1 Linear Modelling

Let us assume a car c_1 starts driving at time $t = 0h$ and runs with $50\frac{km}{h}$. A second car c_2 runs with $90\frac{km}{h}$ but starts 2 hours later at time $t = 2h$ at the same position as car c_1 did.

1. Describe the positions $c_1(t)$ and $c_2(t)$ of the cars as affine linear function of time, i.e., $c_i(t) = m_i t + b_i$ for some appropriate $m_i, b_i \in \mathbb{R}$, $i = 1, 2$.
2. At which time t do the cars meet?

Solution:

We know, that car 1 starts at $t = 0$ with $50\frac{\text{km}}{\text{h}}$ and car 2 starts at $t = 2$ with $90\frac{\text{km}}{\text{h}}$. Now we want to model the position of car i at time t with the function

$$c_i(t) := m_i t + b_i \quad [km].$$

1. We already know $m_1 = 50, m_2 = 90$ and

$$\begin{aligned} &1)\ 0 = c_1(0) = m_1 \cdot 0 + b_1 \quad \Rightarrow \quad b_1 = 0, \\ &2)\ 0 = c_2(2) = \underbrace{m_2}_{=90} \cdot 2 + b_2 \quad \Rightarrow \quad b_2 = -180. \end{aligned}$$

2. We want to find a $\hat{t}$, where the car meet:

$$\begin{aligned} & c_1(\hat{t}) = c_2(\hat{t}) \\ \Leftrightarrow & 50 \cdot \hat{t} = 90 \cdot \hat{t} - 180 \\ \Leftrightarrow & \hat{t} = \frac{180}{40} = 4.5 \end{aligned}$$

So we conclude, that the cars will meet after 4.5 hours or 270 minutes.