

# Time And Space

## Complexity

### Linear Search

for  $n$  elements

data  $\rightarrow$

0	1	2	3	4	5	6	7	8
5	7	3	11	18	-2	8	2	25

$$T(n) = K + T(n-1) \rightarrow \text{for } n-1 \text{ elements}$$

checking  
operation

$$T(n-1) = K + T(n-2)$$

$$T(n-2) = K + T(n-3)$$

$$T(1) = K + T(0)$$

$$T(n) = K + \dots \text{ times}$$

$$= n \cdot K \Rightarrow T(n) = O(n)$$

$$cn \geq K \cdot n$$

Generally for computing Time complexities,

Best :  $\Omega \rightarrow \Omega(1)$

Average :  $\Theta \rightarrow \Theta(n/2)$

Worst :  $\Omega \rightarrow O(n)$

### Binary Search:

3	5	7	11	25	30	40	41	42	45	50
---	---	---	----	----	----	----	----	----	----	----

while (left < right) {

    // find mid

    if (a[mid] < data)

        else if (a[mid] > data)

    else

m terms

$$\left. \begin{array}{l} T(n) = K + T(n/2) \\ T(n/2) = K + T(n/4) \\ T(n/4) = K + T(n/8) \\ T(n/8) = K + T(n/16) \\ \vdots \\ T(n/2^m) = K + T(0) \end{array} \right\} \text{m terms}$$
$$T(n) = K + \dots + K \quad [m \text{-terms}]$$
$$= mK$$
$$= (1 + \log_2 n)K$$
$$\boxed{T(n) = O(\log n)}$$

For Factorial

$$T(n) = \underbrace{K' + T(n-1) + K''}_{\substack{\text{condition} \\ T(n) \in K' + T(n-1)}} \rightarrow \text{for } n-1 \text{ elements} \rightarrow \text{multiplying with } (n-1)$$

$$K' + K'' = K$$

$$T(n-1) = K + T(n-2)$$

$$T(n-2) = K + T(n-3)$$

$$T(1) = K + T(0)$$

$$T(n) = \underbrace{K + K + \dots + K}_{n \text{ times}}$$

=  $nK$

$$\boxed{T(n) = O(n)}$$

## Power linear

public static int power(int x, int n) {

if (n == 0)

return 1;

int xpn = power(x, n - 1);

int xpn = x \* xpn;

return xpn;

}

$$T(n) = K + T(n-1) + K$$

$$T(n) = O(n)$$

## Power logarithmic

public static int power2(int x, int n) {

if (n == 0)

return 1;

int xpn = power2(x, n/2) \* power2(x, n/2)

if (xpn \* 2^3 == 0)

xpn \*= 2;

return xpn;

}

$$\therefore T(n) = K + T(n/2) + T(n/2) \Rightarrow K + 2T(n/2)$$

$$2^1 \cdot T(n/2) = 2K + 2T(n/4) \rightarrow [ \text{multiply with } 2^1 \text{ on both sides} ]$$

$$2^2 \cdot T(n/4) = 4K + 8T(n/8) \rightarrow [ \text{multiply with } 2^2 \text{ on both sides} ]$$

$$2^3 \cdot T(n/8) = 8K + 16T(n/16) \rightarrow 2^3$$

$$T(1) = K + 2T(0)$$

$$T(0) = \frac{K + 2K + 4K + 8K}{x-1} = K(1 + 2^0 + 2^1 + \dots + 2^{x-1})$$

$$= K \left( 1 \times 2^{x-1} \right) = K \cdot 2^{x-1} \approx K \cdot 2^x \quad (n \approx 2^x)$$

$$= K \cdot n$$

$$T(n) = O(n)$$

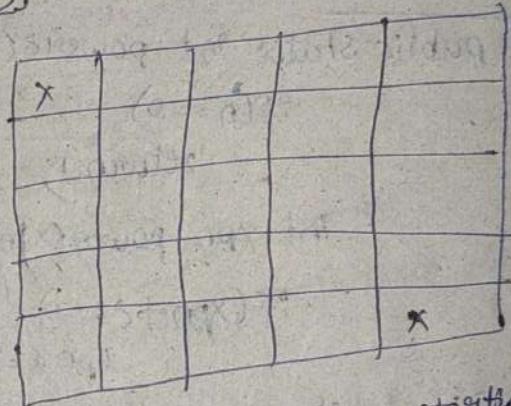
→ Other Approach

```
public static int power3(int x, int n) {  
    if (n == 0)  
        return 1;  
    int xpnd2 = power(x, n/2);  
    int xpnd = xpnd2 * xpnd2;  
    if (n % 2 == 0)  
        xpndx = x;  
    return xpnd;  
}
```

$$T(n) = K + T(n/2)$$

$$T(n) = O(\log n)$$

TC for Maze Paths



n v steps, m h steps  $(0, 0) \rightarrow n+m$  — starting position

n v steps; (m-1) h steps  $(0, 1) \rightarrow n+m-1$

n-1 v steps, m h steps  $(1, 0) \rightarrow n+m-1$

$$T(n+m) = T(n+m-1) + T(n+m-1) + K$$

$$\text{Let } n+m = x$$

$$T(x) = 2T(x-1) + K$$

$$2T(x-1) = 4T(x-2) + 2K \quad [x_2]$$

$$4T(x-2) = 8T(x-3) + 2^2 K \quad [x_2^2]$$

$$8T(x-3) = 16T(x-4) + 2^3 K \quad [x_2^3]$$

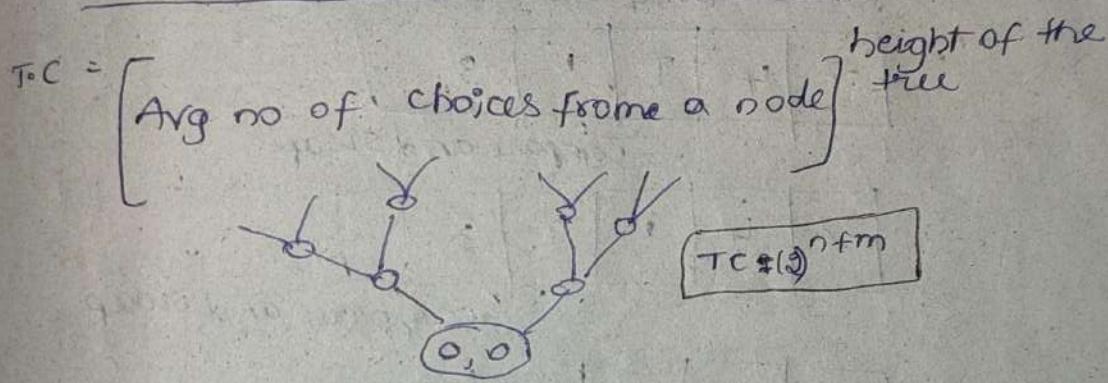
$$\boxed{T(x) = 2T(x-1) + K}$$

$$T(n) = \underbrace{K + 2K + 2^2 K + 2^3 K}_{\text{x - terms}} + \dots$$

x - terms

$$\begin{aligned}
 T(n) &= K * (1 + 2 + 4 + 8 + \dots + 2^{k-1}) \\
 &= K \cdot \left( \frac{2^x - 1}{2 - 1} \right) \\
 &= K \cdot 2^x - 1 \\
 &\approx K \cdot 2^x \\
 &\approx K \cdot 2^{l+m} \\
 &\boxed{T(n) \approx K \cdot 2^{l+m}}
 \end{aligned}$$

### Recursion Tree - RamBantareek Technique



### Bubble Sort

<u>Sample Input</u>	<u>Sample Output</u>	<u>Constraints</u>
5	Comparing -2 and 7	
7	Swapping -2 and 7	Swapping 3 and 4
-2	Comparing 1 and 7	Comparing 1 and -2
4	Swapping 4 and 7	Comparing 3 and 1
1	Comparing 1 and 7	Comparing 1 and -2
3	Swapping 1 and 7	-2
	Comparing 3 and 7	1
	Swapping 3 and 7	3
	Comparing 4 and -2	4
	Comparing 1 and 4	7
	Swapping 1 and 4	
	Comparing 3 and 4	

# Dog Run; Foo Bubble Sort

1<sup>st</sup>-itg

7	-2	4	1	3
---	----	---	---	---

compare and swap

-2	7	4	1	3
----	---	---	---	---

compare and swap

-2	4	7	1	3
----	---	---	---	---

compare and swap

-2	4	1	7	3
----	---	---	---	---

compare and swap

End of 1<sup>st</sup>  
itg

-2	4	1	3	7
----	---	---	---	---

-2	4	1	3	7
----	---	---	---	---

-2	4	1	3	7
----	---	---	---	---

compare and swap

2<sup>nd</sup>-itg

-2	1	4	3	7
----	---	---	---	---

compare and swap

-2	1	3	4	7
----	---	---	---	---

compare

End of  
2<sup>nd</sup>-itg

-2	1	3	4	7
----	---	---	---	---

3<sup>rd</sup> itt

-2	1	3	4	7
----	---	---	---	---

comparing 1, -2

-2	1	3	4	7
----	---	---	---	---

comparing

4<sup>th</sup> itt

-2	1	3	4	7
----	---	---	---	---

comparing

steps:

1. Take Outer Loop which traverses through the array
2. Inside take another loop which traverses through the array with decreasing the outer loop [because, in each outer loop, we will get the  $n^{\text{th}}$  largest elements, so we will skip those].
3. Check if the elements are smaller, then Apply swapping.

Program:

```
public static void bubbleSort(int[] arr) {  
    for (int i = 1; i <= arr.length - 1; i++) {  
        for (int j = 0; j < arr.length - i; j++) {  
            if (isSmaller(arr, j + 1, j)) {  
                swap(arr, j + 1, j);  
            }  
        }  
    }  
}
```

}

3

## 2. Selection Sort

### Sample Input

5 - no of elements  
 7  
 -2  
 4  
 1  
 3

### Sample Output

comparing -2 and 7  
 comparing 4 and -2  
 comparing 1 and -2  
 comparing 3 and -2  
 swapping 7 and -2  
 comparing 4 and 7  
 comparing 1 and 4  
 comparing 3 and 4  
 swapping 7 and 1  
 comparing 7 and 4  
 comparing 3 and 4

swapping 4 and 3  
 comparing 4 and 3  
 swapping 7 and 4

-2

1

3

4

7

### Constraints

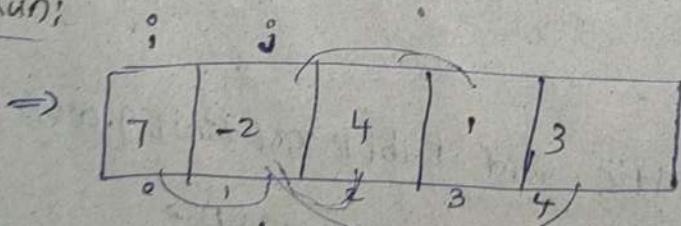
$1 \leq N \leq 100,000$

$-10^9 \leq \text{arr}[i] \leq 10^9$

### Algorithm- Steps

1. To travel through the array, assume min index = i
2. Go through the rest of elements and check any element is minimum then min index element, if yes redesign min index to smallest element and swap the element.
3. This process continues till all elements are covered and finally swapping is done.

### Dry Run:

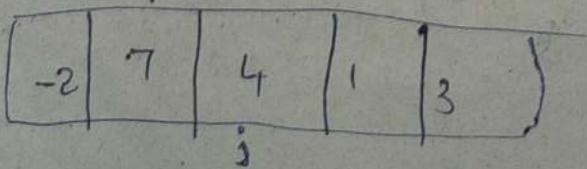


$\min = 0$ , comparing -2 and 7,

update  $\min = -2$ , comparing 4 and -2

comparing 3 and -2

swap arr[0], arr[min]



$min=1$ ,

update  $min \leftarrow$  comparing 4 and 7

update  $min=3 \leftarrow$  comparing 1 and 4  
 $\leftarrow$  comparing 3 and 1

swap  $arr(i), arr(min)$

0	1	2	3	4
-2	1	4	7	3

$i$        $j$

$min=2$

comparing 7 and 4

update  $min=4 \leftarrow$  comparing 3 and 4 , swap  $arr(i), arr(min)$

0	1	2	3	4
-2	1	3	7	4

$i$        $j$

$min=3$

update  $min=4 \leftarrow$  comparing 4 and 7

swap  $arr(i), arr(min)$

0	1	2	3	4
-2	1	3	4	7

$i$        $j$

→ sorted

Program:

```

public static void selectionSort(int[] arr) {
    for(int i=0; i<arr.length-1; i++) {
        int min=i;
        for(int j=i+1; j<arr.length; j++) {
            if (isSmaller(arr, j, min)) {
                min=j;
            }
        }
        swap(arr, i, min);
    }
}

```

### 3. Insertion Sort

#### Sample Input

5 → no of elements  
 7  
 -2  
 4  
 1  
 3

#### Sample Output

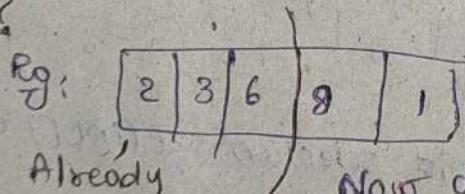
Comparing -2 and 7  
 swapping 7 and -2  
 Comparing 4 and 7  
 swapping 7 and 4  
 Comparing 4 and -2  
 Comparing 1 and 7  
 swapping 7 and 1  
 Comparing 1 and 4  
 swapping 4 and 1  
 Comparing 1 and -2  
 Comparing 3 and 7  
 swapping 7 and 3

Comparing 3 and 7  
 swapping 4 and 3  
 Comparing 3 and -2  
 -2  
 1  
 3  
 4  
 7

#### Steps:

- Insertion Sort is like Arranging a deck of cards.
- 1. Go through the array from first element
- 2. Take another loop and check backwards from index  $i$ , if element is greater than  $j$ , swap  $i$  with  $j$  else Apply Break

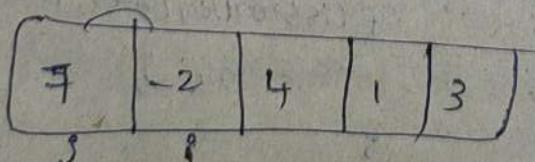
#### Why Break?



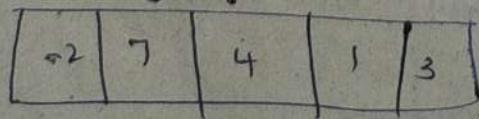
Already  
sorted

Now checking element 8  
as it is greater  
than 6, No need  
to check 3, 2 → so  
Break

#### Dry Run:



Comparing and swapping  
 $j$        $i$



→ Comparing and  
swapping

$j$	$j+1$
-2   4   7   ,   3	comparing and swapping

$j$	$j+1$
-2   4   ,   7   3	comparing and swapping

$j$	$j+1$
-2   ,   4   7   3	comparing

$j$	$j+1$
-2   ,   4   7   3	comparing 7 and 3 swapping

$j$	$j+1$
-2   ,   4   3   7	comparing and swapping

comparing and break  
 $i = 5 \times$

Program:  
public static void insertionSort(int[] arr) {

    for (int i=1; i<arr.length; i++) {

        for (int j=i-1; j>=0; j--) {

            if (isGreater(arr, j, j+1)) {

                swap(arr, j, j+1);

            }

        else {

            break;

        }

}

{

#### 4. Merge Two Sorted Arrays

##### Input

4 — no of elements  
in 1st array

-2

5

9

11

3 — no of elements  
in 2nd array

4

6

8

##### Output

-2

4

5

6

8

9

11

##### Constraints

$i \leq N \leq 10^8$

$-10^9 \leq a(i), b(i) \leq 10^9$

##### Steps:

1. The resultant Array length will be sum of both arrays.
2. Iterate through both arrays, check if element is greater than first smaller element in new array and increment the indexes.
3. If still either  $a(i)$  or  $b(j)$  less their respective arrays length store the remaining elements in new array.

##### Dry Run:

A<sub>1</sub>:

i	
-2	
5	
9	
11	

A<sub>2</sub>:

j	
4	
6	
8	

Res:

-2	4	5	6	8	9	11
----	---	---	---	---	---	----

:  $i < A_1$ .length,  $j < A_2$ .length ✓ ,  $K = 0$

$-2 >= 4$  X , Res( $K$ ) = -2 ,  $K++$ ,  $i = 1$

$1 < 4$ ,  $0 < 3$  ✓

$5 >= 4$  ✓

Res( $K$ ) = 4,  $K++$ ,  $j = 1$

$1 < 4$ ,  $1 < 3$  ✓

$5 >= 6$  X , Res( $K$ ) = 5,  $K++$ ,  $i = 2$

$2 < 4, i < 3 \checkmark$

$j = 6 \checkmark \quad \text{Res}(k) = 6, j++, j = 3$

$2 < 4, 2 < 3 \checkmark$

$j = 8 \checkmark \quad \text{Res}(k) = 8, j++, j = 3$

$2 < 4, 3 < 3 X$

Filling Remaining Elements  
from [Element] Array.

$2 < 4 \checkmark$

$\text{Res}(k) = 9, i++, k++$

$3 < 4 \checkmark$

$\text{Res}(k) = 11, i++, k++$

$4 < 4 X$

From 2nd Array

$3 < 3 X \Rightarrow \text{Res}$

2	4	5	6	8	9	11
---	---	---	---	---	---	----

Program:

public static int merge Two Sorted Arrays (int[] a, int[] b)  
int[] ans = new int[a.length + b.length];

int i = 0;

int j = 0;

int k = 0;

while ( $i < a.length \& j < b.length$ ) {

if ( $a(i) \geq b(j)$ ) {

ans[k] = b[j];

j++;

k++;

} else {

ans[k] = a[i];

i++;

k++;

}

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1) Remaining Elements from 1st Array

while ( $i < a.length$ ) {

$ans[k] = a[i];$

$i++;$

$k++;$

}

2) Remaining Elements from 2nd Array

while ( $j < b.length$ ) {

$ans[k] = b[j];$

$j++;$

$k++;$

}

return ans;

}

## 5. Merge Sort

Sample Input

5

7

-2

4

1

3

Output

Merging these two arrays

left array  $\rightarrow [$

right array  $\rightarrow [$

Merging these two arrays

left array  $\rightarrow [$

right array  $\rightarrow [$

Merging these two arrays

left array  $\rightarrow [$

right array  $\rightarrow [$

Merging these two arrays

left array  $\rightarrow [$

right array  $\rightarrow [$

Sorted array  $\rightarrow [$

Constraints

$1 \leq N \leq 100000$

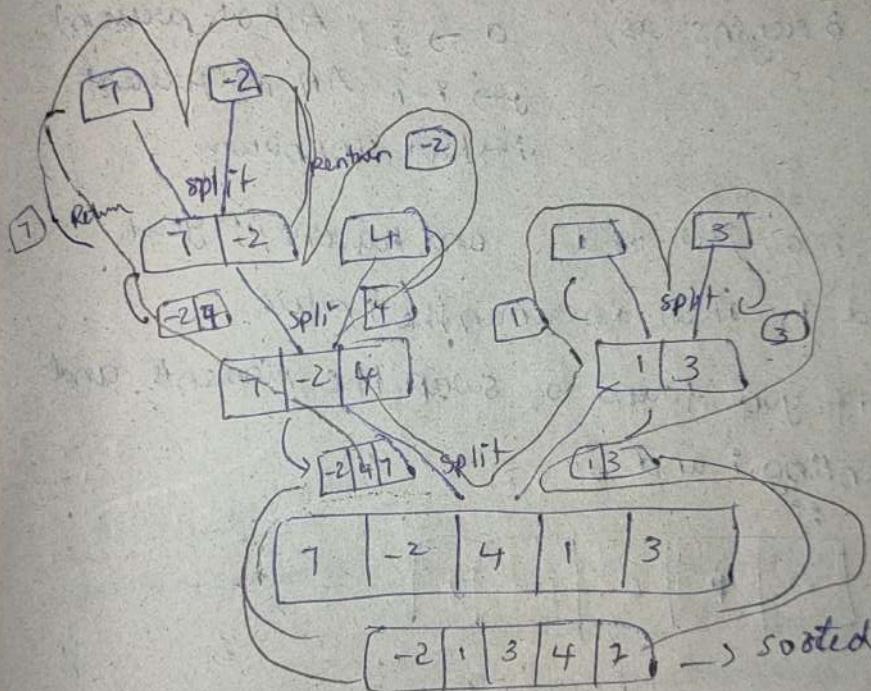
$-10^9 \leq arr[i] \leq 10^9$

Time Complexity:  $n \log n$

steps:

1. split the Array into halves by mid
2. sort the two Arrays by using merging two sorted Arrays (previous problem).
3. if  $low == high$ , then there will be single element return the single element with array

Dry Run:



Program:

```

public static int[] mergeSort(int[] arr, int low, int hi)
{
    if (low == hi) {
        int[] bs = new int[1];
        bs[0] = arr[low];
        return bs;
    }

    int mid = (low + high) / 2;
    int[] a = mergeSort(arr, low, mid);
    int[] b = mergeSort(arr, mid + 1, high);

    int[] ans = mergeTwoSortedArrays(a, b);
    return ans;
}

```

## 6. Sort 01

Sample Input

5  
0  
1  
0  
1  
0

Sample Output

Swapping index 0 and index 0

Swapping index 2 and index 1

Swapping index 4 and index 2

0  
0  
0  
1

Constraints

$1 \leq N \leq 10000$   
 $\text{ans}(i) = 0, 1$

Steps:

- 1) Consider 3 regions as;  
 $0 \rightarrow j-1$  All 0's present  
 $j \rightarrow i-1$  All 1's present  
 $i \rightarrow last$  unknown
- 2) Initially  $i=0, j=0$  and traverse, if you found 1, then increment the  $j$  value.
- 3) Else if you found 0, swap the elements and incrementing  $i$  and  $j$ .

Dry Run:

0	*	0	1	0
---	---	---	---	---

$i=0, j=0$

$\text{ans}(i)=0$ , swap  $\Rightarrow$  inc  $i, j$

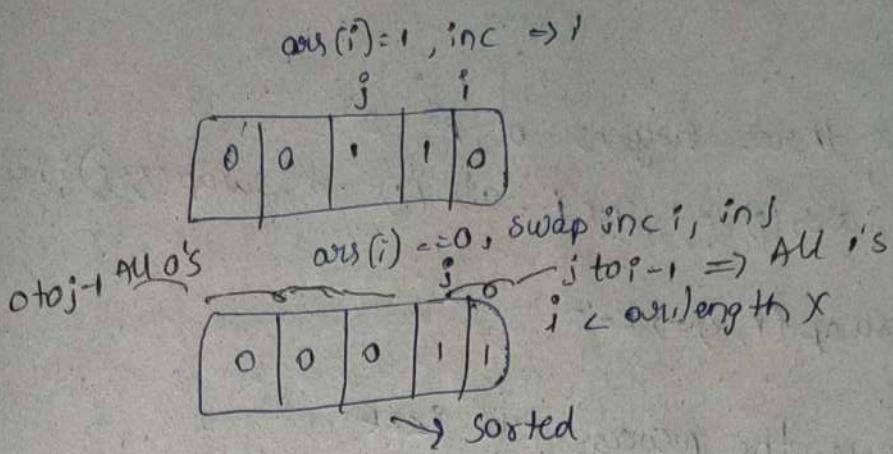
0	1	0	1	0
---	---	---	---	---

$\Rightarrow$  so inc  $i$

0	1	0	1	0
---	---	---	---	---

$\text{ans}(i)=0$ , swap  $\Rightarrow$  inc  $i, j$

0	0	1	1	0
---	---	---	---	---



Program:

```
public static void sort01(int[] arr) {
    int i = 0;
    int j = 0;
    while (i < arr.length) {
        if (arr[i] == 1)
            i++;
        else {
            swap(arr, i, j);
            i++;
            j++;
        }
    }
}
```

## 7. Sort 012

### Sample Input

10 → No of elements  
1  
0  
2  
2  
1  
0  
2  
1  
0  
2

### Sample Output

Swapping index 1 and index 0  
Swapping index 2 and index 9  
Swapping index 2 and index 8  
Swapping index 2 and index 1  
Swapping index 3 and index 7  
Swapping index 5 and index 2  
Swapping index 6 and index 6

0  
0  
1  
1  
2  
2  
2  
2  
2

Constraints  
 $1 \leq N \leq 10000$   
 $\text{arr}(i) = 0, 1, 2$

## Steps:

1. Make three regions  $0^*$ ,  $1^*$ ,  $2^*$
2. If  $\text{arr}(i) = 1$ , inc  $i$ ,  $\text{arr}(i) = 0$ , swap( $i, j$ ),  $i++$ ,  $j++$
3. else swap( $i, k$ )  $\Rightarrow k--$
4. Continue the process till  $i \leq k$

## Dry Run:

$i$	$j$	$k$
1	0	2
2	1	0
1	0	2
1	0	2

$i$	$j$	$k$
1	0	2
2	1	0
1	0	2
1	0	2

$i$	$j$	$k$
1	0	2
2	1	0
1	0	2
1	0	2

$i$	$j$	$k$
0	1	2
2	1	0
1	0	2
1	0	2

$i$	$j$	$k$
0	1	2
0	2	1
1	0	2
1	2	0

$i$	$j$	$k$
0	0	1
1	2	0
0	2	1
1	2	0

0	0	1	2	1	0	2	2	2	2
---	---	---	---	---	---	---	---	---	---

0	0	1	1	1	0	2	2	2	2
---	---	---	---	---	---	---	---	---	---

0	0	1	1	1	0	2	2	2	2
---	---	---	---	---	---	---	---	---	---

0	0	0	1	1	1	2	2	2	2
---	---	---	---	---	---	---	---	---	---

0	0	0	1	1	2	2	2	2	2
---	---	---	---	---	---	---	---	---	---

✓ Sorted

) Next  
K > i X

### Program:

```
public static void solve(int[] arr) {
    int i = 0;
    int j = 0;
    int k = arr.length - 1;

    while (i <= k) {
        if (arr[i] == 0) {
            swap(arr, i, j);
            i++;
            j++;
        } else if (arr[i] == 1) {
            i++;
        } else {
            swap(arr, i, k);
            k--;
        }
    }
}
```

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## 8. Partition An Array

## Sample Input

$5 \rightarrow$  No of elements

7

-2

4

1

3

$3 \rightarrow \text{pivot}$

## Sample Output

Swapping -2 and 1

Swapping 1 and 7

Swapping 3 and 4

-2 1 3 7 4

## Constraints

$$1 \angle = N \angle = 100000$$

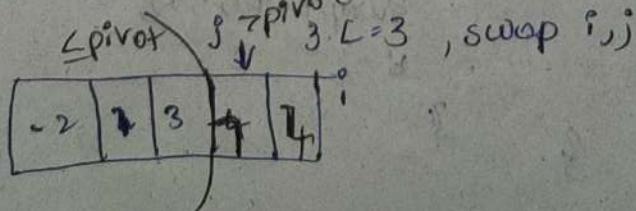
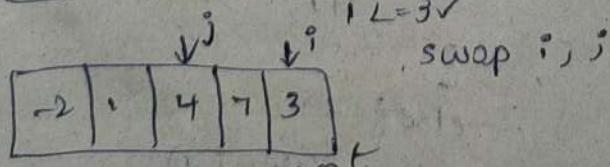
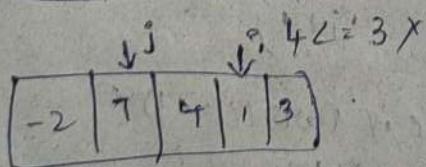
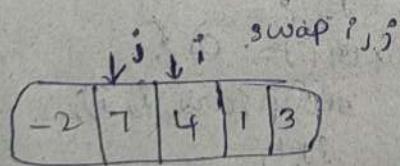
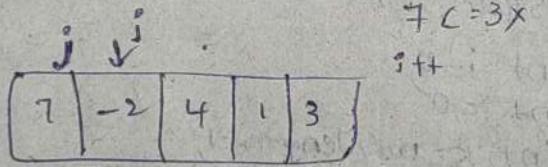
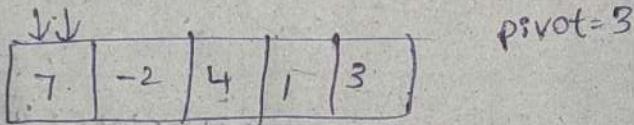
$$10^9 \angle = 0.001 \text{ rad} \angle = 10^{-9}$$

$$\delta_9 \angle = \text{pivot} \angle = 10^{\circ} 9$$

## Steps:

1. Traverse through the array, if the array element is smaller than pivot, swap the elements and increment the  $i,j$  indices.
  2. Then increase the  $j$  indice.

Dry Run; 9.9



Program:

```

public static void partition(int arr[], int pivot) {
    int i = 0;
    int j = 0;
    while (i < arr.length) {
        if (arr[i] <= pivot) {
            swap(arr, i, j);
            j++;
            j++;
        } else {
            i++;
        }
    }
}

```

### 9. Quick Sort

#### Sample Input

5  
7  
-2  
4  
1  
3

#### Sample Output

pivot → 3  
 swapping -2 and 7  
 swapping 1 and 7  
 swapping 3 and 4  
 pivot index → 2  
 pivot → 1  
 swapping -2 and -2  
 swapping 1 and 1  
 pivot index → 1  
 pivot → -2  
 swapping -2 and -2  
 pivot index → 0  
 pivot → 4  
 swapping 4 and 7  
 pivot index → 3  
 pivot → 7  
 swapping 7 and 7  
 pivot index → 4

#### Constraints

$1 \leq N \leq 100000$   
 $-10^9 \leq arr[i] \leq 10^9$

-2 1 3 4 7

### Steps:

1. Get the pivot element, by assigning to arr[high]
2. Then Partition the array based on pivot index which Returns the pivot index.
3. All elements lesser than pivot index will go to Recursive calls of quick sort and all the other half.
4. The base condition will be as  $i > h$  till then the process continues

### Time Complexity:

$$\begin{aligned}
 T(n) &= 2T\left(\frac{n}{2}\right) + n + K \\
 2T\left(\frac{n}{2}\right) &= 2T\left(\frac{n}{4}\right) + 2n/2 + 2K \quad \times 2 \\
 2^2T\left(\frac{n}{4}\right) &= 2^3T\left(\frac{n}{8}\right) + 4\frac{n}{4} + 4K \quad \times 4 \\
 2^3T\left(\frac{n}{8}\right) &= 2^4T\left(\frac{n}{16}\right) + 16\frac{n}{16} + 8K \quad \times 8 \\
 \frac{n}{2^x} = 1 \quad n = 2^x \quad T &= 2T(0) + n + K \\
 (x = \log_2 n) \quad \underbrace{T(n)}_{\text{x times}} &= \underbrace{n + n + \dots + n}_{\text{x times}} + \underbrace{K + 2K + 4K + \dots}_{\text{x times}} \\
 &= n \log_2 n + x(1 + 2 + \dots + 2^{x-1}) \\
 &= n \log_2 n + K(2^x - 1) \propto 2^x \\
 &= n \log_2 n + n \\
 \boxed{T(n) = O(n \log n)}
 \end{aligned}$$

### Program:

```

public static void quickSort(int[] arr, int lo, int hi) {
    if (lo > hi)
        return;
    int pivot = arr[hi];
    int pi = partition(arr, pivot, lo, hi);
    quickSort(arr, lo, pi - 1);
    quickSort(arr, pi + 1, hi);
}

```

## 10. Quick Select

Sample Input  
 $5 \rightarrow$  no of elements  
 7  
 -2  
 4  
 1  
 3  
 $3 \rightarrow K$

### Sampleout

pivot  $\rightarrow 3$   
 swapping -2 and 7  
 swapping 1 and 7  
 swapping 3 and 4  
 pivot index  $\rightarrow 2$   
 3

### Constraints

$$1 \leq N \leq 100000$$

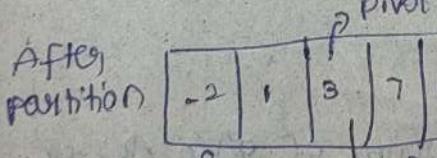
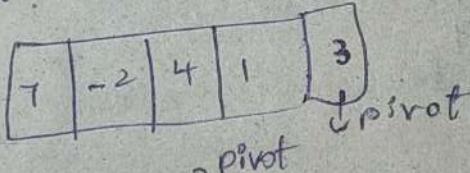
$$-10^9 \leq arr[i] \leq 10^9$$

$$1 \leq K \leq N$$

### Steps:

- 1) Assign pivot as hi (last element)
- 2) Then do the partition, Get the pivot index
- 3) Now compare the pivot index with k
- 4) If k is smaller than pivot index search in the first half
- 5) If k is greater than pivot index, search in the second half. (3) gives  $K^{th}$  smallest value
- 6) Else return the arr(pi) which gives the  $K^{th}$  smallest element in the array.

### Dry Run:



$\hookrightarrow (3) \Rightarrow$  satisfies 3rd smallest is 3

### Program:

```
public static int quickSelect (int[] arr, int lo, int hi, int k) {
    int pivot = arr[hi];
    int pi = partition (arr, pivot, lo, hi);
    if (k < pi)
        return quickSelect (arr, lo, pi-1, k);
    else if (k > pi)
        return quickSelect (arr, pi+1, hi, k);
    else
        return arr[pi];
```

Worst case; smallest, last before index  $\Rightarrow O(n^2)$   
 $T(n) = n + k + T(n-1)$

Best/Avg case;  $T(n) = n + k + T(n/2)$

$$\begin{aligned} &= n + n/2 + T(n/4) \\ &= n + n/2 + n/4 + \dots + n/2 \\ &= n \left( 1 + \frac{1}{2} + \frac{1}{4} + \dots + \frac{1}{2^{\lfloor \log_2 n \rfloor}} \right) \\ &= n \left( \frac{1}{1 - \frac{1}{2}} \right) = 2n \end{aligned}$$

↓ by G series

Note:

For Array Sorts

Small Elements  
Insertion sort

Large Elements  
Dual pivot  
Quick sort

## 11. Count Sort

Input

5  
7  
-2  
4  
1  
3

Sample Output

-2  
1  
3  
4  
7

Constraints

1.  $4 \leq N \leq 1000000$   
2.  $-10^8 \leq arr[i] \leq 10^8$

Steps:

1. Take the Given Array
2. Construct the frequency array of given Array  
(by Max, Min element from Array + 1)
3. Construct the prefix sum array from frequency array
4. Construct the Ans Array by traversing in reverse order from original array , with subtracting element from min (As index starts from 0).
5. Get the position, and place the element in

ans array with pos-1.

6. Decrement the value of prefixsum at that index.

7. Copy the entire ans Array  $\rightarrow$  Original Array.

DryRun:

Original Array:

7	4	-3	2	-1	3	-1	e	.3	-1	6	4	-1	6
0	1	2	3	4	5	6	7	8	9	10	11	12	13

Construct Freq Array: length = ( $\max - \min + 1$ )

$$= (-7 - (-3) + 1) = 10$$

1	0	4	0	0	2	2	2	0	2	1			
0	1	2	3	4	5	6	7	8	9	10			
(-3)	(-2)	(-1)	(0)	(1)	(2)	(3)	(4)	(5)	(6)	(7)			
+1	+2	+3	+4	+5	+6	+7	+8	+9	+10	+11			

Construct PrefixSum Array

0	1	5	10	15	20	25	30	35	40	45	50	55	60
0	1	2	3	4	5	6	7	8	9	10			

$$\text{prefixsum}(k) = \text{arr}(0) + \dots + \text{arr}(k-1)$$

$$\text{arr}(0) = 6, \frac{6 - m^{\text{in}}}{6 - (-3)} = 6 + 3 = 9$$

Construct Ans Array

$$\text{pos} = \text{pos} - 1$$

$$\text{pos} = 12$$

$$\text{ans}(\text{pos})_{\text{array}}^{C_i}$$

-3	-1	-1	-1	-1	2	2	3	3	4	4	6	6	7
0	1	2	3	4	5	6	7	8	9	10	11	12	13

## Program:

```
public static void countSort(int[] arr, int min, int max)
{
    int[] freq = new int[max - min + 1];
    // Create Freq array
    for (int i = 0; i < arr.length; i++) {
        freq[arr[i] - min]++;
    }
    // Create prefix sum
    int[] psum = new int[freq.length];
    psum[0] = freq[0];
    for (int i = 1; i < freq.length; i++) {
        psum[i] = psum[i - 1] + freq[i];
    }
    // Create ans array
    int[] ans = new int[arr.length];
    for (int i = arr.length - 1; i >= 0; i--) {
        int k = arr[i] - min;
        int pos = psum[k];
        ans[pos] = arr[i];
        psum[k]--;
    }
    // Modifying Array.
    for (int i = 0; i < ans.length; i++) {
        arr[i] = ans[i];
    }
}
```

