1 Interpolation

Let $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) be points in the real plain \mathbb{R}^2 . If $x_i \neq x_j$ for $i \neq j$ there is exactly one parabola $f \colon \mathbb{R} \to \mathbb{R}$ (i.e., there are uniquely determined coefficients $a, b, c \in \mathbb{R}$) which satisfies

$$f(x_i) = ax_i^2 + bx_i + c = y_i$$
 for all $i = 1, 2, 3$. (1)

- 1. Please implement a function interpolate(data) which takes the points (x_i, y_i) for i = 1, 2, 3 as arguments and returns the coefficients a, b, c, for which (1) holds.
- 2. Implement a function parabola(x, coeff) which evaluates a parabola depending on an argument x and some coefficients a,b,c. Then randomly generate some data points (x_i,y_i) for i=1,2,3, determine the corresponding coefficients a,b,c with interpolate(data) and create a plot which contains the pairs (x_i,y_i) for i=1,2,3 and the associated fitted function f from (1) with the determined coefficients.
- 3. Assume the measuring points x_1, x_2 and x_3 are fixed and we want to obtain many interpolations for different values of y_1, y_2, y_3 . What could be changed in your implementation to possibly make it more efficient?

Hint: Interpret task (i) as linear system of equations and solve it with linalg.solve.

Solution: