

### Compute Determinants

Compute the determinants of the following matrices and check whether they are invertible.

$$A = \begin{pmatrix} 1 & \pi & 2 & 12 \\ 0 & \frac{1}{5} & \frac{1}{\sqrt{2}} & 17 \\ 0 & 0 & 5 & \frac{1}{3} \\ 0 & 0 & 0 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 5 & 3 \\ 1 & -2 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & \frac{1}{2} & 2 \\ -\frac{1}{2} & 0 & 7 \\ -2 & -7 & 0 \end{pmatrix}.$$

### Solution:

1.  $\det(A) \stackrel{[A \text{ is upper triangular}]}{=} 1 \cdot \frac{1}{5} \cdot 5 \cdot 4 = 4$

2.  $\det(B) = 5 \cdot (-2) - 3 \cdot 1 = -13$

3.

$$\begin{aligned} \det(C) = \det\left(\begin{pmatrix} 0 & \frac{1}{2} & 2 \\ -\frac{1}{2} & 0 & 7 \\ -2 & -7 & 0 \end{pmatrix}\right) &\stackrel{[\text{Sarrus' Rule}]}{=} (0 \cdot 0 \cdot 0) + \left(\frac{1}{2} \cdot 7 \cdot (-2)\right) + \left(2 \cdot \left(-\frac{1}{2}\right) \cdot (-7)\right) \\ &\quad - (-2 \cdot 0 \cdot 2) - (-7 \cdot 7 \cdot 0) - \left(0 \cdot \left(-\frac{1}{2}\right) \cdot \frac{1}{2}\right) = 0 \end{aligned}$$