## 12. Radix Sort

<u>Sample Input:</u>	<u>Sample Output</u>	<u>Constraints</u>
5 7 2 4 1 3	After sorting on 1 place $\rightarrow 1 \ 2 \ 3 \ 4 \ 7$ 12 3 4 7	$O(C) = N \cdot L = 10000$ $O(L) = \text{arr}(i) \cdot L = 10^8$

### Steps:

1. Get the maximum element from Array
2. Now pass Array to countsort in multiples of exponent, till it doesn't exceed the max value of array.
3. Modify the count sort code

### Dry Run:

eg:	1 2	1 4	1 4	1 2 1	1 4 0
	1 2	1 4	1 0	1 2 1	1 0 0
	1 2	1 5	1 5	1 2 1	1 5 5
	1 2	1 8	1 8	1 2 1	1 2 7
	1 2	1 9	1 9	1 2 1	1 2 7
	1 2	1 9	1 9	1 2 1	1 9 9
	1 2	1 7	1 7	1 2 1	1 9 9
	1 2	1 7	1 7	1 2 1	1 9 9

count sort on unit place

count sort on 100's place	0 0 0	5
0 5	count sort	9
0 9	→ on	2 7
1 2 1	100's place	4 8
1 2 7		1 0 0
1 2 7		1 2 1
1 4 0		1 2 7
1 4 8		1 4 0
1 9 9		1 9 9

For getting units place

$(arr[i] \% 10) / 10$ , tensplace:  $(arr[i] / 10) / 10$

Hundreds place:  $(arr[i] / 100) / 10$

Program:

```
public static void radixSort(int[] arr) {
```

```
    int max = arr[0];
```

```
    for (int i = 1; i < arr.length; i++) {
```

```
        if (max < arr[i]) {
```

```
            max = arr[i];
```

```
}
```

```
    int exp = 1;
```

```
    while (exp <= max) {
```

```
        countSort(arr, exp);
```

```
        exp = exp * 10;
```

```
}
```

```
}
```

```
public static void countSort(int[] arr, int exp) {
```

```
    int[] freq = new int[10];
```

```
    for (int i = 0; i < arr.length; i++) {
```

```
        freq[arr[i] / exp % 10]++;
```

```
}
```

```
int[] psum = new int[freq.length];
```

```
psum[0] = freq[0];
```

```
for (int i = 1; i < freq.length; i++) {
```

```
    psum[i] = psum[i - 1] + freq[i];
```

```
}
```

int[] ans = new int[arr.length];

for (int i = arr.length - 1; i >= 0; i--) {

    int k = arr[i];

    int pos = psum((k / exp) \* 10);

    pos = pos - 1;

    ans[pos] = arr[i];

    psum((k / exp) \* 10) --;

}

for (int i = 0; i < ans.length; i++) {

    arr[i] = ans[i];

}

System.out.print("After sorting on "+ exp + " place->");

print(arr);

}

### 13. Sort Dates

#### Sample Input

5

12041996

20101996

05061997

12071989

11081987

#### Sample Output

11081987

12041989

12041996

20101996

05061997

#### Constraints

1 ≤ N ≤ 10000

All dates are between year 0 to year 2500.

- Steps:
1. Apply the count sort through Days wise then month's wise, then years wise
  2. As input is in string format, parse them into integer format (use parameter as 10, to get in decimal format).

## Dry Run:

12	04	1996
20	10	1996
05	06	1997
12	04	1989
11	08	1987

sort date wise

05	06	1997
11	08	1987
12	04	1996
12	04	1989
20	10	1996

sort month wise

12	04	1996
12	04	1989
05	06	1997
14	08	1987
20	10	1996

sort year wise

=>

11	08	1987
12	04	1989
12	04	1996
20	10	1996
05	06	1997

To get Date:  $(arr[0] / 1000000) \cdot 100$   
month:  $(arr[0] / 10000) \cdot 100$   
years:  $(arr[0] / 1) \cdot 10000$

freq Array Range should be greater than days (32)  
 As Array starts with 0, [1-32], 11y Months,  
 11y for Years

## Program:

```
public static void sortDates (String arr) {
    countSort (arr, 1000000, 100, 32); // 11 days
    countSort (arr, 10000, 100, 13); // 11 months
    countSort (arr, 1, 10000, 250); // 11 years
```

}

```
public static void countSort (String [] arr, int div, int
mod, int range) {
    int[] freq = new int [range];
    for (int i = 0; i < arr.length; i++) {
        freq [Integer.parseInt (arr[i]) / div % mod]++;
    }
}
```

}

```
int[] psum = new int(freq.length);
psum[0] = freq[0];
for (int i = 1; i < freq.length; i++) {
    psum[i] = psum[i - 1] + freq[i];
}
```

```
String[] ans = new String(ans.length);
```

```
for (int i = ans.length - 1; i >= 0; i--) {
    int k = Integer.parseInt(ans[i], 10);
    int pos = psum[(k / div) * mod];
    pos = pos + 1;
    ans[pos] = ans[i];
    psum[(k / div) * mod] -= 1;
}
```

```
for (int i = 0; i < ans.length; i++) {
    ans[i] = ans[i];
}
```

```
}
```