Compute the eigenvalues of the following matrices.

1.
$$A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$$

2.
$$B = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 1 & 3 \\ 2 & 0 & 1 \end{pmatrix}$$

3.
$$C = \begin{pmatrix} \pi & 0 & 0 & 0 & 0 \\ 1 & -7 & 0 & 0 & 0 \\ 2 & 0 & i & 0 & 0 \\ 3 & 0 & 8 & 0 & 0 \\ 4 & 7 & 9 & 0 & \frac{1}{2} \end{pmatrix}$$

Solution:

1.
$$A - \lambda I = \begin{pmatrix} \sqrt{2} - \lambda & \pi \\ 0 & 1 - \lambda \end{pmatrix} \quad (1P) \quad \Rightarrow \quad 0 = \det(D - \lambda I) = (\sqrt{2} - \lambda)(1 - \lambda) \quad (1P)$$
$$\Rightarrow \quad \sigma(A) = \{\sqrt{2}, 1\} \quad (2P)$$

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Alternatively: Argue via triangular matrix (2P)

2.
$$B - \lambda I = \begin{pmatrix} -\lambda & 0 & 0 \\ 1 & (1-\lambda) & 3 \\ 2 & 0 & (1-\lambda) \end{pmatrix} \begin{matrix} -\lambda & 0 \\ 1 & (1-\lambda) \\ 2 & 0 \end{matrix} \quad (1P)$$

$$\Rightarrow \quad 0 = \det(E - \lambda I) = -\lambda(1-\lambda)^2 \quad (1P)$$
$$\Rightarrow \quad \sigma(B) = \{0, 1\} \quad (2P)$$