

Image formation

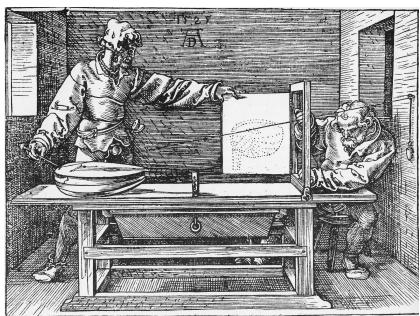
370: Intro to Computer Vision

Subhransu Maji

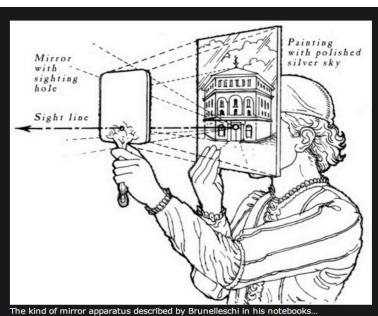
Feb 13 & 18

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COMPUTER SCIENCES  UMASS
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Cameras



Albrecht Dürer early 1500s



Brunelleschi, early 1400s

Overview

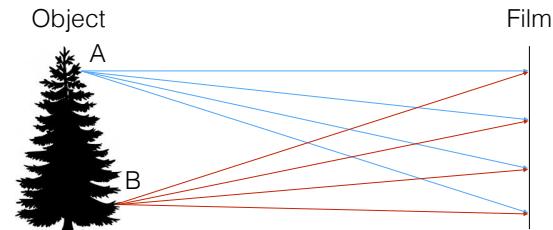
The pinhole projection model

- Qualitative properties

Cameras with lenses

- Depth of focus
- Field of view
- Lens aberrations

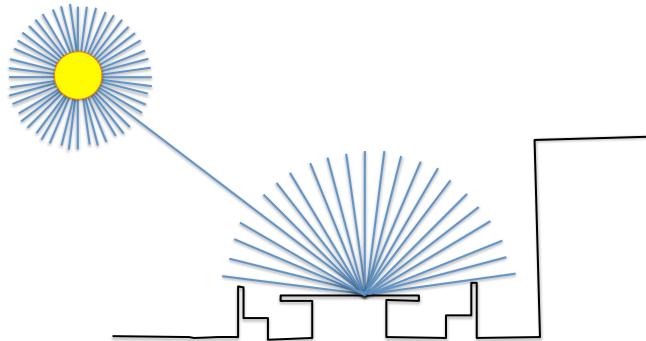
Lets design a camera



Let's put a film in front of an object

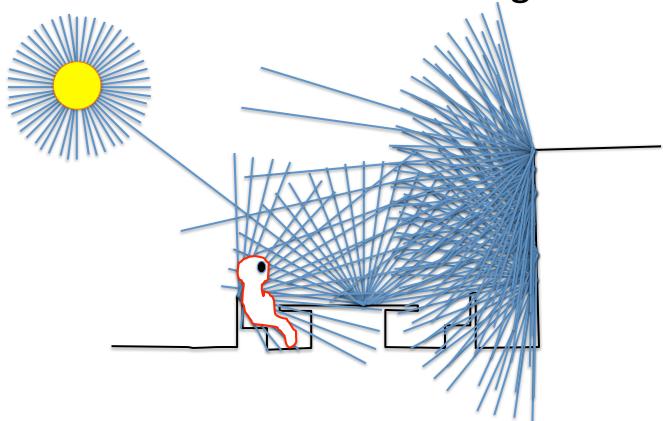
Question: What image do we expect on the film?

