Compute eigenvalues

Compute the eigenvalues of the following matrices.

1. $$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

2. $$B = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & -9 \\ 0 & 1 & 6 \end{bmatrix}$$

3. $$C = \begin{bmatrix} \pi & 3 & i^2 & 6 \\ 0 & 4 & 1 & e^3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{bmatrix}$$

Solution:

1. By applying the determinant rule for 2×2 matrices we find
$$0 = \det(A - \lambda I) = \lambda^2 + 1 \quad \Leftrightarrow \quad \lambda \in \{-i, +i\}.$$

2. By applying the Sarrus rule for 3×3 matrices we find
$$0 \overset{!}{=} \det(A - \lambda I) = \det \begin{pmatrix} -\lambda & 0 & 0 \\ 1 & -\lambda & -9 \\ 0 & 1 & 6-\lambda \end{pmatrix} \begin{matrix} -\lambda & 0 \\ 1 & -\lambda \\ 0 & 1 \end{matrix} \overset{\text{[Sarrus]}}{=} \lambda^2(6-\lambda) - 9\lambda$$
$$\Leftrightarrow \quad 0 = \lambda(\lambda(6-\lambda) - 9) = \lambda(-\lambda^2 + 6\lambda - 9) = -\lambda(\lambda-3)^2$$
$$\Leftrightarrow \quad \lambda = 0 \text{ or } \lambda = 3 \ (\sigma(A) = \{0, 3\})$$

3. By applying the determinant rule for triangular matrices we find
$$\det(A - \lambda I) = (\pi - \lambda)(4 - \lambda)(1 - \lambda)\left(\frac{1}{2} - \lambda\right)$$
$$\Rightarrow \quad \sigma(A) = \{\pi, 4, 1, \frac{1}{2}\}$$

(Short: A triangular $(\Rightarrow \ \sigma(A) = \ \text{diag}(A))$)