Chris Wallace
Problem Set 10

1. $$\begin{array}{ccccc} u(t) & \rightarrow & \boxed{\dot{y}+y=x} & \rightarrow & (1-e^{-t})u(t) \\ \downarrow & & & & \uparrow \\ X(s) & \rightarrow & H(s) & \rightarrow & H(s)X(s) \end{array}$$

$$\dot{y}+y=x \rightarrow \mathcal{L} = sY(s)+Y(s)=X(s) \qquad \frac{Y(s)}{X(s)} = \frac{1}{s+1} = H(s)$$

$$u(t) \rightarrow \mathcal{L} = \frac{1}{s} = X(s)$$

$$X(s) \times H(s) = \frac{1}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1} \qquad B=-1 \quad A=1 \qquad X(s)H(s) = \boxed{\frac{1}{s} + \frac{-1}{s+1}} \rightarrow \mathcal{L}^{-1}$$

$\rightarrow u(t) - u(t)e^{-t} = u(t)(1-e^{-t})$ These are equivalent. Verified!

2A. DC Gain

$$\frac{Y}{Y_{sp}} = \frac{K_I/s\, H(s)}{1+\frac{K_I}{s}H(s)} \times \frac{s}{s} = \frac{K_I H(s)}{s+K_I H(s)} \qquad \lim_{s\to 0} \text{ of } \frac{K_I H(s)}{s+K_I H(s)} = 1$$

DC Gain = 1

2B $\cancel{H(s) = \frac{\frac{1}{T}}{s+\frac{1}{T}} \qquad K(s) = \frac{K_I}{s}}$

$$\frac{Y(s)}{Y_{sp}(s)} \quad \mathcal{L} \quad \cancel{\frac{\frac{K_I}{sT}}{\frac{K_I}{s^2}+\frac{K_I}{sT}}} \quad \cancel{\frac{\frac{K_I}{sT}}{K_I+\frac{K_I}{sT}}} \quad \cancel{\frac{K_I}{sTK_I+K_I}}$$

$$\frac{\frac{K_I}{sT}}{1+\cancel{\frac{\frac{K_I}{sT}}{K_I+\frac{K_I}{sT}}}}$$

2B $H(s) = \dfrac{\frac{1}{T}}{s+1/T}$ $\quad H(s)K(s) = \dfrac{\frac{K_I}{T}}{s^2+\frac{s}{T}} = \dfrac{K_I}{s^2T+s}$

$K(s) = \dfrac{K_I}{s}$

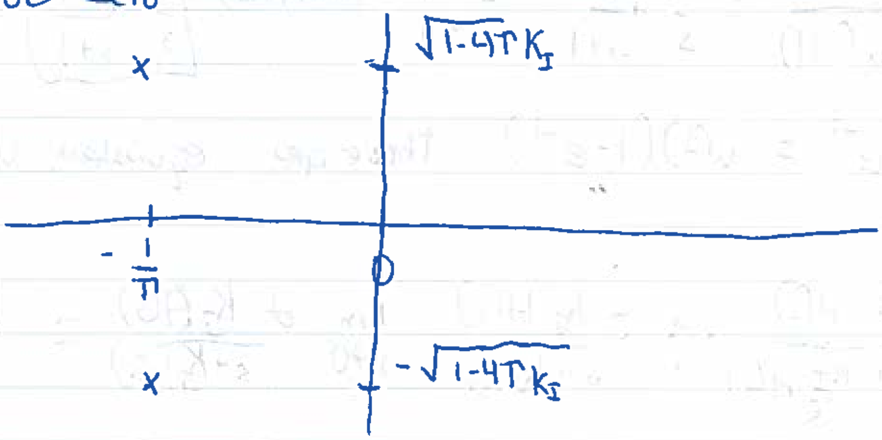
$$\frac{Y(s)}{Y_{sp}(s)} = \frac{\frac{K_I}{s^2T+s}}{1+\frac{K_I}{s^2T+s}} = \frac{K_I}{s^2T+s+k_I}$$

$$\boxed{\frac{Y(s)}{Y_{sp}(s)} = \frac{K_I}{s^2T+s+K_I}}$$

Find Poles:

$$\text{Poles} = \frac{-1 \pm \sqrt{1-4TK_I}}{T}$$

Pole Zero:



3. See Attached Document

4A See Attached Figure

Blacks Law: $\frac{Y(s)}{Y_{sp}(s)} = \frac{HK}{1+HK}$

4B. $H(s) = \frac{1}{s^2 - .01s + 1}$ $\quad \frac{Y(s)}{Y_{sp}(s)} = \frac{k}{s^2 - .01s + 1 + k}$

$K(s) = k$

See figure for pole zero map, You ~~cannot~~ can't stabilize the system.

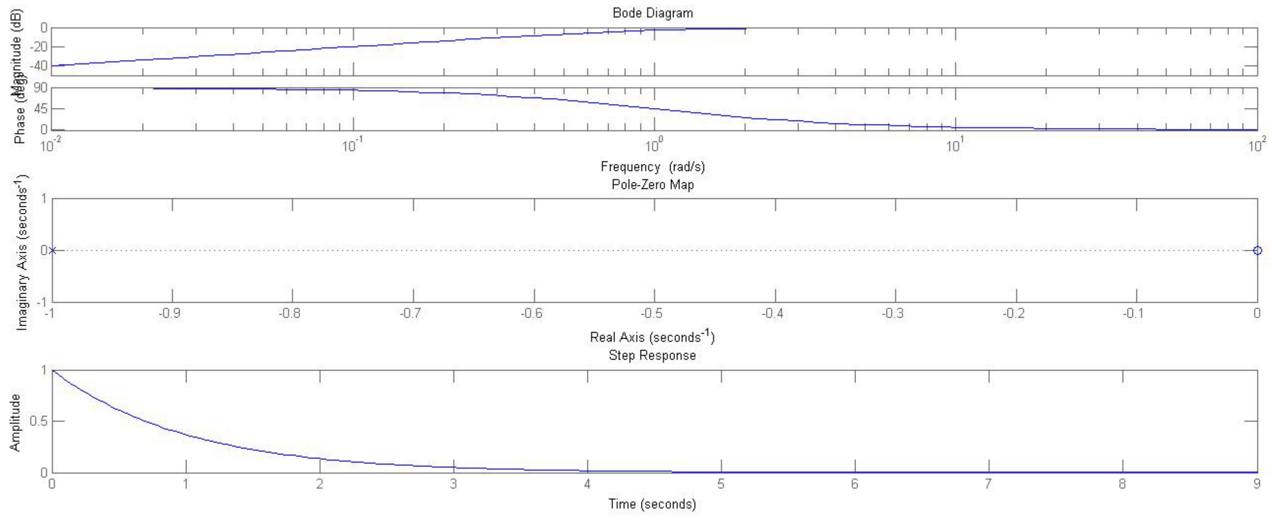
4C. $K(s) = \frac{1}{ks}$ $\quad \frac{Y(s)}{Y_{sp}(s)} = \frac{1}{ks^3 - .01ks^2 + ks + 1}$

See figure Cannot stabilize

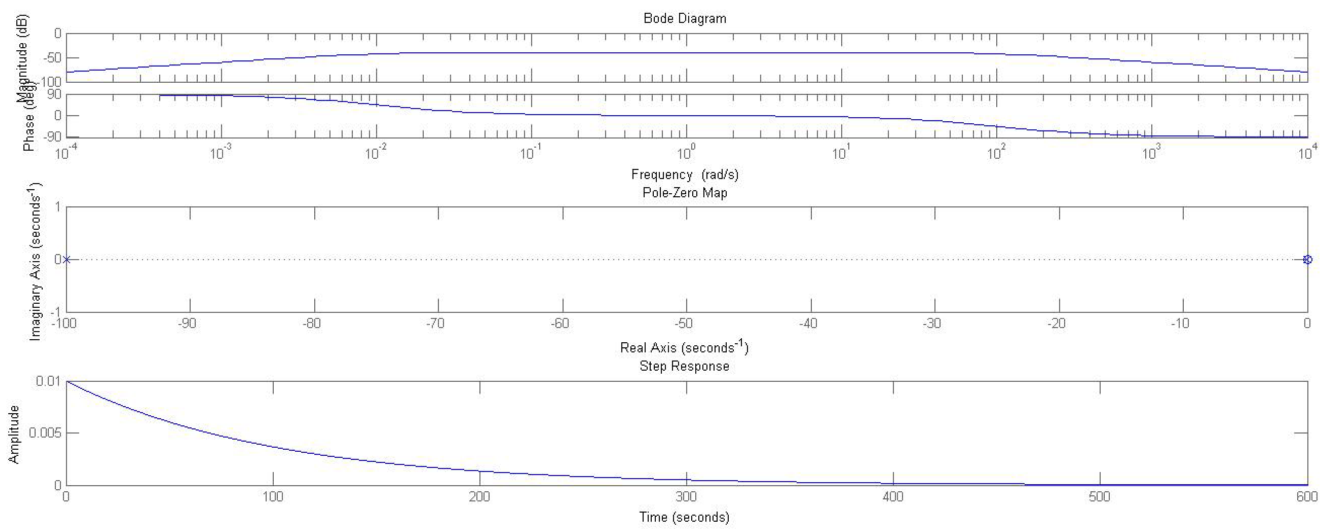
4D. $K(s) = ks$ $\quad \frac{Y(s)}{Y_{sp}(s)} = \frac{ks}{ks^2 - .01ks + k + ks} = \cancel{\frac{ks}{ks^2 + sk}} = \frac{ks}{ks^2 + ks + .01ks + k}$

See figure, You can stabilize & do so effectively

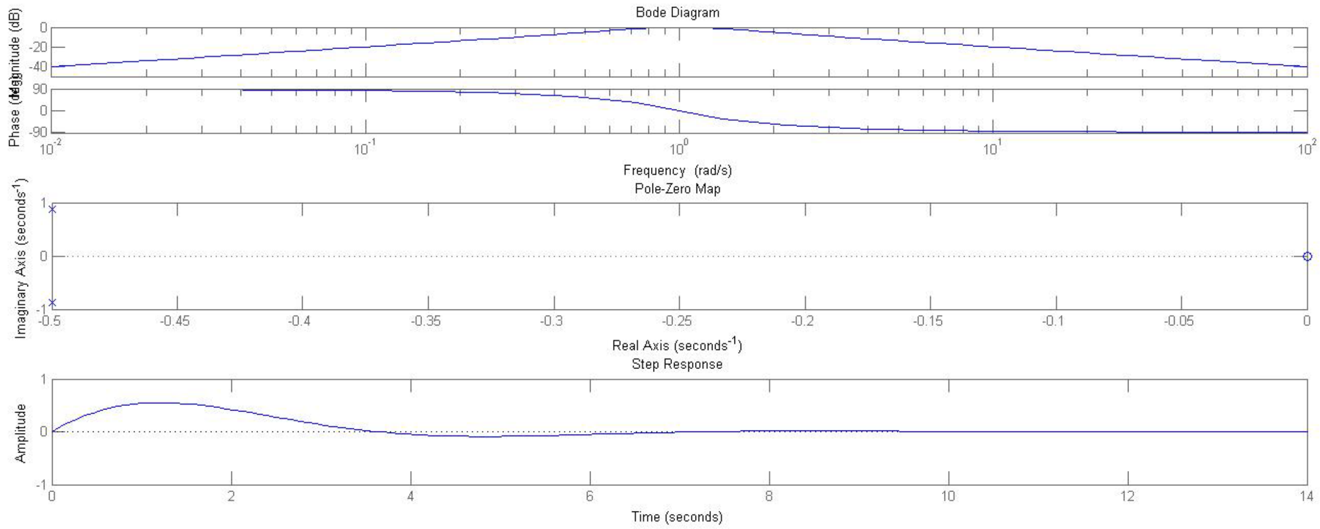
3A



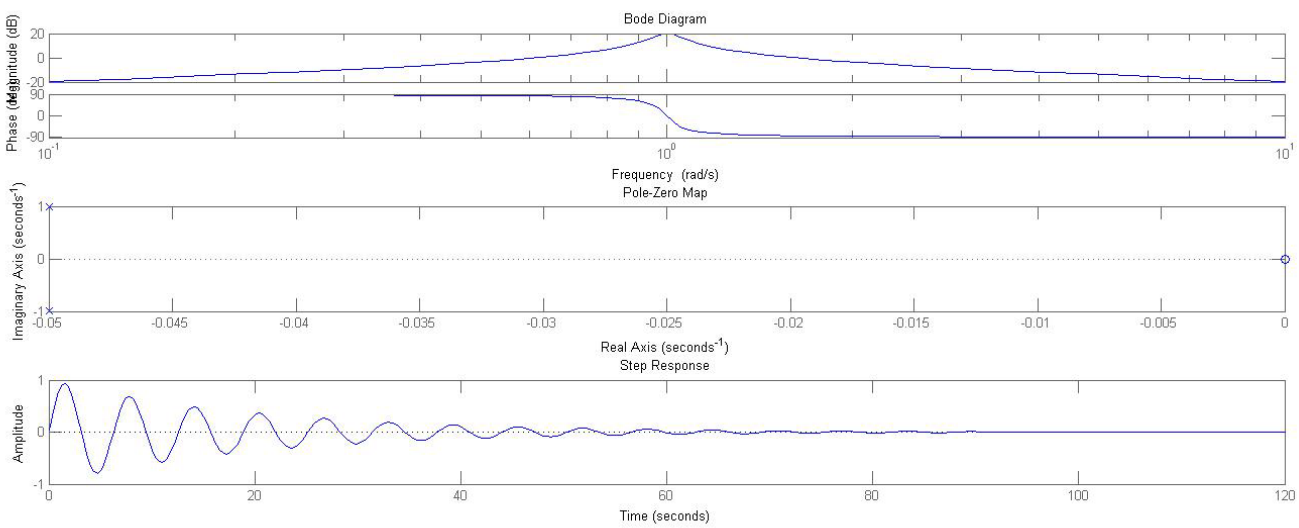
There is a pole at -1+0j. This tells us that there will be no oscillations in the step response and the system will exponentially decay. Since there is zero at the origin of the pole-zero graph we can tell that this system is a high pass filter. Looking at the Bode Plot confirms this.



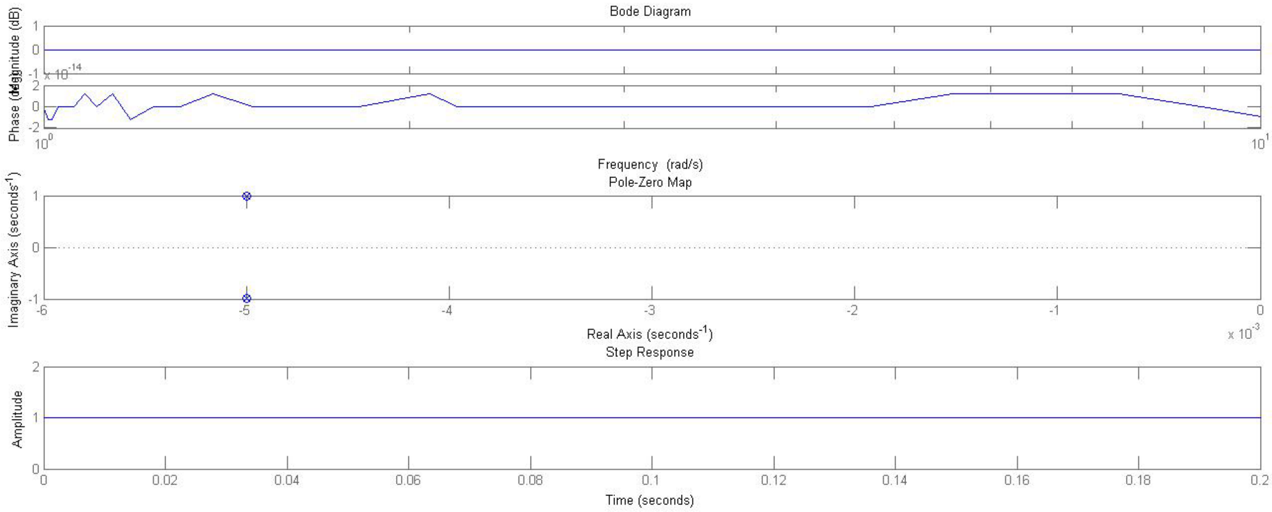
This system is a band pass filter. Poles are going to act to decrease the slope of the bode plot, while zeros will increase the slope. Since there is a zero at the origin the slope of the bode plot will increase initially. However, the pole that is at around 1/100 causes the slope to go from increasing to flat. Now at 100 there is another pole which will cause the slope to go to a decreasing trend.



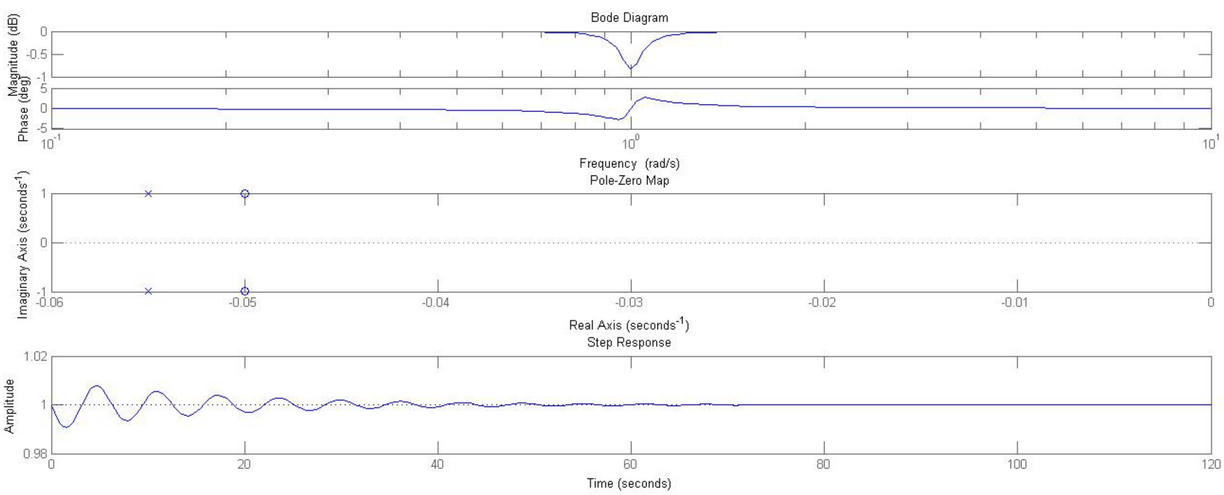
This system contains poles at imaginary values so there will be an oscillation in the step response. However, since the poles are still less than zero in their real component the system will experience exponential decay. There is a zero at the origin so the bode plot starts off increasing, but the two poles that occur at -.5 cause the slope of the Bode plot to quickly become decreasing.



This systems general behavior is similar to the system above, and the three plots relate in the same way. However, the increased angle of the poles causes the system to be less stable and oscillate for much longer.

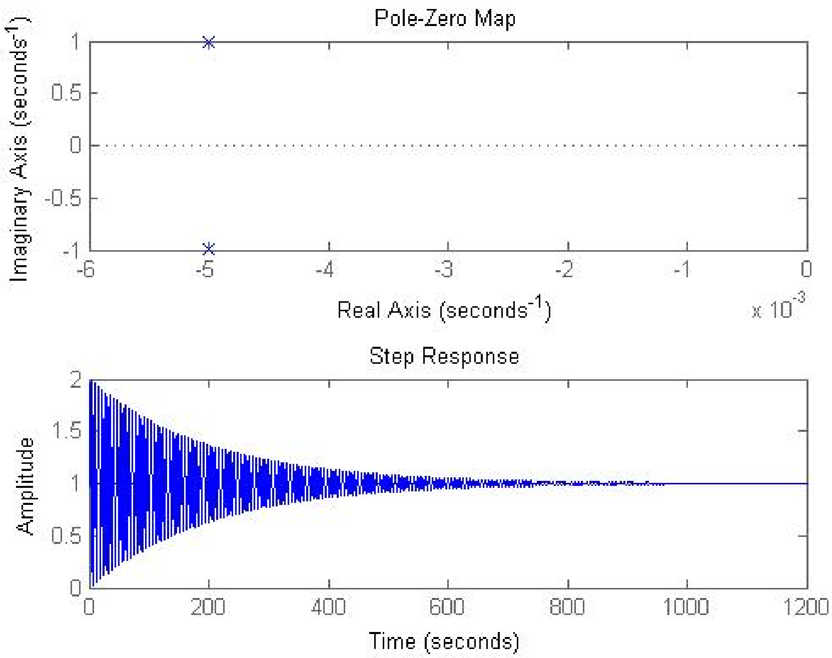


This system is odd. Since there is no pole or zero at the origin the slope of the Bode plot will start at zero. There are also poles and zeros that occur at identical points, so they in effect cancel each other out and the system remains constant.



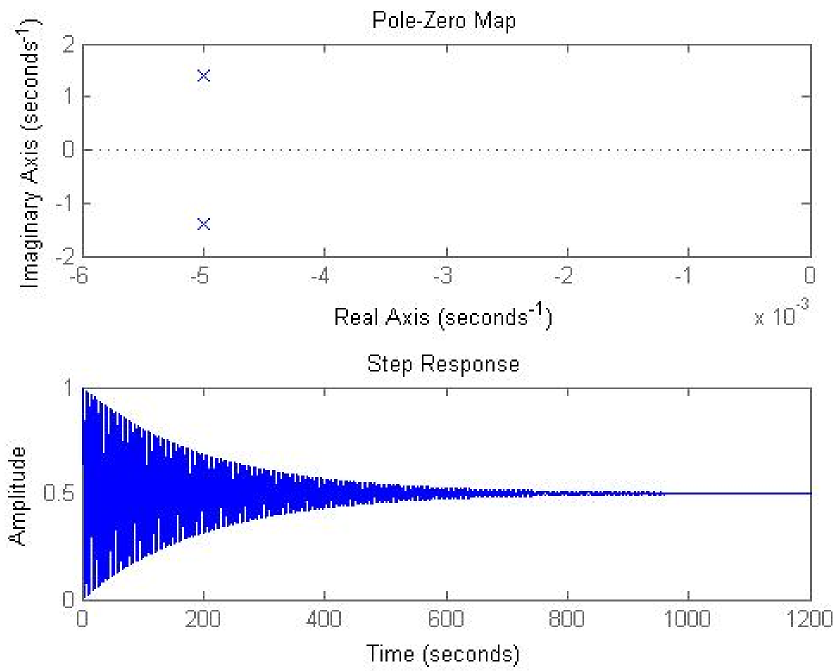
Since the system as negative, imaginary poles the system will oscillate and decay exponentially. The poles and zeros are very close to each other on the real plane so as a result there is very little amplitude change in the Bode plot, but there is a change that exists.

4A.

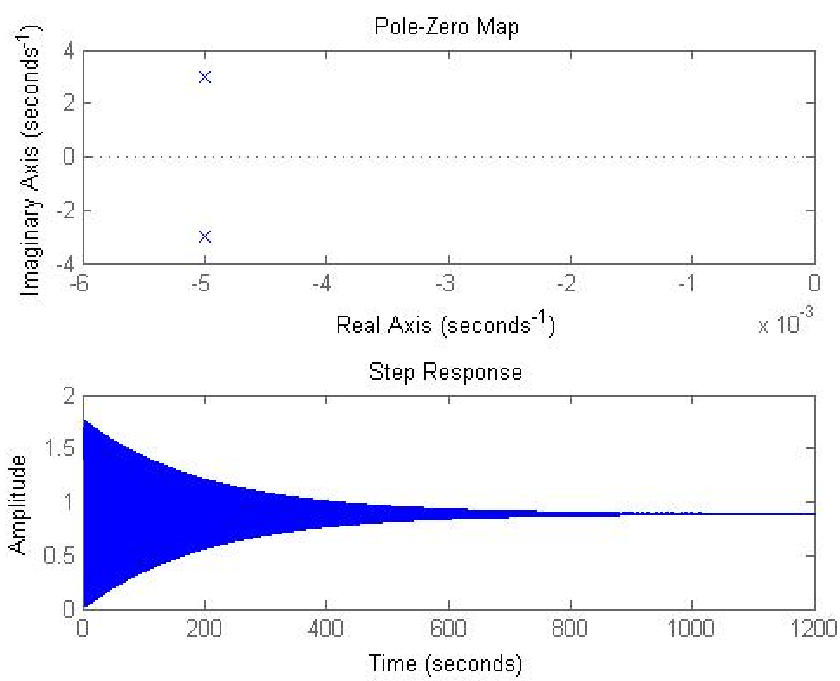

Pole-Zero Map
Imaginary Axis (seconds-1)
1
0.5
0
-0.5
-1
-6
-5
-4
-3
-2
-1
0
Real Axis (seconds-1)
x 10-3
Step Response
Amplitude
2
1.5
1
0.5
0
0
200
400
600
800
1000
1200
Time (seconds)

4B.

