Lecture Notes: Image Formation

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1 Overview

This lecture will introduce the fundamentals of image formation. Key topics include the pinhole projection model, lens-based cameras, depth of focus, field of view, and lens aberrations. Understanding how images are formed, how lenses work, and their limitations is essential for image processing and computer vision.

2 Pinhole Camera Model

Light reflects off a surface in all directions, causing repeated scattering. As a result, simply placing a film in front of an object does not produce a clear image. A pinhole camera controls this by allowing only a single path for light from each point in the scene to reach the film. This prevents blurring caused by overlapping light rays, resulting in a sharp image. However, this comes at the cost of reduced light efficiency, as most of the incoming light is blocked—more on this later.

The geometry of a pinhole camera causes light passing through a small aperture to project an inverted image onto a surface. The position of the projected point on the screen depends on the camera's parameters, such as the focal length (f), and the location of the point in the 3D scene. Figure 1 illustrates this concept: a point $P = (x, y, z)$ in the scene is projected to a point $P' = (x', y', z')$ whose coordinates are given by:

$$x' = -f\left(\frac{x}{z}\right) \quad (1)$$

$$y' = -f\left(\frac{y}{z}\right) \quad (2)$$

$$z' = -f \quad (3)$$

2.1 Qualitative Properties

The basic perspective projection formulas lead to several important phenomena:

1. The projection process transforms a 3D scene into a 2D image. It preserves straight lines and incidences but distorts angles and lengths.

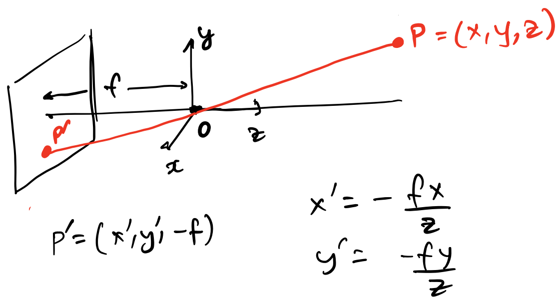

Figure 1: The pinhole camera model and perspective projection equations.

2. The apparent size of an object is inversely proportional to its distance, as seen in the $\frac{1}{z}$ dependence in the projection equations.

3. *Vanishing points* – Lines extending to infinity appear to converge at a single point in the image, known as the vanishing point. All lines parallel in a given direction share the same vanishing point. A useful exercise is to prove this mathematically.

4. *Vanishing lines* – Planes extending infinitely appear to converge to a line. Another way to express this is that all vanishing points of lines lying in a plane form a vanishing line. A special case is the horizon line, which represents the vanishing line of the ground plane.

5. The projection and its associated distortions serve as useful cues for estimating the relative heights and distances of objects.

6. *Orthographic projection* (or parallel projection) is a special case of perspective projection where the object is infinitely far from the image plane. This type of projection is commonly used in games and design visualizations.

3 Lens and Optics

Shrinking the aperture to a very small point reduces the brightness of the image because the amount of light reaching the film is proportional to the aperture's area. One way to compensate for this is by increasing the exposure time. However, this introduces motion blur if either the scene or the camera moves during exposure. This presents a dilemma: reducing the aperture improves sharpness but makes the camera less efficient, while increasing the aperture allows more light but results in a blurry image. The solution to this problem lies in using a lens.

By placing a convex lens at the optical center of the camera, aligned with its axis, we allow multiple paths for light from a single point in the scene to reach the same point on the film. In contrast, a pinhole camera only permits light to travel along a single straight-line path between the source and destination. Mathematically, a thin convex lens has the following properties:

- Light rays passing through the lens center remain undeviated. Thus, the lens center behaves like a pinhole camera.

- *Focal Point and Focal Length*: Parallel rays converge at a specific point known as the *focal point*. The distance between the focal point and the optical center of the lens is called the *focal length*.

