What is the determinant of

$$A = \begin{pmatrix} 0 & -e^{-i\pi} & i^2 & 0 & -\pi \\ e^{-i\pi} & 0 & -2 & 0 & 0 \\ 1 & 2 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & -3 \\ \pi & 0 & 0 & 3 & 0 \end{pmatrix}?$$

**Solution:** 

$$A = \begin{pmatrix} 0 & -e^{-i\pi} & i^2 & 0 & -\pi \\ e^{-i\pi} & 0 & -2 & 0 & 0 \\ 1 & 2 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & -3 \\ \pi & 0 & 0 & 3 & 0 \end{pmatrix}$$

$$\text{since } i^2 = -1, \ A^T = -A \ \ (1P)$$

$$\Rightarrow \ \det(A) = \det(A^T) = \det(-A) = (-1)^5 \det(A) \ \ \ (1P)$$

$$\Rightarrow \ \det(A) = 0 \ \ \ \ \ (1P)$$