Math Problem Set #4

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6.1

Given $x, y \in \mathbb{R}^n$, $a, b, \in \mathbb{R}$, and $A \in M_n(\mathbb{R})$, choose $w \in \mathbb{R}^n$ in order to

$$\begin{aligned} &\text{minimize} && -e^{-w^T x} \\ &\text{subject to} && w^T x \leq w^T A w - w^T A Y + a \\ &&& y^T w = w^T x + b \end{aligned}$$

6.5

Let $x, y \in \mathbb{R}$ be number of milk bottles and knobs, respectively. The standard form optimization problem is:

$$\begin{aligned} &\text{minimize} && -(0.07x + 0.05y) \\ &\text{subject to} && -(0.04x + 0.03y) \geq -240 \\ &&& -(\frac{x}{30} + \frac{y}{60}) \geq -100 \end{aligned}$$

6.6

First order conditions are:

$$\frac{\partial f}{\partial x} = 6xy + 4y^2 + y = 0$$
$$\frac{\partial f}{\partial y} = 8xy + 3x^2 + x = 0$$

The critical points solving these equations are: $(-\frac{1}{3}, 0); (-\frac{1}{9}, -\frac{1}{12}); (0, -\frac{1}{4}); (0, 0)$. The Hessian is:

$$H = \begin{bmatrix} 6y & 6x + 8y + 1 \\ 8y + 6x + 1 & 8x \end{bmatrix}$$

From the determinants of the Hessian evaluated at the critical points, we have that $(-\frac{1}{3}, 0)$ and $(0, -\frac{1}{4})$ are saddle points, $(-\frac{1}{9}, -\frac{1}{12})$ is a local maximum, and $(0, 0)$ is inconclusive.

6.11

The first and second derivatives of any quadratic function $f(x) = ax^2 + bx + c$ are:

$$\frac{\partial f}{\partial x} = 2ax + b$$
$$\frac{\partial^2 f}{\partial x^2} = 2a$$

The first order condition gives $x^* = \frac{-b}{2a}$. For any initial x_0, the Newton method gives:

$$x_{k+1} = x_0 - \frac{2ax_0 + b}{2a} = \frac{-b}{2a}$$

Thus, the Newton method converges to the unique minimizer of f after one iteration.