Econ, Problem Set #5, DSGE

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1.1 We guess for the policy function $K_{t+1} = Ae^{z_t}Kt^{\alpha}$ and plug into toe euler equation (60) :

$$\frac{1}{e^{z_t}K_t^{\alpha} - K_{t+1}} = \beta E_t \left\{ \frac{\alpha e^{z_{t+1}} K_{t+1}^{\alpha-1}}{e^{z_{t+1}}K_{t+1}^{\alpha} - K_{t+2}} \right\}$$

$$\frac{1}{e^{z_t}K_t^{\alpha} - Ae^{z_t}K^{\alpha}} = \beta E_t \left\{ \frac{\alpha e^{z_{t+1}} K_{t+1}^{\alpha-1}}{e^{z_{t+1}}K_{t+1}^{\alpha} - Ae^{z_{t+1}}K_{t+1}^{\alpha}} \right\}$$

$$\frac{1}{e^{z_t}K_t^{\alpha} - Ae^{z_t}K^{\alpha}} = \beta E_t \left\{ \frac{\alpha K_{t+1}^{\alpha-1}}{K_{t+1}^{\alpha} - AK_{t+1}^{\alpha}} \right\}$$

$$\frac{1}{(1-A)e^{z_t}K_t^{\alpha}} = \beta \frac{\alpha K_{t+1}^{\alpha-1}}{(1-A)K_{t+1}^{\alpha}}$$

$$\frac{1}{e^{z_t}K_t^{\alpha}} = \beta \frac{\alpha K_{t+1}^{\alpha-1}}{K_{t+1}^{\alpha}}$$

$$\frac{1}{e^{z_t}K_t^{\alpha}} = \beta \frac{\alpha}{K_{t+1}} \qquad \text{plugging in the guess once more gives}$$

$$\frac{1}{e^{z_t}K_t^{\alpha}} = \beta \frac{\alpha}{Ae^{z_t}K_t^{\alpha}}$$

$$A = \beta\alpha$$

1.2 For the specific functions the equilibrium equations are the following:

$$\frac{1}{c_t} = \beta E_t \left\{ \frac{1}{c_{t+1}}[(r_{t+1} - \delta)(1-\tau) + 1] \right\}$$

$$a\frac{1}{1-l_t} = \frac{w_t(1-\tau)}{c_t}$$

$$r_t = \alpha e^{z_t} \left(\frac{K_t}{L_t} \right)^{\alpha-1}$$

$$w_t = (1-\alpha)e^{z_t} \left(\frac{K_t}{L_t} \right)^{\alpha}$$

1.4 Characterizing equations are as follows:

$$c_t = (1-\tau)[w_t l_t + (r_t - \delta)k_t] + k_t + T_t - k_{t+1}$$

$$c_t^{-\gamma} = \beta E_t[c_t^{-\gamma}((r_{t+1} - \delta)(1-\tau) + 1)]$$

$$a(1-l_t)^{-\xi} = c_t^{-\gamma} w_t (1-\tau)$$

$$r_t = e^{z_t}\alpha[\alpha K_t^{\eta} + (1-\alpha)L_t^{\eta}]^{\frac{1}{\eta}-1} K_t^{\eta-1}$$

$$w_t = e^{z_t}(1-\alpha)[\alpha K_t^{\eta} + (1-\alpha)L_t^{\eta}]^{\frac{1}{\eta}-1} L_t^{\eta-1}$$

$$T_t = \tau[w_t l_t + (r_t - \delta)k_t]$$

$$z_t = (1-\rho)\overline{z} + \rho z_{t-1} + \epsilon_t^2$$

1.5 Solving for the steady States $\bar{r}$ and $\bar{K}$ and plugging in the parameter values from the exercise gives

$$\bar{r} = \frac{1-\beta}{\beta(1-\tau)} + \delta = 0.124$$
$$\bar{k} = \left(\bar{r}^{\frac{1}{\alpha-1}}\right) \frac{1}{\alpha} = 7.287$$