1 Triangle Inequality

Let $x, y \in \mathbb{R}$. Please show the triangle inequality and the reverse triangle inequality.

1. $|x+y| \leq |x| + |y|$
2. $|x-y| \geq ||x| - |y||$

Solution:

1. First we note that as

$$|x| = \begin{cases} x : x \geq 0 \\ -x : x < 0 \end{cases}$$

$x \leq |x|$.
Case 1 ($x = -y$): $\surd$
Case 2 ($x > -y$):

$$|x+y| = x + y \leq |x| + |y|.$$

Case 3 ($x < -y \Leftrightarrow -x > -(-y)$):

$$|x+y| = |-x-y| = -x - y \leq |-x| + |-y| = |x| + |y|.$$

2.

$$\begin{aligned} |x| = |x - y + y| &\leq |x-y| + |y| \\ \Leftrightarrow |x| - |y| &\leq |x-y| \end{aligned}$$

By interchanging x and y we hence obtain the assertion.