Problem Set #2, Set 1

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Exercise 1

The spectral radius less than 1 implies a unique solution. Using successive approximations (see code in `exercises1.py`), we find the answer converges to:

$$x = \begin{bmatrix} -0.89552239 \\ 13.34328358 \\ 45.64179105 \end{bmatrix}$$

Matrix algebra confirms the result:

$$\begin{aligned} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} &= \begin{bmatrix} 0.6 & 0.1 & -0.3 \\ 0.5 & -0.4 & 0.2 \\ 1.0 & -0.2 & 1.1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 12 \\ 10 \\ -1 \end{bmatrix} \\ \implies x &\approx \begin{bmatrix} -0.89552239 \\ 13.34328358 \\ 45.64179105 \end{bmatrix} \end{aligned}$$

Exercise 2

Suffices to show that the equation is a contraction map. For any x_1, x_2,

$$\begin{aligned} &|c(1-\beta) + \beta \sum_{k=1}^{K} \max\{w_k, x_1\} p_k - c(1-\beta) + \beta \sum_{k=1}^{K} \max\{w_k, x_2\} p_k| \\ &= |\beta \sum_{k=1}^{K} \max\{w_k, x_1\} p_k - \beta \sum_{k=1}^{K} \max\{w_k, x_2\} p_k| \\ &= \beta | \sum_{k=1}^{K} \max\{w_k, x_1\} p_k - \sum_{k=1}^{K} \max\{w_k, x_2\} p_k| \\ &= \beta | \sum_{k=1}^{K} (\max\{w_k, x_1\} - max\{w_k, x_2\} | p_k \\ &\leq \beta \sum_{k=1}^{K} |(\max\{w_k, x_1\} - max\{w_k, x_2\} | p_k \\ &\leq \sup | \max\{w_k, x_1\} - max\{w_k, x_2\}| \\ &\leq \beta |x_1 - x_2| \end{aligned}$$

Yes, the function is a contraction, so has unique fixed point solution. The function applied to itself forms a Cauchy sequence that converges to the fixed point, so the unique solution can be computed by iterating over the sequence.

Exercise 3

See code in `exercises1.py`. Figure 1 shows the positive relationship between unemployment compensation and reservation wages, in line with intuition.

Figure 1: Unemployment compensation and reservation wages