Let $b \in \mathbb{R}^m$ and $A \in \mathbb{R}^{m \times n}$. Assume you are given the least squares problem

$$\min_{x \in \mathbb{R}^n} \|Ax - b\|_2^2.$$

1. Which equation does a solution $\hat{x}$ of the above least squares problem solve? Give formula and name of the equation.
2. Assume you are given the following data

z	-2	-1	0	1	2
y	3,5	2,5	1	0,5	-2,5

.

   Solve the curve fitting problem

$$\min_{c_0, c_1 \in \mathbb{R}} \sum_{i=1}^{5} (c_0 + c_1 z_i - y_i)^2,$$

   i.e., determine the minimizing parameters c_0 and c_1.

Solution:

1. $\hat{x}$ solves the normal $(1P)$ equation: $A^T A \hat{x} \overset{(1P)}{=} A^T y$
2. In this case: $A = \begin{pmatrix} 1 & -3 \\ 1 & -1 \\ 1 & 0 \\ 1 & 1 \\ 1 & 3 \end{pmatrix} \quad (2P),$

$$A^T A = \begin{pmatrix} 5 & 0 \\ 0 & 20 \end{pmatrix}, \quad (2P)$$

$$A^T y = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ -3 & -1 & 0 & 1 & 3 \end{pmatrix} \begin{pmatrix} 3,5 \\ 2,5 \\ 1 \\ 0,5 \\ -2,5 \end{pmatrix} = \begin{pmatrix} 5 \\ -20 \end{pmatrix} \quad (2P)$$

   Normal equation:

$$\begin{pmatrix} 5 & 0 \\ 0 & 20 \end{pmatrix} \begin{pmatrix} c_0 \\ c_1 \end{pmatrix} = \begin{pmatrix} 5 \\ -20 \end{pmatrix} \quad \Leftrightarrow \quad c_0 = 1, \ c_1 = -1 \quad (1 + 1P)$$

   $\Rightarrow \quad \hat{x} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$