The structure of ambient light



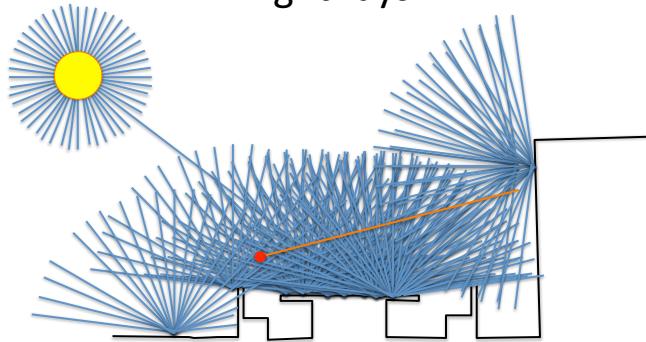
Source: Bill Freeman

The structure of ambient light



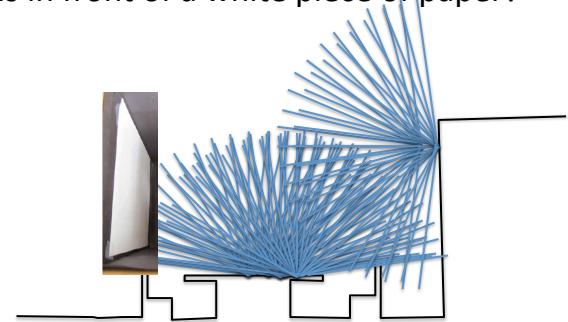
Source: Bill Freeman

All light rays



Source: Bill Freeman

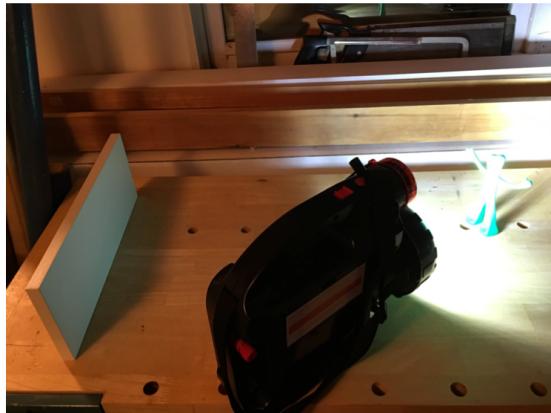
Why don't we generate an image when an object is in front of a white piece of paper?



Why is there no picture appearing on the paper?

Source: Bill Freeman

Let's check, do we get an image?



Source: Bill Freeman

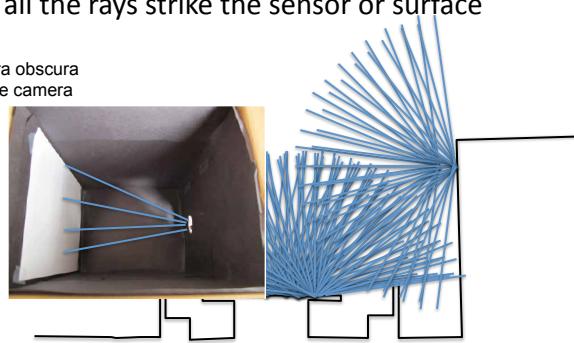
Let's check, do we get an image? No



Source: Bill Freeman

To make an image, we need to have only a subset
of all the rays strike the sensor or surface

The camera obscura
The pinhole camera



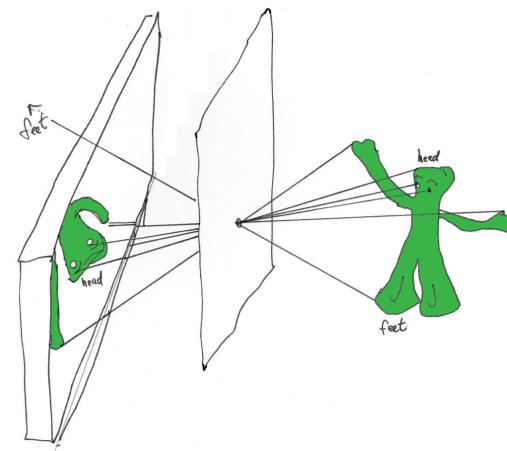
Let's try putting different occluders in between the object
and the sensing plane



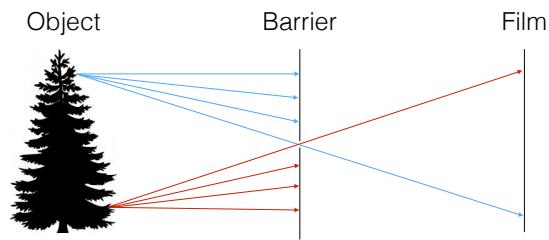
light on wall past pinhole



image is inverted

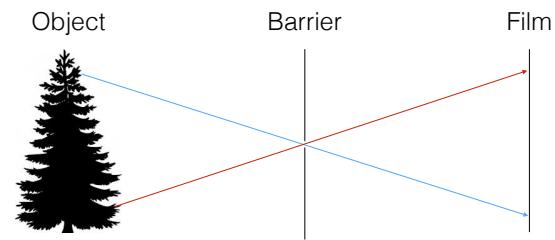


Pinhole camera



Add a barrier to block off most rays

Pinhole camera



- Captures pencil of rays - all rays through a single point: **aperture**, **center of projection**, **focal point**, **camera center**
- The image is formed on the **image plane**

Camera obscura



Gemma Frisius, 1558

Basic principle known to Mozi (470-390 BCE), Aristotle (384-322 BCE)
Drawing aids for artists: described by Leonardo Da Vinci (1452-1519 AD)

"Camera obscura" Latin for "darkened room"

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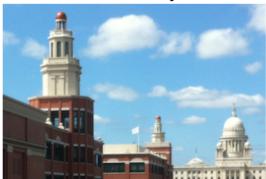
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Accidental pinhole cameras

My hotel room,
contrast enhanced.



