

DYNAMIC PROGRAMMING

Lab Exercises:

1. Write a Program to implement 0/1 knapsack using dynamic programming technique and analyze the complexity.

```
#include <iostream>
#include <bits/stdc++.h>
using namespace std;
```

```
int knapsack ( int c, int W[], int P[], int n ) {
    int i, f;
    int k[2][c+1];
    for( i=0; i<=n; i++ ) {
        for( f=0; f<=c; f++ ) {
            if( i==0 || f==0 )
                k[i%2][f] = 0;
            else if( W[i-1] <= f ) {
                k[i%2][f] = max( P[i-1] + k[(i-1)%2]
                    [f-W[i-1]], k[(i-1)%2][f] );
            } else
                k[i%2][f] = k[(i-1)%2][f];
        }
    }
}
```

```
    return k[n/2][c];  
}
```

```
int main() {
```

```
    int n;
```

```
    cout << "Enter total no. of elements: ";
```

```
    cin >> n;
```

```
    cout << endl;
```

```
    int w[n], c, p[n];
```

```
    cout << "Enter Capacity: ";
```

```
    cin >> c;
```

```
    cout << "Enter the weights: ";
```

```
    for( int i=0; i<n; i++ ) {
```

```
        cin >> w[i]; }
```

```
    cout << "Enter the profits: ";
```

```
    for( int i=0; i<n; i++ ) {
```

```
        cin >> p[i]; }
```

```
    cout << knapsack(c,w,p,n);
```

```
}
```

O/p
Enter total no. of elements : 3
Enter capacity : 10
Enter the weight : 3 5 7
Enter the profits : 20 10 5

$\Rightarrow 30$

Time Complexity Analysis

The time complexity of the 0/1 knapsack problem using dynamic programming is $O(nW)$, where n is the number of items and W is the maximum weight that the knapsack can carry.

This is because we create a table of size $(n+1) \times (W+1)$ to store the solutions to all subproblems.

2. Write a program to find the optimal order of multiplication in a chain of matrices and analyze the complexity.

```
#include <iostream>
#include <bits/stdc++.h>
using namespace std;
```

```
int MatrixChainOrder(int p[], int i, int j) {
    if (i == j)
        return 0;
    int k;
    int mini = INT_MAX;
    int count1;
    for (k = i; k < j; k++) {
        count1 = MatrixChainOrder(p, i, k) +
        MatrixChainOrder(p, k + 1, j) + p[i - 1] * p[k] * p[j];
        mini = min(count1, mini);
    }
    return mini;
}
```

```
int main() {
    int n;
    cout << "Enter the no. of elements: ";
```

DATE []

```
cin >> n;
int arr[n];
cout << "Enter the elements: ";
for( int i=0 ; i<n ; i++)
    cin >> arr[i];
cout << "Minimum no. of multiplication is "
    << MatrixChainOrder( arr, 1, n-1 );
return 0;
```

Output

Enter the number of elements : 5

Enter the num element : 40 20 30 10 30

Minimum number of multiplications is 26000.

Time Complexity Analysis

The time complexity of matrix chain multiplication using dynamic programming (DP) is $O(n^3)$ where n is the number of matrices in the chain.

3. Write a program to find the shortest path between every pair of vertices in a given graph using dynamic programming technique.

```
#include <bits/stdc++.h>
```

```
using namespace std;
```

```
void printSolution (int dist[][20], int V) {
```

```
    cout << "The following matrix shows the  
shortest " distances " between every  
pair of vertices \n ";
```

```
    for (int i = 0; i < V; i++) {
```

```
        for (int j = 0; j < V; j++) {
```

```
            if (dist[i][j] == 999)
```

```
                cout << "INF"
```

```
                << " " ;
```

```
        else
```

```
            cout << dist[i][j] << " " ;
```

```
    }
```

```
    cout << endl ;
```

```
    }
```

```
    }
```

shortest path
given
technique.

, int V) {
shows the
every

```
void floyd_marshall ( int dist[20][20] , int V ) {
    int i, j, k ;
    for( k=0 ; k < V ; k++ ) {
        for( i=0 ; i < V ; i++ ) {
            for( j=0 ; j < V ; j++ ) {
                if( dist [i][j] > (dist [i][k] + dist [k][j]) )
                    && ( dist [k][j] != 999 && dist [i][k] != 999 ) )
                    dist [i][j] = dist [i][k] + dist [k][j] ;
    }
    printSolution ( dist, V );
}
```

```
int main () {
    int V ;
    cout << "Enter no. of vertices: " ;
    cin >> V ;
    int graph [20][20] ;
    cout << "Enter the distances, Enter 999 if
path does not exist " ;
    for( int i=0 ; i < V ; i++ ) {
        for( int j=0 ; j < V ; j++ ) {
            cin >> graph [i][j] ;
```

```
floydWarshall(graph, V);
return 0;
```

4

Output :

Enter number of vertices : 4

Enter the distances, Enter 999 if path
doesn't exist : 0 5 999 10

999 0 3 999

999 999 0 1

999 999 999 0

The following matrix shows the shortest distances
between every pair of vertex .

