

Show by induction that for any $n \in \mathbb{N}$ it holds that

$$\sum_{k=1}^n k = \frac{n(n+1)}{2}.$$

Solution:

Show: $\sum_{k=1}^n k = \frac{n(n+1)}{2}$

Proof:

Induction Basis ($n = 1$)

$$1 = 2 = \frac{1(1+1)}{2} \quad \checkmark \quad (2P)$$

Induction Step ($n \mapsto n+1$)

$$\sum_{k=1}^{n+1} k = (n+1) + \sum_{k=1}^n k \stackrel{[I.A.](2P)}{=} (n+1) + \frac{n(n+1)}{2} = (n+1) + \left(\frac{n}{2} + 1\right) = \frac{(n+1)(n+2)}{2} \quad \checkmark \quad (4P)$$