The view from my window



Accidental pinholes produce images that are unnoticed or misinterpreted as shadows

A. Torralba and W. Freeman, [Accidental Pinhole and Pinspeck Cameras](#), CVPR 2012

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Pinhole cameras are everywhere



Tree shadow during a solar eclipse

photo credit: Nils van der Burg

<http://www.physicstogo.org/index.cfm>

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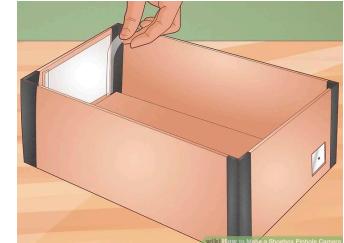
Slide by Steve Seitz

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Home-made pinhole camera



<https://kids.nationalgeographic.com/books/article/pinhole-camera>



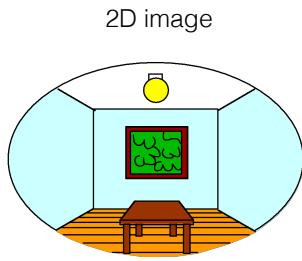
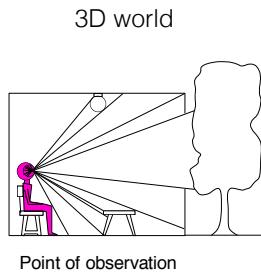
<https://www.wikihow.com/Make-a-Shoebox-Pinhole-Camera>

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Dimensionality reduction: 3D to 2D



Point of observation

- **What is preserved?**
 - Straight lines, incidence
- **What is not preserved?**
 - Angles, lengths

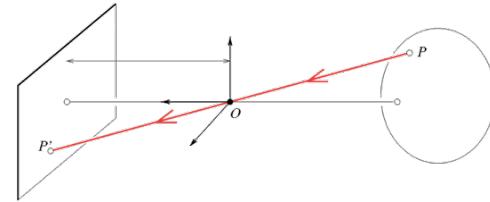
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Slide by A. Efros

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Modeling projection



To compute the projection P' of a scene point P , form a **visual ray** connection P to the camera center O and find where it intersects the image plane

- All scene points that lie on this visual ray have the same projection on the image

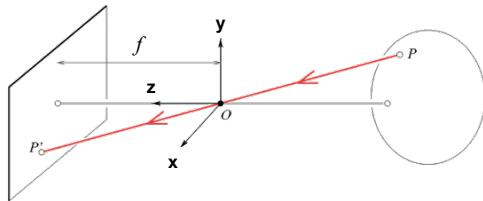
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Modeling projection



The coordinate system

- The optical center (O) is at the origin
- The image plane is parallel to the xy -plane (perpendicular to the z axis)

Projection equations

- Derive using similar triangles

$$(x, y, z) \rightarrow \left(-f \frac{x}{z}, -f \frac{y}{z} \right)$$

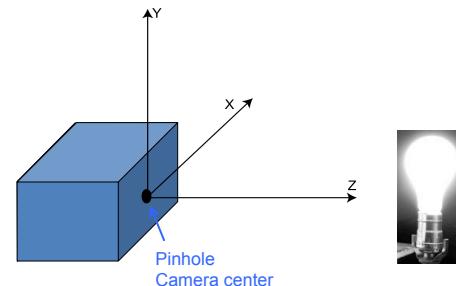
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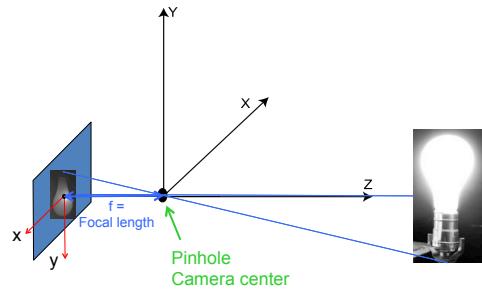
Slide by Steve Seitz

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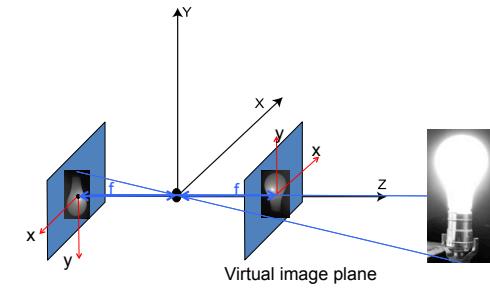
Real and virtual imaging planes



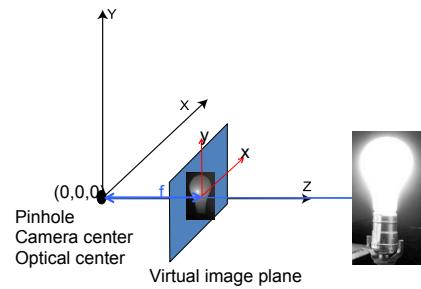
Real and virtual imaging planes



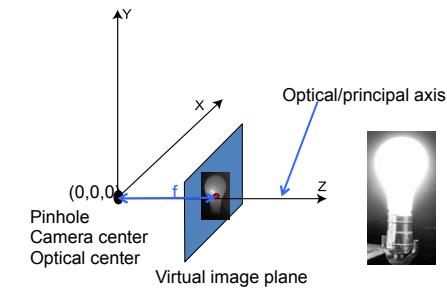
Real and virtual imaging planes



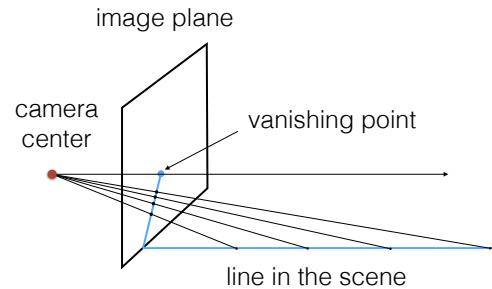
Real and virtual imaging planes



Real and virtual imaging planes



Projection of a line



Question: What if we add another line parallel to the first one?

