1 Linear Iterations: Example

Let $A \in \mathbb{R}^{n \times n}$ be a matrix with spectral radius $\rho(A) < 1$.

1. What is the limit of the iteration
$$x^{k+1} = Ax^k + b$$
for $k \to \infty$?

2. Implement a numerical experiment with a random matrix $A \in \mathbb{R}^{2\times 2}$ which satisfies $\rho(A) < 1$ and some $b \in \mathbb{R}^2$ to check your answer in (i) and draw the iterates $x^k \in \mathbb{R}^2$ into a plot.
Hint: For constructing such a random matrix A recall that $\sigma(\alpha A) = \alpha\sigma(A)$ for any $\alpha \in \mathbb{R}$, i.e., the eigenvalues of the scaled matrix are the scaled eigenvalues of A.

Solution:

```
import numpy as np
import matplotlib.pyplot as plt


# Generate a random 2x2 matrix with \rho(A)<1
A = np.random.rand(2,2)
norm_A = np.abs(np.sort(np.linalg.eig(A)[0])[-1]) # spectral radius of A
A = A/(norm_A +1)
b = np.array([0.1,0.1])

# Iteration
n = 50
X = np.zeros((n,2))
X[0] = np.random.rand(2)
for k in range(n-1):
    X[k+1] = (A@X[k]).T + b

# Test
print("(I-A)x =",(np.eye(2)-A).dot(X[-1]),"\nb =", b)
# Plot iterates into 2d Plot
plt.plot(X[:,0], X[:,1], "ro-")
plt.axhline(0)
plt.axvline(0)
plt.show()
```