Let $A \in \mathbb{F}^{n \times n}$ be invertible and λ some eigenvalue of A.

1. Please show that $\lambda \neq 0$.
2. Please show that $\frac{1}{\lambda}$ is an Eigenvalue of A^{-1}.

Solution:

Let (λ, v) be eigenvalue-eigenvector-pair of A.
$\Leftrightarrow \quad Av = \lambda v \quad \Leftrightarrow \quad BAv = \lambda Bv \quad \Leftrightarrow \quad BA\underbrace{B^{-1}B}_{=I}v = \lambda Bv$

$\Leftrightarrow \quad (\lambda, Bv)$ is eigenvalue-eigenvector-pair of B.