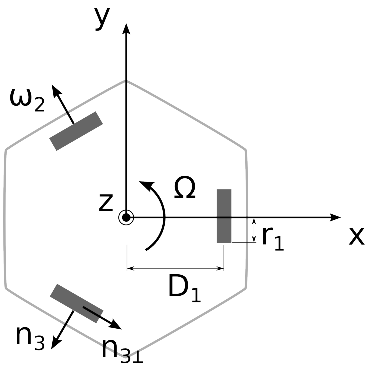
Holonomic Robot Wheel Mixing Functions

Patrick Spieler *Florian Reinhard*

patrick.spieler@epfl.ch florian.reinhard@epfl.ch

1 Definitions



ω_i wheel i rotational speed [rad/s]

D_i distance from wheel to center [m]

r_i wheel radius [m]

$\overrightarrow{n_i}$ axis unit vector of the wheel i

$\overrightarrow{n_{i\perp}}$ unit vector perpendicular to $\overrightarrow{n_i}$ ($+\pi/2$ from $\overrightarrow{n_i}$)

$\overrightarrow{v}$ robot velocity [m/s]

Ω robot angular velocity [rad/s]

$$\overrightarrow{\omega_i} = \omega_i \overrightarrow{n_i} \tag{1}$$

$$\overrightarrow{D_i} = D_i \overrightarrow{n_i} \tag{2}$$

2 Robot velocity to wheel rotational speed conversion

The velocity at the contact point of the wheel is:

$$\overrightarrow{v_i} = \overrightarrow{v} + \overrightarrow{\Omega} \times \overrightarrow{D_i} \tag{3}$$

The rotational speed of the wheel is obtained by projecting the velocity at the contact point of the wheel on $\overrightarrow{n_{i\perp}}$:

$$\omega_i = -\frac{\overrightarrow{v_i} \cdot \overrightarrow{n_{i\perp}}}{r_1} \tag{4}$$

TODO: drawing

Writing these equations with the components of the vectors:

$$\overrightarrow{n_i} = \begin{bmatrix} cos\beta_i \\ sin\beta_i \\ 0 \end{bmatrix}, \overrightarrow{n_{i\perp}} = \begin{bmatrix} -sin\beta_i \\ cos\beta_i \\ 0 \end{bmatrix} \tag{5}$$

$$\overrightarrow{v_i} = \begin{bmatrix} v_x \\ v_y \\ 0 \end{bmatrix}, \ \overrightarrow{\Omega} = \begin{bmatrix} 0 \\ 0 \\ \Omega \end{bmatrix} \tag{6}$$

$$\overrightarrow{v_i} = \overrightarrow{v} + \overrightarrow{\Omega} \times \overrightarrow{D_i} = \begin{bmatrix} v_x \\ v_y \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \Omega \end{bmatrix} \times D_i \begin{bmatrix} cos\beta_i \\ sin\beta_i \\ 0 \end{bmatrix} = \begin{bmatrix} v_x - \Omega D_i sin\beta_i \\ v_y + \Omega D_i cos\beta_i \\ 0 \end{bmatrix} \tag{7}$$

$$\begin{aligned} \omega_i &= -\frac{\overrightarrow{v_i} \cdot \overrightarrow{n_{i\perp}}}{r_i} \\ &= -\frac{1}{r_i} \begin{bmatrix} v_x - \Omega D_i sin\beta_i \\ v_y + \Omega D_i cos\beta_i \\ 0 \end{bmatrix} \cdot \begin{bmatrix} -sin\beta_i \\ cos\beta_i \\ 0 \end{bmatrix} \\ &= -\frac{1}{r_i}(-v_x sin\beta_i + \Omega D_i sin^2\beta_i + v_y cos\beta_i + \Omega D_i cos^2\beta_i) \\ &= -\frac{1}{r_i}(\Omega D_i - v_x sin\beta_i + v_y cos\beta_i) \end{aligned} \tag{8}$$

The mixing function can be expressed as a matrix multiplication:

$$\begin{bmatrix} -D_1 & sin\beta_1 & -cos\beta_1 \\ -D_2 & sin\beta_2 & -cos\beta_2 \\ -D_3 & sin\beta_3 & -cos\beta_3 \end{bmatrix} \begin{bmatrix} \Omega \\ v_x \\ v_y \end{bmatrix} = \begin{bmatrix} r_1\omega_1 \\ r_2\omega_2 \\ r_3\omega_3 \end{bmatrix} \tag{9}$$

3 Wheel rotational speed to robot velocity conversion

Converting wheel speed to robot velocity is the inverse of the above conversion:

$$\begin{bmatrix} -D_1 & sin\beta_1 & -cos\beta_1 \\ -D_2 & sin\beta_2 & -cos\beta_2 \\ -D_3 & sin\beta_3 & -cos\beta_3 \end{bmatrix}^{-1} \begin{bmatrix} r_1\omega_1 \\ r_2\omega_2 \\ r_3\omega_3 \end{bmatrix} = \begin{bmatrix} \Omega \\ v_x \\ v_y \end{bmatrix} \tag{10}$$

The inverse of the conversion matrix is (using WolframAlpha[1]):

$$\begin{bmatrix} -D_1 & sin\beta_1 & -cos\beta_1 \\ -D_2 & sin\beta_2 & -cos\beta_2 \\ -D_3 & sin\beta_3 & -cos\beta_3 \end{bmatrix}^{-1} = \frac{1}{D_3 sin(\beta_1 - \beta_2) - D_2 sin(\beta_1 - \beta_3) + D_1 sin(\beta_2 - \beta_3)}$$

$$\begin{bmatrix} cos\beta_2 sin\beta_3 - cos\beta_3 sin\beta_2 & cos\beta_3 sin\beta_1 - cos\beta_1 sin\beta_3 & cos\beta_1 sin\beta_2 - cos\beta_2 sin\beta_1 \\ D_3 cos\beta_2 - D_2 cos\beta_3 & D_1 cos\beta_3 - D_3 cos\beta_1 & D_2 cos\beta_1 - D_1 cos\beta_2 \\ D_3 sin\beta_2 - D_2 sin\beta_3 & D_1 sin\beta_3 - D_3 sin\beta_1 & D_2 sin\beta_1 - D_1 sin\beta_2 \end{bmatrix} \tag{11}$$

The first row in the above matrix can be expressed with the sin of the angle difference using:

$$sin(\alpha \pm \beta) = sin(\alpha)cos(\beta) \pm cos(\alpha)sin(\beta) \tag{12}$$

This gives the following matrix (without the scaling factor):

$$\begin{bmatrix} sin(\beta_3 - \beta_2) & sin(\beta_1 - \beta_3) & sin(\beta_2 - \beta_1) \\ D_3 cos\beta_2 - D_2 cos\beta_3 & D_1 cos\beta_3 - D_3 cos\beta_1 & D_2 cos\beta_1 - D_1 cos\beta_2 \\ D_3 sin\beta_2 - D_2 sin\beta_3 & D_1 sin\beta_3 - D_3 sin\beta_1 & D_2 sin\beta_1 - D_1 sin\beta_2 \end{bmatrix} \tag{13}$$

[1] inv([[-D1, sin(b1), -cos(b1)],[-D2, sin(b2), -cos(b2)],[-D3, sin(b3), -cos(b3)]])