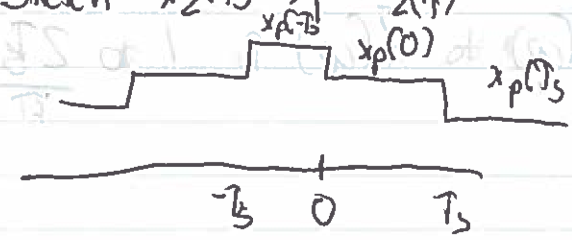
Chris Wallace

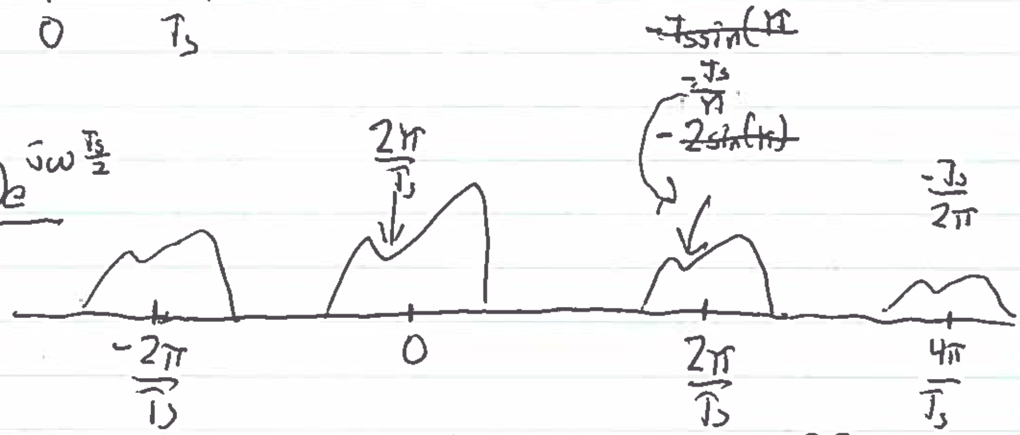
Problem Set 8

1g. Sketch $x_z(t) = x_p * z(t)$



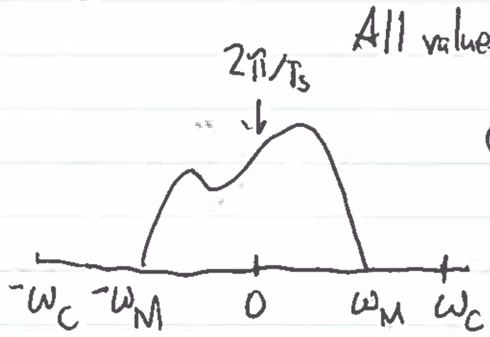
h. Sketch $X_z(\omega)$

$Z(\omega) = \dfrac{2\sin(\omega \frac{T_s}{2}) e^{j\omega \frac{T_s}{2}}}{\omega}$

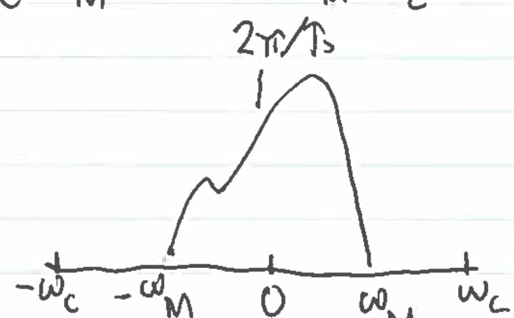


All values are $Z(\omega) \times \frac{2\pi}{T_s}$

i. $\bar{X}(\omega) = X_z(\omega) H(\omega)$ (sinc)



$\hat{X}(\omega) = X_p(\omega) H(\omega)$



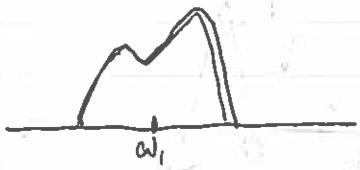
j. $\bar{X}(\omega)$ is going to diminish because there is a sinc function in its Fourier Transform. They are the same only at $\omega = 0$. Outside of that $\bar{X}(\omega)$ becomes less than $\hat{X}(\omega)$ at a rate of $\dfrac{2\sin(\omega \frac{T_s}{2}) e^{j\omega T_s/2}}{\omega}$

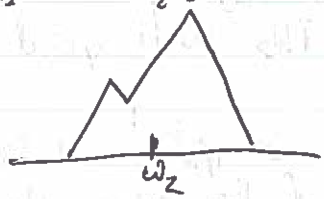
K.

$$\omega_M = \frac{\pi}{T_s} = \frac{2\sin\left(\omega \frac{T_s}{2}\right)e^{j\omega\frac{T_s}{2}}}{\omega} \quad \frac{2\sin\left(\frac{\pi}{2}\right)e^{j\frac{\pi}{2}}}{\pi} T_s \rightarrow \frac{2T_s j}{\pi}$$