3.1 Thin Lens Formula

For a given point in the scene, there exists a specific distance at which the point appears in focus. This relationship is governed by the thin lens equation, which relates the object distance (D), the image distance (D'), and the focal length (f):

$$\frac{1}{D'} + \frac{1}{D} = \frac{1}{f} \tag{4}$$

The formula can be derived by applying similar triangles to the geometric setup shown in Figure 2.

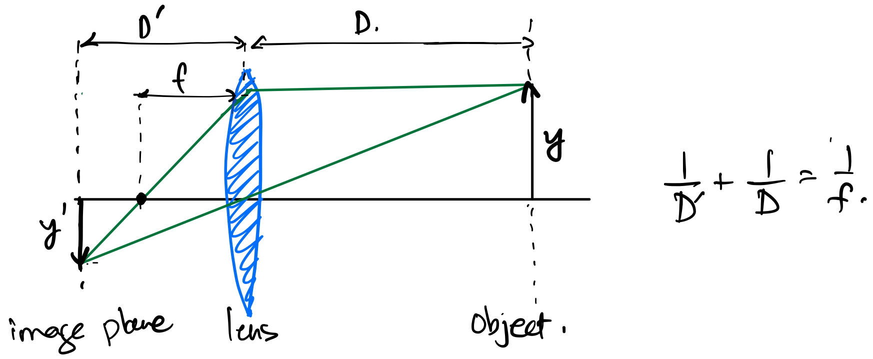

Figure 2: Thin lens geometry.

Points that do not satisfy the thin lens formula project to a circle, also known as the *circle of confusion*. The size of this circle (or blur) depends on how far the point deviates from the ideal depth that satisfies the thin lens formula.

3.2 Lens Phenomenon

The *depth of field* of a camera refers to the range of depth values for which objects appear acceptably sharp in the image. A pinhole camera has an *infinite depth of field*, meaning that objects at any depth appear perfectly sharp. However, a camera with a lens has a *finite depth of field*. The depth of field is inversely related to the *focal length*—cameras with a small focal length (e.g., microscopes) have a narrow depth of field, while those with a large focal length (e.g., telescopes) have a large depth of field.

Independently of the focal length, one can control the depth of field by adjusting the *aperture size*. Reducing the aperture size increases the depth of field, and in the limiting case, it approximates a pinhole camera, which has an *infinite depth of field*. A fun fact is that *pinhole glasses* can be used as an alternative to prescription glasses. They remove the lens of your eye from the equation, allowing for clear vision (on a bright day).

Another important concept is the *field of view*, which is the angular extent of the scene captured by the camera. Cameras with a large focal length have a small field of view. By adjusting the focal length while moving the camera, one can maintain a constant field of view, creating the *Dolly Zoom* effect.

3.3 Lens Flaws

Simple convex lenses do not perfectly obey the thin lens formula and exhibit several optical flaws, including:

- *Chromatic Aberration*: Color fringing occurs due to different refractive indices for different wavelengths of light. For example, blue light bends more than red light.
- *Spherical Aberration*: Blur results from imperfections in the lens shape, causing light rays to focus at different points.
- *Radial Distortion*: Warping effects such as barrel, pincushion, or mustache distortion alter the image geometry.

High-quality camera lenses (e.g., Carl Zeiss Tessar) use a complex arrangement of concave and convex lenses, along with mirrors, to approximate a thin lens with a fixed focal length. Adjustable zoom lenses incorporate even more intricate optical designs. These compound lenses are expensive to manufacture but are essential in applications such as cameras, telescopes, and microscopes.

4 Optional readings

- Modern cameras are highly sophisticated, integrating both optical elements and image processing techniques to capture the best possible image.

- *Light field* cameras capture not only the intensity of light but also its direction, unlike traditional cameras that integrate light from all directions. This enables post-capture adjustments to the *depth of field* and other image properties. The field of *computational photography* explores techniques that combine optics, sensors, and computational methods to enhance imaging capabilities.