Show by induction that for any $n \in \mathbb{N}$ it holds that

$$\sum_{k=1}^{n} k(k+1) = \frac{1}{3}n(n+1)(n+2).$$

Solution:

Show: $\sum_{k=1}^{n} k(k+1) = \frac{1}{3}n(n+1)(n+2)$
Proof:
Induction Basis $(n=1)$

$$1(1+1) = 2 = \frac{1}{3}1(1+1)(1+2) \ \checkmark \quad (2P)$$

Induction Step $(n \mapsto n+1)$

$$\begin{aligned}
\sum_{k=1}^{n+1} k(k+1) &= (n+1)(n+1+1) + \underbrace{\sum_{k=1}^{n} k(k+1)}_{\overset{[\text{I.A.}](2P)}{=} \frac{1}{3}n(n+1)(n+2)} \\
&= (n+1)(n+2)\underbrace{\left(1+\frac{1}{3}n\right)}_{=\frac{1}{3}(3+n)} \\
&= \frac{1}{3}(n+1)((n+1)+1)((n+1)+2) \ \checkmark \quad (4P)
\end{aligned}$$