Chris Wallace Problem Set 6

Question 1:

Earlier in the course, you saw how the recorded audio signal of a gun being fired in a shooting range can be convolved with a violin recording to approximate how the violin would sound if played in a shooting range. Please explain this using what you know about the impulse and impulse responses.

This works because:

1. We are assuming the gunshot is an impulse, this is generally a good assumption.

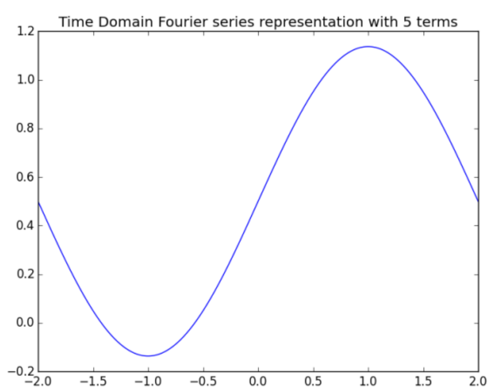
2. The recording of the impulse is thus the impulse response of the firing range.

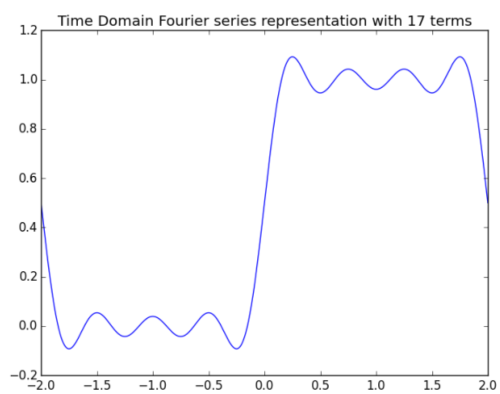
3. Since the sound response of a firing range is an LTI system the effect it has on the gunshot and the violin recording will be the same. Convolving a signal with an impulse response will modify the original signal by the impulse response which in our case is the sound characteristics of a gun range, so the sound of the violin will change so it sounds like it was recorded in a firing range.

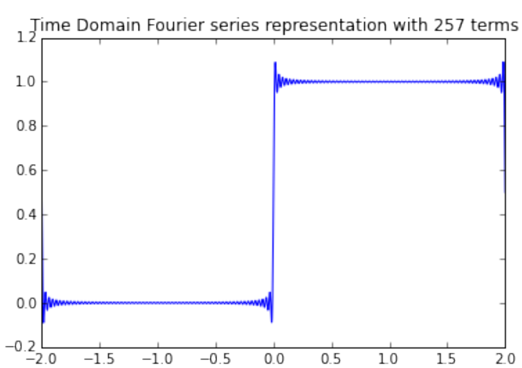
2) See lined paper

3a) see lined paper

3b) 3b) Fourier series representation of square wave





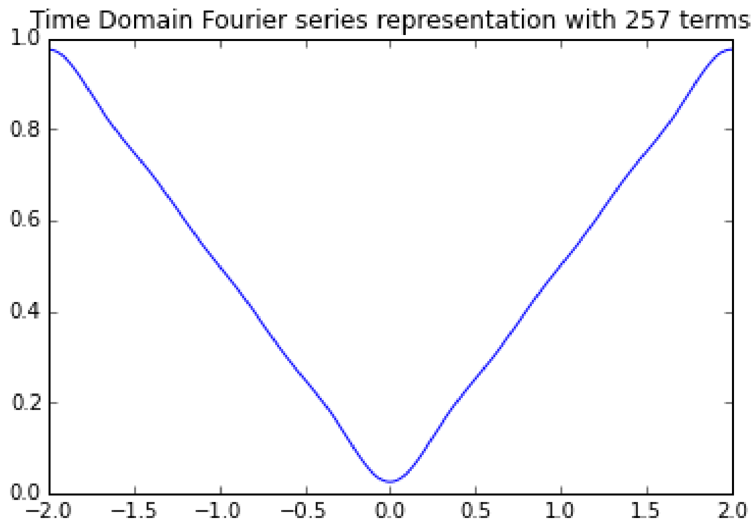


3c) Describe what you see at the point of discontinuity:

At each point of discontinuity there are high frequency sine waves with amplitudes that slowly get less and less. The reason for this is that at a discontinuity there are a lot of high frequency components, so many components that we can not truly represent this in the analog world. As you add more high frequencies the effect gets less and less, supporting number ten in that as k increases the error approaches zero. The error does approach zero, it just can not quite make zero because the discontinuity in effect creates a component with an infinity frequency.

4) if y(t) is a delayed version of x(t) and Ck is the Fourier coefficients for x(t) what are the fourier coefficients for y(t)

The coefficients are going to be the same, since the signal is periodic it does not matter where you evaluate it from as long as there is one period that is evaluated



The answer I found accurately depicts a triangle wave, this makes sense since my conclusion was that the coefficients should be identical.

