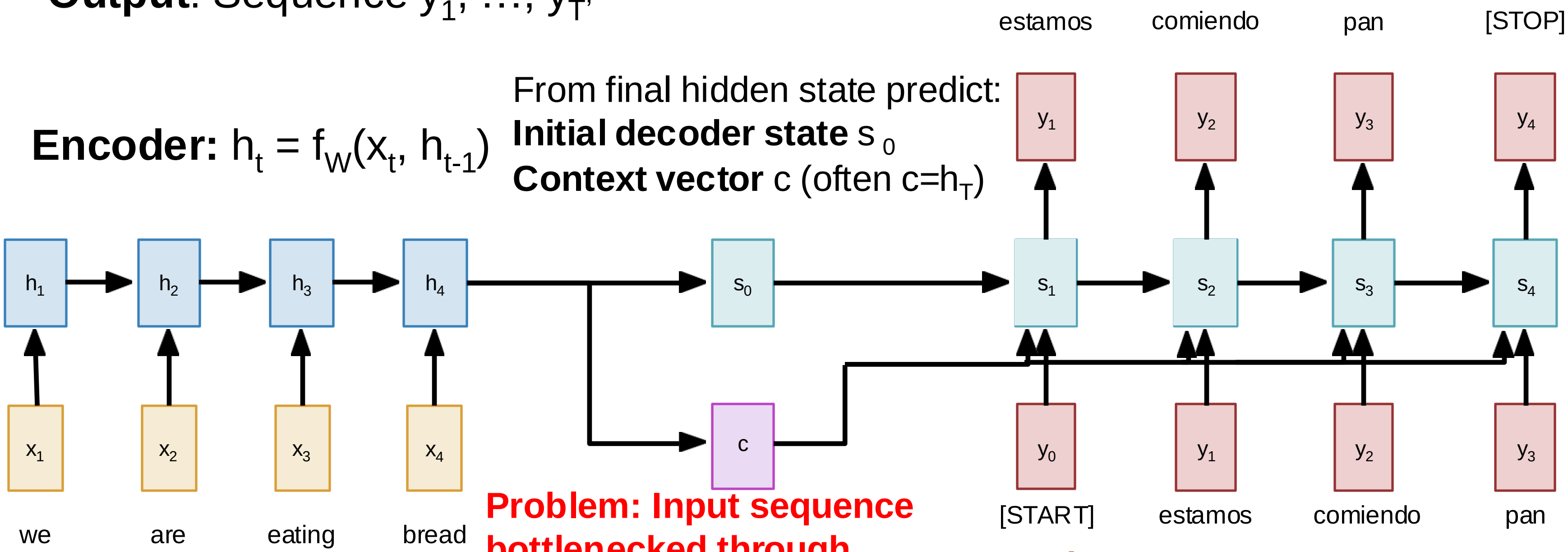


Problem: Input sequence bottlenecked through fixed-sized vector. What if

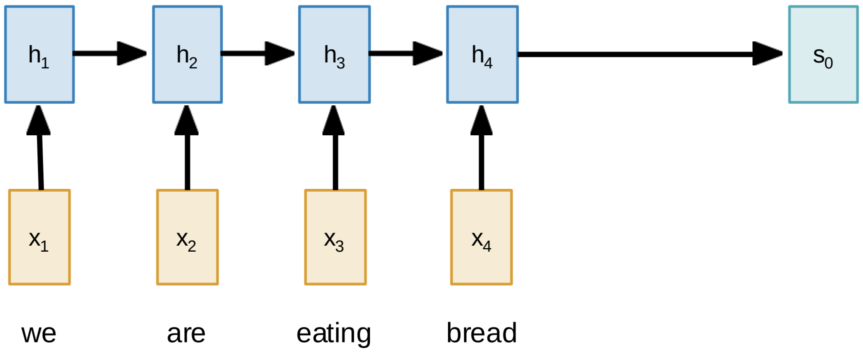
Idea: use new context vector at each step of decoder!

Sequence to Sequence with RNNs and Attention

Input: Sequence $x_1, \ldots x_T$
Output: Sequence $y_1, \ldots, y_{T'}$

Encoder: $h_t = f_W(x_t, h_{t-1})$

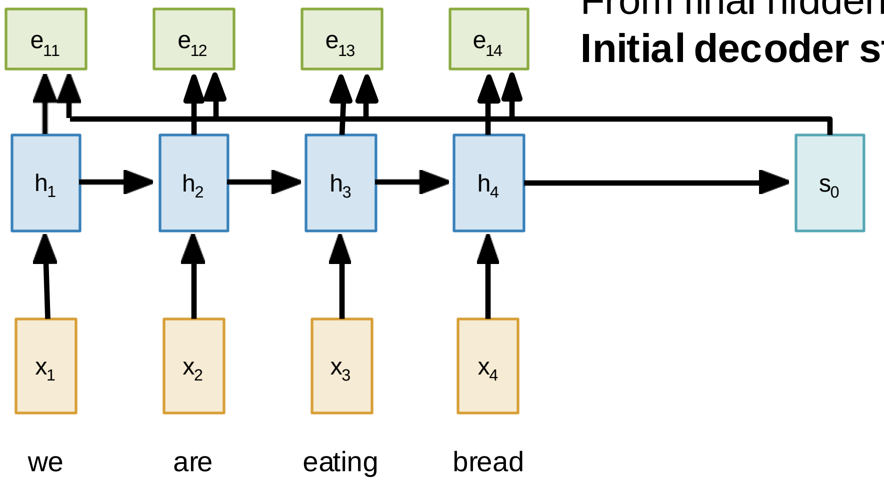
From final hidden state:
Initial decoder state s_0



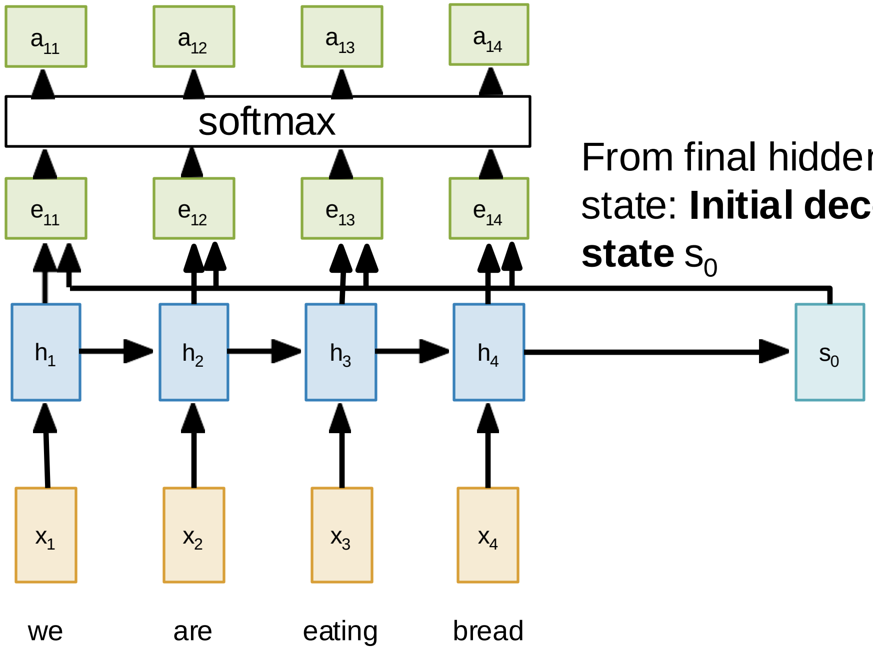
Sequence to Sequence with RNNs and Attention

Compute (scalar) **alignment scores**
$e_{t,i} = f_{att}(s_{t-1}, h_i)$ (f_{att} is an MLP)

From final hidden state:
Initial decoder state s_0



Sequence to Sequence with RNNs and Attention

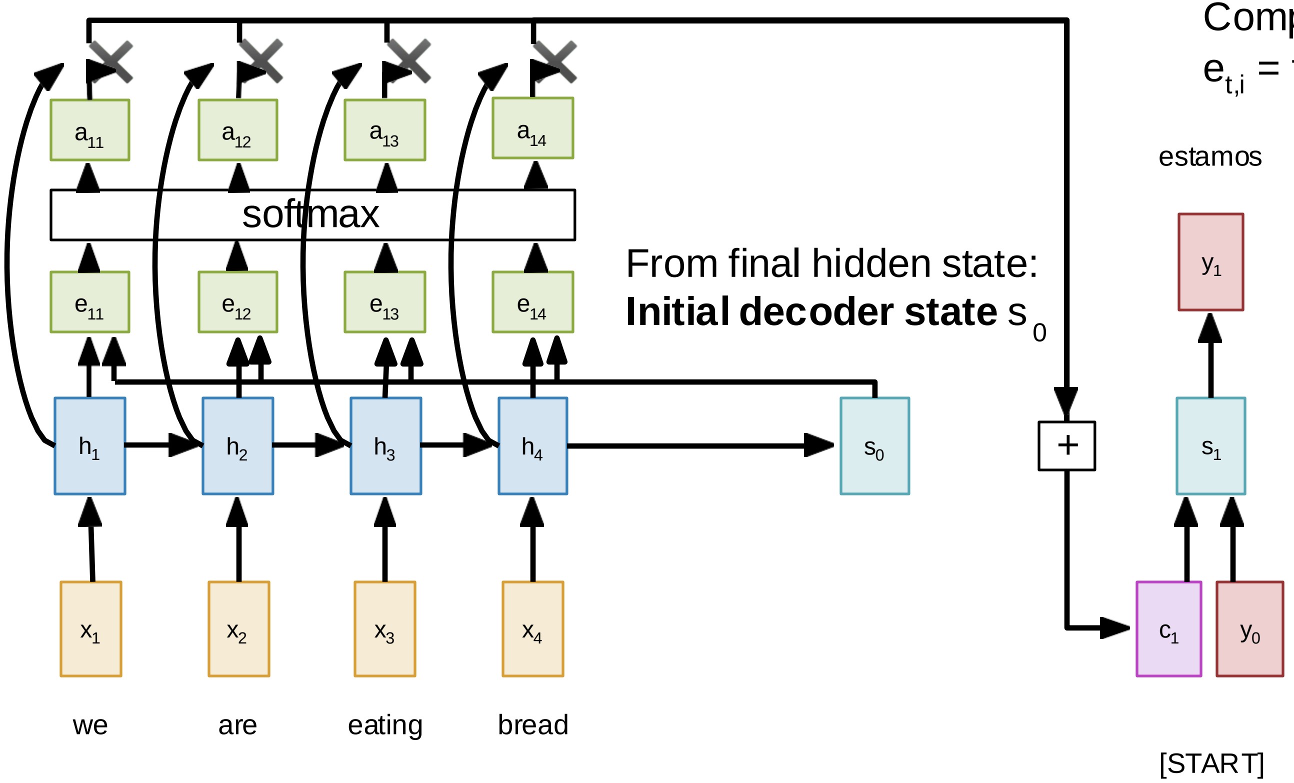


From final hidden state: **Initial decoder state** s_0

Compute (scalar) **alignment scores**
$e_{t,i} = f_{att}(s_{t-1}, h_i)$ (f_{att} is an MLP)

Normalize alignment scores to get **attention weights**
$0 < a_{t,i} < 1 \quad \sum_i a_{t,i} = 1$

Sequence to Sequence with RNNs and Attention



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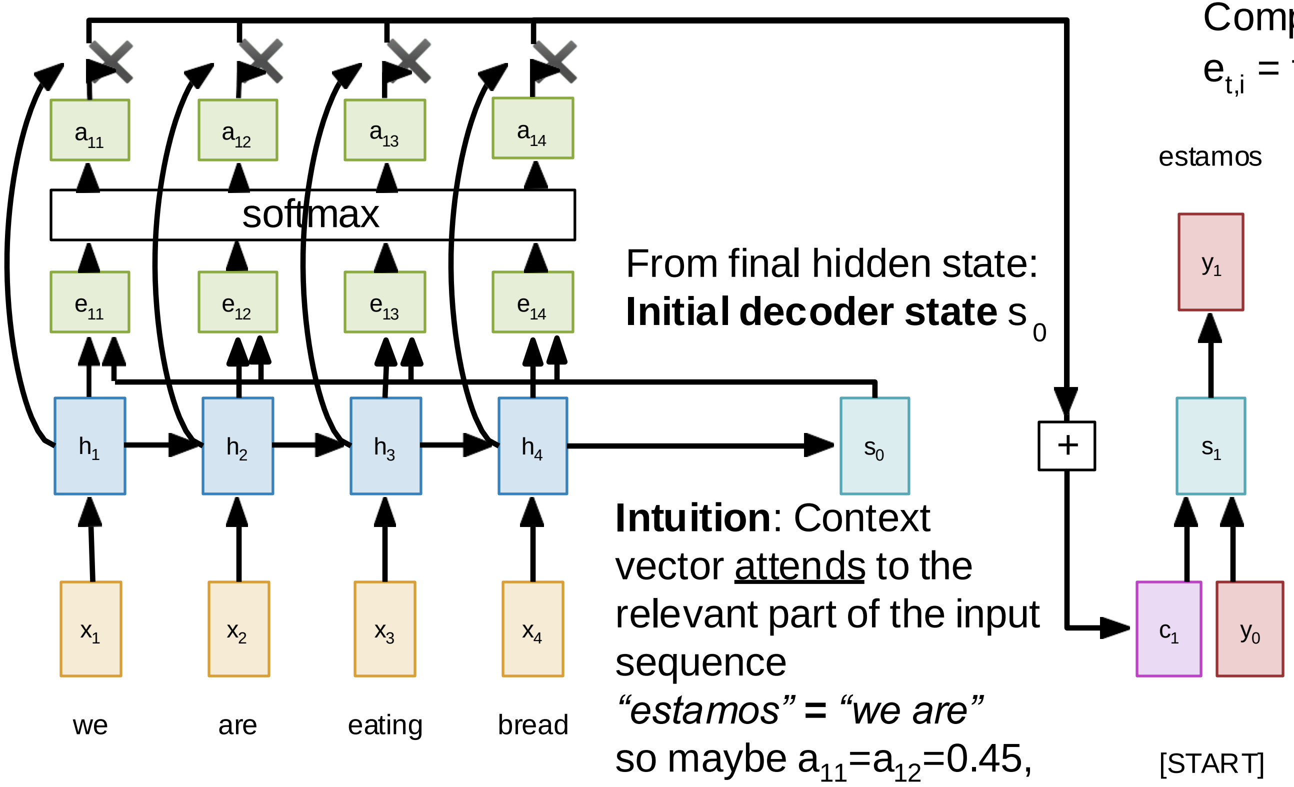
From final hidden state:
Initial decoder state s_0

Normalize alignment scores to get **attention weights**
$0 < a_{t,i} < 1$ $\sum_i a_{t,i} = 1$

Compute context vector as linear combination of hidden states
$c_t = \sum_i a_{t,i} h_i$