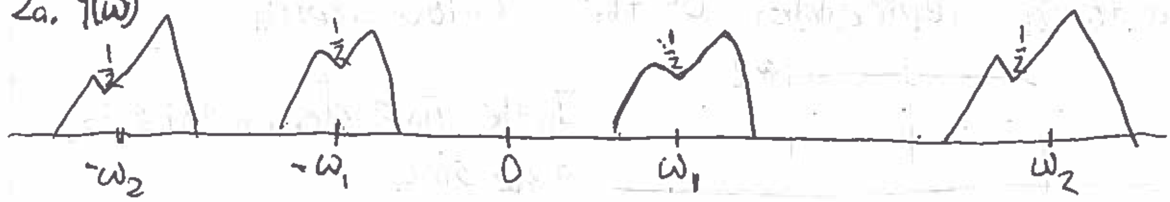
At ω_M the ratio of $\tilde{X}(\omega)$ to $\bar{X}(\omega)$ is 1 to $\frac{2T_s j}{\pi}$

2. $x_1(t)\cos(\omega_1 t)$ $x_2(t)\cos(\omega_2 t)$



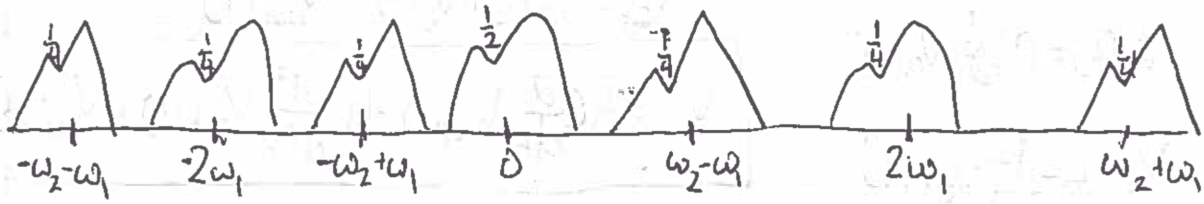


2a. $Y(\omega)$



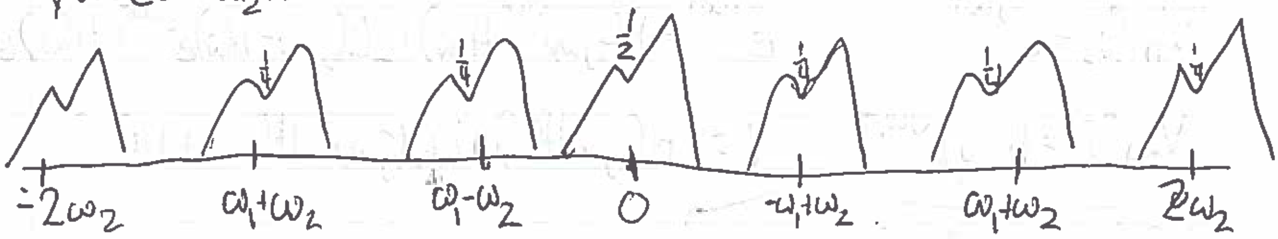
The ending of each frequency band is always centerpoint $\pm\omega_M$. It is just terrible to write that at each cluster.

2b. $y(t)\cos(\omega_1 t)$

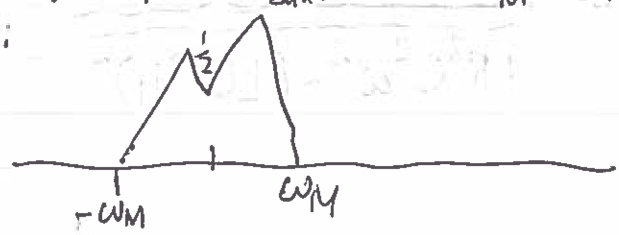


Same comment as above

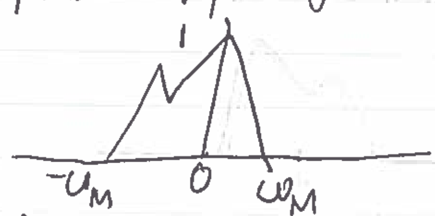
$y(t)\cos(\omega_2 t)$



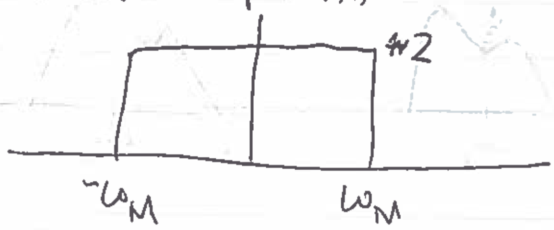
2c. To recover each ~~multiply by~~ $x_1(t)$ multiply signal by $\cos(\omega_1 t)$ to recover $x_2(t)$ multiply by $\cos(\omega_2 t)$. Sketches showing this above. A band pass filter ~~ø~~ with a band of $\pm\omega_M$ should be applied leaving the signal:



To fully recover the signal we can amplifier with a gain of 2 from $\pm\omega_M$. This will yield:



The previous steps can be applied in one step ~~with a sig~~ if the filter has a Fourier representation of the below sketch



In the time domain this is a ~~sy~~ sinc

3. $i(t) = C\frac{d}{dt}V_{out}(t)$ $\quad V_L(t) = \frac{d}{dt}i(t)$

a. Dif Eq V_{out} to V_{in}:

$$V_{in} = V_R(t) + V_L(t) + V_{out}(t)$$

$$V_R(t) = RC\frac{d}{dt}V_{out}(t)$$

$$V_L = LC\frac{d^2}{dt^2}V_{out}(t)$$

$$\boxed{V_{in} = RC\frac{d}{dt}V_{out}(t) + LC\frac{d^2}{dt^2}V_{out}(t) + V_{out}(t)}$$

b. Find $H(\omega)$

$$\cancel{e^{j\omega t} = RCj\omega e^{j\omega t} + LC(j\omega)^2}$$

$$V_{in}(t) = e^{j\omega t}$$

$$V_{out}(t) = H(\omega)e^{j\omega t}$$

$$e^{j\omega t} = RCj\omega e^{j\omega t}H(\omega) + LC(j\omega)^2 H(\omega)e^{j\omega t} + H(\omega)e^{j\omega t}$$

$$1 = RCj\omega H(\omega) + LC(j\omega)^2 H(\omega) + H(\omega)$$

$$\boxed{H(\omega) = \frac{1}{RCj\omega - LC\omega^2 + 1}}$$

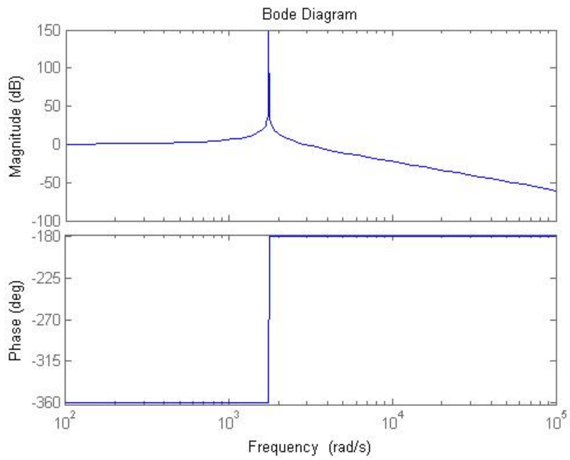
c. Magnitude: $\dfrac{1}{\sqrt{R^2C^2\omega^2 - (LC\omega^2+1)^2}}$

d. There is a 100 % chance I would make an error doing this calculus out by hand so I used MATLAB symbolic to do it. Attached is code.
I took derivative of $H(w)$ then solved to find its max which is:

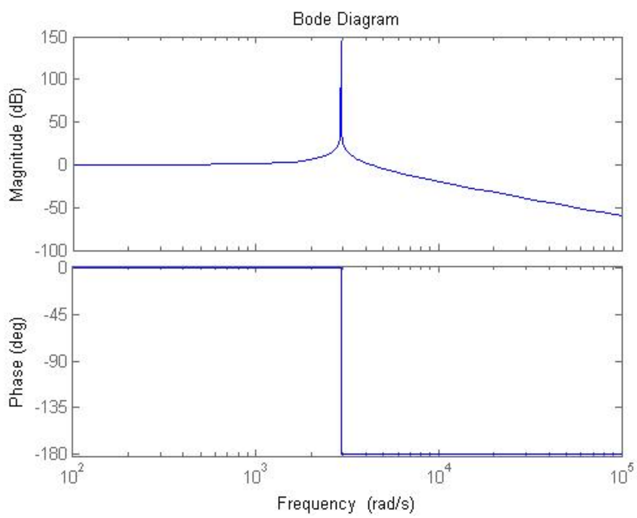
$$\sqrt{2}\ \frac{\sqrt{-2C(-CR^2+2L)}}{2CL}$$

e. See next page for bode plots:

i. Bode plot 1



ii. Bode plot 2



Matlab Code for 3d

```
syms C L R w
f = 1/(R^2*C^2*w^2-(L*C*w^2+1)^2)
g = diff(f,'w')
solve(g)
%VALUES
%                                            0
%  (2^(1/2)*(-C*(- C*R^2 + 2*L))^(1/2))/(2*C*L)
% -(2^(1/2)*(-C*(- C*R^2 + 2*L))^(1/2))/(2*C*L)
```

Code for 3e

```
C = 10E-7
L = 10E-2
R = 50
H = tf([1],[L*C,R*C*i,1]);
bode(H)
```