1 Spectral Radius Iterations

Let $A \in \mathbb{R}^{n \times n}$ be a matrix with spectral radius $\rho(A) < 1$.

1. What is the limit of the iteration

$$x^{k+1} = Ax^k + b$$

for $k \to \infty$?

2. Implement a numerical experiment with a random matrix $A \in \mathbb{R}^{2 \times 2}$ which satisfies $\rho(A) < 1$ and some $b \in \mathbb{R}^2$ to check your answer in (i) and draw the iterates $x^k \in \mathbb{R}^2$ into a plot.

Hint: For constructing such a random matrix A recall that $\sigma(\alpha A) = \alpha \sigma(A)$ for any $\alpha \in \mathbb{R}$, i.e., the eigenvalues of the scaled matrix are the scaled eigenvalues of A.

Solution: