

Copper, Silver and Gold Alloys

Let us assume we are given the alloys M_1 , M_2 and M_3 , which are built of copper, silver and gold with the following percental proportions:

	M_1	M_2	M_3
Copper	20	70	50
Silver	60	10	50
Gold	20	20	0

Is it possible to mix these alloys to form a new alloy which consists of 40% copper, 50% silver and 10% gold.

Hint: Cast the problem into a linear system $Ax = b$.

Solution:

We want to find $M_4 = x_1 M_1 + x_2 M_2 + x_3 M_3$, so that $M_4 = \begin{pmatrix} 40 \\ 50 \\ 10 \end{pmatrix}$:

$$\begin{aligned}
 & \left(\begin{array}{ccc|c} 20 & 70 & 50 & 40 \\ 60 & 10 & 50 & 50 \\ 20 & 20 & 0 & 10 \end{array} \right) \begin{matrix} \text{(I)} \\ \text{(II)} \\ \text{(III)} \end{matrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \\
 & \xrightarrow{\text{(II)' = (II) - 3(I)}} \left(\begin{array}{ccc|c} 20 & 70 & 50 & 40 \\ 3 & -200 & -100 & -70 \\ 20 & 20 & 0 & 10 \end{array} \right) \begin{matrix} \text{(I)} \\ \text{(II)'} \\ \text{(III)} \end{matrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \\
 & \xrightarrow{\text{(III)' = (III) - (I)}} \left(\begin{array}{ccc|c} 20 & 70 & 50 & 40 \\ 3 & -200 & -100 & -70 \\ 1 & -50 & -50 & -30 \end{array} \right) \begin{matrix} \text{(I)} \\ \text{(II)'} \\ \text{(III)'} \end{matrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \\
 & \xrightarrow{\text{(III)'' = (III)' - \frac{50}{200} (II)'}} \left(\begin{array}{ccc|c} 20 & 70 & 50 & 40 \\ 3 & -200 & -100 & -70 \\ 1 & \frac{1}{4} & -25 & -12,5 \end{array} \right) \begin{matrix} \text{(I)} \\ \text{(II)'} \\ \text{(III)''} \end{matrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \\
 & \text{(III)''} \Rightarrow -25x_3 = -12,5 \Rightarrow x_3 = \frac{1}{2} \\
 & \text{(II)'} \Rightarrow -200x_2 - 100\left(\frac{1}{2}\right) = -70 \Rightarrow x_2 = -\frac{1}{200}(-70 + 50) = 0,1 \\
 & \text{(I)} \Rightarrow 20x_1 + 70 \cdot 0,1 + 50\frac{1}{2} = 40 \Rightarrow x_1 = \frac{1}{20}8 = 0,4
 \end{aligned}$$

Answer: Yes!, $x = \begin{pmatrix} 0,4 \\ 0,1 \\ 0,5 \end{pmatrix}$