Risk-Based Portfolio Optimization in the Cryptocurrency World*

Tobias Burggraf†

October 31, 2019

Abstract

This study explores the performance of seven state-of-the-art risk-based portfolio optimization strategies from the perspective of a cryptocurrency investor. Analyzing the inverse volatility, minimum variance, l2-norm constrained minimum variance, l2-norm constrained maximum decorrelation, maximum diversification and risk parity portfolio, we find that most strategies systematically outperform individual cryptocurrencies and the equally-weighted benchmark portfolio. Further, a bull and bear market performance comparison as well as tail, extreme risk, and diversification analyses reveal that these strategies provide significant downside risk reduction. The results are robust to using different estimation windows, rebalancing periods and covariance estimation methodologies. Finally, our empirical results indicate that the maximum decorrelation portfolio is the worst strategy in terms of risk-adjusted return, while the long-only minimum variance portfolio is the best performing strategy.

Keywords: Portfolio Optimization, Cryptocurrencies, Investments

JEL Classification: G15, G41

*I am grateful to Markus Rudolf and Mei Wang for their encouragement and support. I thank Markus Schmid, Marc Oliver Rieger, Frank Riedel, Ralf Korn, Nabil Alkafri, Toan Luu Duc Huynh as well as seminar participants of the 4th Symposium "Quantitative Finance and Risk Analysis" for helpful comments and discussions. I am responsible for remaining errors.

†WHU – Otto Beisheim School of Management, Allianz Endowed Chair of Finance, Vallendar, Germany. Email: Tobias.burggraf@whu.edu.

1 Introduction

The 2018 cryptocurrency crash has been devastating for many investors. After an unprecedented rally led by the world's largest cryptocurrency Bitcoin in 2017, Bitcoin prices peaked on December 17, 2017 at $19,783.06. Only five days later, Bitcoin plunged below $11,000, down 45%. And the sell-off continued – From 6 January to 6 February Bitcoin fell by another 65% to $6,000, with almost all other cryptocurrencies following. Overall, Bitcoin has lost more than 80% of its market value since its peak, making the crash worse than the 2000 Dot-com bubble burst[1]. Throughout 2018, cryptocurrency prices remained at relatively low levels and stabilized only recently. Figure 1 shows the evolution of Bitcoin over time.

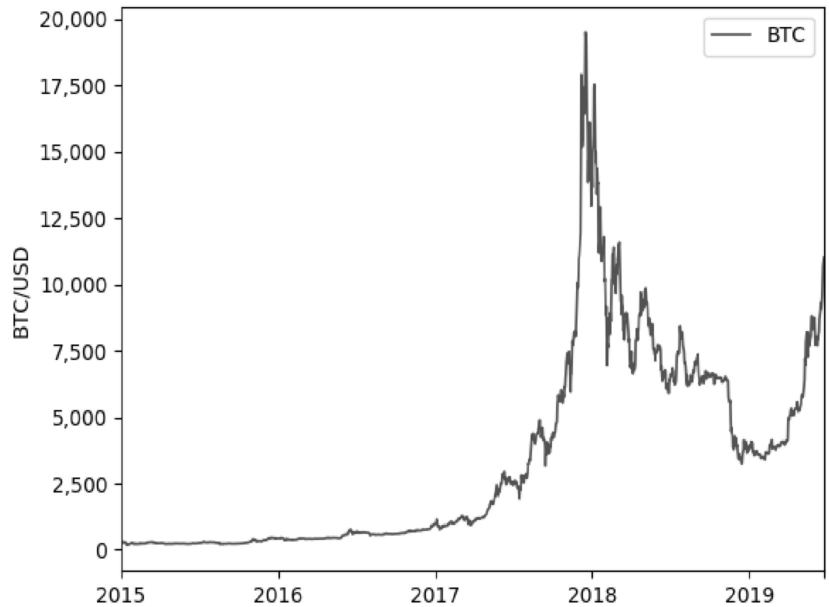


Figure 1: Bitcoin price evolution and 2018 cryptocurrency crash. The figure plots the evolution of the exchange rate for the Bitcoin (BTC) cryptocurrency to US Dollar (USD) for the period January 1, 2015 to June 24, 2019.

The disastrous results reminded many investors of the 2008 financial crisis. Back then, investors had to question themselves what went wrong and whether the impact on portfolio performance could have been less severe if portfolio allocation schemes other than those following the theories of modern portfolio theory (MPT) would have been applied. As a result, classical models such as mean-variance optimization (MVO) or the 60/40 portfolio[2] have been put on trial by practitioners and critics alike for their apparent underdiversification and accused failure to provide risk con-

[1]The Nasdaq Composite Index declined 78% from its peak after the dot-com bubble burst in the year 2000 (Patterson, 2008).
[2]The 60/40 portfolio relates to a portfolio consisting of 60% stocks and 40% bonds.

trol, with some practitioners even claiming to displace MPT altogether (W. Lee, 2011; Liesching, 2010). What went wrong – the same question must be asked today. While we are sceptic that many cryptocurrency investors construct their portfolios by means of classical Markowitz mean-variance optimization, the large market capitalization of Bitcoin relative to other cryptocurrencies indicates that many investors held largely underdiversified portfolios and that better risk and diversification control may have helped to mitigate one of the worst financial crashes of all time.

In this study, we examine seven alternative portfolio construction schemes, often labeled "risk-based asset allocation". A property these "new paradigm" solutions have in common is their focus on reducing risk while increasing diversification. Instead of relying on expected return estimations, risk-based approaches rely on estimates of the covariance matrix only. This makes the portfolio construction totally insensitive to errors in the expected returns estimates. Several previous studies show that these portfolios outperform on a risk-adjusted basis both the market capitalization-weighted portfolio and portfolios that are constructed to be ex-ante mean-variance optimal using the Markowitz optimization algorithm (see, for example, Behr et al., 2008; R. Clarke et al., 2011; Deguest et al., 2013; DeMiguel et al., 2007).

While there is a rich body of literature on the performance of the various risk-based portfolio strategies applied to traditional asset classes, there is surprisingly little research for other assets and even less for alternative investments. This study attempts to fill this gap. To the best of our knowledge, this is the broadest study to examine risk-based asset allocation strategies from a cryptocurrency investor perspective. With previous studies applying risk-based optimization to a global universe of assets, we propose applying them to a portfolio of cryptocurrencies only. By shifting the focus from traditional assets towards pure cryptocurrency investments, we believe that investors can better exploit their low linear dependency structure and correlation with each other. In the next step, investors could then test whether adding an optimized cryptocurrency portfolio to a portfolio of traditional assets improves its performance.

This study contributes to the literature in the following ways. First, we provide an extensive analysis of a portfolio of thirteen cryptocurrencies, exploring both their individual risk-return characteristics as well as their diversification benefits in a portfolio context. Second, we provide a detailed comparison of the seven most popular risk-based asset allocation schemes. While previous studies have covered the analytical part, the focus of this study is more on the economic intuition behind each strategy and their relationship to mean-variance optimized portfolios. Third, we compare the performance of the risk-based asset allocation strategies by evaluating each strategy using a large number of well-established risk, return and diversification metrics. Finally, in order to test the robustness of our results, we also compare the results to daily, monthly and quarterly rebalancing; two, three and four years covariance estimation windows; as well as alternative covariance estimation techniques including shrinkage and constant correlation estimation.

The remainder of this study is structured as follows. Section 2 briefly reviews the empirical literature. Section 3 gives an overview on the various risk-based optimization methodologies. Section 4 introduces the data and discusses the descriptive statistics. Section 5 covers the empirical results from our risk-based portfolio optimizations, including tail- and extreme risk analysis, portfolio diversification metrics as well as bull and bear market comparison. Section 6 provides robustness tests and Section 7 concludes.

2 Literature Review

In this section, we briefly review the empirical literature on risk-based asset allocation approaches and cryptocurrency portfolio optimization strategies. In the next sections, we then investigate whether combining both strands can help to improve the overall portfolio performance of a cryptocurrency investor.

2.1 Risk-based asset allocation

The mean-variance approach of Markowitz (1952) has been the standard asset allocation approach in the asset management industry for decades. Despite its theoretical appeal and rationality, it does not hold up well in practice, leaving investors with ex-ante efficient portfolios rather than ex-post efficient portfolios (Broadie, 1993; DeMiguel et al., 2007). The disappointing results often are consequences of estimation errors in the input parameters, especially in the expected returns, which are notoriously difficult to estimate. Because it has been shown that even small changes in the expected return input assumptions lead to very different efficient portfolios, the Markowitz mean-variance optimization approach has been referred to as an "Error Maximization" procedure (Michaud, 1989) and is the major reason it is very rarely applied in practice.

As a consequence, a growing amount of literature has suggested replacing modern portfolio theory and traditional mean-variance optimization by alternative asset allocation solutions. One thing these "new paradigms" have in common is that they focus on risk and diversification, rather than on estimating expected returns. The most well-known alternative to the mean-variance portfolio is the equally-weighted portfolio, which assigns the same weight to all the assets considered in the portfolio. These portfolios are widely used in practice (Benartzi & Thaler, 2001; Windcliff & Boyle, 2004) and have shown promising out-of-sample results (DeMiguel et al., 2007; Plyakha et al., 2015). According to their studies, the performance of the equally-weighted portfolio is significantly better than that of a value-weighted portfolio, and no worse than that of portfolios from a number of optimal portfolio selection models.

Another popular approach includes the minimum variance portfolio (R. Clarke et al., 2011; Markowitz, 1952). Although it can be derived from the mean-variance optimization approach (Jagannathan & Ma, 2003), its unique property of not requiring information about the expected returns makes it easy to compute and particularly popular among market practitioners. An extensive body of literature proves the advantage of the minimum variance portfolio over other asset allocation strategies. R. G. Clarke et al. (2006) and Behr et al. (2008) find that the minimum variance portfolio outperforms the market capitalization-weighted index, demonstrating higher returns, lower volatility and therefore better risk-adjusted performance.

The maximum decorrelation portfolio (Christoffersen et al., 2012) builds upon the minimum variance portfolio, but instead of minimizing variance, it aims at minimizing the correlation among the assets in the portfolio. This reduces the number of input parameters required for the optimization, and thereby the problem of estimation errors.

Another closely related but more heuristic approach is the inverse volatility portfolio. Due to its analytical simplicity and its intuitive appeal – it assigns a weight for each asset that is inversely proportional to its volatility – it is sometimes referred to as "naive" risk parity. While it is one of the most popular approaches in practice, the academic literature on this approach is relatively limited, with Maillard et al. (2008) being the major reference studying the theoretical properties of inverse volatility portfolios.

An additional, more recent approach is the maximum diversification portfolio introduced by Choueifaty et al. (2013). The authors interpret diversification as the ratio of the weighted average of the volatilities of assets to the volatility of the portfolio of the same assets. The maximum diversification portfolio maximizes the diversification ratio. They find that the maximum diversification portfolio significantly outperforms the market capitalization-weighted portfolio as well as the minimum variance and equally-weighted portfolio, delivering both higher returns and lower volatilities.

Finally, risk parity, or the so-called equally-weighted risk contribution portfolio, puts the diversification of risk at its core. First introduced by Qian (2005), it attempts to equalize the risk contribution from each portfolio component, therefore maximizing (ex-ante) risk diversification. Maillard et al. (2010) argue that the risk parity portfolio is an attractive alternative to the minimum variance and equally-weighted portfolio and that it might be considered a good trade-off between those two approaches in terms of absolute level of risk, risk budgeting and diversification. While risk parity strategies have become a popular tool for investors due to their easy-to-understand and intuitive narrative, they also gained significant traction in the portfolio literature (see, for example, Bruder & Roncalli, 2012; Foresti & Rush, 2010; Galane, 2014; Levell, 2010; Maillard et al., 2010; Qian, 2005; Roncalli, 2016), which for the most part report convincing results in terms of risk and diversification compared to alternative risk-based approaches.

Generally speaking, the studies presented aim to minimize risk while achieving the benefits of diversification. Despite these characteristics that are common at the heart of all the different portfolio construction methods, the above risk-based approaches rely on different meanings and definitions of that concept, which often result in significantly different portfolio compositions and come with their own benefits and drawbacks. In addition, the literature finds significantly different results in terms of portfolio performance – while one study favors a particular method, another finds that it actually underperforms. Accordingly, the goal of this paper is to compare different risk-based asset allocation strategies in the context of cryptocurrencies and determine which method to use in order to guarantee an efficient risk-return allocation. Table 1 provides an overview of the risk-based asset allocation literature, including seminal publications as well as a summary of the main advantages and disadvantages of each strategy.

Table 1: Overview of the risk-based asset allocation literature

Year	Author(s)	Method	(+)	(−)
1952	Markowitz	Mean-variance	Theoretically and analytically sound; guaranteed optimal solution due to the convex nature of the problem	Expected returns are notoriously difficult to estimate; disappointing out-of-sample performance
1952	Markowitz; Jagannathan & Ma; R. Clarke et al.	Minimum variance	Merely requires an estimate of the variance-covariance matrix as the only input; only (ex-ante) risk-based strategy on the efficient frontier	Often yields highly concentrated and poorly diversified portfolios; sensitive to estimation errors in both volatilities and correlations
2001	Benartzi & Thaler; Windcliff & Boyle	Equally-weighted	No objective function; can be uniquely determined; easy to implement; least concentrated portfolio; often beats optimized portfolios out-of-sample; useful to use as a benchmark	Heuristic; leads to risk concentrations if the individual risks are very different; ex-ante efficient only under the (unrealistic) assumption of equal expected returns, equal volatilities and uniform correlations
2005	Qian; Maillard et al.; Bruder & Roncalli	Risk parity	Less sensitive to small changes in the covariance matrix than the MV portfolio; often yields more reasonable and intuitive portfolios as investors can specify preferred risk contribution ax-ante	No guarantee that the numerical risk parity portfolio solution achieves global optimality, therefore somewhat heuristic
2008	Maillard et al.	Inverse volatility	Intuitiv and computationally simplistic; avoids estimation errors in the correlation matrix	Disregards differences in the structure of the cross-correlation; concentration in low volatility assets

Continued on the next page.

Table 1: Overview of the risk-based asset allocation literature (cont.)

Year	Author(s)	Method	(+)	(–)
2012	Christoffersen et al.	Maximum decorrelation	Exploits risk reduction effects from low correlations, rather than reducing risk by concentrating in low volatility assets	Can result in high asset loadings by focussing on assets with low correlations to other assets; ignores individual volatilities
2013	Choueifaty et al.	Maximum diversification	Only strategy that takes diversification directly into consideration; idea of creating a portfolio that is as diversified as possible is widely accepted and an essential consideration for every investor	Diversification is subject to interpretation and different definitions; can be relatively concentrated when judged by alternative definitions of diversification; no clear investment objective function

Note: This table presents an overview of the risk-based asset allocation literature in chronological order. Year is the year when the method has been first published, Author(s) are the authors, where the first name corresponds to the author who has established the methodology. Other authors are included when they made major contributions to a particular strategy. Method is the main method or idea, (+) is a summary of the main advantages and (–) of the main disadvantages of the respective asset allocation strategy.

2.2 Cryptocurrency portfolio optimization

With cryptocurrencies being considered a distinct asset class and an alternative to traditional assets (Glas, 2019; Krueckeberg & Scholz, 2018), cryptocurrency markets are also becoming increasingly attractive for investors looking for alternative sources of investment income. A natural question then arises about the performance of cryptocurrencies both on a standalone basis and in a portfolio management context. Early research mainly focused on the performance of Bitcoin. For example, Briere et al. (2015) analyze a Bitcoin investment from the standpoint of an US investor with a diversified portfolio including both traditional assets and alternative investments. They find that the inclusion of even a small proportion of Bitcoin dramatically improves portfolio performance. Similarly, Eisl et al. (2015) show that including Bitcoin in optimized portfolios improves portfolio diversification and risk-adjusted returns. More recent studies focused on the effects of adding cryptocurrencies to traditional asset portfolios. Chuen et al. (2017) were among the first to investigate the inclusion of cryptocurrencies represented by the Cryptocurrency Index (CRIX) to a portfolio that consists of traditional assets such as S&P 500, private equity (PE), real estate investment trusts (REITs) and gold. They find that adding CRIX significantly improves overall portfolio risk–return performance. Trimborn et al. (2017) considered including cryptocurrencies into a portfolio consisting of S&P 100, US Bonds and commodities. Introducing Liquidity Bounded Risk-return Optimization (LIBRO), they show that the optimized portfolio efficiently protects investors from the risk of an inability to trade due to low trading volume. Finally, Petukhina et al. (2018) study whether adding cryptocurrencies in addition to five traditional asset classes (equities, fixed-income, fiat currencies, commodities and real estate) and subsequently optimizing that portfolio improves its out-of-sample performance, clearly finding that cryptocurrencies are important, non-redundant additions to their investment universe.

While several studies covered the risk and diversification effects of adding cryptocurrencies to a portfolio of traditional assets, to the best of our knowledge, there has been no study yet that analyzes the potential of portfolio optimization from the perspective of a *pure cryptocurrency investor* aiming at exploiting their well-documented characteristic of low linear dependency with each other (Elendner et al., 2018). This study attempts to fill this gap by applying a variety of risk-based asset allocation strategies to a portfolio of cryptocurrencies. There are three major reasons that prompt us to use risk-based strategies over traditional risk-return-based strategies. First, cryptocurrencies are a new asset class that is dominated by noisy data, large volatility and short time-series. This makes the already challenging task of estimating expected returns even more difficult. Avoiding estimating expected returns altogether in order to reduce the number of inputs is one common approach in the investment literature that promises more robust weight allocations and less sensitive efficient portfolios. Second, the academic literature shows that risk-based strategies often outperform standard asset allocation models such as the Markowitz mean-variance model on an out-of-sample basis, which is partly due to unrealistic normality assumptions and high sensitivity to estimation errors in the expected returns. Third, given the well-documented, risk-related properties of cryptocurrencies such as long memory and stochastic volatility (Phillip et al., 2019), price clustering (Urquhart, 2017), non-normal higher moments, extreme tail risk (Gkillas & Katsiampa, 2018), as well as the general feature of being considered speculative (Fry & Cheah, 2016; Urquhart, 2016), selecting risk-based models that explicitly aim to reduce risk is a natural choice which we believe will deliver significantly improved portfolio performance. However, academic literature in this field is extremely limited. This is particularly surprising given the vast amount of literature confirming that risk-based strategies applied to other asset classes often outperform on an out-of-sample basis (see, for example, Choueifaty & Coignard, 2008; R. G. Clarke et al., 2006; DeMiguel et al., 2007)

In this section, we reviewed both strands of empirical literature and motivated why risk-based strategies are a powerful tool for cryptocurrency investors. In the next section, we go into detail of the various asset allocation models, where the focus will be on the economic intuition. For more rigorous mathematical derivations, please refer to the references in Table 1.

3 Methodology

Traditionally, portfolios are constructed by maximizing expected return based on a given level of investor risk. Despite its simplicity and intuitive appeal, portfolio managers have found traditional mean-variance asset allocation models (MV) difficult to use in practice because of the difficulty of estimating expecting returns, which often yield unstable and unreasonable portfolio weights. In contrast, risk-based asset allocation strategies rely on estimates of the covariance matrix as the only input parameter. The level of information required for the optimization problems is different, however, and depends on whether the investor trusts his own parameter estimates. While minimum variance-, maximum diversification- and risk parity portfolios require information on standard deviations and correlations, inverse volatility portfolios only require information on standard deviations. Equally weighted portfo-

Table 2: Overview risk-based optimization strategies

Abbr.	Strategy	Optimization
EW	Equally weighted	$w_{ew} = \frac{1}{N}, \forall i = 1, ..., N$
IV	Inverse volatility	$w_{iv} = \frac{1/\sigma_1}{\sum_{i=1}^{N} 1/\sigma_i}, ..., \frac{1/\sigma_N}{\sum_{i=1}^{N} 1/\sigma_i}$
MV	Minimum variance	$w_{mv} = \underset{w \in \mathbb{R}}{\arg\min}\ w' \Sigma w$
MVN	l2 Minimum variance	$w_{mvn} = \underset{w \in \mathbb{R}}{\arg\min}\ w' \Sigma w \quad s.t. \|\|w\|\|_2^2$
MCN	l2 Maximum decorrelation	$w_{mcn} = \underset{w \in \mathbb{R}}{\arg\min}\ w' \Omega w \quad s.t. \|\|w\|\|_2^2$
MD	Maximum diversification	$w_{md} = \underset{w \in \mathbb{R}}{\arg\max}\ \frac{\sum_{i=1}^{N} w_i \sigma_i}{\sqrt{w' \Sigma w}}$
RP	Risk parity	$w_{rp} = \underset{w \in \mathbb{R}}{\arg\min} \sum_{i=1}^{N} \sum_{j=1}^{N} (w_i(\Sigma w)_i - w_j(\Sigma w)_j)^2$

Note: The table provides an overview of the risk-based portfolio optimization strategies. For each optimization, the strategy's abbreviation (Abbr.), the strategy's name (Strategy) and the optimization algorithm (Optimization) is provided. All optimizations are subject to no-short selling ($w_i \geq 0$) and budget ($\sum_{i=1}^{N} w_i = 1$) constraints. MVN and MCN are also subject to l2-norm constraints ($||w||_2^2 = \sum_{i=1}^{N} w_i^2 \leq 3/N$).

lios require neither estimates about expected returns nor about standard deviations or correlations.

We employ seven risk-based portfolio construction strategies, including inverse volatility portfolios (IV), minimum variance portfolios (MV), minimum variance under norm constraints portfolios (MVN), maximum decorrelation under norm constraints portfolios (MCN), maximum diversification portfolios (MD), and risk parity / equal risk contribution portfolios (RP). Their performance is tested against the equally weighted (EW) cryptocurrency portfolio. All strategies are subject to no-short selling constraints ($w_i \geq 0$) and budget constraints ($\sum_{i=1}^{N} w_i = 1$). Following Kremer et al. (2017), MVN and MCN are also subject to l2-norm constraints ($||w||_2^2 = \sum_{i=1}^{N} w_i^2 \leq 3/N$)[3]. Adding this non-linear constraint comes with the advantage of (i) increasing the number of constituents in the portfolio and (ii) dealing better with the problem of multicollinearity. A summary of all methodologies, their corresponding abbreviations and optimization objectives is given in Table 2.

[3] L2 regularization, or ridge regularization, as it is known in the machine learning literature (Hastie et al., 2005; Robert, 2014), is a technique to reduce model complexity and prevent over-fitting. This introduces a small bias, but at the same time reduces the variance of the estimates.

In the following, we briefly discuss the characteristics of the seven risk-based portfolio construction methodologies applied in this study. We limit ourselves to theoretically sound and published asset allocation approaches.

3.1 Equally weighted portfolio

The equally weighted portfolio assigns equal weights to each asset in the investment universe. Because it is straightforward to implement, it is sometimes referred to the "1 over N" or "naive" portfolio and is constructed as follows:

$$w_i = \frac{1}{N}, \forall i = 1, ..., N \tag{1}$$

where w_i is the weight allocated to the ith asset and N is the number of assets in the portfolio. Since the EW portfolio invests an equal amount of wealth in each of the N assets, it is the least concentrated portfolio when it comes to portfolio weights. However, as Braga (2015) points out, the assumption that using the simple $1/N$ heuristic guarantees diversification can be misleading – for example, if the risk of the individual assets is extremely different, the EW portfolio can lead to highly concentrated risk loadings. Furthermore, from (1), it follows that the $1/N$ portfolio coincides with the *ex-ante* Markowitz mean-variance efficient portfolio only if all assets have equal expected returns, equal volatilities and uniform correlations. Although it is extremely difficult to make the case for ex-ante optimality, a long strand of literature found that the EW strategy consistently outperforms more sophisticated strategies *ex-post*. For example, DeMiguel et al. (2007) find that no portfolio weighting strategy consistently performs better than the $1/N$ rule in terms of Sharpe ratio, certainty-equivalent return, or turnover. They conclude that there are still many "miles to go" before the gains promised by optimal portfolio choice can actually be realized out-of-sample.

Strictly speaking, the EW portfolio is not a risk-based strategy, as it requires no parameter estimates for risk or correlations nor an objective function to be optimized. However, in line with the literature, we include the EW strategy and classify it risk-based for one major reason. That is, investors typically consider the EW portfolio a defense strategy by allocating wealth equally across assets, therefore trying to improve diversification and consequently reduce risk. This goal is in line with the general objective of risk-based portfolio optimization strategies and the reason why it will serve as our benchmark strategy.

3.2 Inverse volatility portfolio

The inverse volatility portfolio sets each asset weight of the N assets proportional to the inverse of their volatility and is then normalized such that the portfolio weights sum to one. Therefore, the weight for each asset is obtained by

$$w_i = \frac{1/\sigma_i}{\sum_{i=1}^{N} 1/\sigma_i}, \forall i = 1, ..., N \tag{2}$$

where w_i is the weight allocated to the ith asset and σ_i is the volatility of asset i. One major advantage of the IV portfolio stems from its computationally simplicity and its intuitive appeal – From (2), we can easily see that the IV strategy allocates more (less) weight to asset i, if the volatility of that asset is low (high). Hence, the aim of the IV strategy is to control portfolio risk. However, since IV relies on volatility estimates while disregarding the correlation matrix as an input parameter, investors cannot exploit the differences between cross-correlations in the weighting of their portfolio (Kremer et al., 2018). Maillard et al. (2008) show that this can have significant effects on individual weight allocations, especially when both negative and positive correlations exist. Hence, IV only provides true homogeneity in asset risk contributions for two specific cases, namely when pairwise correlations across assets are equal and ignoring correlations has no impact, i.e. if $\rho_{i,j} = \rho$ for all $i \neq j$; or when the portfolio consists of only two assets.

3.3 Minimum variance portfolio

The minimum variance portfolio allocates the weights of the N assets such that the portfolio variance σ_P^2 is minimized. It follows from the Markowitz mean-variance framework if expected return estimates are disregarded and the weights are restricted to be larger than zero. It is located at the left most end of the efficient frontier and is the only portfolio on the efficient frontier which does not require knowledge about expected returns, but can be determined by performing an optimization merely using an estimate of the covariance matrix. The minimum variance portfolio is computed by solving the following optimization problem:

$$\begin{aligned} w_i = \underset{w \in \mathbb{R}}{\arg\min}\ w'\Sigma w \\ s.t. \\ w_i \geq 0 \\ \sum_{i=1}^{N} w_i = 1 \end{aligned} \tag{3}$$

where w is the $N \times 1$ vector of asset weights and Σ is the $N \times N$ variance-covariance matrix. The optimization in (3) is a quadratic programming problem with inequality constraints, which can only be solved numerically. While it can be easily seen that (3) should yield the portfolio with the lowest possible volatility, one disadvantage of the MV portfolio comes from the fact that most risk reduction is accomplished by concentrating in assets with low volatility and / or low correlations with other assets. This results in extreme loadings to specific assets, which often yield highly concentrated and poorly diversified portfolios (for more details, see for example R. Clarke et al., 2011; Elton et al., 2009; Scherer, 2011). As a result, the MV strategy is particularly sensitive to volatilities and correlations estimates and small estimation errors in the covariance matrix input parameter can have deteriorating impact on the performance of the optimized portfolio.

3.4 Minimum variance under norm constraints portfolio

In order to address the problem of extreme weight allocation, the minimum variance under norm constraints portfolio adds the non-linear l2 constraint that the number of active positions must be at least one-third[4] of the total number of constituents in the portfolio. Therefore, the portfolio weights are obtained by solving the following minimization problem:

$$\begin{aligned} w_i = \arg\min_{w \in \mathbb{R}} \; & w' \Sigma w \\ s.t. & \\ w_i \geq\; & 0 \\ \sum_{i=1}^{N} w_i =\; & 1 \\ ||w||_2^2 = \sum_{i=1}^{N} w_i^2 \leq\; & \frac{3}{N} \end{aligned} \tag{4}$$

where w is the $N \times 1$ vector of asset weights, Σ is the $N \times N$ variance-covariance matrix and $|| \cdot ||_2$ is the flexible concentration constraints on the 2-norm. From (4), it is straightforward to see that the MVN strategy is closely related to the MV strategy but adds an additional l2 constraint to the optimization. This additional constraint works as a flexible lower bound on the effective number of constituents and aims to improve on the well-known problem of high asset concentration by shrinking the portfolio weights closer towards zero and promoting sparse solutions. In addition,

[4]The number of active positions can be arbitrarily selected. In this study, we follow Kremer et al. (2018) and set $||w||_2^2 = 1/3$.

as Kremer et al. (2018) indicate, the use of flexible l2-norm constraints, in contrast to rigid upper and lower bounds on individual weights, allow for a better use of the correlation structure and improves out-of-sample risk and return properties.

3.5 Maximum decorrelation under norm constraints portfolio

The maximum decorrelation portfolio is closely related to the minimum variance portfolio, but attempts to reduce the number of input parameters. Instead of using the variance-covariance matrix Σ, the strategy assumes that individual asset volatilities are identical and solely uses the correlation matrix Ω as its main input. Hence, the optimal portfolio weights are given by the following optimization problem:

$$\begin{aligned} w_i = \underset{w \in \mathbb{R}}{\arg\min}\ w'\Omega w \\ s.t. \\ w_i \geq 0 \\ \sum_{i=1}^{N} w_i = 1 \\ ||w||_2^2 = \sum_{i=1}^{N} w_i^2 \leq \frac{3}{N} \end{aligned} \tag{5}$$

where w is the $N \times 1$ vector of asset weights, Ω is the $N \times N$ correlation matrix and $|| \cdot ||_2$ is the flexible concentration constraints on the 2-norm. Therefore, in contrast to the MV and MVN portfolio, which attempts to reduce risk by concentrating in low volatility constituents, the MDN portfolio rather tries to exploit risk reduction effects stemming from investing in assets with low correlations. While it avoids concentration in certain assets by ignoring differences in individual volatilities, the strategy can still result in high asset loadings by focusing on assets with low correlations with other assets. Therefore, we again use flexible l2-norm constraints for the optimization problem to improve upon the problem of high weight concentration.

3.6 Maximum diversification portfolio

The maximum diversification portfolio aims at constructing a portfolio that is as diversified as possible. More precisely, it attempts to maximize the diversification ratio, which is defined as the ratio of the weighted average of volatilities divided by the portfolio volatility (Choueifaty & Coignard, 2008). Mathematically, the maximum diversification portfolio is constructed by solving the following optimization problem:

$$
\begin{aligned}
w_i = \underset{w \in \mathbb{R}}{\arg\max}\ \frac{w'\sigma}{\sqrt{w'\Sigma w}} \\
s.t. \\
w_i \geq 0 \\
\sum_{i=1}^{N} w_i = 1
\end{aligned}
\tag{6}
$$

where w is the $N \times 1$ vector of asset weights, Σ is the $N \times N$ covariance matrix and σ is the $N \times 1$ vector of asset volatilities. The term in the numerator in (6) is the weighted average volatility of the assets, the term in the denominator is the volatility of the portfolio. Because the portfolio's standard deviation is always less or equal to the weighted average volatility, the diversification ratio of any long-only portfolio will be strictly greater than one for all portfolios that include not perfectly correlated assets. In essence, the MD portfolio quantifies how much higher the risk of the portfolio would be if all constituents were perfectly correlated. This comes from the fact that the numerator of the diversification ratio computes portfolio volatility as if all pair-wise correlations were equal to one (Braga, 2015). Furthermore, and in contrast to the MV and MVN portfolio, which measures diversification in absolute terms, the MD portfolio quantifies diversification in relative terms, which eventually results in different weight allocations. Finally, from (6), it follows that the maximum diversification portfolio is mean-variance efficient if the Sharpe ratios for all assets are equal, i.e. the expected excess returns are proportional to their volatility.

3.7 Risk Parity portfolio

The risk parity portfolio or equally-weighted risk contributions (ERC) portfolio is the portfolio for which all assets' contributions to portfolio risk are equalized. It mimics the diversification effect of the EW portfolio while taking into account single and joint risk contributions of the assets. Or, to put it simply: The goal of the RP portfolio is that no asset contributes more to the total risk of the portfolio than any other asset. The theoretical foundations of the RP approach have been extensively investigated in the portfolio literature (Maillard et al., 2010; Neukirch, 2008; Qian, 2005). It starts with the assets' marginal risk contribution (MRC) to portfolio risk, which is defined as follows:

$$MRC_i = \frac{\partial \sigma_p}{\partial w_i} = \frac{w_i \sigma_i^2 + \sum_{i \neq j} w_j \sigma_{i,j}}{\sigma_p} \tag{7}$$

where w_i is the weight of asset i, σ_p is the portfolio volatility and $\sigma_{i,j}$ is the covariance between asset i and j. Hence, MRC gives the change in volatility of the portfolio induced by an infinitesimal increase in the weight of one asset. Premultiplying (7) with w_i, we get[5]

$$TRC_i = \sigma_{p,i} = w_i \frac{\partial \sigma_p}{\partial w_i} \tag{8}$$

which is the total risk contribution (TRC) of the ith asset to portfolio risk. We then obtain the total portfolio risk as follows:

$$\sigma_p = \sum_{i=1}^{N} \sigma_{p,i} = \sum_{i=1}^{N} w_i \frac{\partial \sigma_p}{\partial w_i} = \sum_{i=1}^{N} TRC_i \tag{9}$$

Hence, the volatility of a portfolio is simply the sum of the total risk contributions of its constituents. Starting from (8), the idea of RP strategies is to find a risk-balanced portfolio such that the risk contribution is the same for all assets of the portfolio (Maillard et al., 2010). While equations (7) – (9) allow for easy and intuitive interpretation of the RP approach, they do not provide a closed-form solution. We therefore need to apply a numerical algorithm that implements an iterative process. Following Maillard et al. (2010), we solve the optimization problem using the following Sequential Quadratic Programming (SQP) algorithm[6]:

$$\begin{aligned} w^* = \arg\min f(x) \\ s.t. \\ w_i \geq 0 \\ \sum_{i=1}^{N} w_i = 1 \end{aligned} \tag{10}$$

where

[5]The method of risk decomposition applied here builds upon Euler's Homogeneous Function Theorem. It states that, if a function is continuously differentiable and (positive) homogeneous of degree k such that $f(\lambda x) = \lambda^k f(x)$ for all $\lambda > 0$, then it satisfies the equation $x \nabla f(x) = \sum_{i=1}^{N} x_i \frac{\partial f}{\partial x_i} = k f(x)$ (Roncalli, 2016; Tasche, 2007).

[6]This method is an iterative procedure which models nonlinear optimization problems for a given iterate $x^k, k \in \mathbb{N}_0$, by a Quadratic Programming (QP) subproblem, solves that problem, and then uses the solution to construct a new iterate x^{k+1}. Therefore, SQP produces a sequence of solutions $(x^k)_{k \in \mathbb{N}_0}$ that converges to the local minimum x^* as $k \to \infty$ (Boggs & Tolle, 1995).

$$f(x) = \sum_{i=1}^{N}\sum_{j=1}^{N}(w_i(\Sigma w)_i - w_j(\Sigma w)_j)^2 \tag{11}$$

From (11), we note that the existence of the RP portfolio is ensured when the condition $w_i(\Sigma w)_i = w_j(\Sigma w)_j$ for all assets in the portfolio holds. Note that the term in parentheses in (11) can be rewritten as $(w_i\frac{\partial\sigma_p}{\partial w_i} - w_j\frac{\partial\sigma_p}{\partial w_j})^2$, which equals the TRC from (9). Therefore, the algorithm minimizes the distance between all TRCs such that each asset contributes the same amount of risk to the total risk of the portfolio. Finally, Maillard et al. (2010) show that the volatility of the equal-risk-contribution portfolio is located between those of the minimum variance portfolio and the equally weighted portfolio, i.e. $\sigma_{mv} \leq \sigma_{rp} \leq \sigma_{ew}$. If all assets have the same volatility and if all pairwise correlations are uniform, the equal-risk-contribution portfolio becomes the $1/N$ portfolio.

4 Data

Daily coin closing prices quoted in reference to the USD for the top 100 largest cryptocurrencies measured by market capitalization are collected from Coinmarketcap[7] for the period January 1, 2015 to June 24, 2019 for a total of 1,636 daily observations. Coins with non-continuous time-series price data or which have been initialized after January 2015 were excluded, resulting in a final sample of 13 coins. Table 3 provides an overview of the cryptocurrency investment universe. Brief descriptions of each cryptocurrency including price, market cap, all time high and total supply information are provided in Appendix A. Logarithmic returns are calculated by

$$return_{i,t} = \ln\left(\frac{P_{i,t}}{P_{i,t-1}}\right) \tag{12}$$

where $P_{i,t}$ is the price of coin i in month t, $P_{i,t-1}$ the price of coin i in month $(t-1)$ and $return_{i,t}$ the corresponding logarithmic price change of coin i in month t.

Table 4 reports descriptive statistics. While all cryptocurrencies reward investors with positive returns over the considered time period, annualized mean returns are extremely heterogeneous, ranging from 20% for MAID to 119% for XVG. Similarly, annualized standard deviations range from 62% for BTC to 258% for XVG. Over the sample period, BTC exhibits the highest risk-adjusted return (Sharpe ratio) of 0.89 while MAID rewards investors with a Sharpe ratio of only 0.19. The average return, volatility and Sharpe ratio across cryptocurrencies for the sample period

[7]https://coinmarketcap.com/.

Table 3: Cryptocurrency investment universe

Name	Abbreviation
Bitcoin	BTC
Ripple	XRP
Litecoin	LTC
Stellar	XLM
Monero	XMR
Dash	DASH
Dogecoin	DOGE
Bitshares	BTS
Monacoin	MONA
Digibyte	DGB
Bytecoin	BCN
Verge	XVG
Maidsafecoin	MAID

Note: The table displays the cryptocurrency investment universe and the corresponding abbreviations. The sample consists of 13 cryptocurrencies for the period January 01, 2015 – June 24, 2019.

considered is 61.9%, 126.2% and 0.51, respectively. Except for BTC and MAID, all returns are positively skewed and exhibit heavy tails (excess kurtosis). Therefore, the Jarque–Bera test of normality is rejected at the 1% level for all coins. In addition, according to the Augmented Dickey-Fuller (ADF) and Phillips–Perron (PP) tests of stationarity, all variables are stationary at level. Table 5 shows the correlation coefficients between the 13 cryptocurrencies. Correlations fall in the range 0.09 – 0.60 with lowest and highest values observed for XVG-BCN and LTC-BTC, respectively. The average correlation across all coins is 0.32. Because the correlations between the cryptocurrencies are positive, but well below unity, we expect some diversification benefits from our risk-based asset allocation strategies.

In the next step, we apply our risk-based portfolio optimization strategies to our cryptocurrency portfolio[8]. Input parameters are estimated using a rolling window of 252 daily observations[9] (one year). To ensure replicability of our results, we use the sample variance-covariance matrix as the input parameter for the optimization. More sophisticated covariance estimation techniques such as the constant correlation model and the shrinkage covariance matrix estimation model are applied in the robustness test section. The results remain consistent regardless of how the covariance is estimated. The optimal portfolios are rebalanced on a daily basis. Therefore, our backtest is initiated on September 10, 2015, providing 1,384 out-of-sample observations.

[8]The code is available upon request.

[9]In this study, we follow the 252-trading day convention.

Table 4: Descriptive statistics

	BTC	XRP	LTC	XLM	XMR	DASH	DOGE	BTS	MONA	DGB	BCN	XVG	MAID
N	1635	1635	1635	1635	1635	1635	1635	1635	1635	1635	1635	1635	1635
Mean	0.548	0.456	0.604	0.485	0.852	0.695	0.449	0.229	0.706	0.884	0.744	1.194	0.204
SD	0.618	1.097	0.944	1.210	1.079	0.934	1.001	1.172	1.352	1.553	1.802	2.579	1.063
SR	0.887	0.416	0.639	0.401	0.790	0.744	0.448	0.195	0.522	0.569	0.413	0.463	0.192
Skew	-0.336	3.057	0.713	2.072	0.870	0.840	0.973	0.852	2.430	2.455	3.578	1.479	-0.081
Kurt	8.567	46.059	16.344	20.585	10.244	8.931	15.955	10.845	22.899	26.487	52.354	18.655	6.031
JB	214***	130***	120***	220***	378***	259***	120***	439***	290***	390***	170***	170***	628***
DF	40.1***	40.5***	39.9***	38.0***	41.7***	42.1***	37.6***	38.6***	37.7***	40.2***	48.7***	51.5***	44.6***

Note: The table presents descriptive statistics for the thirteen cryptocurrencies for the period January 1, 2015 to June 24, 2019. N is the number of observations, Mean is the annualized mean return, SD is the annualized standard deviation, SR is the Sharpe ratio, Skew is skewness, Kurt is excess kurtosis, JB is the Jarque-Bera test of normality (tests the joint null hypothesis of the skewness being zero and the excess kurtosis being zero) and DF is the augmented Dickey-Fuller test (tests the null hypothesis that a unit root is present in the time series sample). JB test statistics are at least e+01, DF test statistics are in absolute values. In addition to the augmented Dickey-Fuller test, we apply the Phillips–Perron test (tests the null hypothesis that the time series is integrated of order 1), which builds on ADF but makes a non-parametric correction to the t-test statistic. The null is also rejected at the 1% level for all cryptocurrencies. All series are in logarithmic first differences. *** $p < 0.01$; ** $p < 0.05$; * $p < 0.10$.

Table 5: Correlation matrix

	BTC	XRP	LTC	XLM	XMR	DASH	DOGE	BTS	MONA	DGB	BCN	XVG	MAID
BTC	1.00												
XRP	0.33	1.00											
LTC	0.60	0.35	1.00										
XLM	0.38	0.55	0.37	1.00									
XMR	0.52	0.30	0.42	0.37	1.00								
DASH	0.50	0.26	0.41	0.30	0.46	1.00							
DOGE	0.53	0.43	0.50	0.47	0.37	0.38	1.00						
BTS	0.45	0.46	0.42	0.52	0.38	0.38	0.54	1.00					
MONA	0.30	0.17	0.24	0.19	0.22	0.24	0.20	0.20	1.00				
DGB	0.36	0.28	0.29	0.36	0.29	0.28	0.40	0.38	0.16	1.00			
BCN	0.26	0.14	0.19	0.21	0.15	0.15	0.32	0.17	0.10	0.28	1.00		
XVG	0.24	0.11	0.16	0.15	0.20	0.21	0.26	0.24	0.10	0.22	0.09	1.00	
MAID	0.48	0.31	0.39	0.39	0.43	0.42	0.39	0.42	0.24	0.33	0.17	0.20	1.00

Note: The table presents Pearson correlations for the thirteen cryptocurrencies for the period January 1, 2015 to June 24, 2019. All correlation coefficients are statistically significant at the 1% level (H0: correlation coefficient = 0).

5 Empirical results

In this section, we evaluate the performance of our optimized portfolios. We start with a risk and return analysis of the risk-based portfolios against the equally-weighted benchmark. Then, we focus on quantifying tail and extreme risk, as investors are typically more concerned about the risks in the left tail of the distribution. We complement our analysis by introducing a conditional performance analysis by investigating portfolio performance in different market regimes.

5.1 Risk and return analysis

We begin our analysis by comparing the performance of our risk-based portfolio optimization strategies against the performance of the individual constituents. In the next step, we compare the strategies against the equally-weighted benchmark. Table 6 reports the out out-of-sample risk and return measures of our risk-based optimization strategies. Annualized returns from our risk-based strategies are higher than those of the individual constituents, with an average return of 68.7% (ranging from 56.8% to 76.4%) against 61.9% (ranging from 20.4% to 119.4%). More importantly, the annualized standard deviation is significantly reduced, with an average volatility of 67.9% for the optimized portfolios and 126.2% for the constituents. The reduction in risk leads to an improvement in the average Sharpe ratio from 0.51 to 1.03. Hence, diversifying across cryptocurrencies significantly improves the risk-adjusted performance.

