

1 Height of a tree

Let $a, b, c \in \mathbb{R}^2$ be vectors as illustrated in the figure below. We know from the lecture that the angle α between two vectors a, c is defined by

$$\cos(\alpha) = \frac{a^\top c}{\|a\|_2 \|c\|_2}.$$

Use this equality to compute the height $\|b\|_2$ of the tree in the figure below. The angle $\alpha := 36.87^\circ$ and the distance to the tree $a := (4, 0)^\top$ are given.

Solution:

$$a = (4, 0), b = (0, h), c = (4, h)$$

$$\cos(\alpha) = \frac{a^\top c}{\|a\| \|c\|}$$

$$\Rightarrow \|c\| = \frac{4}{\cos(\alpha)}$$

$$\Rightarrow \|c\| = 5$$

$$\Rightarrow 25 = \|c\|^2 = \|a\|^2 + \|b\|^2 = 16 + h^2$$

$$\Rightarrow h = 3$$