1 Properties of A^TA and AA^T

Let $A \in \mathbb{R}^{m \times n}$ be any matrix. Please show:

1. number example
2. Name two sufficient conditions for the invertibility of A^TA and AA^T.

Solution:

1. For example:
   i) Let A have independent columns ("full column rank").
   This is equivalent to $\underbrace{\ker(A)}_{=\ker(A^TA)} = \{0\}$, thus also the columns of A^TA are independent, which implies that A^TA is invertible.
   ii) Let A^TA be positive definite.
   Then its eigenvalues are strictly positive. Since A^TA is symmetric we can use its eigendecomposition to conclude that A^TA is invertible.