Sequence to Sequence with RNNs and Attention



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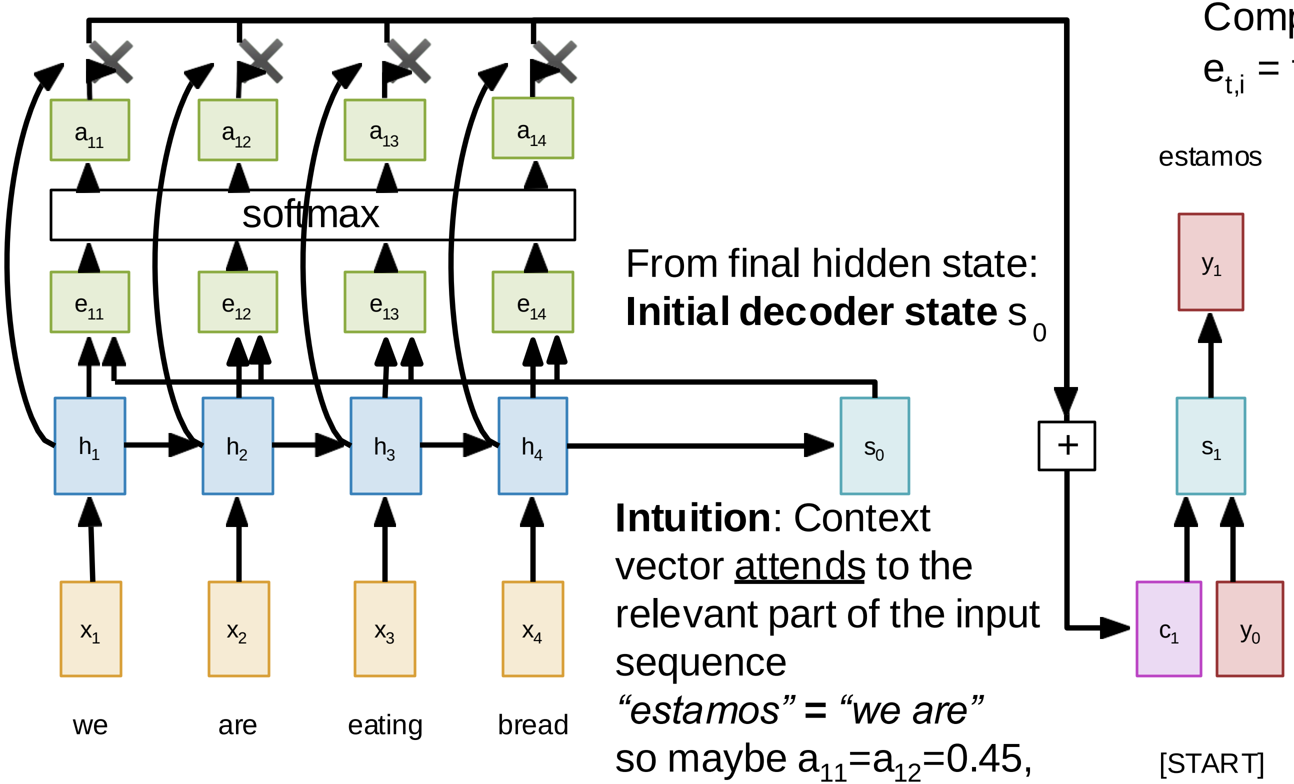
Compute context vector as linear combination of hidden states
$c_t = \sum_i a_{t,i} h_i$

Use context vector in decoder: $s_t = g_U(y_{t-1}, s_{t-1}, c_t)$

From final hidden state: **Initial decoder state** s_0

Intuition: Context vector attends to the relevant part of the input sequence
"estamos"* = *"we are"
so maybe $a_{11}=a_{12}=0.45$, $a_{13}=a_{14}=0.05$

Sequence to Sequence with RNNs and Attention



From final hidden state: **Initial decoder state s_0**

Intuition: Context vector <u>attends</u> to the relevant part of the input sequence
"estamos" **=** *"we are"*
so maybe $a_{11}=a_{12}=0.45$, $a_{13}=a_{14}=0.05$

Compute (scalar) **alignment scores**
$e_{t,i} = f_{att}(s_{t-1}, h_i)$ (f_{att} is an MLP)

Normalize alignment scores to get **attention weights**
$0 < a_{t,i} < 1$ $\sum_i a_{t,i} = 1$

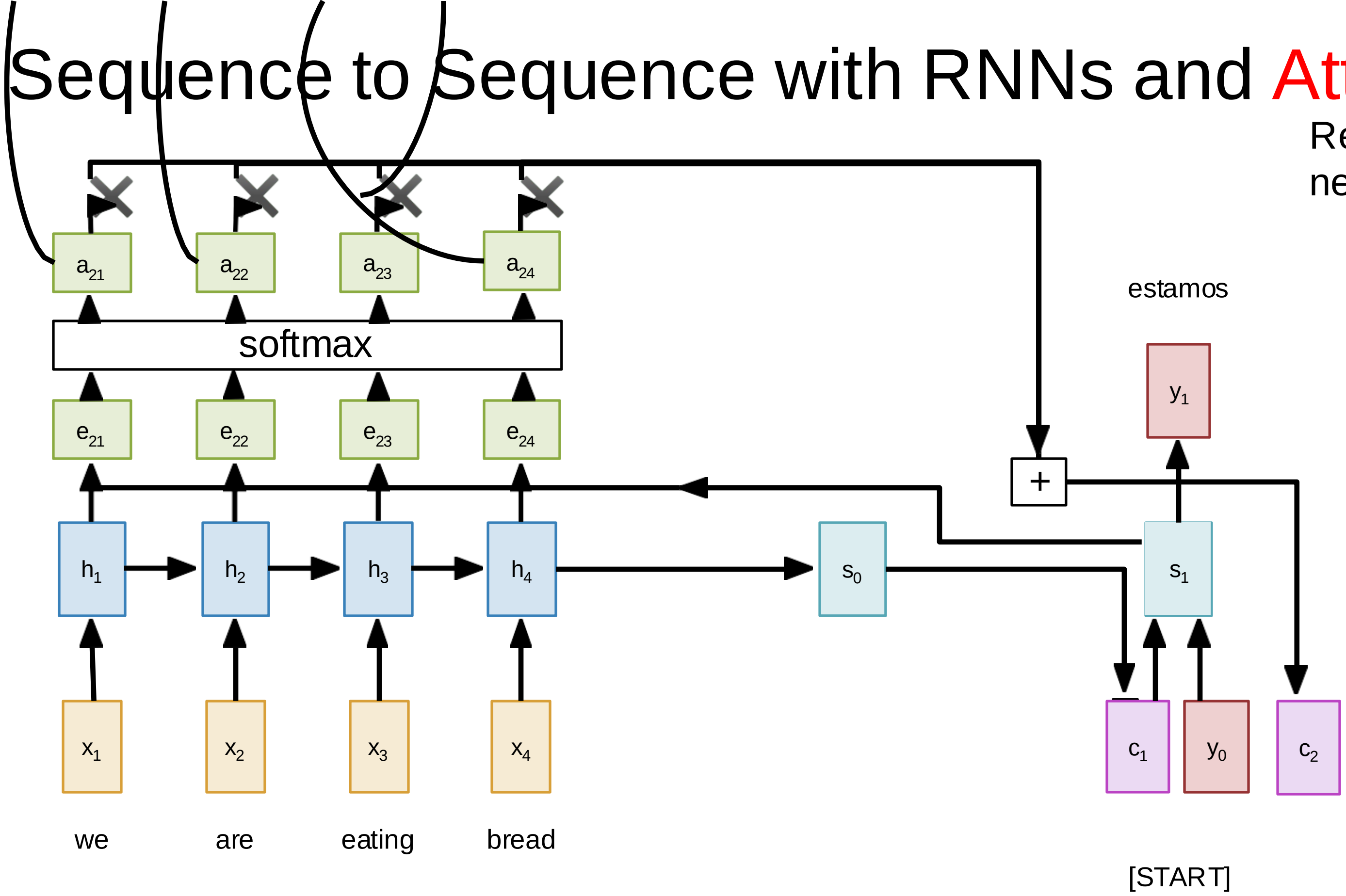
Compute context vector as linear combination of hidden states
$c_t = \sum_i a_{t,i} h_i$

Use context vector in decoder: $s_t = g_U(y_{t-1}, s_{t-1}, c_t)$

This is all differentiable! No supervision on attention weights – backprop through everything

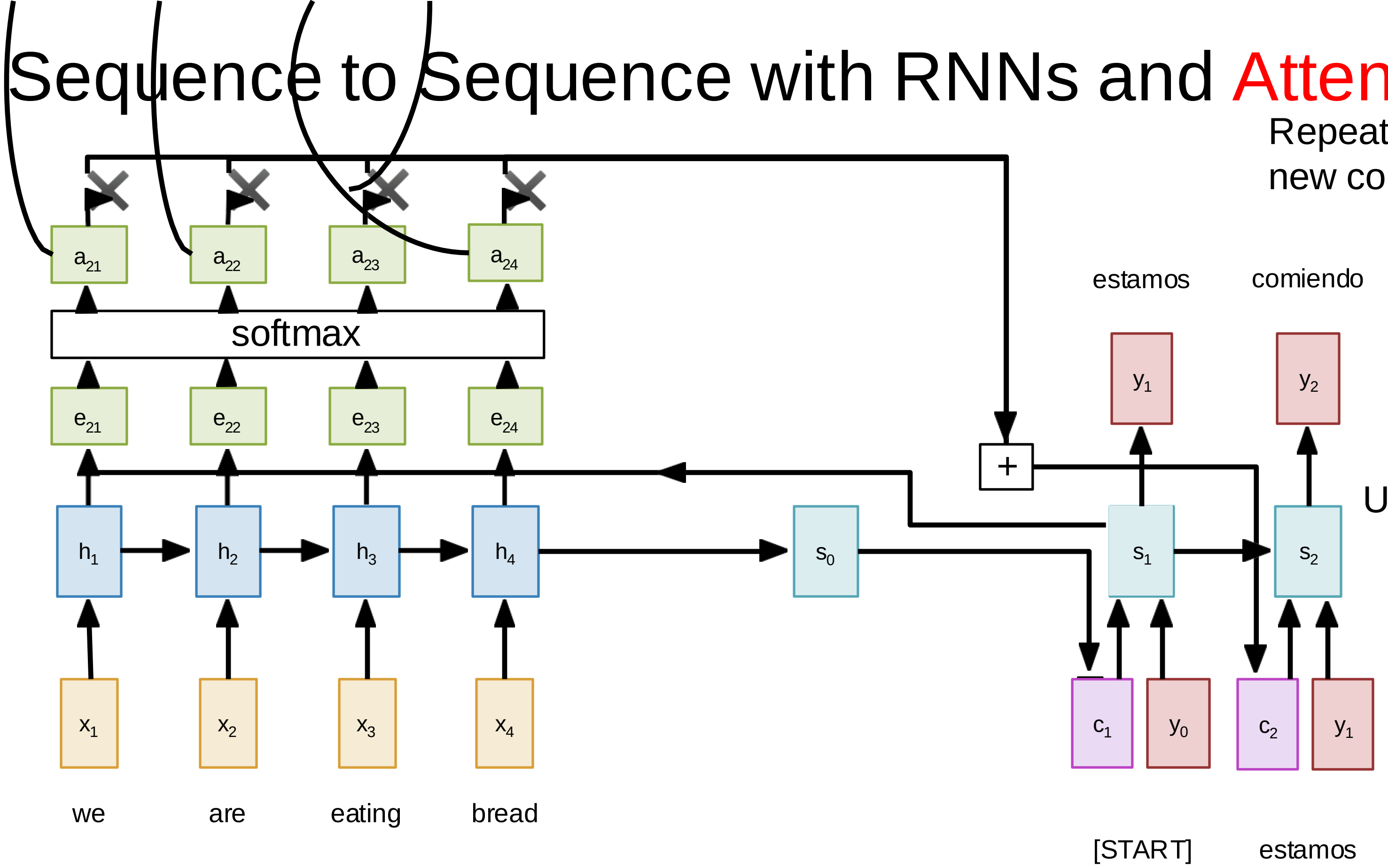
Sequence to Sequence with RNNs and Attention

