1 Binomial Coefficient

The binomial coefficient is defined by

$$\binom{n}{k} = \frac{n!}{k!(n-k)!},$$

where $n! = n \cdot (n-1) \cdot (n-2) \cdot \cdots \cdot 1$. Pascal's triangle states the equation

$$\binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}$$

for any $0 \leq k < n$. Please prove this equation.

Solution:

$$\binom{n}{k} + \binom{n}{k+1} = \frac{n!}{k!(n-k)!} + \frac{n!}{(k+1)!(n-(k+1))!} = \frac{n!(k+1) + n!(n-k)}{(k+1)!(n-k)!} =$$
$$\frac{n!k + n! + n!\,n - n!\,k}{(k+1)!(n-k)!} = \frac{(n+1)!}{(k+1)!(n+1-1-k)!} = \binom{n+1}{k+1}$$