Show by induction that for any $n \in \mathbb{N}$ it holds that

$$\sum_{k=0}^{n} (2k+1) = (n+1)^2.$$

Solution:

Show: $\sum_{k=0}^{n}(2k+1) = (n+1)^2$
Proof:
Induction Basis $(n = 0)$

$$2 \cdot 0 + 1 = 1 = (0+1)^2 \ \checkmark \quad (2P)$$

Induction Step $(n \mapsto n+1)$

$$\sum_{k=0}^{n+1}(2k+1) = 2(n+1)+1+\sum_{k=0}^{n}(2k+1) \overset{\text{[I.A.]}(2P)}{=} 2(n+1)+1+(n+1)^2$$
$$= 2n+2+1+n^2+2n+1 = ((n+1)+1)^2 \ \checkmark \quad (4P)$$