Repeat: Use s_1 to compute new context vector c_2



Sequence to Sequence with RNNs and Attention

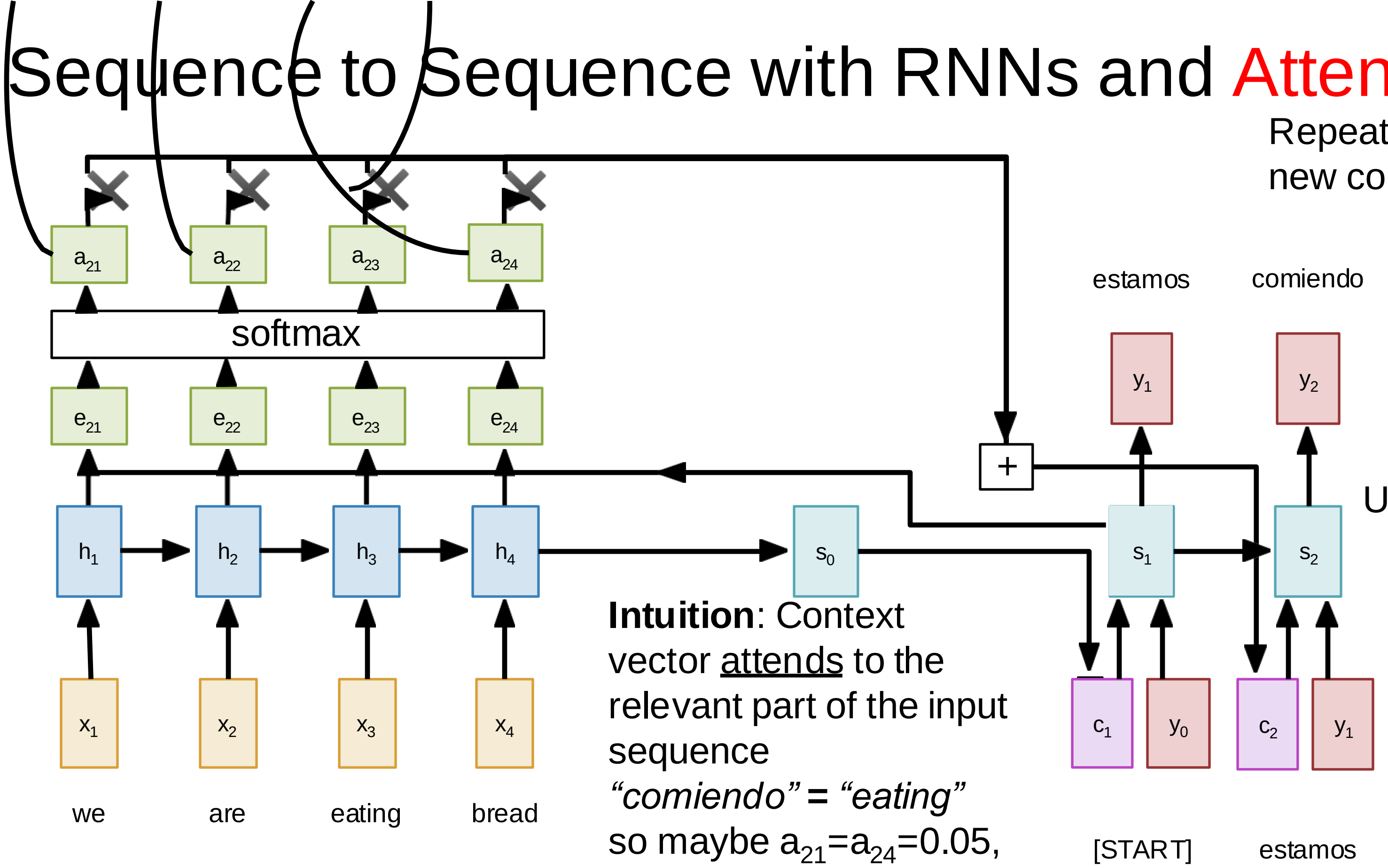
Repeat: Use s_1 to compute new context vector c_2



