

Holonomic Robot Wheel Mixing Functions

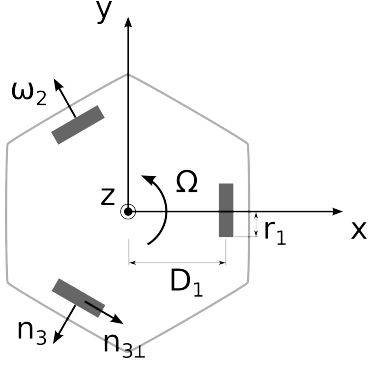
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1 Definitions



ω_i wheel i rotational speed [rad/s]

D_i distance from wheel to center [m]

r_i wheel radius [m]

\vec{n}_i axis unit vector of the wheel i

$\vec{n}_{i\perp}$ unit vector perpendicular to \vec{n}_i ($+\pi/2$ from \vec{n}_i)

\vec{v} robot velocity [m/s]

Ω robot angular velocity [rad/s]

$$\vec{\omega}_i = \omega_i \vec{n}_i \quad (1)$$

$$\vec{D}_i = D_i \vec{n}_i \quad (2)$$

2 Robot velocity to wheel rotational speed conversion

The velocity at the contact point of the wheel is:

$$\vec{v}_i = \vec{v} + \vec{\Omega} \times \vec{D}_i \quad (3)$$

The rotational speed of the wheel is obtained by projecting the velocity at the contact point of the wheel on $\vec{n}_{i\perp}$:

$$\omega_i = -\frac{\vec{v}_i \cdot \vec{n}_{i\perp}}{r_i} \quad (4)$$

TODO: drawing

Writing these equations with the components of the vectors:

$$\vec{n}_i = \begin{bmatrix} \cos\beta_i \\ \sin\beta_i \\ 0 \end{bmatrix}, \quad \vec{n}_{i\perp} = \begin{bmatrix} -\sin\beta_i \\ \cos\beta_i \\ 0 \end{bmatrix} \quad (5)$$

$$\vec{v}_i = \begin{bmatrix} v_x \\ v_y \\ 0 \end{bmatrix}, \quad \vec{\Omega} = \begin{bmatrix} 0 \\ 0 \\ \Omega \end{bmatrix} \quad (6)$$

$$\vec{v}_i = \vec{v} + \vec{\Omega} \times \vec{D}_i = \begin{bmatrix} v_x \\ v_y \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \Omega \end{bmatrix} \times D_i \begin{bmatrix} \cos\beta_i \\ \sin\beta_i \\ 0 \end{bmatrix} = \begin{bmatrix} v_x - \Omega D_i \sin\beta_i \\ v_y + \Omega D_i \cos\beta_i \\ 0 \end{bmatrix} \quad (7)$$

$$\begin{aligned} \omega_i &= -\frac{\vec{v}_i \cdot \vec{n}_i}{r_i} \\ &= -\frac{1}{r_i} \begin{bmatrix} v_x - \Omega D_i \sin\beta_i \\ v_y + \Omega D_i \cos\beta_i \\ 0 \end{bmatrix} \cdot \begin{bmatrix} -\sin\beta_i \\ \cos\beta_i \\ 0 \end{bmatrix} \\ &= -\frac{1}{r_i} (-v_x \sin\beta_i + \Omega D_i \sin^2\beta_i + v_y \cos\beta_i + \Omega D_i \cos^2\beta_i) \\ &= -\frac{1}{r_i} (\Omega D_i - v_x \sin\beta_i + v_y \cos\beta_i) \end{aligned} \quad (8)$$

The mixing function can be expressed as a matrix multiplication:

$$\begin{bmatrix} -D_1 & \sin\beta_1 & -\cos\beta_1 \\ -D_2 & \sin\beta_2 & -\cos\beta_2 \\ -D_3 & \sin\beta_3 & -\cos\beta_3 \end{bmatrix} \begin{bmatrix} \Omega \\ v_x \\ v_y \end{bmatrix} = \begin{bmatrix} r_1 \omega_1 \\ r_2 \omega_2 \\ r_3 \omega_3 \end{bmatrix} \quad (9)$$

3 Wheel rotational speed to robot velocity conversion

Converting wheel speed to robot velocity is the inverse of the above conversion:

$$\begin{bmatrix} -D_1 & \sin\beta_1 & -\cos\beta_1 \\ -D_2 & \sin\beta_2 & -\cos\beta_2 \\ -D_3 & \sin\beta_3 & -\cos\beta_3 \end{bmatrix}^{-1} \begin{bmatrix} r_1 \omega_1 \\ r_2 \omega_2 \\ r_3 \omega_3 \end{bmatrix} = \begin{bmatrix} \Omega \\ v_x \\ v_y \end{bmatrix} \quad (10)$$

The inverse of the conversion matrix is (using WolframAlpha¹):

$$\begin{aligned} \begin{bmatrix} -D_1 & \sin\beta_1 & -\cos\beta_1 \\ -D_2 & \sin\beta_2 & -\cos\beta_2 \\ -D_3 & \sin\beta_3 & -\cos\beta_3 \end{bmatrix}^{-1} &= \frac{1}{D_3 \sin(\beta_1 - \beta_2) - D_2 \sin(\beta_1 - \beta_3) + D_1 \sin(\beta_2 - \beta_3)} \\ &\begin{bmatrix} \cos\beta_2 \sin\beta_3 - \cos\beta_3 \sin\beta_2 & \cos\beta_3 \sin\beta_1 - \cos\beta_1 \sin\beta_3 & \cos\beta_1 \sin\beta_2 - \cos\beta_2 \sin\beta_1 \\ D_3 \cos\beta_2 - D_2 \cos\beta_3 & D_1 \cos\beta_3 - D_3 \cos\beta_1 & D_2 \cos\beta_1 - D_1 \cos\beta_2 \\ D_3 \sin\beta_2 - D_2 \sin\beta_3 & D_1 \sin\beta_3 - D_3 \sin\beta_1 & D_2 \sin\beta_1 - D_1 \sin\beta_2 \end{bmatrix} \end{aligned} \quad (11)$$

¹ inv([[-D1, sin(b1), -cos(b1)],[-D2, sin(b2), -cos(b2)],[-D3, sin(b3), -cos(b3)]])