# **An Improved Least Square Channel Estimation Algorithm for Underwater Acoustic OFDM Systems**

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Abstract—Channel estimation is vital for the performance of underwater acoustic (UWA) orthogonal frequency division multiplexing (OFDM) systems. In this paper, we first analyze the conventional least square (LS) channel estimation algorithm, which is very sensitive to noise and frequency synchronization errors, and then we develop a novel LS algorithm. It utilizes the channel frequency response of one pilot symbol to revise that of the following symbol with weighted average, so that it can suppress the error diffusion that results from the former error symbol. Simulation results show that the proposed LS algorithm achieves better bit error rate (BER) and mean square error (MSE) performance compared to the conventional one.

Keywords-OFDM; underwater acoustic communication; channel estimation; least-square

#### I. INTRODUCTION

Underwater acoustic (UWA) channel is characterized by multi-path phenomenon with characteristics as time-varying, narrow-band and frequency-selective fades, which are the main obstacle of the underwater signal's reliable, high-rate and long range transmission [1]-[2]. Multicarrier modulation in the form of the orthogonal frequency division multiplexing (OFDM) can convert a frequency selective fading channel into several almost flat fading channels in order to avoid deeply fading as well as improve the channel bandwidth utilization, so it is preferred to be established in UWA communication. And to achieve coherent modulation, real-time, accurate and low complex channel estimation is an important part of UWA OFDM systems [3].

Different approaches on channel estimation for wireless OFDM systems have been studied over recent years [4]-[7], but those for UWA OFDM systems are not that popular until recently, most of which are based on the pilot symbol assisted modulation (PSAM). PSAM requires the transmitter to insert pilot symbols or training sequences that have been known to a fixed position, and at the receiving end these pilot symbols or training sequences will be used for channel estimation according to some algorithms. Therefore algorithms of high accuracy, low latency and low complexity are needed to cope with the highly scattered, multipath fading UWA channel.

One conventional algorithm is least square (LS), which does not need knowledge on channel statistics or the noise statistics, and it is very easy to be implemented with less computational complexity. However, it is too sensitive to noise and frequency synchronization errors. In addition to LS, several other algorithms have been proposed. Reference [8], for instance, presents a method of transform-domain channel estimation based on minimum mean squared error (MMSE), a process of FFT and IFFT added, and it improves the performance of the OFDM system in UWA channels. Reference [9] proposes a sparse channel estimation method that extracts the portion of the time domain signal corresponding to pilot symbols and its simulation results verify that the algorithm is robust even for highly challenging UWA channel. However, these approaches have their drawbacks. Although they have better performance over LS and can effectively reduce the intercarrier interference (ICI), they are excessively complex and often require the channel statistics accurately, which is too difficult to realize in practical underwater environments [10].

In this paper, we propose an improved PSAM based LS channel estimation algorithm for UWA OFDM systems. It utilizes the channel frequency response of one pilot symbol to revise that of the following symbol with weighted average, and it can suppress the error diffusion due to the former error symbol. Simulation results show that its performance is superior to that of the conventional methods.

The rest of the paper is organized as follows. Section II describes a baseband model of UWA OFDM systems. Section III first analyses the conventional LS channel estimation algorithm and then presents a novel LS algorithm. The performance of the proposed algorithm is evaluated and compared with the conventional LS algorithm in section IV and some concluding remarks are shown in section V.

## II. SYSTEM DESCRIPTION

Fig. 1 shows a baseband model of UWA OFDM systems based on PSAM [11]. Perfect frequency offset correction are assumed. The binary information data are first grouped and mapped according to the modulation in signal mapper block. The mapped signal are converted into parallel, which is more efficient for high data rate communication.

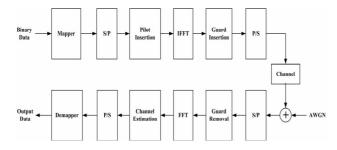


Figure 1. Baseband model of UWA OFDM systems

After pilot insertion either to all sub-carriers with a specific period or uniformly between the information data sequence, the modulated data  $\{X(k)\}$  of length N are sent to inverse fast Fourier transform (IFFT) block and are transformed and multiplexed into time domain signal  $\{x(n)\}$  as

$$x(n) = IFFT\{X(k)\} \qquad n = 0, 1, ..., N - 1$$
  
=  $\sum_{j=1}^{N-1} X(k)e^{j2\pi kn/N}$  (1)

Then, guard time whose length is longer than the expected delay spread is inserted to prevent possible intersymbol interference (ISI). This guard time includes the cyclically extended part of OFDM symbol in order to eliminate ICI. The resultant samples  $x_f(n)$  are given as

$$x_f(n) = \begin{cases} x(N+n), & n = -N_g, -N_g + 1, \dots, -1 \\ x(n), & n = 0, 1, \dots, N - 1 \end{cases}$$
 (2)

where  $N_g$  is the length of guard time. Signals described in (2) can be received at the receiver after passing through the frequency selective time varying fading channel, and added with additive white Gaussian noise (AWGN). The received signal can be expressed as

$$y_f(n) = x_f(n) \otimes h(n) + \omega(n)$$
 (3)

where  $\otimes$  denotes the convolution and  $\omega(n)$  is AWGN, and h(n) is the channel impulse response that can be viewed as

$$h(n) = \sum_{i=0}^{L-1} h_i \delta(n - \tau_i)$$
 (4)

where L is the total number of paths,  $h_i$  denotes the complex impulse response of the i th path, and  $\tau_i$  is the i th-path delay time normalized by sampling time. After passing to discrete domain through A/D and low-pass filter, guard time is removed. When  $\tau_i$  is assumed not to exceed the length of guard interval, signals that will get through the FFT block can be described as

$$y(n) = y_r(n+N_g)$$
  $n = 0,1,...,N-1$  (5)