0	5	8	9
INF	0	3	4
INF	INF	0	1
INF	INF	INF	0

Time Complexity Analysis:

The time complexity of the Floyd-Warshall algorithm is $O(n^3)$, where n is the number of vertices in the graph. This is because the algorithm needs to consider all possible pairs of vertices in order to and for each pair, it needs to consider all possible intermediate vertices in order to compute the shortest path between them.

BACKTRACKING :LAB EXERCISES :

1. Write a program to find the Maximum clique of a graph using backtracking and analyze the complexity.

```
# include <bits/stdc++.h>
```

```
Using namespace std;
```

```
const int MAX = 100;
```

```
int store[MAX], n;
```

```
int graph[MAX][MAX];
```

```
int d[MAX];
```

```
bool is_clique(int b) {
```

```
    for (int i = 1; i < b; i++) {
```

```
        for (int j = i + 1; j < b; j++) {
```

```
            if (graph[store[i]][store[j]] == 0)
```

```
                return false; }
```

```
}
```

```
return true;
```

```
}
```

```
int max
```

```
int
```

```
for (int
```

```
sto
```

```
if (is
```

```
m
```

```
?
```

```
4
```

```
return r
```

```
4
```

```
int main
```

```
int
```

```
cout
```

```
cin
```

```
int e
```

```
cout <
```

```
for (int
```

```
for
```

```
ci
```

```
4
```

```
int maxCliques (int i, int l) {  
    int max_ = 0;  
    for (int j = i + 1; j <= n; j++) {  
        store [l] = j;  
        if (is_clique (l + 1)) {  
            max_ = max (max_, l);  
            max_ = max (max_, maxCliques (j, l + 1));  
        }  
    }  
    return max_;  
}
```

```
int main () {  
    int n;  
    cout << "Enter the number of nodes: " << endl;  
    cin >> n;  
    int edges [n] [2];  
    cout << "Enter edges relation: "  
    for (int i = 0; i < n; i++) {  
        for (int j = 0; j < 2; j++) {  
            cin >> edges [i] [j];  
        }  
    }
```

```

for (int i = 0; i < size; i++) {
    graph[edges[i][0]][edges[i][1]] = 1;
    graph[edges[i][1]][edges[i][0]] = 1;
    d[edges[i][0]]++;
    d[edges[i][1]]++;
}
cout << maxCliques(0, 1);
return 0;

```

and calls
given the
The 'is-'
it needs
graph.

Output:

Enter the number of nodes : 4

Enter edges relation : 1 2 2 3 3 1 4 2

= 3

Time Complexity Analysis

The time complexity of this algorithm is $O(2^n * n^2)$, where 'n' is the number of vertices in the graph. This is because 'max cliques' is a recursive function that calculates all possible subsets of the vertices

and calls 'is-clique' function to check if the given set of vertices form a clique or not. The 'is-clique' function takes $O(n^2)$ time as it needs to check for all the edges in the graph.

{
[1] = D;
[0] = 1;

4
2 3 3 1 4 2

algorithm is
the number
this is because
function that
sets of the vertices

2.

Implement Travelling Sales Person problem
using backtracking technique.

```
#include <bits/stdc++.h>
```

```
using namespace std;
```

```
#define V 100
```

```
void tsp( int graph[V][V], bool v[], int currpos,  
int n, int count, int cost, int &ans, int verts)  
{
```

```
if (count == n && graph[currpos][0])  
{
```

```
ans = min (ans, cost + graph[currpos][0]);  
return;
```

```
for( int i=0 ; i<n ; i++ ) {
```

```
if( !v[i] && graph[currpos][i] )
```

```
{
```

```
v[i] = true;
```

```
tsp( graph, v, i, n, count+1, cost + graph[currpos][i]  
.ans, verts);
```

```
v[i] = false;
```

```
}
```

```
y
```

```
y
```

```
y;
```

```
int main() {  
int verts;  
cout << "N"  
cin >> vert  
int graph[  
cout << "E"
```

```
for( int i =  
for( int
```

```
cin
```

```
?
```

```
y
```

```
bool v[V]
```

```
for( int i = 0
```

```
v[i] =
```

```
v[0] = +
```

```
int ans =  
tsp( graph  
cout <<
```

```
int main() {  
    int verts;  
    cout << "Number of Cities" << endl;  
    cin >> verts;  
    int graph[V][V];  
    cout << "Enter adj matrix of cities" << endl;
```

```
for( int i=0 ; i < verts ; i++ ) {  
    for( int j=0 ; j < verts ; j++ ) {
```

```
        cin >> graphs[i][j];
```

```
    }  
}  
bool v[V];
```

```
for( int i=0 ; i < verts ; i++ ) {  
    v[i] = false;  
    v[0] = true;
```

```
int ans = INT_MAX;
```

```
tsp( graph, v, 0, verts, 1, 0, ans, verts );
```

```
cout <<
```

Output:

The time complexity of the backtracking approach for TSP is $O(n!)$, where n is the number of cities. There are $n!$ possible permutations of the cities to visit, and the backtracking algorithm will explore all of them in the worst case.

NUMBER OF CITIES

4

enter adj matrix of cities

0 10 15 20

10 0 35 25

15 35 0 30

20 25 30 0

→ 60

3
Find the optimal backtracking techniques.

#include <iostream>
using namespace

int knapsack (int cw, int cp, int i)

if (cw > c)

if (cp > max)

for (int i = n - 1; i >= 0; i--) {
int ub = c - cw;
if (ub < r[i])

cw += w[i];
cp += v[i];

if (cw <= c)
knapsack (cw, cp, i);

cw -= w[i];
cp -= v[i];

4

3

Find the optimal packing of the knapsack using backtracking technique use proper bounding functions.

```
#include <iostream>
```

```
using namespace std;
```

```
int knapsack ( int w[], int v[], int n, int c, int cp,
    int cw, int &maxp )
```

```
{
```

```
    if (cw > c) return 0;
```

```
    if (cp > maxp) maxp = cp;
```

```
    for ( int i=n ; i>=1 ; i-- ) {
```

```
        int ub = cp + (c-cw)*v[i]/w[i] ;
```

```
        if (ub < maxp) return 0;
```

```
        cw += w[i] ;
```

```
        cp += v[i] ;
```

```
        if (cw <= c)
```

```
            knapsack (w,v,i-1,c,cp,cw,maxp);
```

```
        cw -= w[i] ;
```

```
        cp -= v[i] ;
```

DATE

--	--	--	--	--	--	--

return maxp;
}

```
int main () {  
    int n, c;  
    cout << "Enter the no. of items: ";  
    cin >> n;  
    int w[n+1], v[n+1];  
    cout << "Enter the weights and values of  
the items: \n";  
  
    for( int i=1; i<=n; i++)  
        cin >> w[i] >> v[i];  
    cout << "Enter the capacity of the knapsack: ";  
    cin >> c;  
  
    int maxp = 0;  
    knapsack (w, v, n, c, 0, 0, maxp);  
    cout << "The maximum profit is: "  
        << maxp << endl;  
  
    return 0;  
}
```

OUTPUT

Enter th

Enter th

2 10

3 20

4 30

5 40

Enter the

The maxim

Time Comp

The time

using ba

the num

either inc

Therefore,

of items -

OUTPUT:

Enter the no. of items: 4

Enter the weight and values of the items:

2 10

3 20

4 30

5 40

Enter the capacity of the knapsack: 7

The maximum profit is: 50.

Time Complexity:

The time complexity of the knapsack algorithm using backtracking is $O(2^n)$, where n is the number of items. There are two choices either include the item in the knapsack or exclude it. Therefore, there are 2^n possible combinations of items that can be considered.

4. Implement container loading problem using backtracking technique.

```
# include <iostream>
```

```
using namespace std;
```

```
const int MAX_N = 100;
```

```
int num_boxes, capacity;
```

```
int boxes[MAX_N];
```

```
void backtracking (int cur_capacity, int cur_box)
```

```
{
```

```
cout << "Solution: " ;
```

```
for (int i=0; i < cur_box; i++) {
```

```
cout << boxes[i] << " " ;
```

```
}
```

```
cout << endl;
```

```
return;
```

```
if (cur_capacity + boxes[cur_box] <= capacity) {
```

```
boxes[cur_box] = - boxes[cur_box];
```

```
backtracking (cur_capacity + abs(boxes[cur_box]),
```

```
cur_box + 1);
```

```
boxes[cur_box]
```

```
backtracking
```

```
}
```

```
int main()
```

```
cout << "
```

```
cin >> cap
```

```
cout << "
```

```
cin >> nu
```

```
cout << "
```

```
for (int i=0; i <
```

```
int bo
```

```
cin >>
```

```
boxes[i]
```

```
}
```

```
backtrackin
```

```
return 0;
```

```
}
```

