Computation Rules for Matrices and Vectors
Below you find a collection of computation rules that are helpful when dealing with matrices and vectors. Feel free to prove them based on the definitions given in the lecture.

Compatibility properties of summing and scaling matrices
Let $A, B \in \mathbb{F}^{m\times n}$ and $r, s \in \mathbb{F}$. Then

$$\begin{array}{ll} i) & (r\cdot s)\cdot A = r\cdot(s\cdot A) \\ ii) & (r+s)\cdot A = r\cdot A + s\cdot A \\ & r\cdot(A+B) = r\cdot A + r\cdot B \\ iii) & 1\cdot A = A \end{array}$$

From which we can derive:

$$\begin{array}{ll} i) & 0\cdot A = 0 \\ ii) & r\cdot 0 = 0 \\ iii) & r\cdot A = 0 \quad \Rightarrow r = 0 \ \vee \ A = 0 \\ iv) & (-1)\cdot A = -A \end{array}$$

Compatibility properties of matrix sum and product
Let $A, \tilde{A} \in \mathbb{F}^{m\times n}$, $B, \tilde{B} \in \mathbb{F}^{n\times l}$, $C \in \mathbb{F}^{l\times t}$, $r \in \mathbb{F}$. Then

$$\begin{array}{rl} i) & (A\cdot B)\cdot C = A\cdot(B\cdot C) \\ ii) & (A+\tilde{A})B = AB + \tilde{A}B \\ iii) & A(B+\tilde{B}) = AB + A\tilde{B} \\ iv) & I_m A = A I_n = A \\ v) & (r\cdot A)\cdot B = r(A\cdot B) = A(r\cdot B) \\ vi) & 0A = A0 = 0 \end{array}$$

Group property of invertible matrices
For two invertible matrices $A, B \in \mathbb{F}^{n\times n}$ we find

$$\begin{array}{rl} i) & (AB)^{-1} = B^{-1}A^{-1} \\ ii) & \left(A^{-1}\right)^{-1} = A \end{array}$$

Transpose matrices
Let $A \in \mathbb{R}^{m\times r}$ and $B \in \mathbb{R}^{r\times m}$. Then

i) $(A^\top)^\top = A$,

ii) $(AB)^\top = B^\top A^\top$,

iii) $(A+B)^\top = A^\top + B^\top$,

iv) for $A \in GL(n,\mathbb{R})$ we have $(A^\top)^{-1} = (A^{-1})^\top$.

Solution: