1 Symmetric Difference

Let X be a set and $A, B \subset X$ be two subsets of X. The symmetric difference of A and B is then defined by

$$A \Delta B := (A \cap B^c) \cup (B \cap A^c).$$

Please show that

$$A \Delta B = (A \cup B) \cap (A \cap B)^c.$$

Hint: Use the De Morgan's rule and exploit the following distributive laws:

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \quad \text{and} \quad A \cup (B \cap C) = (A \cup B) \cap (A \cup C).$$

Solution:

$$\begin{aligned}
&(A \cap B^c) \cup (B \cap A^c) \\
=&((A \cap B^c) \cup B) \cap ((A \cap B^c) \cup A^c) \\
=&\underbrace{((A \cup B) \cap (B^c \cup B))}_{A \cup B} \cap \underbrace{((A \cup A^c) \cap (B^c \cup A^c))}_{B^c \cup A^c} \\
=&(A \cup B) \cap (B^c \cup A^c) \\
=&(A \cup B) \cap (A \cap B)^c
\end{aligned}$$