using

```
    boxes[cur_box] = abs(boxes[cur_box]);  
    }  
    backtracking(cur_capacity, cur_box + 1);  
}
```

```
int main() {  
    cout << "Enter the capacity of the container: ";  
    cin >> capacity;
```

```
    cout << "Enter the number of boxes: ";  
    cin >> num_boxes;
```

```
    cout << "Enter the sizes of the boxes: ";  
    for (int i = 0; i < num_boxes; i++) {
```

```
        int box_size;
```

```
        cin >> box_size;
```

```
        boxes[i] = box_size;
```

```
}
```

```
backtracking(0, 0);
```

```
return 0;
```

```
}
```

OUTPUT:

Enter the capacity of the container: 50

Enter the number of boxes: 4

Enter the sizes of the boxes: 10 20 30 40

Possible loading configurations are:

10 20

10 30

10 40

10

20 30

20

30

40

Time Complexity:

The time complexity is $O(2^n)$, where the n is the number of items. This is because for each item, there are two possible choices: either include it in the container or exclude it.

BRANCH and BOUND

Lab Exercises :

40

1. Write a program to implement container loading problem using Max-Heap branch and bound technique.

```
# include <iostream>
# include <algorithm>
# include <vector>
# include <queue>

using namespace std;

struct item {
    int wt, vol;
    double density;
};

struct node {
    int level, value, wt, volume;
};

struct compare_node {
    bool operator() (const node &a, const node &b) {
        return a.value < b.value;
    }
};

int x, max_wt, max_val;
vector <item> items;
```

```

int containerloading() {
    sort(items.begin(), items.end(), []const item&a,
         const item&b) {
        return a.density > b.density; }
priority_queue<Node, vector<node>, compare_node> pq;

```

```

Node root = {0,0,0,0};
pq.push(root);
int max_value = 0;
while (!pq.empty()) {
    Node current = pq.top();
    pq.pop();

```

```

if (current.level == n) {
    max_value = max(max_value, current.value, container);
}
if (current.wt + items[current.level].volume
    <= max_volume) {
    Node left;
    current.level + 1, current.value, current.wt +
    items[current.level].wt = current.volume +
    items[current.level].volume;
    pq.push(left); }

```

```

return n;
int main()
cin >> items;
for (int i = 0; i < n; i++) {
    item[i] = {double} cin;
    int m = cout << i << endl;
}

```

OUTPUT

4 10

2 3

max

complex

worst

best

```
return max_volume;  
4  
int main() {  
    cin >> x >> max_wt >> max_vol;  
    items.resize(n);  
    for (int i=0; i<n; i++)  
        cin >> items[i].w >> items[i].volume;  
    items[i].density = (double) items[i].volume /  
        (double) items[i].volume  
    int max_value = containerloading();  
    cout << "Max value: " << max_value;  
}
```

OUTPUT:

4 10 20

2 3 4 6 5 7 3 4

max value: 9

Complexity:

worst case: $O(2^n)$

best case: $O(n)$

2. Implement TSP using Branch and Bound.

```
# include <iostream>
using namespace std;
#define V 4
int Tsp( int graph[][][V], int s ) {
    vector<int> vertex;
    for( int i=0; i<V; i++ ) {
        if( i != s )
            vertex.push_back(i);
    }
    int min-path = INT_MAX;
    int curr-pathwt = 0;
    int k = i;
    for( int i = 0; i < vertex.size(); i++ ) {
        current-pathwt += graph[k][vertex[i]];
        k = vertex[i];
    }
    current-pathwt += graph[k][s];
    min-path = min(min-path, curr-path);
    while( next_permutation(vertex.begin(),
                            vertex.end()) );
    return min-path;
}
```

```
int main() {  
    int graph [][][V];  
    graph [][][V] = {{ { 0, 10, 15, 20 },  
                      { 10, 0, 35, 25 },  
                      { 15, 35, 0, 30 },  
                      { 20, 25, 30, 0 } };
```

```
    int i = 0;  
    cout << "Min cost: " << endl;  
    cout << TSP(graph, s);  
    return 0;
```

4

OUTPUT

min cost = 80

Complexity:

$O(n^2)$:

