

Let $A \in \mathbb{R}^{n \times n}$ be a positive definite matrix. Show that the diagonal entries are strictly positive, i.e., $a_{ii} > 0$ for all $1 \leq i \leq n$.

Solution:

Consider the unit vectors $e_i = (0, \dots, 1, \dots, 0)^\top \in \mathbb{R}^n$. Then

$$0 < e_i^\top A e_i = a_{ii}.$$