

Variational Autoencoders

Learning generative models with latent representations

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Deep Generative Models - SoSe 2020

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Inference through optimization

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- Idea: rephrase inference as optimization

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- f is deterministic, \mathcal{N} enables optimization

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- Characterizes the "distance" between distributions

¹In a mathematical, strict sense, for practical purposes

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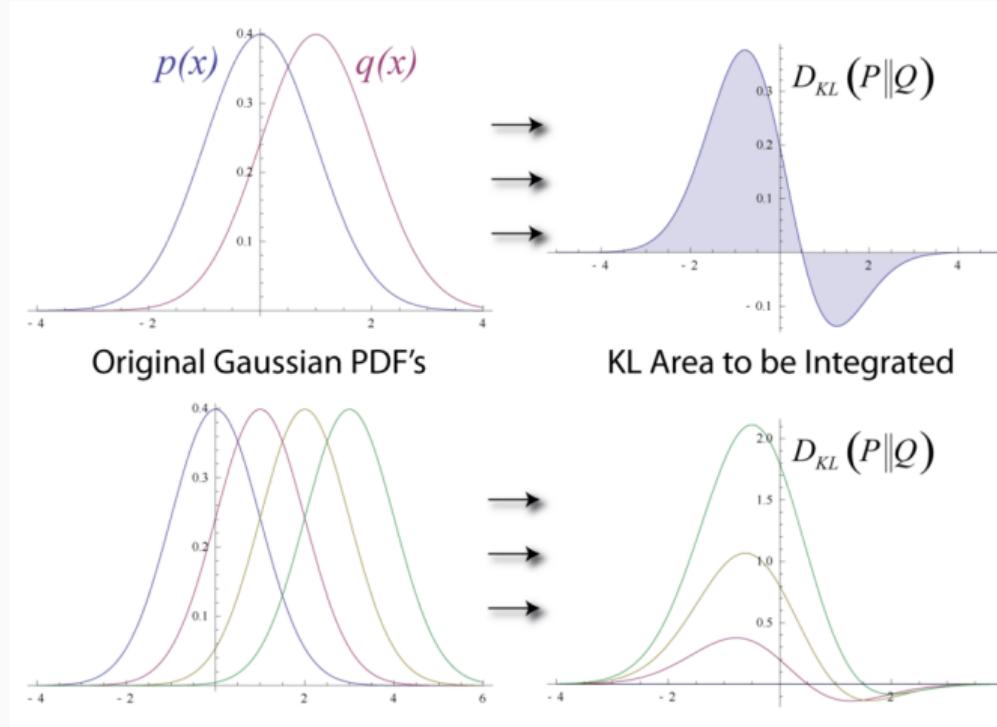


Figure 1: Taken from "Kullback–Leibler divergence", Wikipedia, CC BY-SA 3.0

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- ... so we can choose an entry which depends on x

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- $P(z|x)$ will hopefully be approximated well by $Q(z|x)$. (discussion later)

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- Are we finished now?

Variational Autoencoders

What is an autoencoder?

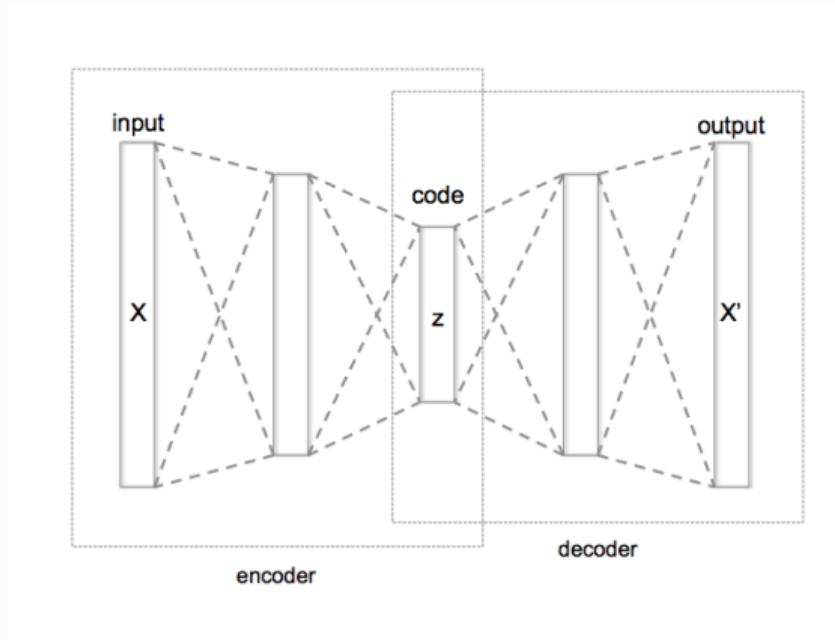


Figure 2: Taken from

https://commons.wikimedia.org/wiki/File:Autoencoder_structure.png, (CC BY-SA 4.0)

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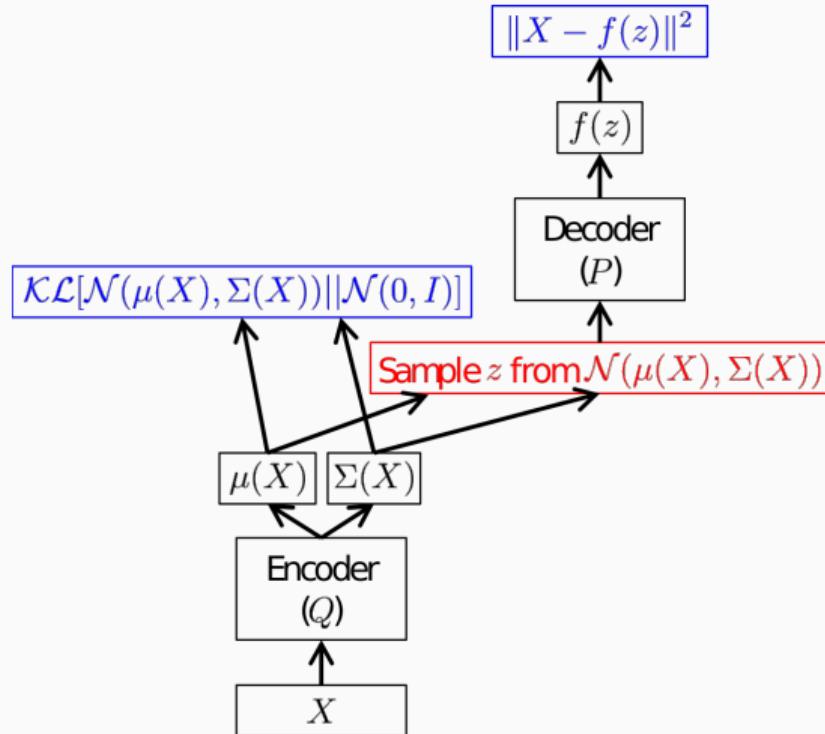
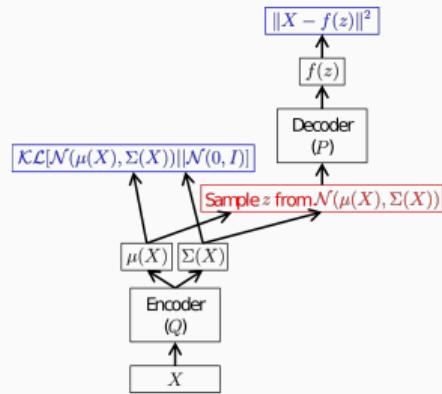
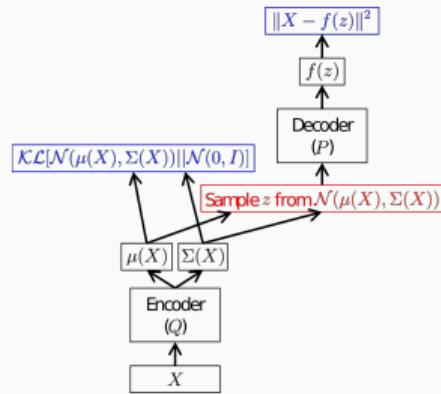


Figure 3: Taken from "Tutorial on Variational Autoencoders", Doersch, 2016

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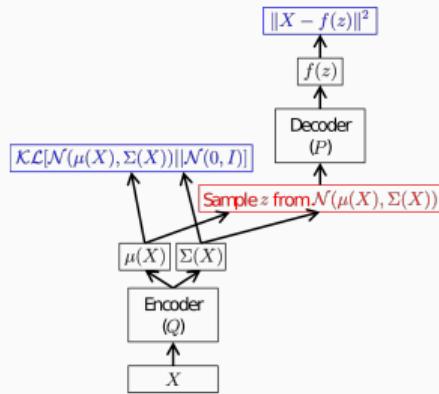


