1 Result for Injective Funtions

Let $f : X \to Y$ be an injective function and $A, B \subset X$. Please show that

$$f(A) \cap f(B) = f(A \cap B).$$

Hint: Split up the equality "$=$" into the parts "$\subset$" and "$\supset$". One direction is straightforward the other one requires, that f is injective.

Solution:

- $\underline{f(A \cap B) \subset f(A) \cap f(B):}$
  We have
  $$f(A \cap B) = f(A \cap B) \cap f(B \cap A) \subset f(A) \cap f(B).$$
- $\underline{f(A) \cap f(B) \subset f(A \cap B):}$
  Let $y \in f(A) \cap f(B)$,
  $\Rightarrow \quad \exists \; x_a \in A, x_b \in B : \; f(x_a) = y = f(x_b).$

  Since f is injective, we find $x_a = x_b$
  $\Rightarrow \quad x_a, x_b \in A \cap B$
  $\Rightarrow \quad y \in f(A \cap B).$