But how do our optimizations hold up against our benchmark portfolio, the equally-weighted portfolio? To test that, we compare the cross-section of portfolio performances. The equally-weighted portfolio generates an average annual return of 72.5%. Comparison across strategies reveals that only one strategy outperforms the benchmark on an absolute return basis – the minimum variance portfolio. With a return of 76.4%, it achieves a 3.9% higher return than the equally-weighted benchmark portfolio. The minimum variance portfolio is followed by the inverse volatility, risk parity and maximum diversification portfolio with 72.4%, 69.2% and 67.1% annualized return, respectively. Hence, the equally-weighted benchmark portfolio provides slightly better absolute return. Interestingly, a two-sample t-test on the difference in return reveals that the difference is statistically insignificant. This finding is in favor of our risk-based strategies, since the explicit goal is to minimize risk rather than maximizing expected returns.

So how do our portfolios behave in terms of risk reduction? Extremely good. While the average annualized volatility of the equally-weighted portfolio is 75.1%,

Table 6: Risk-based strategies performance

	EW	**IV**	**MV**	**MVN**	**MCN**	**MD**	**RP**
Mean	0.725	0.724	0.764	0.668	0.568	0.671	0.692
SD	0.751***	0.698***	0.590***	0.613***	0.797	0.613***	0.690***
SR	0.966	1.037	1.294	1.090	0.713	1.094	1.002
Min	-0.305	-0.294	-0.223	-0.247	-0.293	-0.241	-0.289
Max	0.190	0.180	0.221	0.221	0.213	0.221	0.186
Skew	-0.337	-0.522	-0.084	-0.282	0.024	-0.162	-0.492
Kurt	7.538	8.102	8.622	8.698	6.479	8.346	8.099

Note: The table presents the performance of our risk-based portfolio optimization strategies against our benchmark portfolio, the equally weighted portfolio, for the period September 10, 2015 to June 24, 2019. Mean is the annualized mean return, SD is the annualized standard deviation, SR is the Sharpe ratio, Min is the minimum value, Max is the maximum value, Skew is skewness and Kurt is excess kurtosis. Returns are in logarithmic first differences. Portfolios are rebalanced on a daily basis. We perform tests on the difference in mean returns (H0: $\mu_{ew} = \mu_{rb}$) and standard deviations (H0: $\sigma_{ew} = \sigma_{rb}$) between the equally weighted benchmark and the other risk-based (rb) strategies. *** $p < 0.01$; ** $p < 0.05$; * $p < 0.10$.

all other strategies except the l2 norm-constrained maximum decorrelation portfolio were able to substantially reduce portfolio risk. The minimum variance portfolio again is the best portfolio with an annualized volatility of 59.0%. The l2 norm constrained minimum variance portfolio and the maximum diversification portfolio both yield 61.3% annualized standard deviation, and the risk parity portfolio 69.0%. In order to test whether the reduction in risk is statistically significant, we perform a two-sample t-test on the difference in volatility (H0: Standard deviations are equal). The null hypothesis that the standard deviation of the equally-weighted benchmark and the volatility of the other strategies is equal is rejected at the 1% level. This confirms that our risk-based strategies significantly reduce the volatility of the portfolio.

Finally, comparison across strategies reveals that five out of six risk-based strategies outperform the benchmark on a risk-adjusted basis. The minimum variance portfolio is the best performing portfolio with a Sharpe ratio of 1.29, followed by maximum diversification, inverse volatility and risk parity with 1.09, 1.03 and 1.00, respectively. The equally-weighted portfolio has a Sharpe ratio of 0.96. Only the maximum decorrelation portfolio performed worse than the benchmark portfolio with a Sharpe ratio of 0.71. As we will see later in the robustness test section, these results are consistent for different model adjustments, such as alternative covariance estimation techniques, covariance estimation windows or rebalancing periods.

To further compare the various strategies, and especially to find out where the somewhat surprising underperformance of the maximum decorrelation portfolio

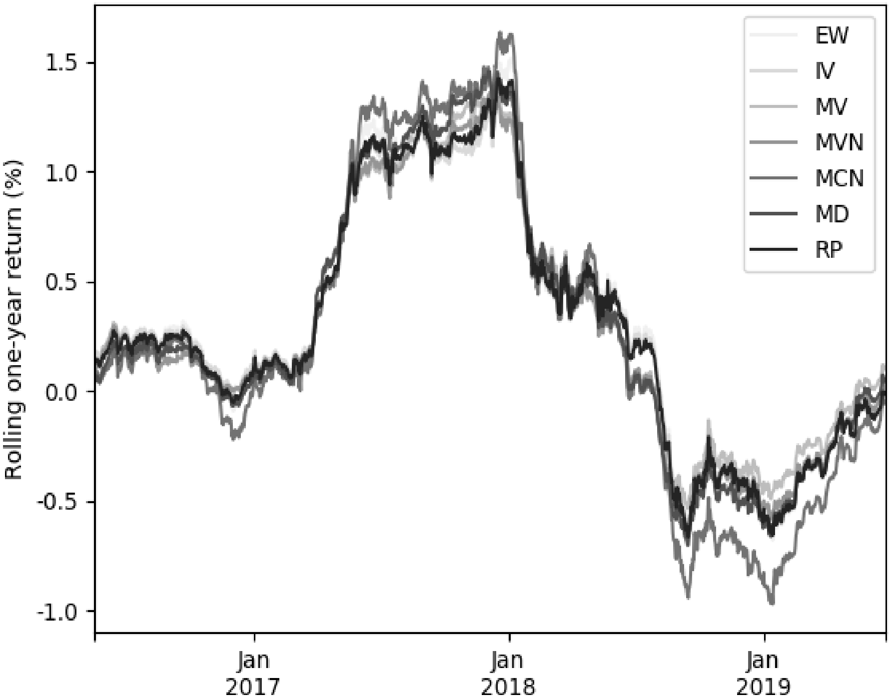


Figure 2: Rolling one-year portfolio returns. The figure plots the rolling one-year (252 days) portfolio returns for the seven risk-minimization strategies for the period May 18, 2016 to June 24, 2019. Returns are in logarithmic first differences.

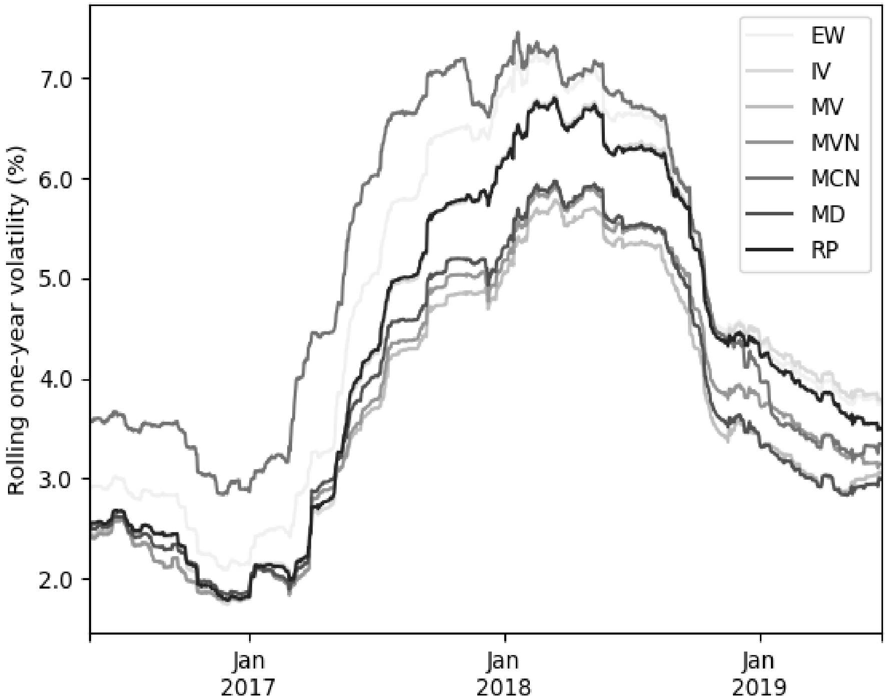
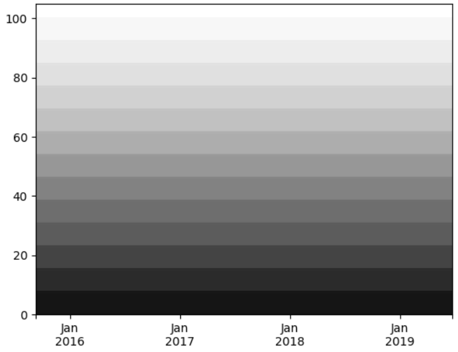


Figure 3: Rolling one-year portfolio volatilities. The figure plots the rolling one-year (252 days) portfolio volatilities for the seven risk-minimization strategies for the period May 18, 2016 to June 24, 2019. Returns are in logarithmic first differences.

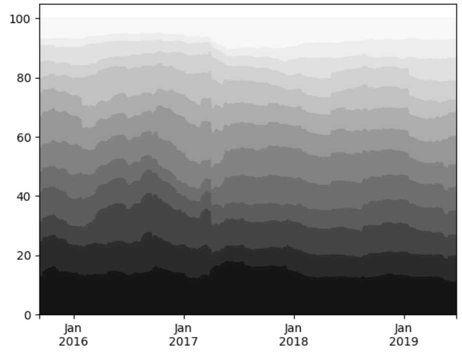
might come from, we next investigate the performance of the strategies graphically. Figure 2 and 3 plot the one-year (252 days) rolling returns and volatilities of the risk-minimization strategies versus the equally-weighted benchmark portfolio for the period May 18, 2016 to June 24, 2019. From a graphical view, we find that the returns of the strategies closely follow the benchmark and that on average, there is very little variation between strategies on an absolute return basis. This confirms our previous finding that the difference in mean between the equally-weighted portfolio and our risk-based strategies is statistically insignificant. We also identify in which periods the underperformance of the maximum decorrelation portfolio falls. It seems to be among the top portfolios during favourable times, namely in the period from January 2017 through December 2018, when the cryptocurrency market was up. Unfortunately, this changes during unfavourable market conditions, i.e. during the cryptocurrency market downturn. During these times, the maximum decorrelation portfolio is the only strategy that significantly breaks out, with a particularly large fall during the market turmoil in the beginning of 2019.

While returns are more or less the same across strategies and time, volatilities on the other hand differ significantly, and the difference across strategies remains comparatively stable over time, with the equally-weighted portfolio having the largest and the minimum variance portfolio the lowest volatility over all periods. While it seems that the risk-based strategies work particularly well during market downturns (the period at the beginning of 2018), i.e. in times when they are needed most, it is also obvious that they also work during calmer market phases. While the difference in rolling volatilities between the equally-weighted and the other risk-based strategies decreases during these periods, it is still large enough to have a noticeable impact on portfolio performance.

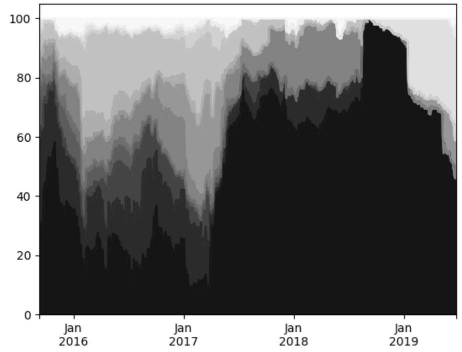
To further test where the difference in portfolio performance comes from, we need to know in which cryptocurrencies the strategies invest. Figure 4 shows the portfolio weight decomposition of the thirteen cryptocurrencies for each strategy over time. This does not only allow us to draw conclusions about to which cryptocurrencies the models allocate weights, but also about the frequency and amplitude of weight rotations, where a high turnover leads to additional costs which might render a previously profitable strategy unprofitable. Unsurprisingly, the equally-weighted strategy continuously allocates one-thirteenth or 7.7% to each cryptocurrency. This results in the lowest turnover for all strategies. The inverse volatility and risk parity strategy are invested in each of the thirteen cryptocurrencies at all times. Similar to the equally-weighted portfolio, both allocate roughly one-thirteenth to each cryptocurrency, but tilt towards certain cryptocurrencies. Minimum variance exhibits



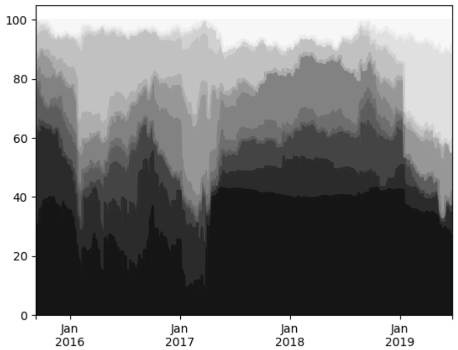
(a) Equally weighted portfolio



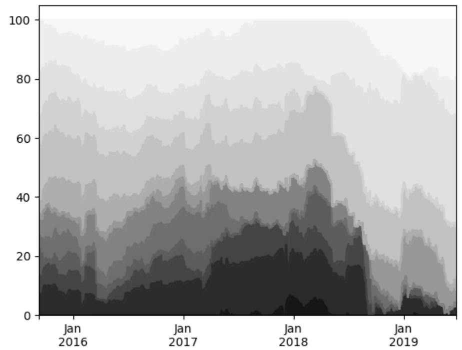
(b) Inverse volatility portfolio



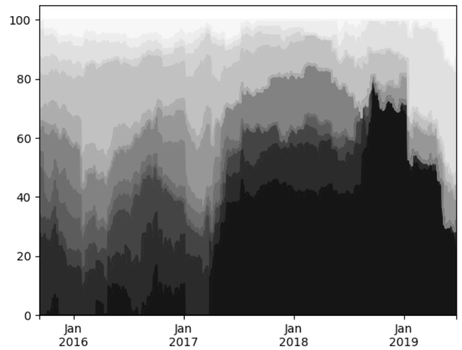
(c) Minimum variance portfolio



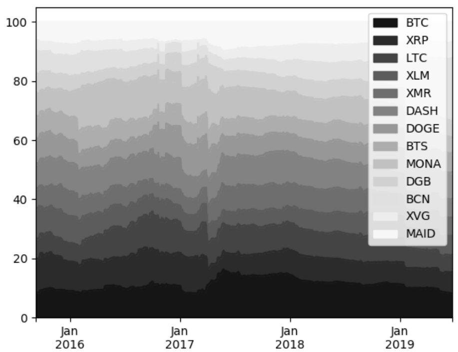
(d) l2 Minimum variance portfolio



(e) l2 Maximum decorrelation portfolio



(f) Maximum diversification portfolio



(g) Risk parity portfolio

Figure 4: Portfolio weight allocations across strategies. The figure plots the weight decomposition over time for the seven risk minimization asset allocation strategies. The values represent the investment period from September 10, 2015 to June 24, 2019, considering a rolling window approach of one year (252 days). The y-axis shows the weight allocation on a scale from 0% to 100%.

strong diversification for the period January 2016 to February 2017, but starts allocating about 80% to Bitcoin from March 2017 onwards and from November 2018 almost 100%, resulting in the most concentrated portfolio among all strategies. The allocation is reasonable, however, as Bitcoin is the least volatile cryptocurrency, particularly after the cryptocurrency market crash, as we will see later in the conditional performance analysis. In addition, it shows significant variation in weight composition, leading to the highest turnover among all portfolios. In contrast, the minimum variance norm-constrained strategy shows stronger weight diversification. This is due to the addition of the l2 norm constraint, which makes the portfolio less sensitive to changes in the covariance matrix and serves as a flexible upper bound on the portfolio weights. However, this does not necessarily come with better risk-adjusted returns, as it does not allow to move away from underperforming cryptocurrencies and forces the portfolio to be less invested in Bitcoin, which has been one of the major drivers for the last few months of the sample period. The maximum decorrelation portfolio does exactly what it is supposed to do, namely investing in assets which are least correlated with the other assets, which results in allocations to the less "popular" cryptocurrencies. This also explains where the underperformance comes from. In contrast to the other strategies, which often overweight BTC, maximum decorrelation hardly allocates to BTC at all and even moves out completely at the end of the sample period. This results in maximum decorrelation being by far the worst performing portfolio among all strategies, both in terms of absolute return and risk. Finally, maximum diversification shows similar weight allocations as the minimum variance portfolio but without the significant BTC tilt, leading to slightly worse performance.

We have seen that portfolio reallocation can be significantly different depending on the risk minimization strategy. This leads to very different turnover values. In order to test what impact the frequency of portfolio readjustments has on performance, we take transaction costs into consideration by applying a simple linear model. Following Kremer et al. (2018), we assume equal costs for buying and selling cryptocurrencies and consider three cost regimes – no, low and high cost regimes, which correspond to transaction costs of 0, 35 and 50 basis points (bps). The total daily transaction costs for each strategy are then calculated as the sum of the multiple of each assets' turnover with the corresponding transaction cost. Averaging and multiplying with 252 trading days yields the annualized transaction costs of the strategy.

Table 7 reports annualized transaction costs, mean return and Sharpe ratio for each cost regime and strategy. Naturally, due to their stable weight allocation, the

Table 7: Risk-based strategies performance for different transaction cost regimes

	EW	IV	MV	MVN	MCN	MD	RP
Panel A: No cost regime							
Mean	0.725	0.724	0.764	0.668	0.568	0.671	0.692
SD	0.751	0.698	0.590	0.613	0.797	0.613	0.690
SR	0.966	1.037	1.294	1.090	0.713	1.094	1.002
Panel B: Low cost regime							
Mean	0.725	0.720	0.745	0.648	0.552	0.652	0.686
SD	0.751	0.698	0.590	0.613	0.797	0.613	0.690
SR	0.966	1.032	1.263	1.058	0.692	1.063	0.993
Panel C: High cost regime							
Mean	0.725	0.719	0.737	0.640	0.545	0.644	0.683
SD	0.751	0.698	0.590	0.613	0.797	0.613	0.690
SR	0.966	1.030	1.250	1.045	0.683	1.049	0.990

Note: The table presents the performance of our risk-based portfolio optimization strategies against our benchmark portfolio, the equally weighted portfolio, across different transaction cost regimes for the period September 10, 2015 to June 24, 2019. no, low and high cost regimes correspond to cost rates of 0 bps, 35 bps and 50 bps, respectively. Mean is the annualized mean return, SD is the annualized standard deviation and SR is the Sharpe ratio. Returns are in logarithmic first differences. Portfolios are rebalanced on a daily basis. *** $p < 0.01$; ** $p < 0.05$; * $p < 0.10$.

equally-weighted and inverse volatility portfolio experience no or very minor drops in Sharpe ratio if we include transaction costs. The minimum variance portfolio, in contrast, is affected the most as it frequently reallocates between cryptocurrencies. This is also reflected in its risk-adjusted performance – the Sharpe ratio drops from 1.29 to 1.26 in the 35 bps regime and to 1.25 in the 50 bps regime, which corresponds to a decrease of 2.4% and 3.4%, respectively. Although it reports the highest turnover among all strategies, even after transaction costs it remains the most successful strategy as the increased turnover and the resulting trading costs are more than compensated by the value add that comes from the active rotation within the cryptocurrency investment universe. Therefore, an investor seeking to achieve the highest risk-adjusted return should still follow the minimum variance portfolio approach.

5.2 Tail and extreme risk analysis

Ultimately, risk-based strategies are supposed to protect investors during bad times. Therefore, we investigate tail and extreme risk characteristics[10] of the various strate-

[10] Tail risk is defined as the additional risk of an asset or portfolio of assets moving more than three standard deviations from its current price. It is sometimes defined less strictly, as merely the risk or probability of the occurrence of rare events.

gies in this section. We consider the Worst Loss (WL), Value at Risk (VaR), Conditional Value at Risk (CVaR), Drawdown (DD), Maximum Drawdown (MDD), Average Maximum Drawdown (AMD), Average Maximum Drawdown Squared (AMDS), Lower Partial Moments (LPM) and Higher Partial Moments (HPM). In contrast to volatility, which defines positive and negative deviations from the mean as risk, in this section, risk is defined as either (i) negative deviations from the mean or (ii) the probability of the occurrence of returns in the very left tail of the distribution. The results are reported in Table 8. Detailed descriptions of the various tail and extreme risk measures are provided in Appendix B.

In the previous section, we observed that the equally-weighted allocation performs worse than the risk minimization strategies based on risk-adjusted return. This is also the case for tail and extreme risk. In fact, all strategies, except the maximum decorrelation portfolio, significantly outperform the benchmark in every single tail risk measure. While the worst loss of the equally-weighted portfolio for the whole sample period is 30.5%[11], the average worst loss among the other strategies is 26.5%. We find similar results for VaR and CVaR. The 5% and 1% VaR for the benchmark is 7.0% and 13.6% versus 6.3% and 12.1% for the other strategies[12]. Drawdown, which is the percentage loss between peak and trough, is also worse for the benchmark portfolio with 25.8% versus an average of 25.2% for the risk-based portfolios. In addition, average maximum drawdown, which is the average maximum loss from a peak to a trough, is reduced on average from 35.0% for the equally-weighted portfolio to 30.8% for the risk-based strategies. We find similar results in favour of our risk minimization strategies for the average maximum drawdown squared and lower partial moments, reducing risk from 1.2% and 1.5% to 1.0% and 1.3%, respectively, compared to the equally-weighted portfolio.

While the minimum variance portfolio is the superior strategy on a risk-adjusted basis, it is also the preferred approach with regard to tail risk. The worst loss over the entire sample period of the minimum variance strategy is 22.3%, while the worst loss value of the equally-weighted strategy is 30.5%. In terms of 5% and 1% VaR, minimum variance reduces tail risk significantly compared to the equally-weighted benchmark with values of 5.7% and 11.0% compared to 7.0% and 13.6%. That is, on average, minimum variance reduces the risk of extreme losses in the left tail of the distribution – there is only a 5% chance that a loss of 5.7% or more occurs, compared to 7.0%. The risk reduction effects are especially pronounced in regard to

[11]Here we follow the convention in the finance literature and report tail risk in absolute terms.

[12]Note that this also includes values for the maximum decorrelation portfolio. Excluding the maximum decorrelation portfolio from our sample would result in even better 5% and 1% VaR values of 6.1% and 11.9%, respectively.

Table 8: Tail- and extreme risk statistics

	EW	IV	MV	MVN	MCN	MD	RP
WL	-0.305	-0.294	-0.223	-0.247	-0.293	-0.241	-0.289
VaR (5%)	-0.070	-0.066	-0.057	-0.060	-0.073	-0.057	-0.065
VaR (1%)	-0.136	-0.130	-0.110	-0.114	-0.135	-0.112	-0.128
CVaR (5%)	-0.114	-0.110	-0.089	-0.095	-0.114	-0.093	-0.108
CVaR (1%)	-0.181	-0.175	-0.142	-0.152	-0.183	-0.149	-0.174
DD	-0.258	-0.250	-0.245	-0.237	-0.281	-0.240	-0.256
AMD	-0.350	-0.335	-0.268	-0.287	-0.340	-0.285	-0.331
AMDS	0.012	0.011	0.009	0.009	0.014	0.009	0.010
LPM	0.015	0.013	0.011	0.011	0.016	0.011	0.013
HPM	0.017	0.016	0.014	0.014	0.018	0.014	0.016

Note: The table presents the tail- and extreme risk statistics for the period September 10, 2015 to June 24, 2019. WL is the Worst daily Loss, VaR is Value at Risk, ES is Expected Shortfall, DD is Drawdown, MD is Maximum Drawdown, AMD is average maximum drawdown over n periods, AMDS is average maximum drawdown squared over n periods, LPM is lower partial moments and HPM is higher partial moments. Percentages correspond to the confidence level α. Risk-based asset allocation strategies comprise the equally-weighted portfolio (EW), the inverse volatility portfolio (IV), the minimum variance portfolio (MV), the minimum variance under l2 norm-constraints portfolio (MVN), the maximum decorrelation under l2 norm-constraints portfolio (MCN), the maximum diversification portfolio (MD) and the risk parity portfolio (RP).

Table 9: Portfolio concentration metrics

	EW	IV	MV	MVN	MCN	MD	RP
ANC	13.000	13.000	10.669	11.722	11.705	11.789	12.999
ENC	0.077	0.087	0.410	0.209	0.148	0.229	0.086
DR	5.208	5.898	6.263	6.390	4.452	7.030	6.202
GINI	0.000	0.197	0.786	0.634	0.515	0.627	0.181
Turnover	0.000	0.000	0.002	0.002	0.001	0.002	0.001

Note: The table presents portfolio concentration statistics for the period September 10, 2015 to June 24, 2019. ANC is Average Number of Constituents, ENC is Effective Number of Constituents (or Herfindahl-Hirschman Index), DR is Diversification Ratio, GINI is the Gini coefficient and Turnover is the turnover.

CVaR, which measures the average loss in case losses go beyond the alpha 5% and 1% threshold value. The 5% and 1% CVaR is 8.9% and 14.2% compared to 11.4% and 18.1% of the equally-weighted portfolio. Furthermore, drawdown and average maximum drawdown are significantly reduced, from 25.8% and 35.0% to 24.5% and 26.8%. Therefore, our risk-based optimized portfolios deliver what they promise – in case of rare events, they effectively protect investors from extreme losses.

Table 9 reports portfolio diversification metrics, including the Average Number of Constituents (ANC), Effective Number of Constituents (ENC), or the so-called Herfindahl-Hirschman Index, Diversification Ratio (DR), Gini coefficient (GINI) and portfolio turnover (Turnover). By construction, the equally-weighted portfolio has an average number of constituents of 13.0. As a result, the Gini coefficient, measuring the dispersion among weights, as well as the turnover are equal to zero. Interestingly, the maximum decorrelation portfolio is the most diversified portfolio in terms of diversification ratio with a value of 4.4. However, as seen previously in the case of the equally-weighted portfolio, diversification is not necessarily an indication of improved performance in terms of risk-adjusted return. The minimum variance portfolio is by far the most concentrated portfolio with an average number of constituents of only 10.7 and effective number of constituents of 0.41 (compared to 0.07 for the equally-weighted portfolio), and there is a substantial difference between the minimum variance portfolio and its l2 norm-constraint counterpart, whose effective number of constituents value is reduced to 0.21. Because that additional constraint increases the number of constituents, the minimum variance l2 norm-constraint is invested in more risky cryptocurrencies than the minimum variance portfolio. While the additional cryptocurrencies fail to compensate for the increased risk, they merely increase portfolio volatility and therefore reduce the risk-adjusted return. Therefore, it is important to realize that more constituents within a portfolio is not always better and that the number of risk-reducing assets is as important as the absolute number of assets.

5.3 Conditional performance analysis

We complement our analysis by investigating the portfolio performance across different market scenarios. Therefore, we split our sample into two parts: A favourable market regime for the period January 1, 2015 to December 17, 2017, when Bitcoin's price briefly reached its all-time high of $19,783.06, and an unfavourable market regime for the period after the cryptocurrency market crash, namely from January 1, 2018 to June 24, 2019. Table 10 provides descriptive statistics for the cryptocurrencies for both sub-samples. Table 11 shows the portfolio performance of

Table 10: Descriptive statistics for bull and bear market regimes

	Bull					Bear				
	Mean	SD	SR	Skew	Kurt	Mean	SD	SR	Skew	Kurt
BTC	0.958	0.597	1.604	-0.352	10.647	-0.251	0.654	-0.384	-0.278	5.731
XRP	0.792	1.124	0.704	3.872	58.101	-0.230	1.041	-0.221	1.038	13.480
LTC	1.112	0.972	1.144	0.796	19.755	-0.443	0.880	-0.503	0.454	6.420
XLM	0.905	1.293	0.700	2.320	21.607	-0.352	1.030	-0.341	0.903	10.945
XMR	1.545	1.144	1.350	1.149	10.963	-0.537	0.934	-0.575	-0.289	4.987
DASH	1.479	0.946	1.564	1.129	9.713	-0.862	0.904	-0.954	0.193	6.926
DOGE	0.813	0.988	0.823	1.303	19.968	-0.295	1.025	-0.288	0.415	9.283
BTS	0.732	1.223	0.599	1.269	11.646	-0.827	1.058	-0.781	-0.481	7.104
MONA	1.446	1.365	1.059	2.154	21.489	-0.777	1.322	-0.588	3.057	26.912
DGB	1.556	1.700	0.915	2.740	26.669	-0.452	1.215	-0.372	0.330	8.368
BCN	1.478	1.839	0.804	3.339	42.068	-0.715	1.726	-0.414	4.148	79.124
XVG	2.209	2.995	0.738	1.330	15.134	-0.849	1.452	-0.584	0.884	12.029
MAID	0.584	1.077	0.543	0.158	6.044	-0.629	1.028	-0.611	-0.667	5.938

Note: The table presents descriptive statistics for the thirteen cryptocurrencies for bull and bear market regimes for the period January 1, 2015 to December 17, 2017 and January 1, 2018 to June 24, 2019, respectively. Mean is the annualized mean return, SD is the annualized standard deviation, SR is the annualized Sharpe ratio, Skew is skewness and Kurt is excess kurtosis. All series are in logarithmic first differences.

Table 11: Risk-based strategies performance for bull and bear market regimes

	Bull					Bear				
	Mean	SD	SR	Skew	Kurt	Mean	SD	SR	Skew	Kurt
EW	1.553	0.708	2.195	-0.026	8.305	-0.040	0.605	-0.066	-0.210	5.726
IV	1.540	0.625	2.464	-0.205	9.973	-0.003	0.610	-0.005	-0.285	5.906
MV	1.571	0.549	2.861	0.654	10.518	0.168	0.463	0.362	-0.336	7.455
MVN	1.512	0.556	2.719	0.371	11.062	-0.024	0.506	-0.048	-0.327	6.719
MCN	1.564	0.798	1.961	0.279	6.249	-0.334	0.548	-0.609	0.530	6.026
MD	1.544	0.574	2.692	0.447	9.686	-0.005	0.463	-0.012	-0.100	6.749
RP	1.528	0.627	2.436	-0.127	9.511	-0.058	0.571	-0.102	-0.348	6.123

Note: The table presents the performance of our risk-based portfolio optimization strategies against our benchmark portfolio, the equally weighted portfolio, for bull and bear market regimes for the period January 1, 2015 to December 17, 2017 and January 1, 2018 to June 24, 2019, respectively. Mean is the annualized mean return, SD is the annualized standard deviation, SR is the Sharpe ratio, Skew is skewness and Kurt is excess kurtosis. All series are in logarithmic first differences. Portfolios are rebalanced on a daily basis.

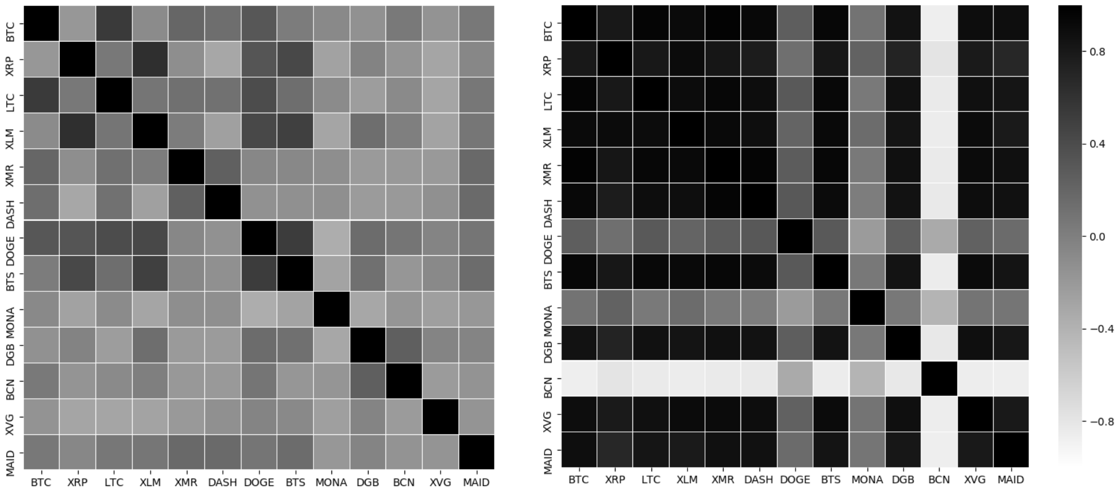
our risk-based strategies, conditional on either the favorable or unfavorable market condition.

While returns and volatilities of the individual constituents fluctuate across both regimes, there is a clear trend that delaminates one regime from another[13]. On an absolute return basis, XVG has been the best performing cryptocurrency in the bull market with 220.9% annualized return, followed by DGB and BCN with 155.6% and 147.8%, respectively. In the bear market regime, XRP, BTC and DOGE have performed best, with annualized returns of -23.0%, -25.1% and -29.5%. Across cryptocurrencies, average return is positive during bull markets with 120.1% and negative during bear markets with -55.5%. In terms of risk, BTC, DASH and LTC are the least volatile cryptocurrencies in our sample, having volatilities of 59.7%, 94.6% and 97.2% in the bull market regime. During bear markets, BTC, LTC and DASH again show the lowest risk with volatilities of 65.4%, 88.0% and 90.4%, respectively. Interestingly, when considering cross-sectional volatility, we find that annualized volatility in the bull market is higher on average than in the bear market with 132.8% compared to 109.8%. Nevertheless, on a risk-adjusted basis, cryptocurrencies perform much better in the bull market regime with an average Sharpe ratio of 0.97 compared to -0.51 in the bear market regime.

The same applies to cryptocurrency contagion. Correlations significantly increase during bear markets with an average correlation of 0.54 across cryptocurrencies versus 0.22 during bull markets. This is in line with traditional asset classes and the finance literature (see, for example, King & Wadhwani, 1990; S. B. Lee & Kim, 1993), which has shown that assets' correlation coefficients increase significantly after an external market shock. However, because cryptocurrencies are far from perfectly correlated, even during unfavorable market conditions, we expect risk-based strategies to exploit the risk reduction effects stemming from the combination of low volatility cryptocurrencies. For example, BCN seems to be a particularly suitable candidate, as it is the only cryptocurrency in the sample that has very low correlations in both regimes and an average correlation of 0.21 during the bear market. Figure 5 presents Pearson correlation heatmaps for both favourable and unfavourable market regimes.

Comparison across risk-based strategies reveal similar results – average annualized return is positive during bull market regimes with 154.3% and negative during bear market regimes with -4.3%. The best performing strategy in both regimes is the minimum variance portfolio. Furthermore, risk is materially reduced for our risk-

[13]We conduct a hypothesis test for the difference in mean and volatility, finding that both mean and volatility are significantly different from zero at the 1% level across the two regimes.



(a) Bull market regime

(b) Bear market regime

Figure 5: Pearson correlations for different market regimes. The figure shows in Panel (a) the correlation coefficients among each of the thirteen cryptocurrencies considered for the period January 1, 2015 to December 17, 2017 (favourable market regime) and in Panel (b) the correlation coefficients for the period January 1, 2018 to June 24, 2019 (unfavourable market regime). The scale ranges from -1.0 (perfect negative correlation, white colour) to +1.0 (perfect positive correlation, black colour). A value of 0.0 means there is no relationship between the variables (grey colour). Detailed correlation matrices are available upon request.

based portfolios. From an average volatility for the individual constituents of 132.8% in bull markets and 109.8% in bear markets, down to 62.1% and 52.7%. However, as with individual cryptocurrencies, volatility is higher in bull markets than in bear markets. This implies that risk minimization strategies are not immune to increasing market volatility conditions. However, as previously shown, they are still able to materially reduce risk compared to individual cryptocurrencies and the equally-weighted benchmark portfolio. This leads to an improvement in Sharpe ratio from 0.97 for the individual cryptocurrencies to 2.20 for the equally-weighted portfolio and to 2.52 for the risk-based strategies in the bull market, and from -51.0 to -0.07 and -0.06 for the equally-weighted and risk-based portfolios, respectively. The best performing individual strategy again in both regimes is the minimum variance portfolio. During the bull market, it yields an extraordinary Sharpe ratio of 2.86, while in the bear market, it is the only portfolio with a non-negative Sharpe ratio of 0.36.

In summary, our findings reveal that individual cryptocurrencies, the equally-weighted benchmark as well as our risk-minimization strategies perform unfavourably during bear market conditions. However, and more importantly, we find that our risk-based strategies can help significantly reduce the negative impact from market downturns. They were able to “hedge” the portfolio, thereby working as an ex-ante risk management tool that helps to improve the performance of the portfolio independently from the current market situation.

6 Robustness test

In order to evaluate the robustness of our results, we perform our risk and return analysis for three additional model specifications. First, we test the sensitivity of the portfolio to weekly, monthly and quarterly rebalancing. Second, we assess our results in regard to the length of the covariance matrix estimation window by performing 500, 750, and 1,000 days (two, three and four years, respectively) rolling windows. Finally, we investigate portfolio performances considering alternative covariance matrix estimation techniques. In particular, following Elton & Gruber (1973) and Ledoit & Wolf (2004b), we test the constant correlation and shrinkage covariance matrix estimation model.

Table 12 reports the portfolio performances for different rebalancing periods. With less frequent rebalancing intervals, both the annualized returns and volatilities increase. The effect on volatility is stronger, however, which leads to different Sharpe ratios across strategies and rebalancing periods – they become exponentially worse with less frequent rebalancing intervals, namely 1.00, 0.88 and 0.64 for weekly, monthly and quarterly rebalancing. While this is a significant reduction in risk-adjusted performance compared to the 1.03 Sharpe ratio of our base-case daily rebalancing strategy, it is important to note that transaction costs are not taken into consideration here, which would slightly improve the performance of the portfolios with less frequent rebalancing intervals. Nevertheless, even for quarterly rebalancing, the performance of the risk-based portfolios is still significantly better than the Sharpe ratio of 0.51 of the individual cryptocurrencies. The cross-strategy comparison reveals that minimum variance again outperforms the other strategies both in terms of return and risk, and therefore also on a risk-adjusted basis. The maximum decorrelation portfolio again is the worst performing strategy, followed by the equally-weighted benchmark portfolio. On average, our risk-based strategies were able to beat the benchmark portfolio for all rebalancing periods. The results are therefore insensitive to different rebalancing periods and consistent with our previous findings, namely that risk-based optimized portfolios outperform individual cryptocurrency investments as well as the benchmark portfolio.

Table 13 reports strategy performances for different covariance matrix estimation windows. We use a rolling window of 252 days (one year) in our base case scenario and additionally test for 500, 750 and 1,000 days (two, three and four years) estimation windows. We find mixed results with increasing estimation windows. Performance tends to improve for longer estimation windows with returns of 118.0% and 138.7% for 500 and 750 days estimation windows, respectively. Interestingly, it then falls sharply for the 1,000 days estimation window to 31.1%. Just as annualized return, standard deviation also increases with growing estimation windows and decreases again for the 1,000 days window. As a result, the risk-adjusted performance tends to improve with increasing estimation windows with average Sharpe ratios of 1.38 and 1.46 for 500 and 750 days estimation windows, respectively, and drops to 0.35 for the 1,000 days window. Again, apart from the longest estimation window, average performance across strategies remains significantly above the performance of individual cryptocurrencies, therefore supporting the evidence from our previous analyses. Further, in terms of cross-strategy performance, we do not find evidence that the results change significantly for different windows with minimum variance remaining the dominant strategy, outperforming both the equally-weighted benchmark as well as the other risk-optimized portfolios.

Table 14 compares the portfolio performance by trying to directly improve the covariance input parameters. We apply robust covariance estimation under the shrinkage and constant correlation model, which tend to be more resistant to outliers and provide better estimates of the population parameters. Firstly, following Ledoit and Wolf (2004), we estimate rolling covariance matrices by shrinking the sample covariance matrix towards a pre-defined target covariance. This comes with the advantage of more stable estimates for the true population parameters and reduced sampling estimation errors. Or, as Ledoit & Wolf (2004a) state, "The beauty of the principle is that by properly combining two 'extreme' estimators one can obtain a 'compromise' estimator that performs better than either extreme." The new, "shrunk" covariance matrix is given by the following estimation:

$$\hat{\Sigma}_{Shrink} = \hat{\delta} S + (1 - \hat{\delta}) F \tag{13}$$

Where $\hat{\delta}$ is the shrinkage constant that is a number between zero and one and measures the weight that is given to the structured covariance estimator or the so-called shrinkage target S, which in our case is a diagonal matrix of only variances, with zeros elsewhere. F is the sample covariance matrix. We set $\delta = 0.3$.

Secondly, Elton & Gruber (1973) argue the large number of correlation estimates needed for even a reasonable-sized portfolio optimization problem, combined with the problem of estimating the effect of movements of stocks on other stocks, makes the use of a less complex model to estimate correlation coefficients imperative. They recommend a model in which every pair of stocks has the same correlation coefficient.

Thus, there are $(N+1)$ parameters to estimate: The N individual variances, and the constant correlation coefficient ρ, which simply is the average of correlations of assets to other assets. Therefore, given a variance-covariance matrix of the form

$$\Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \cdots & \sigma_{nn} \end{bmatrix} \tag{14}$$

The constant correlation variance-covariance matrix is then constructed under the assumption that all asset returns have the same pair-wise correlation such that

$$\begin{gathered} \Sigma = \sigma_{ij} = \begin{cases} \sigma_{ii} = \sigma_i^2 & \text{when } i = j \\ \sigma_{ij} = \rho\sigma_i\sigma_j & \text{when } i \neq j \end{cases} \\ \rho = \rho_{ij} = \frac{1}{n(n-1)/2} \sum_{i<j} \rho_{ij} \end{gathered} \tag{15}$$

Table 14 shows the portfolio performances for the alternative covariance estimation techniques. Regardless of the covariance estimation method, our risk-optimized portfolios still beat the individual cryptocurrencies in terms of annualized return, volatility and Sharpe ratio. In addition, we find that switching to more sophisticated covariance estimation techniques has no major influence on the cross-sectional results – the risk-based strategies still beat the equally-weighted benchmark on a risk-adjusted basis. Nevertheless, it is important to note that shrinkage estimation provides better results than constant correlation estimation, which achieved significant risk reduction, but only at the cost of additional return. The shrinkage covariance in contrast seems to find a good trade-off between reducing risk while maintaining most of the return structure. Nevertheless, it is important to note that both covariance estimators deliver better results than the sample covariance. Therefore, investors are advised to apply more robust methods when estimating the covariance matrix to obtain some extra performance from their risk-based portfolios.

Table 12: Strategy performance for different rebalancing periods

	EW	**IV**	**MV**	**MVN**	**MCN**	**MD**	**RP**
Panel A: Weekly Rebalancing							
N	199	199	199	199	199	199	199
Mean	1.050	1.040	1.100	0.963	0.881	0.984	1.015
SD	1.133	1.035	0.871	0.901	1.222	0.947	1.043
SR	0.927	1.005	**1.262**	1.068	0.721	1.039	0.974
Min	-0.487	-0.469	-0.391	-0.407	-0.481	-0.417	-0.469
Max	0.749	0.629	0.641	0.641	0.922	0.664	0.625
Skew	1.013	0.672	0.613	0.618	1.402	0.865	0.743
Kurt	7.952	6.966	7.513	7.505	9.154	7.789	7.087
Panel B: Monthly Rebalancing							
N	46	46	46	46	46	46	46
Mean	1.060	1.030	1.094	0.978	0.984	1.024	1.014
SD	1.318	1.184	0.966	1.018	1.511	1.119	1.215
SR	0.804	0.870	**1.132**	0.960	0.651	0.915	0.834
Min	-0.549	-0.546	-0.474	-0.504	-0.547	-0.492	-0.543
Max	1.315	1.094	0.916	0.923	1.602	1.062	1.139
Skew	1.369	1.021	0.625	0.663	1.594	0.860	1.080
Kurt	5.980	4.774	4.118	3.899	6.853	4.204	4.864
Panel C: Quarterly Rebalancing							
N	16	16	16	16	16	16	16
Mean	1.016	0.989	1.139	1.015	1.011	1.092	0.990
SD	1.710	1.572	1.441	1.491	1.972	1.627	1.633
SR	0.594	0.629	**0.790**	0.681	0.513	0.671	0.606
Min	-1.145	-1.092	-0.931	-1.003	-1.222	-1.029	-1.098
Max	2.251	2.085	2.073	2.073	2.549	2.263	2.178
Skew	1.057	0.943	0.985	0.972	1.196	1.086	1.032
Kurt	4.698	4.708	4.783	4.792	4.606	4.652	4.749

Note: The table presents risk-based portfolio performances for three different rebalancing periods: Weekly, monthly and Quarterly rebalancing. Mean is the annualized mean return, SD is the annualized standard deviation, SR is the Sharpe ratio, Min is the minimum value, Max is the maximum value, Skew is skewness and Kurt is excess kurtosis. The best performing strategy in terms of risk-adjusted return for each rebalancing period is in bold.