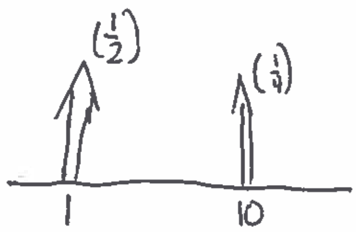
```
from __future__ import print_function
import numpy as np
import matplotlib.pyplot as mplib
get_ipython().magic(u'matplotlib inline')


def fs_triangle(ts, M=3, T=4):
  x = np.zeros(len(ts))

  # if M is even
  if np.mod(M,2) ==0:
    for k in range(-int(M/2), int(M/2)):
      # if n is odd compute the coefficients
      if np.mod(k, 2)==1:
        Coeff = -2/((np.pi)**2*(k**2))
      if np.mod(k,2)==0:
        Coeff = 0
      if n == 0:
        Coeff = 0.5
      x = x + Coeff*np.exp(1j*2*np.pi/T*k*ts)

  # if M is odd
  if np.mod(M,2) == 1:
    for k in range(-int((M-1)/2), int((M-1)/2)+1):
      # if n is odd compute the coefficients
      if np.mod(k, 2)==1:
        Coeff = -2/((np.pi)**2*(k**2))
      if np.mod(k,2)==0:
        Coeff = 0
      if k == 0:
        Coeff = 0.5
      x = x + Coeff*np.exp(1j*2*np.pi/T*k*ts)

  return x

def fs_square(ts, M=3, T=4):
  x = np.zeros(len(ts))

  # if M is even
  if np.mod(M,2) ==0:
    for k in range(-int(M/2), int(M/2)):
##############################################################################################
#################
      ##  change the following line to provide the Fourier series coefficients for the square wave
      if np.mod(k,2) ==1:
        #If k is odd
        #Coeff = 4/(k*np.pi)
```

```
            Coeff = (1/(1j*np.pi*k))
         if np.mod(k,2) ==0:
            #If k is even
            Coeff = 0

         if k == 0:
            Coeff = .5

         x = x + Coeff*np.exp(1j*2*np.pi/T*k*ts)

   # if M is odd
   if np.mod(M,2) == 1:
      for k in range(-int((M-1)/2), int((M-1)/2)+1):

##########################################################################################
#################
         ##  change the following line to provide the Fourier series coefficients for the square wave
         ##  Coeff = ??
         if np.mod(k,2) ==1:
            #If k is odd
            #Coeff = 4/(k*np.pi)
            Coeff = (1/(1j*np.pi*k))
         if np.mod(k,2) ==0:
            #If k is even
            Coeff = 0
         if k == 0:
            Coeff = .5

         x = x + Coeff*np.exp(1j*2*np.pi/T*k*ts)

   return x


ts = np.linspace(-2,2,2048)
T = 4
M = 5
x = fs_square(ts,M,T)
mplib.plot(ts,x)
mplib.title("Time Domain Fourier series representation with %d terms" % (M) )
mplib.show()


# In[18]:

T = 4
M = 17
x = fs_square(ts,M,T)
mplib.plot(ts,x)
```

```
mplib.title("Time Domain Fourier series representation with %d terms" % (M) )
mplib.show()



T = 4
M = 257
x = fs_square(ts,M,T)
mplib.plot(ts,x)
mplib.title("Time Domain Fourier series representation with %d terms" % (M) )
mplib.show()


x = fs_triangle(ts, M=15, T=4)
mplib.plot(ts,x)
mplib.title("Time Domain Fourier series representation with %d terms" % (M) )
mplib.show()
```

2. $y(t) = \frac{1}{2}x(t-1) + \frac{1}{4}x(t-10)$

This channel is going to have two two impulse responses.



It is known as an echo channel because convolving x(t) with that will result in an output where there is a time delay in the first "echo" it is also weaker than initially, and then there will be another echo 1/4 as strong 10 seconds later.

$h(t) = \frac{1}{2}\delta(t-1) + \frac{1}{4}\delta(t-10) \leftarrow \quad x(t) * h\,\delta(t) \to y(t)$

3. Fourier epresentation of square wave:

$f(x) = \cancel{A}\,[H(x/T) - H(x/T-1)]$ $\quad H(x) = \begin{cases} 0 & x<0 \\ \frac{1}{2} & x=0 \\ 1 & x>0 \end{cases}$

$C_k = \frac{1}{T}\int_{T/4}^{5T/4} f(t)\, e^{-j\frac{2\pi k t}{T}}\, dt$

Insert wolfram alpha:

$\cancel{C_k = \frac{4}{k\pi}\begin{cases} 0 \text{ if } n \text{ is even} \\ 1 \text{ if } n \text{ is odd} \end{cases}}$ $\quad C_k = \frac{1}{2}\cdot\begin{cases} \cancel{\frac{1}{\pi k} \text{ if odd}} \\ \cancel{0} \text{ if even} \\ \frac{1}{2} \text{ if } c=0 \end{cases}$

$C_k = \begin{cases} \frac{1}{j\pi k} & k \text{ is odd} \\ 0 & \text{if even} \\ \frac{1}{2} & \text{if } k=0 \end{cases}$

4b.

$ck = \begin{cases} -2/\pi^2 k^2 & \text{if } k \text{ is odd} \\ \frac{1}{2} & \text{if } k=0 \\ 0 & \text{else} \end{cases}$