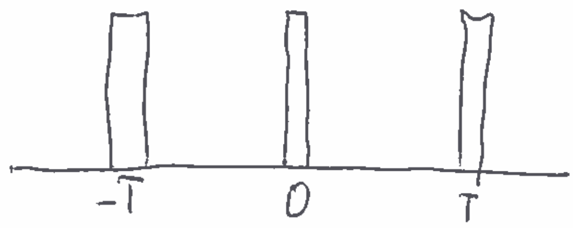
Chris Wallace
Problem Set 7

1a. $p(t) = \sum_{k=-\infty}^{\infty} \delta(t-kT)$

Representation




b. Fourier Series of $p(t)$

$\cancel{C_k}$ $C_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t) e^{-j2\pi kt/T}$

$\int x(t) = 1$ only at $t=0$, therefore: ~~Integral~~ $x(t)$ is non-zero only at 0

$\boxed{C_k = \frac{1}{T}}$

c. $x(t) = \sum_{-\infty}^{\infty} C_k e^{j2\pi kt/T}$ Find $X(\omega)$

$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} d \rightarrow \sum \int C_k e^{-j2\pi kt/T} e^{-j\omega t} dt \rightarrow$

$\rightarrow \sum C_k \int_{-\infty}^{\infty} e^{j2\pi kt/T} e^{-j\omega t}$ → Fourier is impulse centered $2\pi k/T$

$\rightarrow \sum C_k 2\pi \delta(\omega - \omega_0)$ $\boxed{X(\omega) = \sum C_k 2\pi \delta(\omega - \omega_0)}$

d. find $P(\omega)$

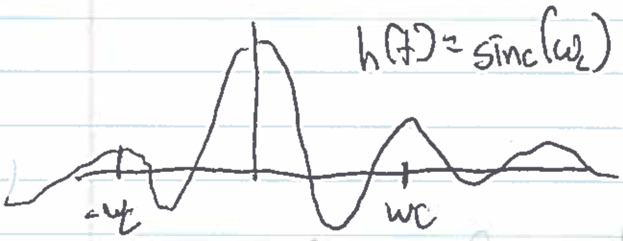
$P(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} \rightarrow \int \frac{1}{T} e^{j\omega t} \rightarrow \frac{1}{T} \cancel{\phi}\pi \delta(\omega - \omega_0)$

$P(\omega) = \frac{\pi}{T}\delta(\omega - \omega_0)$

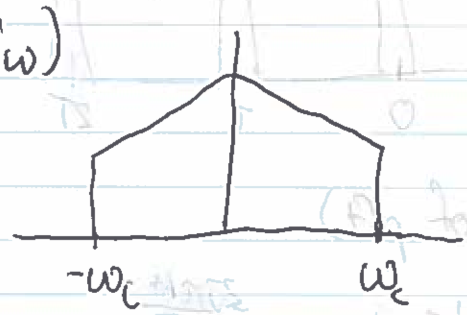
1e. $P(\omega)$


As T increase, the $p(t)$ will become more spaced out, this will be reflete in $P(\omega)$ with lower amplitudes. This what you would expect because there will be fewer instances at each frequency resulting in less amplitude

2 a. $h(t) = \text{sinc}(\omega_c$



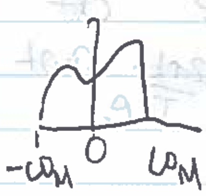

2b. $Y(\omega)$



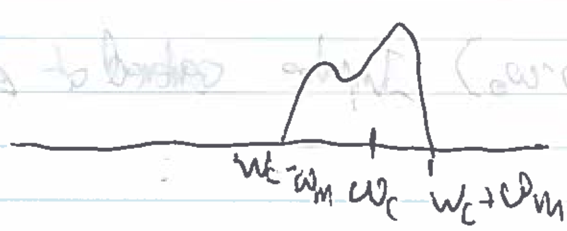

c. This system is known as an ideal low pass filter with cut off frequency $\omega_c$ because filtering is the same as multiplication in the frequency domain and that system makes all frequencies above $\omega_c$ equal to 0.

d. See graph

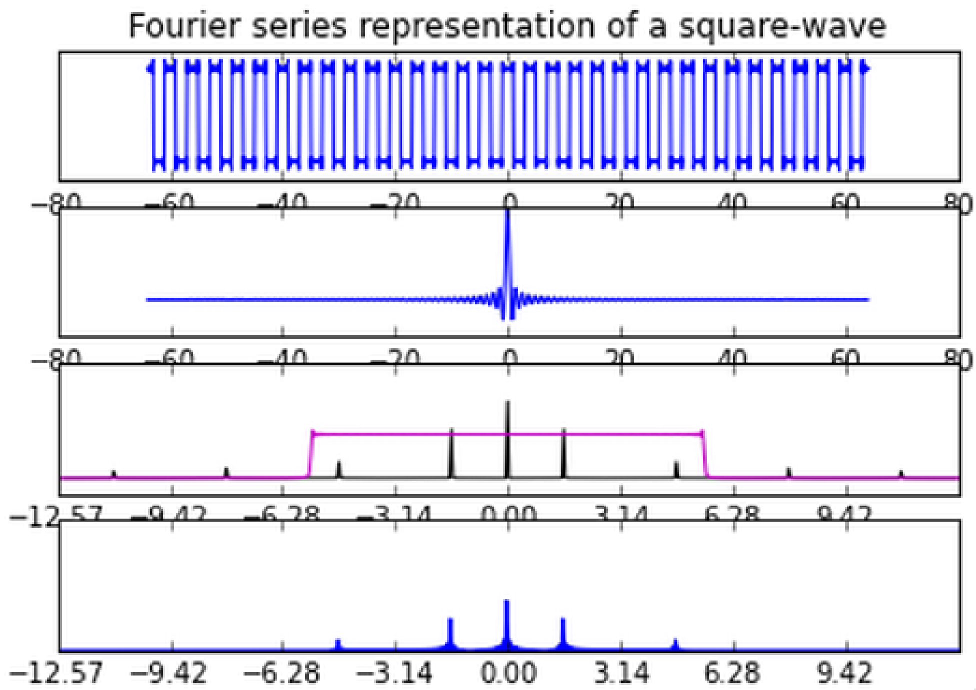
3. $X(\omega)$




Multiplication in time domain = convolution in frequency domain



Wc = 1.75 pi




Wc = .75pi