Linear Dependence

Give an example where a nontrivial combination of three nonzero vectors a_i in $\mathbb{R}^4$ is the zero vector (nontrivial means that not all scaling coefficients are zero). Write your example in the form $Ax = 0$.

Solution:

Take for example

$$A := [a_1, a_2, a_3] := \begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \quad \text{and} \quad x := \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}.$$

Construction recipe: We take two random vectors x, y and form a simple linear combination $x + y$. We know these vectors are linearly dependent, so that we can take those as columns for the example matrix, i.e.,

$$x + y - (x + y) = 0 \quad \text{gives} \quad [x, y, (x + y)] \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = 0.$$