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Vanishing points

Each direction in space has its own vanishing point

- All lines going in the that direction converge at that point
- **Exception:** directions that are parallel to the image plane



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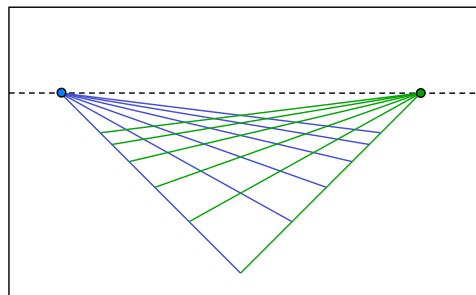
Slide by Steve Seitz

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Vanishing points

Each direction in space has its own vanishing point

- All lines going in the that direction converge at that point
- **Exception:** directions that are parallel to the image plane
- **What happens to the ground plane?**

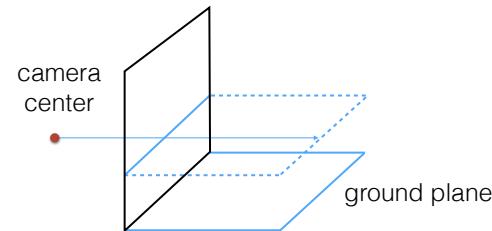


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The horizon



Vanishing line of the ground plane

- All points at the same height of the camera project to the horizon
- Points above the camera project above the horizon
- Provides a way of comparing heights of objects

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The horizon



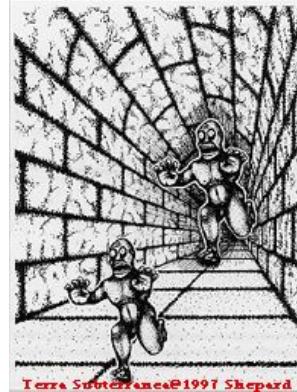
Is the person above or below the viewer?

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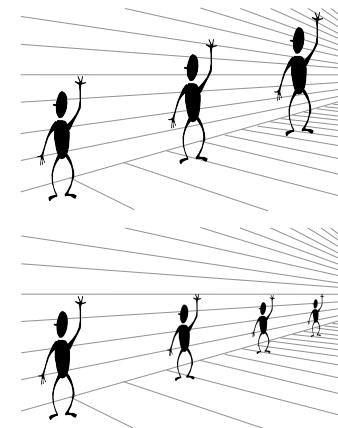
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Perspective cues



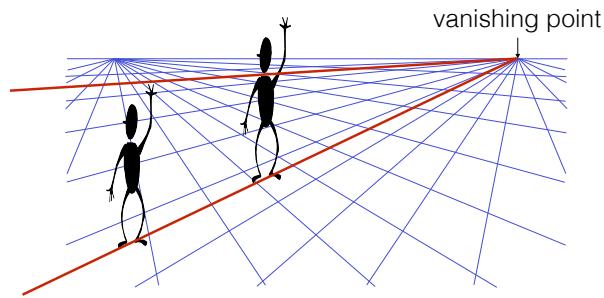
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Comparing heights

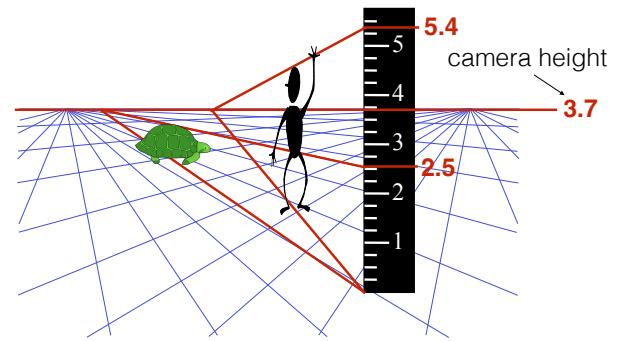


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Measuring heights



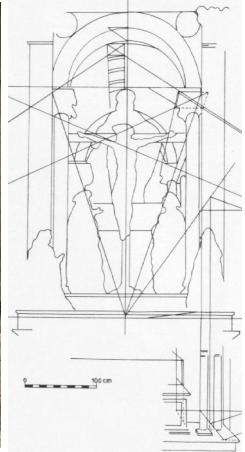
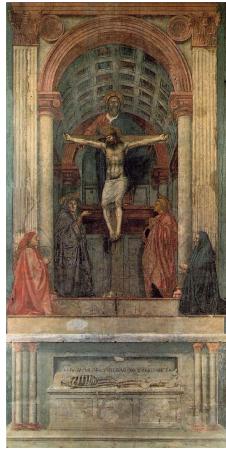
What is the height of the camera?

