Compute the eigenvalues of the matrices or roots of the polynomial.

1. $$A = \begin{pmatrix} \pi & 3 & i^2 & 6 \\ 0 & 4 & 1 & e^3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

2. $$B = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & -9 \\ 0 & 1 & 6 \end{pmatrix}$$

3. $$p(x) = x^3 - 6x^2 + 9x$$

4. $$D = \begin{pmatrix} \sqrt{2} & \pi \\ 0 & 1 \end{pmatrix}$$

5. $$E = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 1 & 3 \\ 2 & 0 & 1 \end{pmatrix}$$

6. $$F = \begin{pmatrix} 1,5 & 0,5 \\ 0,5 & 1,5 \end{pmatrix}.$$

Solution:

1. $$A - \lambda I = \begin{pmatrix} -\lambda & 0 \\ 0 & -\lambda \end{pmatrix} \quad (1P) \quad \Rightarrow \quad 0 = \det(A - \lambda I) = \lambda^2 - \frac{1}{4} \quad (1P)$$
$$\Rightarrow \quad \sigma(A) = \{\pm\frac{1}{2}\} \quad (1P)$$

2. $$0 \overset{!}{=} \det(B - \lambda I) = \det \begin{pmatrix} -\lambda & 0 & 0 \\ 1 & -\lambda & -9 \\ 0 & 1 & 6-\lambda \end{pmatrix} \begin{matrix} -\lambda & 0 \\ 1 & -\lambda \\ 0 & 1 \end{matrix} \quad (1P) \quad \overset{\text{[Sarrus]}}{=} \lambda^2(6-\lambda) - 9\lambda$$
$$\Leftrightarrow \quad 0 = \lambda(\lambda(6-\lambda) - 9) = \lambda(-\lambda^2 + 6\lambda - 9) = -\lambda(\lambda - 3)^2 \quad (1P)$$
$$\Leftrightarrow \quad \lambda = 0 \text{ or } \lambda = 3 \ (\sigma(B) = \{0, 3\}) \quad (1P)$$