Let $A \in \mathbb{R}^{n \times n}$ be matrix for which $A^\top = -A$ (*antisymmetric*). Show that the diagonal entries are zero, i.e., $a_{ii} = 0$ for all $1 \leq i \leq n$.

Solution:

From $A^\top = -A$ it follows that

$$a_{ii} = -a_{ii}$$

for all $1 \leq i \leq n$. This equation is only true for the number 0.