1 Property of a skew-symmetric Matrix

A matrix $C \in \mathbb{R}^{n \times n}$ is called *skew-symmetric* if

$$C^\top = -C.$$

Please show that for a real-valued skew-symmetric matrix $C \in \mathbb{R}^{n \times n}$ it holds

$$x^\top C x = 0$$

for all $x \in \mathbb{R}^n$ (i.e., a vector x is always mapped to a perpendicular vector Cx).

Solution:

Let $C \in \mathbb{R}^{n \times n}$, be a skew-symmetric matrix, i.e. $C^T = -C$.
Now show, that $x^T C x = 0$ holds.

<u>Proof:</u> $x \in \mathbb{R}^n$,

$$x^T C x = (x^T C x)^T = x^T C^T x = -x^T C x$$
$$\Rightarrow \; x^T C x = 0 \;\; (r \in \mathbb{R}, r = -r \Rightarrow r = 0)$$