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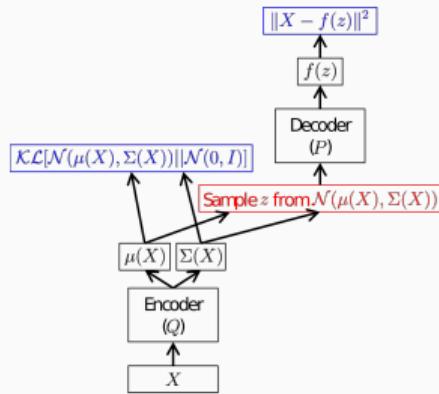
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- You can't propagate through a sampling layer

All parts in detail

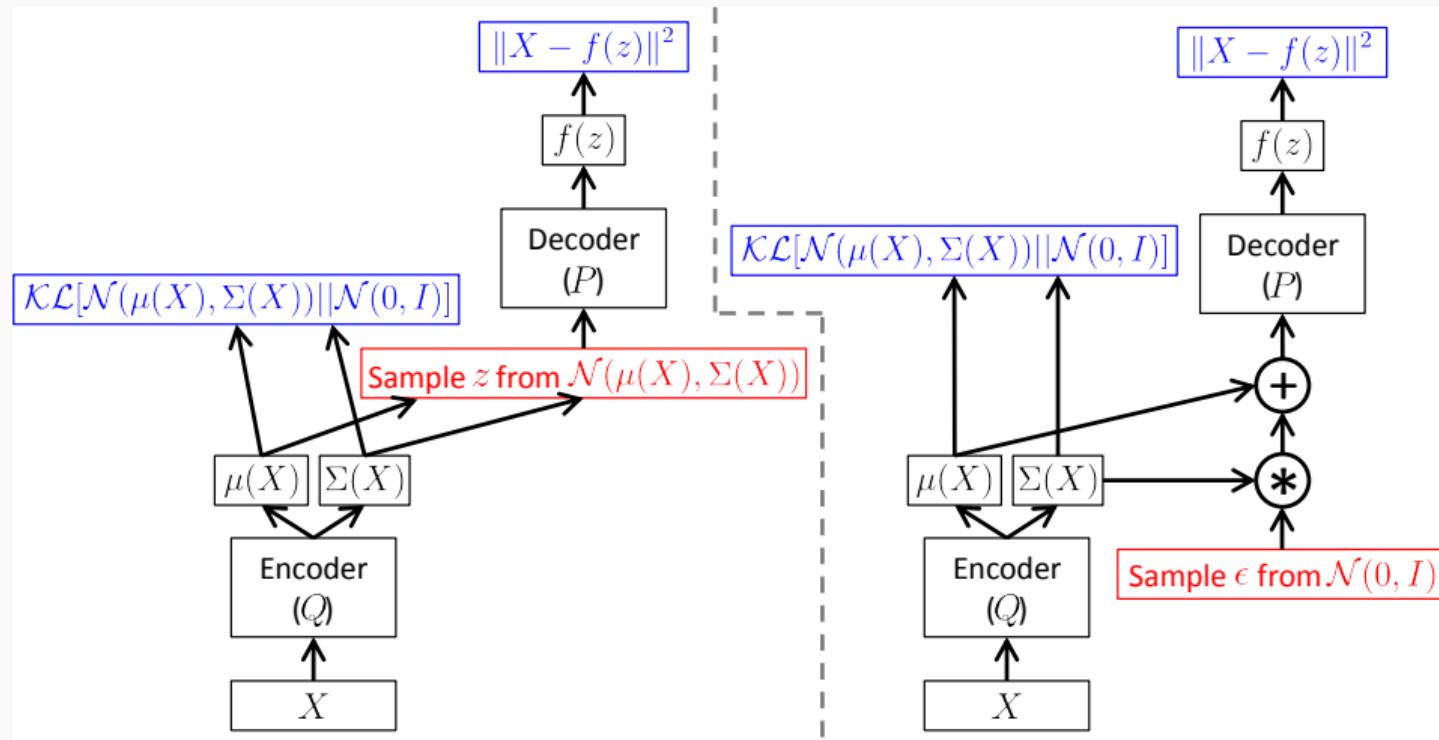


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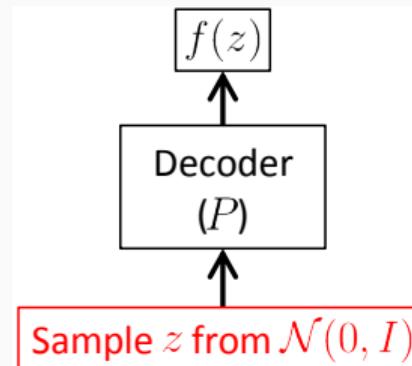


