1 Steepest Descent Method

Consider the following setting:

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 10 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad x_0 = \begin{bmatrix} 4 \\ 1.4 \end{bmatrix}.$$

i) Show that A is spd and determine $\rho(M_\theta)$ and $\hat{\theta}$ from the previous exercise. What is the solution $x^* = A^{-1}b$ here?

ii) Recycle your code from Sheet 8 (see `iter_solve()`) and run the Richardson iteration for the above setting; once with the optimal $\hat{\theta}$ and once with a θ which is almost too large to gaurantee convergence. Extent your code to the *method of steepest descent* (**??**) by choosing the stepsize θ_k from (**??**) in each iteration step and apply it to the above setting. Plot the iterates x_k for all the three runs into the same 2d plot (use three different colors).
Also, plot the function values $f(x_k) = \frac{1}{2}x_k^T A x_k - b^T x_k$ for each iterate of the three runs into a second plot (use three different colors).

Solution: