Let $b \in \mathbb{R}^m$ and $A \in \mathbb{R}^{m\times n}$. Assume you are given the least squares problem

$$\min_{x\in\mathbb{R}^n} \|Ax-b\|_2^2.$$

1. Which equation does a solution $\hat{x}$ of the above least squares problem solve? Give formula and name of the equation.
2. Assume you are given the following data

z	-3	-1	0	1	3
y	-3	-1,5	0	2,5	4

.

   Solve the curve fitting problem

$$\min_{c_0,c_1\in\mathbb{R}} \sum_{i=1}^{5} (c_0 + c_1 z_i - y_i)^2,$$

   i.e., determine the minimizing parameters c_0 and c_1.

Solution:

1. (1+1P): $\hat{x}$ solves the normal equation: $A^T A\hat{x} = A^T y$
2. In this case:

$$(2P): \quad A = \begin{pmatrix} 1 & -3 \\ 1 & -1 \\ 1 & 0 \\ 1 & 1 \\ 1 & 3 \end{pmatrix},$$

$$(2P): \ A^TA = \begin{pmatrix} 5 & 0 \\ 0 & 20 \end{pmatrix},$$

$$(2P): \ A^Ty = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ -3 & -1 & 0 & 1 & 3 \end{pmatrix} \begin{pmatrix} -3 \\ -1,5 \\ 0 \\ 2,5 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ 25 \end{pmatrix}$$

   Normal equation:

$$\begin{pmatrix} 5 & 0 \\ 0 & 20 \end{pmatrix} \begin{pmatrix} c_0 \\ c_1 \end{pmatrix} = \begin{pmatrix} 2 \\ 25 \end{pmatrix} \quad \Leftrightarrow \quad c_0 = \frac{2}{5} = 0,4, \ c_1 = \frac{25}{20} = 1,25$$

$$\Rightarrow \quad (1+1P): \quad \hat{x} = \begin{pmatrix} 0,40 \\ 1,25 \end{pmatrix}, \quad c_0 = 0.40, \quad c_1 = 1.25$$