Table 13: Strategy performance for different covariance estimation windows

	EW	**IV**	**MV**	**MVN**	**MCN**	**MD**	**RP**
Panel A: Estimation Window = 500							
N	1136	1136	1136	1136	1136	1136	1136
Mean	1.175	1.142	1.282	1.147	1.133	1.227	1.148
SD	0.962	0.897	0.767	0.797	1.035	0.808	0.896
SR	1.221	1.274	**1.671**	1.440	1.095	1.518	1.281
Min	-0.305	-0.296	-0.223	-0.247	-0.298	-0.246	-0.289
Max	0.190	0.178	0.195	0.195	0.207	0.195	0.176
Skew	-0.388	-0.558	-0.161	-0.348	0.013	-0.275	-0.527
Kurt	6.984	7.404	7.593	7.627	6.163	7.264	7.221
Panel B: Estimation Window = 750							
N	886	886	886	886	886	886	886
Mean	1.371	1.307	1.423	1.310	1.463	1.471	1.350
SD	1.061	0.992	0.844	0.879	1.137	0.906	0.996
SR	1.292	1.318	**1.687**	1.491	1.287	1.623	1.355
Min	-0.305	-0.295	-0.226	-0.245	-0.304	-0.253	-0.289
Max	0.190	0.186	0.234	0.234	0.207	0.220	0.181
Skew	-0.377	-0.518	-0.208	-0.328	-0.021	-0.322	-0.487
Kurt	6.008	6.308	6.744	6.655	5.481	6.220	6.130
Panel C: Estimation Window = 1,000							
N	636	636	636	636	636	636	636
Mean	0.259	0.287	0.484	0.350	0.141	0.329	0.273
SD	0.981	0.942	0.820	0.855	1.034	0.871	0.946
SR	0.265	0.304	**0.591**	0.410	0.137	0.378	0.288
Min	-0.305	-0.290	-0.228	-0.243	-0.306	-0.257	-0.291
Max	0.190	0.154	0.148	0.143	0.204	0.146	0.153
Skew	-0.445	-0.547	-0.431	-0.481	-0.093	-0.455	-0.520
Kurt	6.343	6.155	5.888	5.864	6.275	6.045	6.113

Note: The table presents risk-based portfolio performances for three different covariance estimation windows: 500 days (1 year), 750 days (2 years) and 1,000 days (3 years). Covariance matrices are estimated using a rolling window. Mean is the annualized mean return, SD is the annualized standard deviation, SR is the Sharpe ratio, Min is the minimum value, Max is the maximum value, Skew is skewness and Kurt is excess kurtosis. The best performing strategy in terms of risk-adjusted return for each rebalancing period is in bold.

Table 14: Strategy performance for different covariance estimation models

	EW	**IV**	**MV**	**MVN**	**MCN**	**MD**	**RP**
Panel A: Shrinkage Estimation							
N	1384	1384	1384	1384	1384	1384	1384
Mean	1.050	1.048	1.027	1.027	0.936	1.006	1.025
SD	0.903	0.840	0.757	0.757	0.910	0.784	0.835
SR	1.162	1.248	**1.356**	1.356	1.029	1.283	1.228
Min	-0.305	-0.294	-0.264	-0.264	-0.304	-0.273	-0.291
Max	0.190	0.180	0.201	0.201	0.207	0.194	0.182
Skew	-0.337	-0.522	-0.487	-0.487	0.064	-0.507	-0.511
Kurt	7.538	8.102	8.495	8.495	8.528	8.288	8.092
Panel B: Constant Correlation Estimation							
N	1384	1384	1384	1384	1384	1384	1384
Mean	0.725	0.724	0.755	0.718	0.568	0.747	0.724
SD	0.751	0.698	0.610	0.636	0.797	0.632	0.698
SR	0.966	1.037	**1.239**	1.129	0.713	1.181	1.036
Min	-0.305	-0.294	-0.248	-0.270	-0.293	-0.270	-0.294
Max	0.190	0.180	0.195	0.195	0.213	0.196	0.181
Skew	-0.337	-0.522	-0.283	-0.447	0.024	-0.464	-0.521
Kurt	7.538	8.102	8.282	8.340	6.479	8.388	8.101

Note: The table presents risk-based portfolio performances for two different alternative covariance estimation models: Shrinkage (Ledoit & Wolf, 2004b) and constant correlation (Elton & Gruber, 1973) estimation. Covariance matrices are estimated using a rolling window. Mean is the annualized mean return, SD is the annualized standard deviation, SR is the Sharpe ratio, Min is the minimum value, Max is the maximum value, Skew is skewness and Kurt is excess kurtosis. The best performing strategy in terms of risk-adjusted return for each rebalancing period is in bold.

7 Conclusion

Risk-based portfolio optimization strategies are highly attractive to investors and have been gaining increased momentum in academia and practice. One characteristic that all the strategies analysed in this study have in common is that the only input required to determine the optimal portfolio allocation is the variance-covariance matrix. This comes with the advantage that portfolio weights are less sensitive to estimation errors in the expected return input, which are extremely difficult to estimate and can badly affect the portfolio performance. In this study, we assessed the performance of seven state-of-the-art risk minimization strategies – namely the inverse volatility, minimum variance, l2-norm constrained minimum variance, l2-norm constrained maximum decorrelation, maximum diversification and risk parity approach. Employed to a portfolio of thirteen cryptocurrencies, our analysis allowed us to compare the resulting portfolio allocations with the individual cryptocurrency performances and across risk minimization strategies, discussing both the advantages and disadvantages of each strategy.

Our main findings can be summarized as follows. First, risk-based strategies outperform individual cryptocurrencies based on both absolute return and volatility. While the average return and volatility of the individual constituents is 61.9% and 126.2%, they are 68.7% and 67.9% for the risk minimization strategies. Second, risk-based strategies (insignificantly) underperform the equally-weighted benchmark in terms of absolute return and (significantly) outperform the benchmark based on volatility. Because the improvement in risk more than compensates for the underperformance in return, the average Sharpe ratio significantly improves from 0.84 of the benchmark portfolio to 1.03. Third, tail risk analyses reveal that risk-based strategies are particularly useful in protecting investors during bad times. Literally every tail and extreme risk metric is improved compared to both individual cryptocurrencies and the equally-weighted benchmark. Fourth, our conditional performance analysis shows that risk-based strategies work especially well during market downturns. They significantly reduce volatility while improving absolute return compared to the individual cryptocurrencies and the benchmark portfolio. Hence, risk-based strategies work best when they are most needed. Lastly, our results are robust to various model specifications, namely to different rebalancing periods, estimation windows and alternative covariance estimation models. Table 15 summarizes the main results from our research.

In summary, our findings suggest that risk-based optimized portfolios have several benefits over both individual cryptocurrencies and heuristic portfolio allocation schemes. We find that they can help significantly improve portfolio properties by

Table 15: Summary of strategy performances

	Individual Cryptocurrencies	**Equally-weighted benchmark**	**Risk-based Strategies**
Panel A: Risk and Return			
Mean	−	+	−
SD	−	−	+
SR	−	−	+
Min	−	−	+
Max	−	−	+
Skew	−	−	+
Kurt	−	−	+
Panel B: Tail and Extreme Risk			
VaR (5%)	−	−	+
VaR (1%)	−	−	+
CVaR(5%)	−	−	+
CVaR (1%)	−	−	+
DD	−	−	+
MD	−	−	+
LPM	−	−	+
Panel C: Conditional Performance			
Bull market	−	−	+
Bear market	−	−	+

Note: The table provides an overview of the performance of the individual cryptocurrencies, equally-weighted benchmark portfolio and the risk-based optimized portfolios. Individual cryptocurrencies include the universe of thirteen individual cryptocurrencies and risk-based strategies include the inverse volatility, minimum variance, l2 norm constrained minimum variance, l2 norm constrained maximum decorrelation, maximum diversification and risk parity portfolio. To evaluate the performance, the mean of the cross-section of the individual cryptocurrencies and the risk-based strategies is calculated. The best performing strategy is in bold. Mean is the annualized mean return, SD is the annualized standard deviation, SR is the Sharpe ratio, Min is the minimum value, Max is the maximum value, Skew is skewness, Kurt is excess kurtosis, VaR is Value at Risk, CVaR is Conditional Value at Risk, DD is Drawdown, MD is Maximum Drawdown and LPM is lower partial moments. Percentages correspond to the confidence level α.

significantly reducing risk while maintaining the overall return profile of the portfolio. Given these results, the high concentration in Bitcoin investments is surprising and worrisome at the same time. For this reason, we encourage cryptocurrency investors who seek to improve the risk-return profile of their portfolio to take a look at risk-based strategies the next time they need to construct or rebalance their portfolio.

Appendix A[14]

Table A.1: Bitcoin (BTC)

Price	9.342,77 USD
Market Cap	166.512.150.578 USD
Total Supply	17.822.562 BTC
All Time High	20.089,00 USD

Bitcoin (BTC) is a consensus network that enables a new payment system and a completely digital currency. Powered by its users, it is a peer to peer payment network that requires no central authority to operate. On October 31st, 2008, an individual or group of individuals operating under the pseudonym "Satoshi Nakamoto" published the Bitcoin Whitepaper and described it as: "a purely peer-to-peer version of electronic cash, which would allow online payments to be sent directly from one party to another without going through a financial institution."

Table A.2: Ripple (XRP)

Price	0,297141 USD
Market Cap	12.648.291.688 USD
Total Supply	99.991.588.101 XRP
All Time High	3,84 USD

Ripple (XRP) is an independent digital asset that is native to the Ripple Consensus Ledger. With proven governance and the fastest transaction confirmation of its kind, XRP is said to be the most efficient settlement option for financial institutions and liquidity providers seeking global reach, accessibility and fast settlement finality for interbank flows.

Table A.3: Litecoin (LTC)

Price	78,58 USD
Market Cap	4.927.277.486 USD
Total Supply	62.701.937 LTC
All Time High	375,29 USD

Litecoin is a peer-to-peer cryptocurrency created by Charlie Lee. It was created based on the Bitcoin protocol but differs in terms of the hashing algorithm used. Litecoin uses the memory intensive Scrypt proof of work mining algorithm. Scrypt allows consumer-grade hardware such as GPU to mine those coins.

[14] Data and descriptions are scraped from https://coinmarketcap.com/ as of July 17, 2019.

Table A.4: Stellar (XLM)

Price	0,077894 USD
Market Cap	1.527.083.216 USD
Total Supply	105.162.759.272 XLM
All Time High	0,938144 USD

The Stellar network is an open source, distributed, and community owned network used to facilitate cross-asset transfers of value. Stellar aims to help facilitate cross-asset transfer of value at a fraction of a penny while aiming to be an open financial system that gives people of all income levels access to low-cost financial services. Stellar can handle exchanges between fiat-based currencies and between cryptocurrencies. Stellar.org, the organization that supports Stellar, is centralized like XRP and meant to handle cross platform transactions and micro transactions like XRP. However, unlike Ripple, Stellar.org is non-profit and their platform itself is open source and decentralized. Through the use of its intermediary currency Lumens (XLM), a user can send any currency that they own to anyone else in a different currency. Stellar was founded by Jed McCaleb in 2014. Jed McCaleb is also the founder of Mt. Gox and co-founder of Ripple, launched the network system Stellar with former lawyer Joyce Kim. Stellar is also a payment technology that aims to connect financial institutions and drastically reduce the cost and time required for cross-border transfers. In fact, both payment networks used the same protocol initially.

Table A.5: Monero (XMR)

Price	72,58 USD
Market Cap	1.241.311.887 USD
Total Supply	17.103.105 XMR
All Time High	495,84 USD

Monero (XMR) is a private, secure, and untraceable cryptocurrency that was launched April 18th, 2014. With Monero, it is said you are in complete control of your funds and privacy no one else can see anyone else's balances or transactions.

Table A.6: Dash (DASH)

Price	98,95 USD
Market Cap	883.273.319 USD
Total Supply	8.926.277 DASH
All Time High	1.642,22 USD

Dash (DASH) describes itself as digital cash that aims to offer financial freedom to everyone. Payments are fast, easy, secure, and with near-zero fees. Built to support real-life use cases, Dash aims to provide a fully-decentralized payments solution. Users can reportedly purchase goods at thousands of merchants and trade it at major exchanges and brokers around the globe.

Table A.7: Dogecoin (DOGE)

Price	0,002860 USD
Market Cap	344.351.840 USD
Total Supply	120.385.986.543 DOGE
All Time High	0,018773 USD

Based on the popular "Doge" Internet meme and featuring a Shiba Inu on its logo, Dogecoin (DOGE) is a cryptocurrency that was forked from Litecoin in Dec 2013. Dogecoin has been used primarily as a tipping system on Reddit and Twitter to reward the creation or sharing of quality content. Dogecoin was created by Billy Markus from Portland, Oregon and Jackson Palmer from Sydney, Australia. Both had envisaged Dogecoin as a fun, light-hearted cryptocurrency that would have greater appeal beyond the core Bitcoin audience.

Table A.8: Bitshares (BTS)

Price	0,040028 USD
Market Cap	109.445.110 USD
Total Supply	2.734.190.000 BTS
All Time High	0,916782 USD

Bitshares (BTS), formerly known as ProtoShares, is a peer-to-peer distributed ledger and network that can issue collateralized market-pegged smart coins known as bitAssets. For instance, it can issue crypto-based assets, denominated by “bitAsset”, that track real-world markets like the USD, such as the bitUSD. Each smart coin has at least 100% of its value backed by the BitShares’ native currency, the BTS, which can be converted at any time at an exchange rate set by a trustworthy price feed. Bitshares was created by Dan Larimer, the co-founder of EOS, Steemit, and Cryptonomex. BitShares also has its own decentralized exchange.
MonaCoin (MONA) is an open source peer-to-peer payment network. It was thrust into the limelight when it was featured on WBS TV Network Tokyo, which reported a man purchasing a plot of land in Nagano with MonaCoin. The idea of MonaCoin was conceived from a Japan-based bulletin board called 2-Channel (2?????) by a user who goes by the handle of ’Mr Watanabe’.

Table A.9: Monacoin (MONA)

Price	1,77 USD
Market Cap	116.104.955 USD
Total Supply	65.729.675 MONA
All Time High	20,23 USD

Table A.10: Digibyte (DGB)

Price	0,010233 USD
Market Cap	123.195.342 USD
Total Supply	12.038.660.043 DGB
All Time High	0,142889 USD

DigiByte (DGB) is a global blockchain focused on cybersecurity for digital payments since 2014. DGB are digital assets that are mined through a combination of cryptographic algorithms with the intent to minimize mining centralization and maximize difficulty stability while reaching consensus at speeds as fast as 15-second block timings. Focused on securing data and pioneering new innovations, the team behind DigiByte claims to have issued a representation of units which hold value on an immutable ledger and cannot be the subject of counterfeiting or hacking, as any computer or device connected to the network may help to relay transactions.