Use c_2 to compute s_2, y_2

Sequence to Sequence with RNNs and Attention

Repeat: Use s_1 to compute new context vector c_2



Use c_2 to compute s_2, y_2

Intuition: Context vector attends to the relevant part of the input sequence

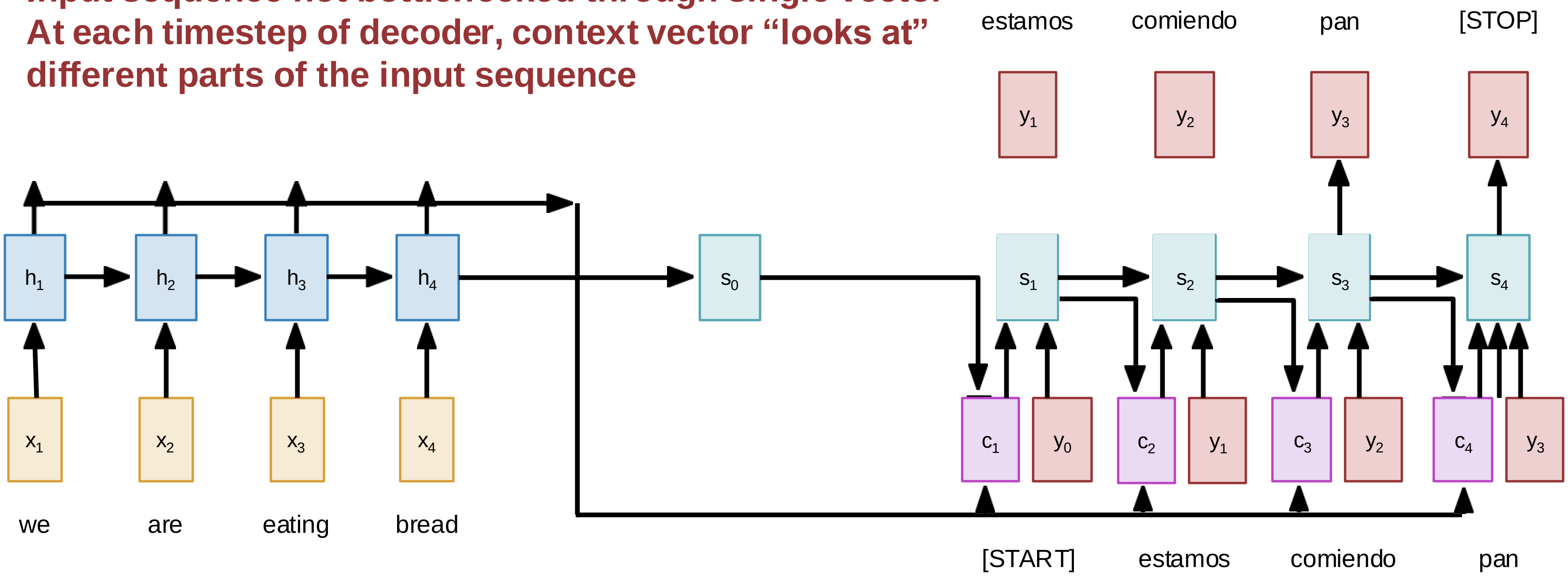
"comiendo" **=** *"eating"*

so maybe $a_{21}=a_{24}=0.05$, $a_{22}=0.1$, $a_{23}=0.8$

Sequence to Sequence with RNNs and Attention

Use a different context vector in each timestep of decoder

- **Input sequence not bottlenecked through single vector**
- **At each timestep of decoder, context vector "looks at" different parts of the input sequence**



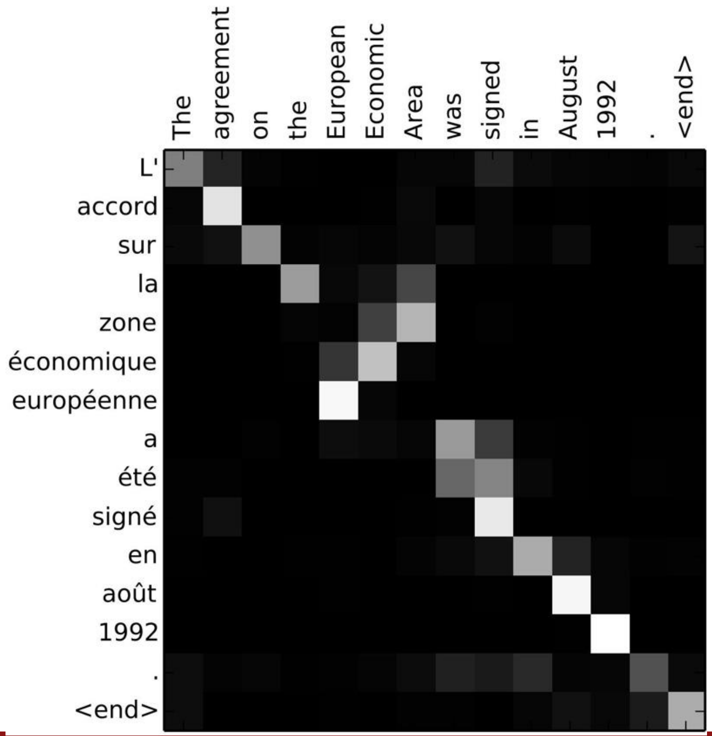
Sequence to Sequence with RNNs and Attention

Example: English to French translation

Input: “The agreement on the European Economic Area was signed in August 1992.”

Output: “L’accord sur la zone économique européenne a été signé en août 1992.”

Visualize attention weights $a_{t,i}$



Sequence to Sequence with RNNs and Attention

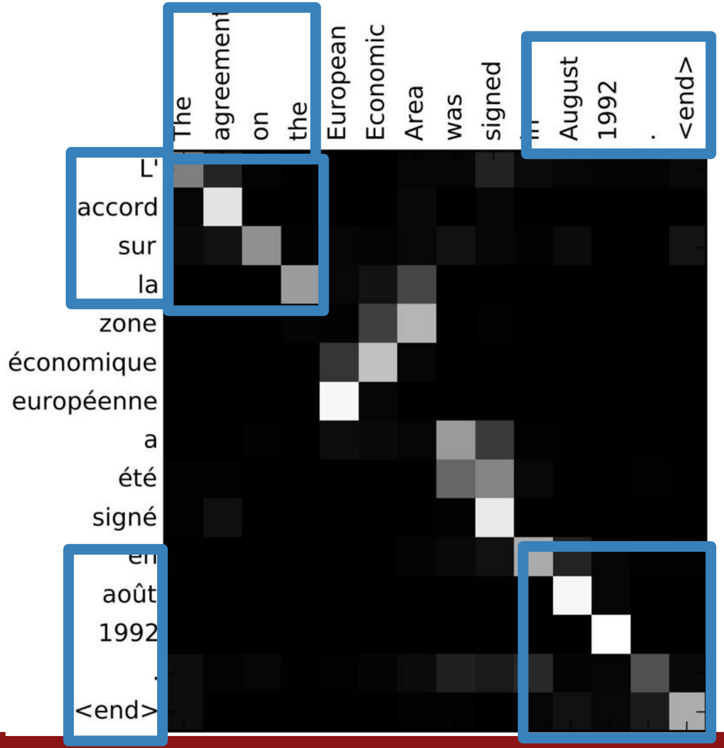
Example: English to French translation

Input: "**The agreement on the** European Economic Area was signed **in August 1992**."

Output: "**L'accord sur la** zone économique européenne a été signé **en août 1992**."

Visualize attention weights $a_{t,i}$

Diagonal attention means words correspond in order



Diagonal attention means words correspond in order

Sequence to Sequence with RNNs and Attention

Example: English to French translation

Input: "The agreement on the European Economic Area was signed in August 1992."

