Computation Rules for Matrices II

Let $A \in \mathbb{R}^{m\times r}$ and $B \in \mathbb{R}^{r\times m}$. Then

i) $(A^\top)^\top = A$,

ii) $(AB)^\top = B^\top A^\top$,

iii) $(A+B)^\top = A^\top + B^\top$,

iv) for $A \in GL(n,\mathbb{R})$ we have $(A^\top)^{-1} = (A^{-1})^\top$.

Solution:

$$C := A \cdot B, C := [c_{ij}]_{ij}, c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}, A^T = [a'_{ij}]_{ij}, B^T = [b'_{ij}]_{ij},$$

$$[C]_{ij} = C_{ij} = \sum_{k=1}^{n} a_{jk} b_{ki} = \sum_{k=1}^{n} a'_{kj} b'_{ik} = \sum_{k=1}^{n} b'_{ik} a'_{kj} = [B^T A^T]_{ji} \Rightarrow C^T = B^T A^T$$

$$A \in GL(n,\mathbb{F}), A^{-1}A = I_n \quad \Rightarrow \quad \underbrace{(A^{-1}A)^T}_{A^T(A^{-1})^T} = I_n^T = I_n$$

$\Rightarrow (A^{-1})^T = (A^T)^{-1}$, because Inverse is unique