Table A.11: Bytecoin (BCN)

Price	0,000589 USD
Market Cap	108.466.460 USD
Total Supply	184.066.828.814 BCN
All Time High	0,030134 USD

Created in 2012, Bytecoin (BCN) describes itself as a private, decentralized cryptocurrency with an open source code. The main goal of the project is to facilitate fast, anonymous, and untraceable transactions. Bytecoin claims to be the first project to implement CryptoNote technology. Its security reportedly comes from using ring signatures to protect a sender's identity and unlinkable addresses to prevent blockchain analysis. Bytecoin claims to have a block time of 2 minutes and adaptive parameters that are designed to make it easy to mine. Recent additions to Bytecoin technology include Auditable Wallets, which reportedly enables secure, publicly observable deposits, and Blockchain Gateways, a means of connecting Bytecoin's blockchain with other blockchains.

Created in 2014 under its original name of DogecoinDark, Verge (XVG) is an open-source privacy coin with a team of international developers. Verge uses the

Table A.12: Verge (XVG)

Price	0,005265 USD
Market Cap	83.331.567 USD
Total Supply	15.827.184.669 XVG
All Time High	0,300588 USD

anonymity tool Tor and an anonymous network layer I2P to hide specific transactions' IP addresses and locations. Transaction speed on Verge is estimated to be at 5 seconds due to the use of Simple Payment Verification (SPV). The Core QT wallet has built-in TOR integration and SSL encryption which obfuscates the IP addresses of users. The introduction of the Wraith Protocol upgrade enables users to send and receive payments privately across the Verge blockchain by enabling stealth addressing services. Verge users are reportedly able to switch between private and public ledgers on the Verge blockchain. Verge offers five different Proof-of-Work algorithms for mining - Lyra2rev2, Scrypt, X17, blake2s and myr-groestl.

Table A.13: MaidSafeCoin (MAID)

Price	0,150204 USD
Market Cap	67.975.374 USD
Total Supply	452.552.412 MAID
All Time High	1,20 USD

MaidSafeCoin (MAID) is the decentralized currency for the SAFE Network, a autonomous and decentralized data network that boasts extra hard disk space, processing power, and data connectivity for its users. The SAFE network is a sharing economy for digital resources, that seeks to "create a secure, autonomous, data-centric, peer-to-peer network as an alternative to the current server-centric model." The network is comprised of two main users: "clients" who access the various features of the network, such as browsing, storing data, or transferring money, and "farmers" who look after the clients' data until it's needed, at which point they might receive a reward for their efforts.

Appendix B

Tail- and Extreme Risk Measures

Value at Risk (VaR). VaR is a measure of downside-risk that provides an estimate about how much an investor might lose with a given probability within a set period of time. VaR is typically used by banks, regulators and insurance companies in order to get an estimate about the amount of assets needed to cover potential losses. In this study, we employ historical simulation VaR, which can be interpreted as follows: A one-day VaR of 1% of $1 million worth of stocks means that there is a 0.01 probability that the value of the portfolio will fall by more than $1 million over the period of one day. In other words, the investor can expect a loss of $1 million or more on 1 day out of 100 trading days.

Expected Shortfall (ES). While VaR describes the expected maximum loss that will not be exceeded within a given period of time with a given probability, the ES calculates the expected loss in the case of a loss that are beyond the confidence level alpha, i.e. it looks at the losses that go beyond the VaR and calculates their average.

Drawdown (DD). Drawdown represents a loss between a high and the subsequent low within a certain period. There may be several drawdowns within one period. Therefore, given historical prices S, the drawdown is the maximum distance between the previous two values S_t and S_{x-t}

$$D(t) = \max\left\{0, \max_{t_i \in (0,t)} \{S_{t_i} - S_{t_i - t}\}\right\} \tag{16}$$

Maximum Drawdown (MD). The Maximum Drawdown represents the cumulative loss that could have occurred within a period if the investor had invested at the peak. It can be thought of as a list of drawdowns calculated from the same historical portfolio values, S, but for different time periods. Thus, the maximum drawdown represents the worst possible outcome of an investment in the period considered.

Average Maximum Drawdown (AMD). Averages the Maximum Drawdown over a specified period of time.

Average Maximum Drawdown Squared (AMDS). Averages the squared Maximum Drawdowns over a specified period of time. Therefore, large Drawdowns receive larger weights.

Lower Partial Moments (LPM). In contrast to volatility, which considers both positive and negative deviations from the mean as risk, lower partial moments consider only deviations below a predefined minimum threshold value τ as risk. For example, one could define negative deviations from the mean as risk only. The lower partial moment of order j from a sample k is then estimated as follows:

$$LPM_j(\tau) = \frac{1}{k}\sum_{i=1}^{k}(\max \tau - r_i, 0)^j \tag{17}$$

where r are the historical returns.

Higher Partial Moments (HPM). Whereas lower partial moments are a measure of downside risk, higher partial moments are a measure of the upside potential of an asset or portfolio given the minimum threshold return tau. HPM is defined as follows:

$$HPM_j(\tau) = \frac{1}{k}\sum_{i=1}^{k}(\max r_i - \tau, 0)^j \tag{18}$$

where r are the historical returns.

Concentration Metrics

Average Number of Constituents. Average number of assets in the portfolio over each rebalancing date.

Effective Number of Constituents. Provides a better sense of the effective diversification of a portfolio than the Average Number of Constituents by not only averaging the number of constituents, but also taking the weighting of the constituents into account. For example, a two asset portfolio which holds 99% of the total weight in asset one and 1% in asset two has an average number constituents of 2, but an "effective number" of just over 1. The ENC has a minimum value of one when the portfolio is completely concentrated in one constituent, and a maximum value of $1/N$ when the portfolio is equally weighted across all constituents. It is calculated as folllows:

$$ENC = \frac{1}{T}\sum_{i=1}^{N} w_i^2 \tag{19}$$

where T is the number of different portfolios and w_i is the weight allocated to the ith asset.

Diversification Ratio. The diversification ratio is the ratio of the weighted variance of the portfolio constituents to the overall portfolio variance. The diversification ratio across all rebalancing periods is calculated as follows:

$$DR = \frac{1}{T}\frac{w'\sigma}{\sqrt{w'\Sigma w}} \tag{20}$$

where T is the number of different portfolios, w is the $N \times 1$ vector of asset weights and σ is the $N \times 1$ vector of asset volatilities.

Gini coefficient. The Gini coefficient is a measure of statistical dispersion. It is the most commonly used measurement of inequality and is defined as

$$G = \frac{\sum_{i=1}^{N}(2i - n - 1)x_i}{n\sum_{i=1}^{N} x_i} \tag{21}$$

where x_i is the observed value, n is the number of values observed and i is the rank of values in ascending order.

Turnover. Following Kremer et al. (2018), we define turnover as the absolute change in the weights of the portfolio such as

$$T = \frac{1}{T}\sum_{t=2}^{T}\sum_{i=1}^{N}|w_{i,t} - w_{i,t-1}| \tag{22}$$

References

Behr, P., Güttler, A., & Miebs, F. (2008). Is minimum-variance investing really worth the while? an analysis with robust performance inference. *EDHEC-Risk working paper*.

Benartzi, S., & Thaler, R. H. (2001). Naive diversification strategies in defined contribution saving plans. *American Economic Review*, *91*(1), 79–98.

Boggs, P. T., & Tolle, J. W. (1995). Sequential quadratic programming. *Acta numerica*, *4*, 1–51.

Braga, M. D. (2015). *Risk-based approaches to asset allocation: Concepts and practical applications*. Springer.

Briere, M., Oosterlinck, K., & Szafarz, A. (2015). Virtual currency, tangible return: Portfolio diversification with bitcoin. *Journal of Asset Management*, *16*(6), 365–373.

Broadie, M. (1993). Computing efficient frontiers using estimated parameters. *Annals of Operations Research*, *45*(1), 21–58.

Bruder, B., & Roncalli, T. (2012). Managing risk exposures using the risk budgeting approach. *Available at SSRN 2009778*.

Choueifaty, Y., & Coignard, Y. (2008). Toward maximum diversification. *The Journal of Portfolio Management*, *35*(1), 40–51.

Choueifaty, Y., Froidure, T., & Reynier, J. (2013). Properties of the most diversified portfolio. *Journal of Investment Strategies*, *2*(2), 49–70.

Christoffersen, P., Errunza, V., Jacobs, K., & Langlois, H. (2012). Is the potential for international diversification disappearing? a dynamic copula approach. *The Review of Financial Studies*, *25*(12), 3711–3751.

Chuen, D. L. K., Guo, L., & Wang, Y. (2017). Cryptocurrency: A new investment opportunity? *The Journal of Alternative Investments*, *20*(3), 16–40.

Clarke, R., De Silva, H., & Thorley, S. (2011). Minimum-variance portfolio composition. *The Journal of Portfolio Management*, *37*(2), 31–45.

Clarke, R. G., De Silva, H., & Thorley, S. (2006). Minimum-variance portfolios in the us equity market. *The Journal of Portfolio Management*, *33*(1), 10–24.

Deguest, R., Martellini, L., & Meucci, A. (2013). Risk parity and beyond-from asset allocation to risk allocation decisions. *Available at SSRN 2355778*.

DeMiguel, V., Garlappi, L., & Uppal, R. (2007). Optimal versus naive diversification: How inefficient is the 1/n portfolio strategy? *The Review of Financial Studies*, *22*(5), 1915–1953.

Eisl, A., Gasser, S., & Weinmayer, K. (2015). Caveat emptor: Does bitcoin improve portfolio diversification? *Available at SSRN 2408997*.

Elendner, H., Trimborn, S., Ong, B., & Lee, T. M. (2018). The cross-section of crypto-currencies as financial assets: Investing in crypto-currencies beyond bitcoin. In *Handbook of blockchain, digital finance, and inclusion, volume 1* (pp. 145–173). Elsevier.

Elton, E. J., & Gruber, M. J. (1973). Estimating the dependence structure of share prices–implications for portfolio selection. *The Journal of Finance*, *28*(5), 1203–1232.

Elton, E. J., Gruber, M. J., Brown, S. J., & Goetzmann, W. N. (2009). *Modern portfolio theory and investment analysis*. John Wiley & Sons.

Foresti, S. J., & Rush, M. (2010). Risk-focused diversification: utilizing leverage within asset allocation. *Wilshire Consulting*.

Fry, J., & Cheah, E.-T. (2016). Negative bubbles and shocks in cryptocurrency markets. *International Review of Financial Analysis*, *47*, 343–352.

Galane, L. C. (2014). *The risk parity approach to asset allocation* (Unpublished doctoral dissertation). Stellenbosch: Stellenbosch University.

Gkillas, K., & Katsiampa, P. (2018). An application of extreme value theory to cryptocurrencies. *Economics Letters*, *164*, 109–111.

Glas, T. N. (2019). Investments in cryptocurrencies: Handle with care! *The Journal of Alternative Investments*, *22*(1), 96–113.

Hastie, T., Tibshirani, R., Friedman, J., & Franklin, J. (2005). The elements of statistical learning: data mining, inference and prediction. *The Mathematical Intelligencer*, *27*(2), 83–85.

Jagannathan, R., & Ma, T. (2003). Risk reduction in large portfolios: Why imposing the wrong constraints helps. *The Journal of Finance*, *58*(4), 1651–1683.

King, M. A., & Wadhwani, S. (1990). Transmission of volatility between stock markets. *The Review of Financial Studies*, *3*(1), 5–33.

Kremer, P. J., Talmaciu, A., & Paterlini, S. (2018). Risk minimization in multi-factor portfolios: What is the best strategy? *Annals of Operations Research*, *266*(1-2), 255–291.

Krueckeberg, S., & Scholz, P. (2018). Cryptocurrencies as an asset class? *Available at SSRN 3162800*.

Ledoit, O., & Wolf, M. (2004a). Honey, i shrunk the sample covariance matrix. *The Journal of Portfolio Management*, *30*(4), 110–119.

Ledoit, O., & Wolf, M. (2004b). A well-conditioned estimator for large-dimensional covariance matrices. *Journal of multivariate analysis*, *88*(2), 365–411.

Lee, S. B., & Kim, K. J. (1993). Does the october 1987 crash strengthen the co-movements among national stock markets? *Review of Financial Economics*, *3*(1), 89–102.

Lee, W. (2011). *Risk-based asset allocation: A new answer to an old question?* Institutional Investor Journals Umbrella.

Levell, C. (2010). Risk parity: in the spotlight after 50 years. *General Research*.

Liesching, R. (2010). Bye bye modern portfolio theory, hello dynamic risk management. *Research notes, Mountain Pacific Review*.

Maillard, S., Roncalli, T., & Teiletche, J. (2008). *Equally-weighted risk contributions: a new method to build risk balanced diversified portfolios.*

Maillard, S., Roncalli, T., & Teïletche, J. (2010). The properties of equally weighted risk contribution portfolios. *The Journal of Portfolio Management*, *36*(4), 60–70.

Markowitz, H. (1952). Portfolio selection. *The journal of finance*, *7*(1), 77–91.

Michaud, R. O. (1989). The markowitz optimization enigma: Is 'optimized'optimal? *Financial Analysts Journal*, *45*(1), 31–42.

Neukirch, T. (2008). Alternative indexing with the msci world index. *Available at SSRN 1106109*.

Patterson, M. (2008). Crypto's 80% plunge is now worse than the dot-com crash. *Bloomberg*. `http://web.archive.org/web/20080207010024/http://www.808multimedia.com/winnt/kernel.htm`. (Retrieved: 2010-09-30)

Petukhina, A., Trimborn, S., Härdle, W. K., & Elendner, H. (2018). Investing with cryptocurrencies–evaluating the potential of portfolio allocation strategies. *Available at SSRN 3274193*.

Phillip, A., Chan, J., & Peiris, S. (2019). On long memory effects in the volatility measure of cryptocurrencies. *Finance Research Letters*, *28*, 95–100.

Plyakha, Y., Uppal, R., & Vilkov, G. (2015). Why do equal-weighted portfolios outperform value-weighted portfolios? In *Working paper, ssrn 2724535.*

Qian, E. (2005). Risk parity portfolios: Efficient portfolios through true diversification. *Panagora Asset Management.*

Robert, C. (2014). *Machine learning, a probabilistic perspective.* Taylor & Francis.

Roncalli, T. (2016). *Introduction to risk parity and budgeting.* Chapman and Hall/CRC.

Scherer, B. (2011). A note on the returns from minimum variance investing. *Journal of Empirical Finance*, *18*(4), 652–660.

Tasche, D. (2007). Capital allocation to business units and sub-portfolios: the euler principle. *arXiv preprint arXiv:0708.2542*.

Urquhart, A. (2016). The inefficiency of bitcoin. *Economics Letters*, *148*, 80–82.

Urquhart, A. (2017). Price clustering in bitcoin. *Economics letters*, *159*, 145–148.

Windcliff, H., & Boyle, P. P. (2004). The 1/n pension investment puzzle. *North American Actuarial Journal*, *8*(3), 32–45.