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Perspective in art



Masaccio,
Trinity, Santa
Maria Novella,
Florence,
1425-28

One of the first
consistent uses
of perspective
in Western art

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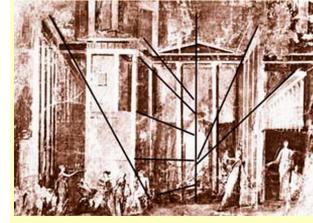
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Perspective in art

(At least partial) Perspective projections in art well before the Renaissance

Several Pompeii wallpaintings show the fragmentary use of linear perspective:



From ottobwiersma.nl

Also some Greek examples,
So apparently pre-renaissance...

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Perspective distortion

What does a sphere project to?



M. H. Pirenne

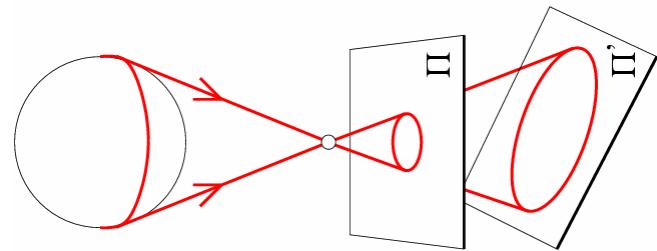
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Slide by Steve Seitz

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Perspective distortion

What does a sphere project to?



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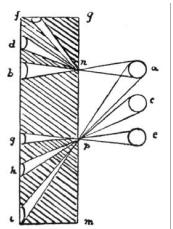
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Perspective distortion

The exterior looks bigger

The distortion is not due to lens flaws

Problem pointed out by Da Vinci



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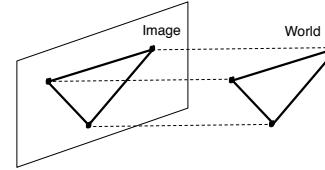
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Orthographic projection

Special case of perspective projection

- Distance of the object from the image plane is infinite
- Also called the “parallel projection”



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Orthographic projection

Special case of perspective projection

- Distance of the object from the image plane is infinite
- Also called the “parallel projection”

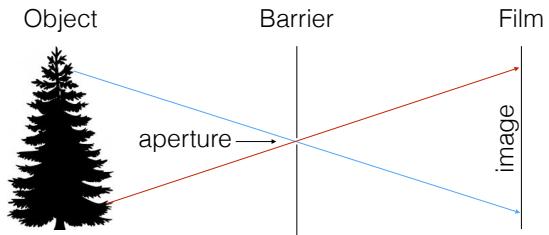


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Pinhole camera

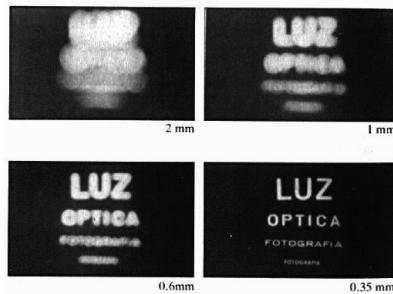


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Shrinking the aperture



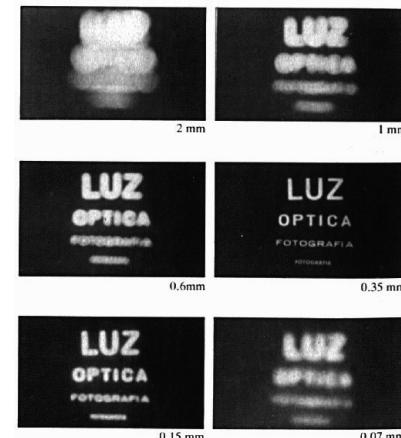
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Shrinking the aperture



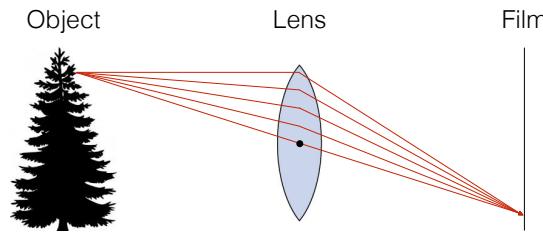
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Adding a lens



A lens focuses light on to the film

Thin lens model:

- Rays passing through the center are not deviated (pinhole projection model still holds)

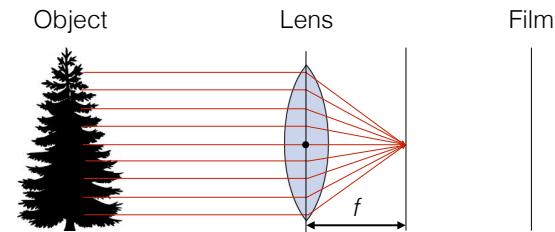
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Slide by F. Durand

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Adding a lens



A lens focuses light on to the film

Thin lens model:

- Rays passing through the center are not deviated (pinhole projection model still holds)
- All parallel rays converge to one point on a plane located at the *focal length* f