Gain = 1

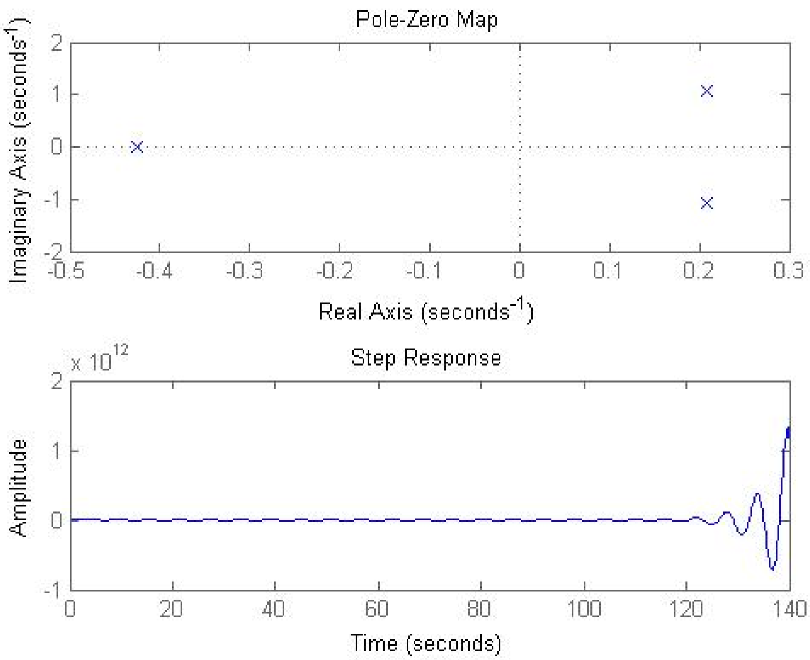


Gain = 8

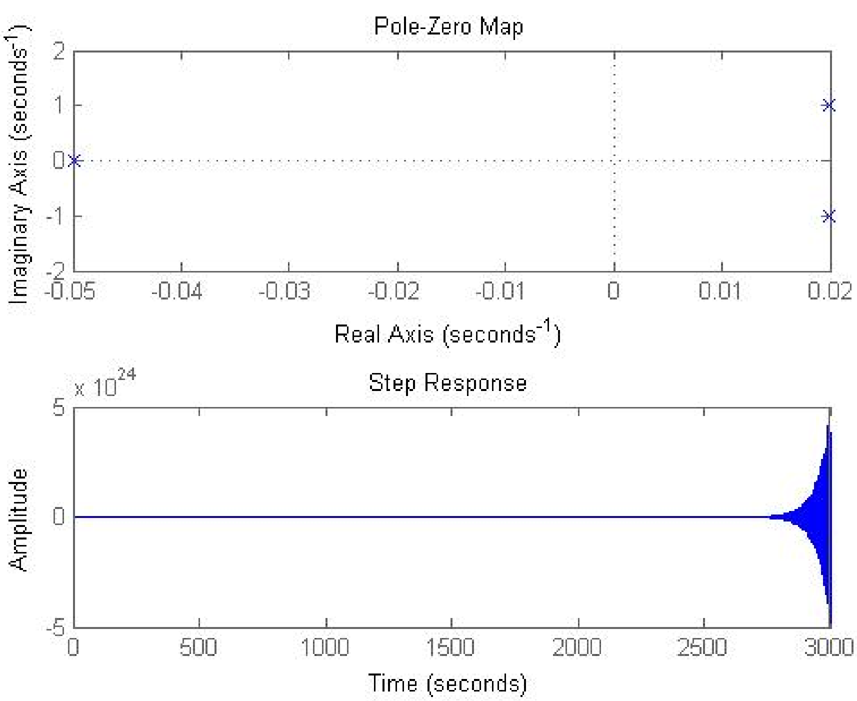


4C

Gain = 2

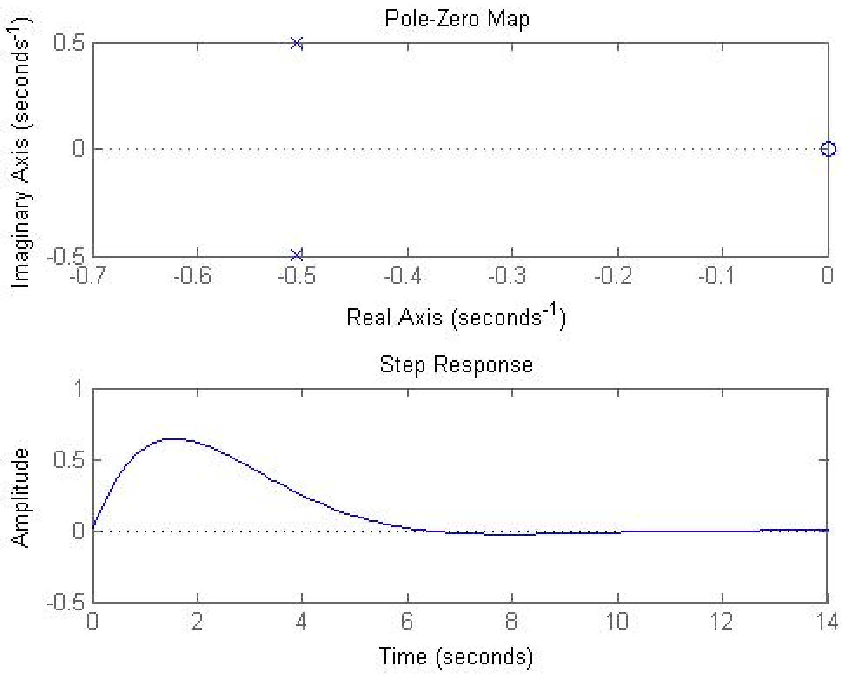


Gain = 20



4D

Gain = 2



Gain = 20