3. Max Clique of a graph

```
#include <stdio.h>
#include <iostream>
#define v 4
```

```
int maxclique size;
bool isSafe (int v, bool graph [v] [v], int
cliquesize, int clique []) {
for (i=0 ; i<cliquesize ; i++) {
if (graph [v] [clique [e]] == false)
return false;
}
return true;
```

4

```
void maxClique (bool graph [v] [v], int clique [],
int index, int cliquesize) {
if (!index == v) {
if (!uniquesize > maxcliquesize)
maxclique = cliquesize;
```

4

```
if (issafe (index, graph, cliquesize, clique))
clique [clique .size ] = index;
maxclique (graph, clique, index+1, cliquesize+1);
if (v-index-cliquesize > maxcliquesize)
maxcliqueutil (graph clique, index+1, cliquesize);
```

```
int maxclique (6
maxcliquesize =
int clique [v];
clique [0] = 0
maxclique util (
return maxclique
```

4

```
int main () {
bool graph [v]
```

```
cout << maxclique
return 0;
```

4

OUTPUT

maxclique = {

Time complexity:
 $O(2^n)$

```

int maxclique (bool graph [v][v]);
maxcliquesize = 1;
int clique [v];
clique [0] = 0;
maxclique util (graph, clique, 1, 1);
return maxcliquesize;

```

4

```

int main () {
    bool graph [v][v] = {{0,1,1,0,0},
                          {1,0,1,1,0},
                          {1,1,0,1,1},
                          {0,1,1,0,1},
                          {0,0,1,1,0}};
    cout << maxclique (graph);
    return 0;
}

```

4

OUTPUT

maxclique = {1, 2, 3, 4}

Time Complexity:

$O(2^n)$

APPROXIMATE ALGORITHMS

Lab Exercises

1. Program to find vertex cover of Minimum size in given directed graph.

```
# include <iostream>
# include <vector>
# include <string>
using namespace std;
#define max 100
vector<int> adj[max];
bool visited [max];
int vertex_cover[max];

void dfs (int u) {
    visited [u] = true;
    for (int v: adj [u]) {
        if (!visited [v]) {
            dfs [v];
            vertex_cover [u] = 1;
            vertex_cover [v] = 1;
        }
    }
}
```

```
int main () {  
    int n, m;  
    for ( i=0 ; i<m ; i++ ) {  
        int u, v;  
        adj [u] . push - back [v];  
        adj [v] . push - back [u];  
        memset ( visited , false , sizeof ( visited ) );  
        memset ( vertex , cover , 0 , sizeof ( vertex - cover ) );  
        for ( int i=1 ; i<=n ; i++ ) {  
            if ( ! visited [i] ) {  
                dfs [i];  
                cout << "vertex cover" ;  
                for ( int i=1 ; i<=n ; i++ ) {  
                    if ( vertex - cover [i] ) {  
                        cout << i ;  
                    }  
                }  
            }  
        }  
        return 0;  
    }  
}
```

OUTPUT :

6 7

1 2

1 3

2 3

2 4

3 4

4 5

4 6

vertex cover : 4

Time Complexity :

$O(n^3)$.

2. Program implementing TSP problem

```
#include <iostream>
#include <vector>
#include <cmath>
using namespace std;
const int INF;
int n;
double adj[20][20];

int findminkey(double key, bool mst-set[]) {
    double min-val = INF;
    int min-idx = -1;
    for (i=0; i<n; i++) {
        if (mst-set[i] && key[i] < min-val) {
            min-val = key[i];
            min-idx = i;
        }
    }
    return min-idx;
}

double tsp-mst() {
    double mst-cost = 0.0;
    vector<double> key (n, INF);
    vector<bool> mst-set (n, false);
    key [0] = 0.0;
```

```
for (i=0; i<n-1; i++)  
    int u = find_minkey(key.data(), mst-set,  
                         data());  
    mst-set[u] = true;  
    for (int v=0; v<n; v++)  
        if (adj[u][v] && mst[v] && adj[v][u]<  
            key[v])  
            key[v] = adj[u][v];  
    }  
    for (int i=1; i<n; i++) {  
        min-cost += adj[i][parent[i]];  
    }  
    return mst-cost;  
}  
  
int main() {  
    for (int i=0; i<n; i++) {  
        for (int j=0; j<n; j++) {  
            cin >> adj[i][j];  
        }  
    }  
    double mst-cost = tspmst();  
    cout << mst-cost;  
    return 0;  
}
```

OUTPUT

0	10	20	30	40
10	0	25	35	15
20	25	0	45	30
30	35	45	0	50
40	13	30	50	0

mst cost = 80

tsp cost = 160

Time Complexity: $O(n^2)$.