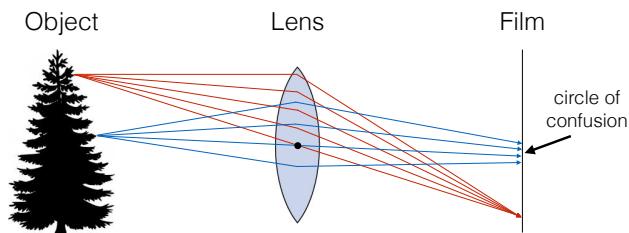
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Adding a lens



A lens focuses light on to the film

- There is a specific distance at which objects are “in focus”
- other points project on to a “circle of confusion” in the image

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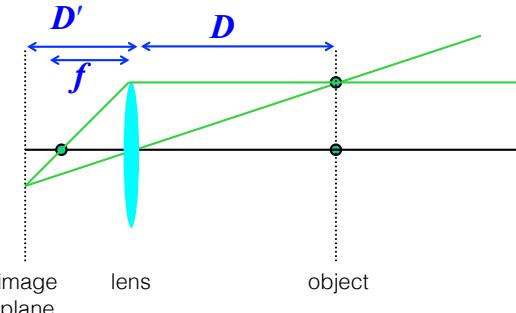
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Thin lens formula

What is the relation between the focal length (f), the distance of the object from the optical center (D) and the distance at which the object will be in focus (D')?



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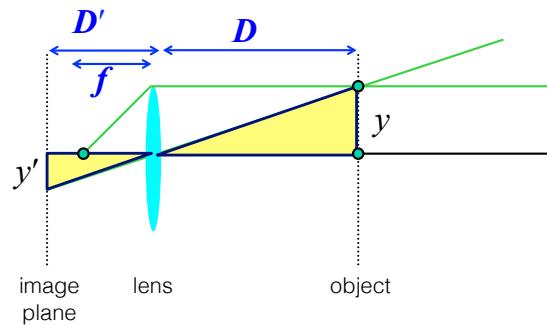
Slide by F. Durand

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Thin lens formula

Similar triangles everywhere!

$$y'/y = D'/D$$



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Slide by F. Durand

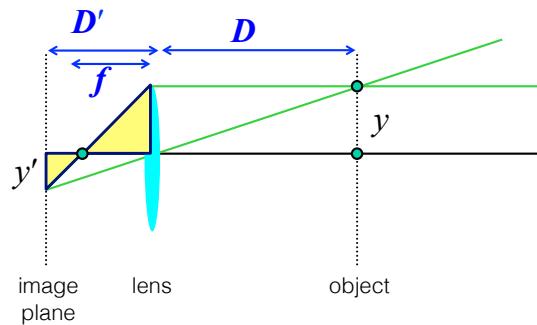
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Thin lens formula

Similar triangles everywhere!

$$y'/y = D'/D$$

$$y'/y = (D' - f)/f$$



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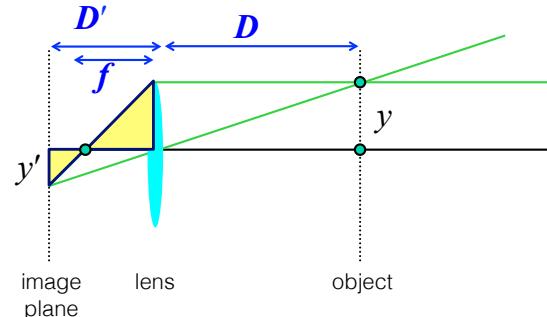
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Thin lens formula

$$\frac{1}{D'} + \frac{1}{D} = \frac{1}{f}$$

Any point satisfying the thin lens equation is in focus



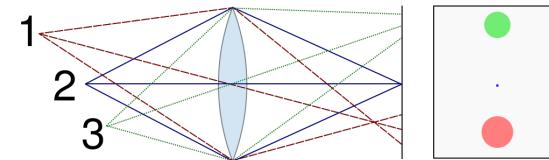
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Depth of field



DOF is the distance between the nearest and farthest objects in a scene that appear acceptably sharp in an image

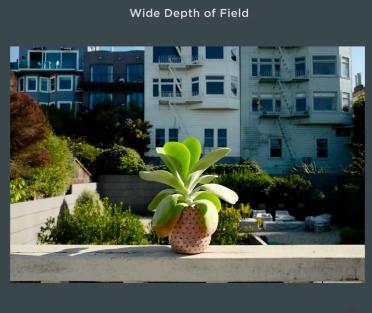
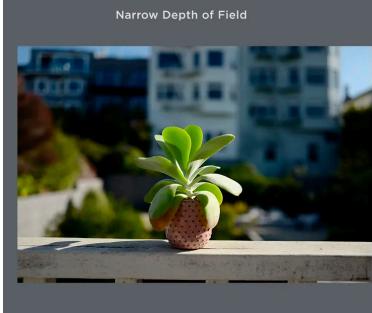
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image credit Wikipedia