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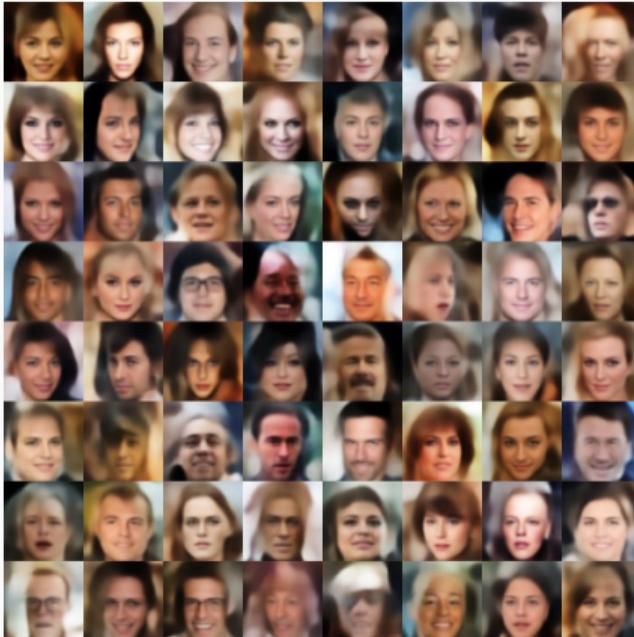
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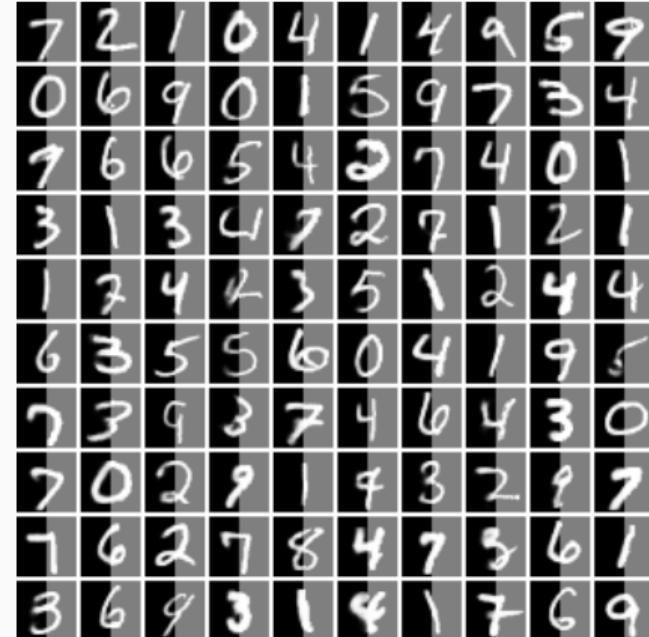
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- There is no mathematical guarantee for choosing a "good" latent
- L2 loss is computationally expensive, also does not capture "visual closeness" well

Applications of VAE/VI

VAE for generating images



(a) Taken from "Tutorial on Variational Autoencoders", Doersch, 2016



(b) Taken from GitHub <https://github.com/yzwxx/vae-celebA>

VAE for translations

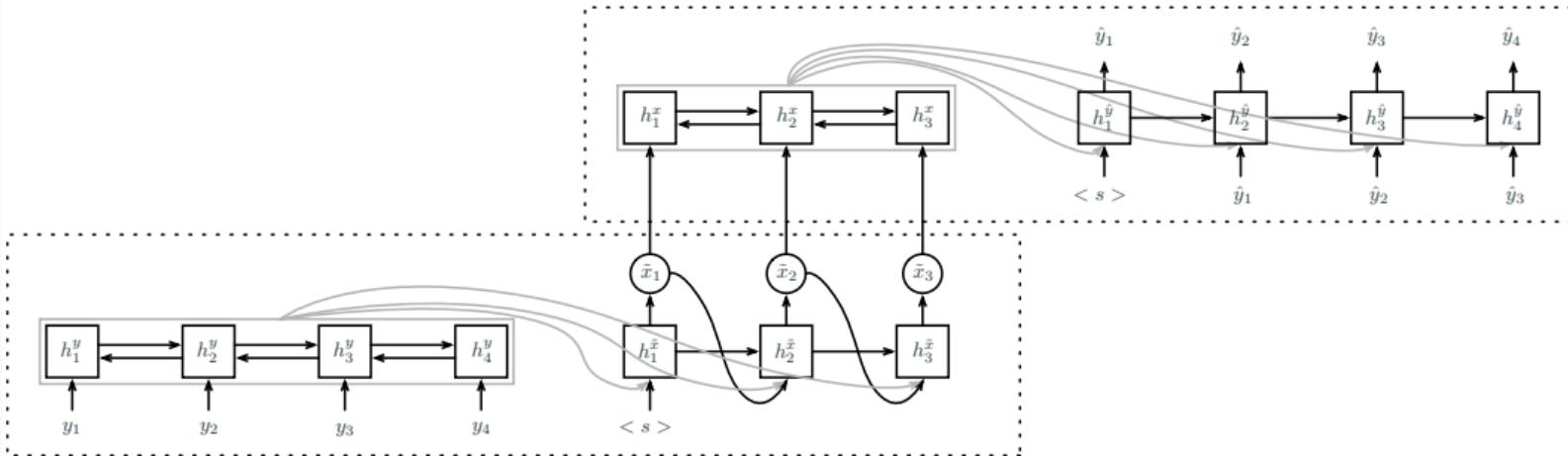


Figure 7: Taken from "Semantic Parsing with Semi-Supervised Sequential Autoencoders", Kočiský et al, 2016

More complex VAE usage

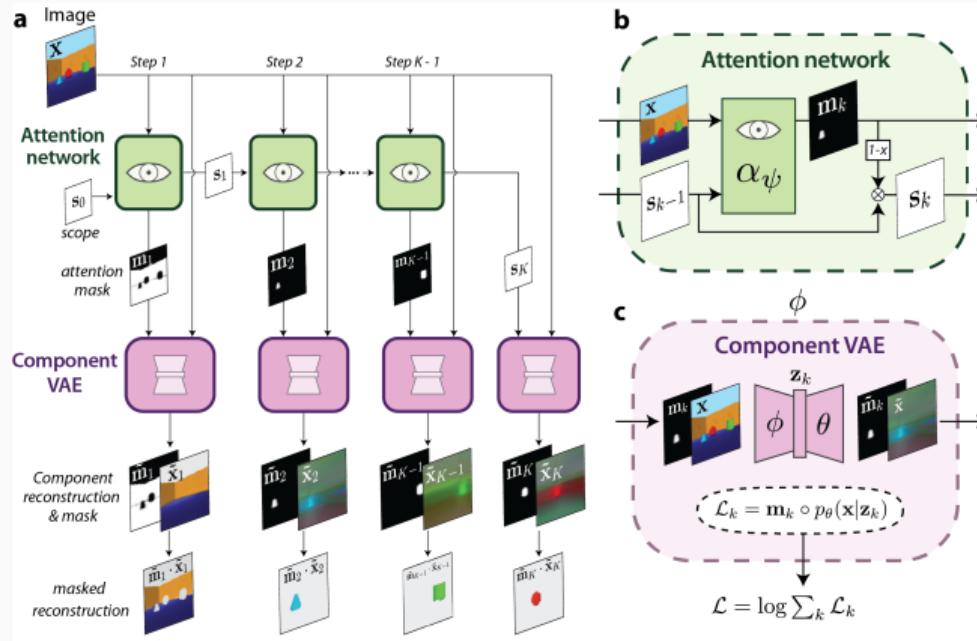


Figure 8: Taken from "MONet: Unsupervised Scene Decomposition and Representation", Burgess et al., 2019

More complex VAE usage

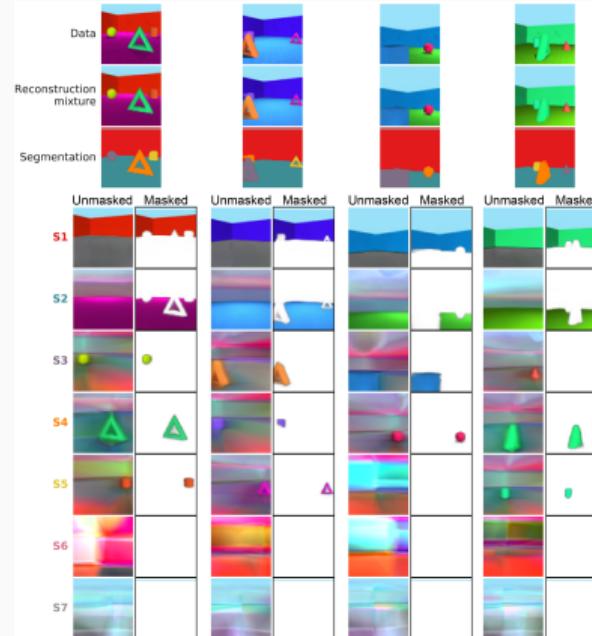
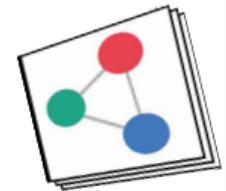


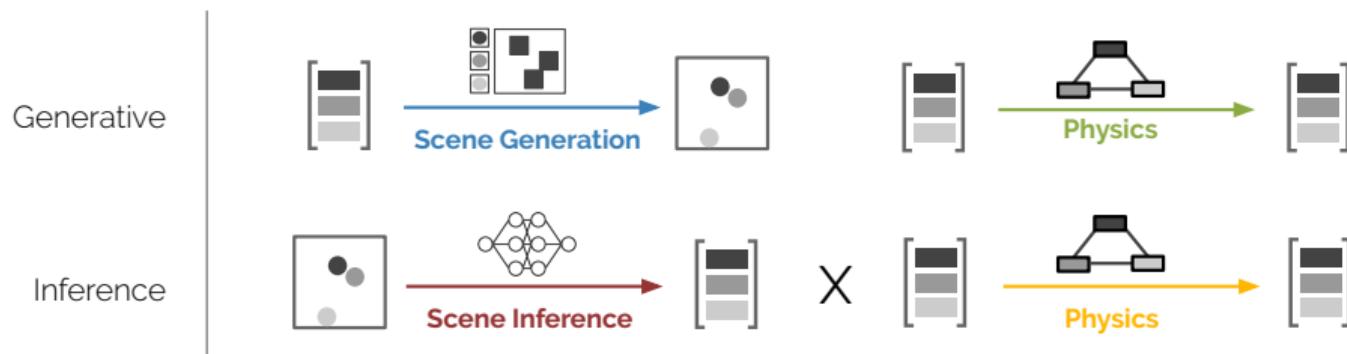
Figure 9: Taken from "MONet: Unsupervised Scene Decomposition and Representation", Burgess et al., 2019

The *STOVE* Model

Maximising the ELBO



$$\log p(x_{1:T}) \geq \mathbb{E}_{q(z_{1:T} | x_{1:T})} \left[\log \frac{p(x_{1:T}, z_{1:T})}{q(z_{1:T} | x_{1:T})} \right] \propto \mathbb{E}_{q(z_{1:T} | x_{1:T})} \left[\sum_{t=1}^T \log \left\{ \frac{p(x_t | z_t)}{q(z_t | x_t)} \frac{p(z_t | z_{t-1})}{q(z_t | z_{t-1})} \right\} \right]$$



Further reading

- "Stochastic Backpropagation and Approximate Inference in Deep Generative Models" by Danilo Rezende et al., 2014
- "Auto-Encoding Variational Bayes" by Durk Kingma and Max Welling, 2014
- "Tutorial on Variational Autoencoders" by Carl Doersch, 2016
- "Variational Inference: A Review for Statisticians" by David Blei et al., 2018
- "Improving Variational Inference with Inverse Autoregressive Flow" by Durk Kingma, 2016
- "Attend, Infer, Repeat: Fast Scene Understanding with Generative Models" by Ali Eslami et al., 2016
- "The Dreaming Variational Autoencoder for Reinforcement Learning Environments" by Per-Arne Andersen, 2018

Questions?

Any remaining questions?