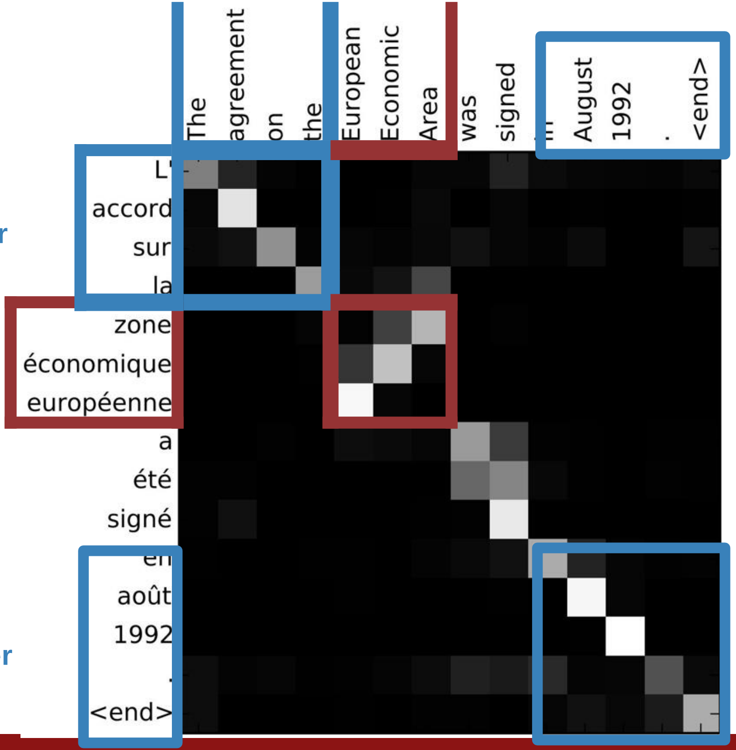
Output: "L'accord sur la zone économique européenne a été signé en août 1992."

Visualize attention weights $a_{t,i}$

Diagonal attention means words correspond in order

Attention figures out different word orders

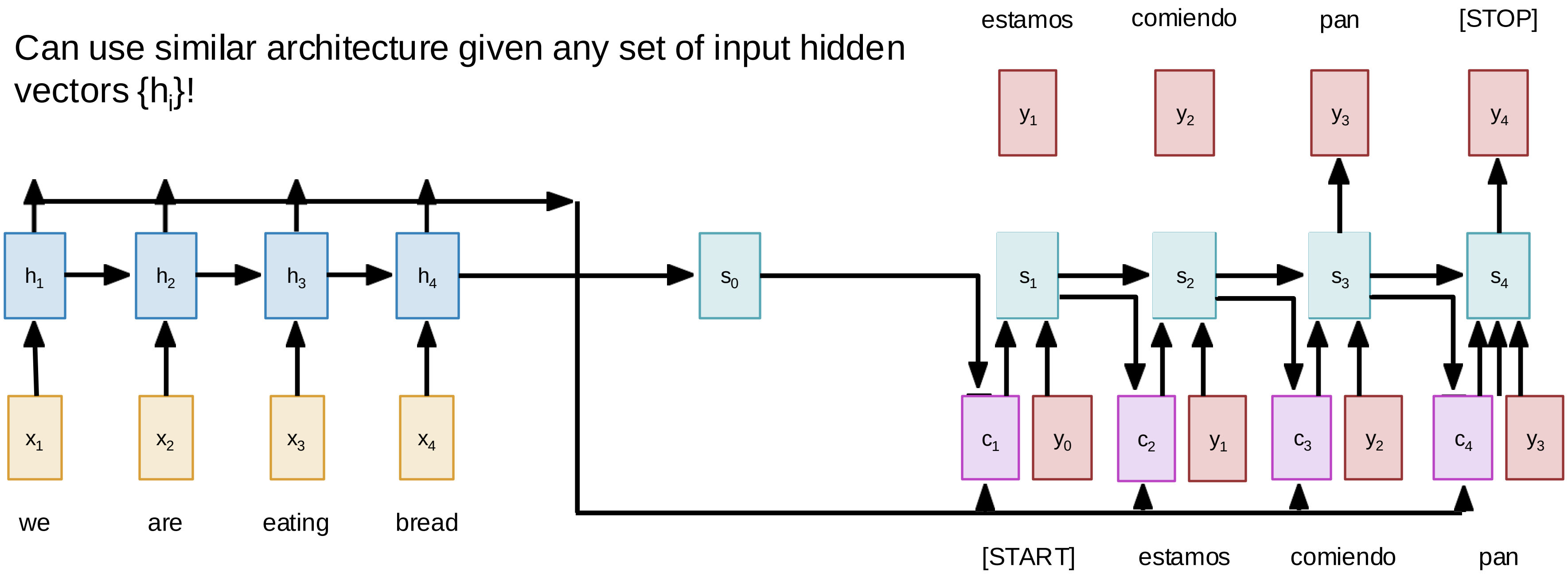
Diagonal attention means words correspond in order



Sequence to Sequence with RNNs and Attention

The decoder doesn't use the fact that h_i form an ordered sequence – it just treats them as an unordered set $\{h_i\}$

Can use similar architecture given any set of input hidden vectors $\{h_i\}$!



Summary

- RNNs allow a lot of flexibility in architecture design
- Vanilla RNNs are simple but don’t work very well
- Common to use LSTM or GRU: their additive interactions improve gradient flow
- Backward flow of gradients in RNN can explode or vanish. Exploding is controlled with gradient clipping. Vanishing is controlled with additive interactions (LSTM)
- Better/simpler architectures are a hot topic of current research
- Better understanding (both theoretical and empirical) is needed.