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Depth of field

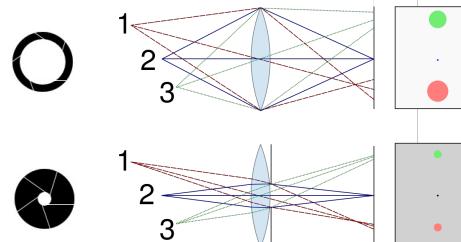


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Controlling depth of field



Changing the aperture size affects the depth of field

- A smaller aperture increases the range in which the object is approximately in focus
- But small aperture reduces the amount of light — need to increase the exposure for contrast
- Pinhole camera has an infinite depth of field

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image credit Wikipedia

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Varying the aperture



Large aperture = small DOF



Small aperture = large DOF

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iPhone portrait mode



<http://www.businessinsider.com/apple-iphone-portrait-mode-explained-2017-10>

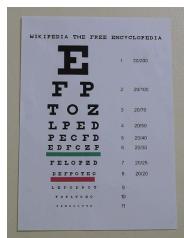
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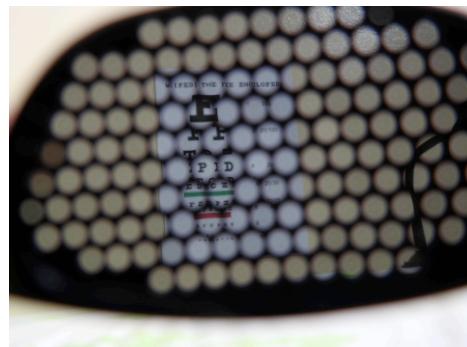
Pinhole glasses



https://en.wikipedia.org/wiki/Pinhole_glasses



Out of focus



In focus, but darker

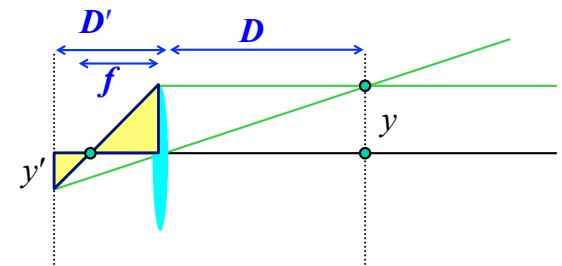
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Controlling depth of field

$$\frac{1}{D'} + \frac{1}{D} = \frac{1}{f}$$



DOF depends on the distance of the object (D) and the focal length (f) of the camera

- Objects far away have a larger depth of field
- Increasing the focal length reduces the depth of field

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image credit Wikipedia

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Fake miniature photography



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"Jodhpur rooftops" by Paul Goyette

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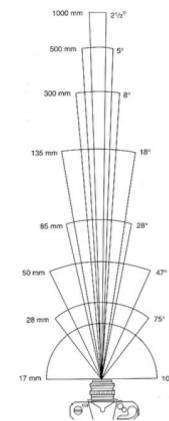


http://www.wallcoo.net/photography/Tilt-shift_Photography_Wallpapers_1920x1080/wallpapers/1600x900/Tallinn_old_town_1920x1080.html



http://www.wallcoo.net/photography/Tilt-shift_Photography_Wallpapers_1920x1080/wallpapers/1366x768/Tilt_Shift_Wallpaper_20_by_leiyagami.html

Field of view



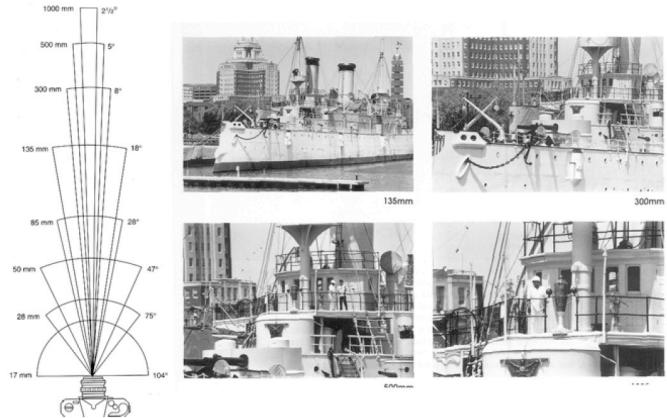
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Field of view



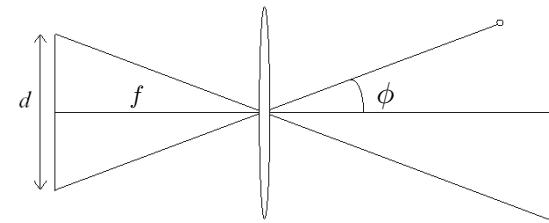
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Field of view



Field of view (FOV) depends on the focal length and the size of the camera retina

$$\phi = \tan^{-1} \left(\frac{d}{2f} \right)$$

Larger focal length = smaller FOV

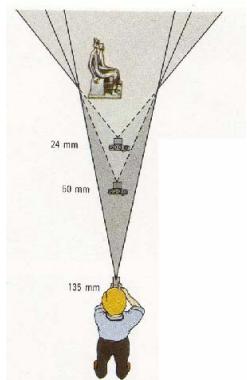
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Slide by A.Efros

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Field of view, focal length



$$\tan(\phi) \times 2f = d$$

$$\sim (\phi) \times 2f = d$$



Large FOV, small f — Camera close to the car



Small FOV, large f — Camera far from the car

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Same effect for faces



wide-angle
(short focus)



standard



telephoto
(long focus)

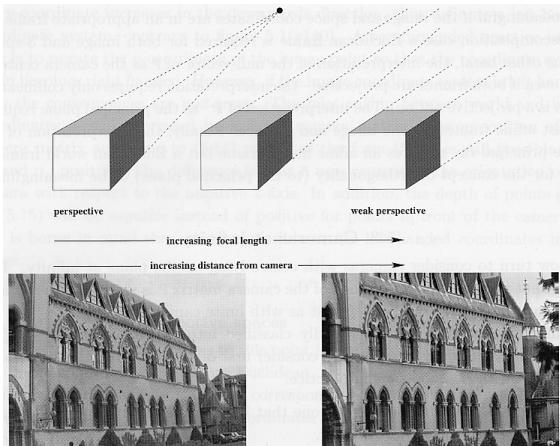
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Approximating an orthographic camera



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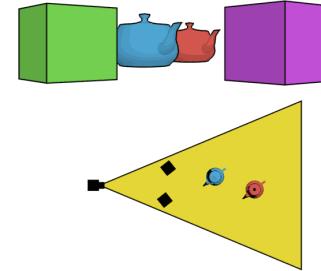
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Source: Hartley & Zisserman

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The dolly zoom

Continuously adjusting the camera focal length while the camera moves away from (or towards) the subject



http://en.wikipedia.org/wiki/Dolly_zoom

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The dolly zoom

Continuously adjusting the camera focal length while the camera moves away from (or towards) the subject

Also called as "Vertigo shot" or the "Hitchcock shot"



[Example of dolly zoom from Goodfellas](#)

[Example of dolly zoom from La Haine](#)

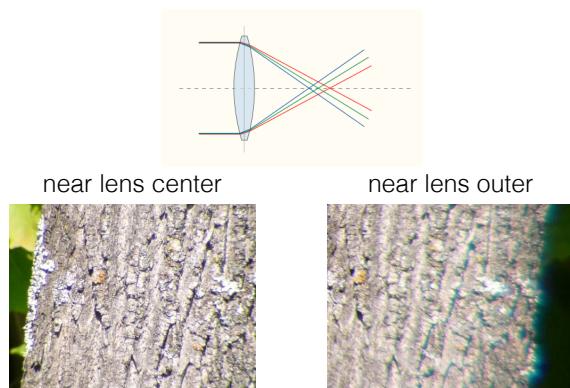
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Lens flaws: Chromatic aberration

Lens have different refractive indices ([Snell's law](#)) for different wavelengths: causes color fringing



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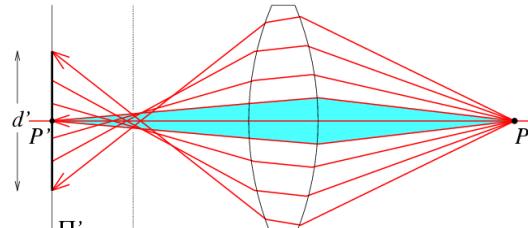
Slide by S.Seitz

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Lens flaws: Spherical aberration

Spherical lenses don't focus light perfectly (thin lens model)

- Rays farther from the optical axis are focussed closer



objects lack sharpness

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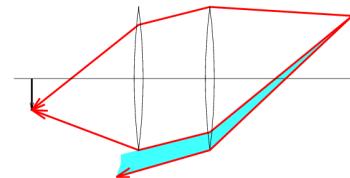
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Lens flaws: Vignetting

Reduction of image brightness in the periphery



Not all rays reach the sensor



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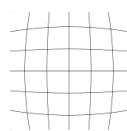
Slide by S.Seitz

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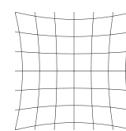
Lens flaws: Radial distortion

Caused by asymmetry of lenses

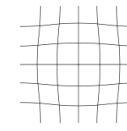
Deviations are most noticeable near the periphery



barrel distortion



pincushion distortion



mustache distortion



<http://clanegezelphotography.blogspot.com/>



<http://parkingandyou.com>

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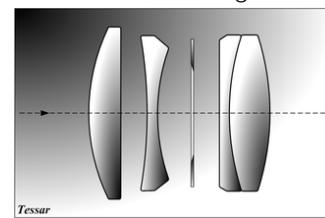
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Real photographic lens

Many uses: cameras, telescopes, microscopes, etc

fixed focal length



Example of a prime lens - Carl Zeiss [Tessar](#)

adjustable zoom



Nikon 28-200 mm zoom lens, extended to 200 mm at left and collapsed to 28 mm focal length at right.

http://en.wikipedia.org/wiki/Zoom_lens

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Lytro camera

Light field camera: capture intensity along each direction of the light

- Traditional cameras integrate light coming from all directions

A captured light field allows you re-render an image post-hoc

- <https://pictures.lytro.com/lytro/collections/41/pictures/1088670>