Getting through FFT block, they finally turn out to be

$$Y(k) = FFT\{y(n)\}$$

$$= \sum_{n=0}^{N-1} y(n)e^{-j(2\pi kn/N)} \qquad k = 0, 1, ..., N-1$$
(6)

Assuming there is no ISI, the demultiplexed samples Y(k) can be expressed as

$$Y(k) = X(k)H(k) + W(k)$$
  $k = 0,1,...,N-1$  (7)

where H(k) is the frequency response of the k th subchannel and W(k) is the Fourier transform of  $\omega(n)$ . After the pilot signals are extracted from the received symbols, the estimated channel for the data sub-channels is obtained in channel estimation block, and then the binary information data is obtained back in demapper block.

#### III. LS CHANNEL ESTIMATION

The first step of PSAM based channel estimation is to estimate the channel frequency response at pilot symbols, and then get the channel frequency response of the entire channel with different interpolation methods. Here we mainly discuss the first step and propose an improved LS channel estimation algorithm.

### A. LS Channel Estimation Algorithm

For simplicity, we express (7) in matrix notation as

$$Y = XFh + W \tag{8}$$

where

$$Y = [Y(0) \ Y(1) \dots Y(N-1)]^{T}$$

$$W = [W(0) \ W(1) \dots W(N-1)]^{T}$$

$$H = [H(0) \ H(1) \dots H(N-1)]^{T} = FFT_{N}\{h\}$$

$$X = diag\{X(0), X(1), \dots, X(N-1)\}$$

$$F = \begin{bmatrix} W_{N}^{00} & \cdots & W_{N}^{0(N-1)} \\ \vdots & \ddots & \vdots \\ W_{N}^{(N-1)0} & \cdots & W_{N}^{(N-1)(N-1)} \end{bmatrix}$$

$$W_{N}^{nk} = e^{-j2\pi nk/N} / \sqrt{N} \qquad k, n = 0, 1, \dots, N-1$$
(9)

The LS channel estimation algorithm in itself is to estimate the parameter H in (7) by LS to minimize the cost function C, which can be represented as

$$C = (Y - XF \hat{h})^{H} (Y - XF \hat{h})$$
 (10)

where

$$\hat{H} = [\hat{H}(0) \ \hat{H}(1) \dots \hat{H}(N-1)]^{T} = FFT_{N}\{\hat{h}\}$$
 (11)

So we only have to make the partial derivative of (10) with respect to  $\hat{h}$  equal to zero as

$$\frac{\partial C}{\partial \hat{h}} = 2F^H X^H X F \, \hat{h} - 2F^H X^H Y = 0 \tag{12}$$

and therefore

$$\hat{H} = F \hat{h} = X^{-1}Y \tag{13}$$

B. Improved LS Channel Estimation Algorithm Furthermore, (13) can be transformed as

$$\hat{H} = X^{-1}Y 
= X^{-1}(XH + W) 
= H + X^{-1}W$$
(14)

If there is no ISI, the orthogonality of various sub-channels will not be affected. So the sub-channels of the OFDM system can be viewed as independent of each other. Now for the k th sub-channel, we introduce the label of pilot sequence m as

$$\hat{H}_{k,m} = H_{k,m} + E_{k,m} \tag{15}$$

where the estimation error  $E_{k,m} = W_{k,m}/X_{k,m}$ .

The improved LS channel estimation algorithm utilizes the channel frequency response of one symbol to revise that of the following symbol with arithmetic mean. However, this method can not be used directly because the channel is timevarying. With  $H_{k+}$  as the benchmark, we take into account

 $\tau_m$ , the offset time of the m th symbol, to obtain

$$\hat{H}_{k,m} = e^{j2\pi k \tau_m/N} H_{k,1} + E_{k,m}$$
 (16)

Likewise,

$$\hat{H}_{k,m-1} = e^{j2\pi k \tau_{m-1}/N} H_{k,1} + E_{k,m-1}$$
 (17)

The two equations above can be arithmetic averaged only if they have the consistent offset time, i.e.

$$H_{k_{m-1}}^{\circ} = e^{j2\pi k(\tau_m - \tau_{m-1})/N} \hat{H}_{k_{m-1}}$$
 (18)

where  $H_{k,m-1}^{\circ}$  is an intermediate variable. Then we can finally obtain the revised channel frequency response  $H_{k,m}^{\mathbb{R}}$  using the algorithm as

$$H_{k,m}^{\Theta} = \alpha \hat{H}_{k,m} + (1 - \alpha) H_{k,m-1}^{\circ}$$
 (19)

where  $\alpha$  is the weighting coefficient.

Since the algorithm is described as (19), now we only have to figure out the unknown quantity  $\Delta \tau = \tau_m - \tau_{m-1}$ . To get  $\Delta \tau$ , we introduce  $P_k = \hat{H}_{k,m} \bullet \hat{H}_{k,m-1}$ , where  $\hat{H}_{k,m-1}$  is the conjugate complex number of  $\hat{H}_{k,m-1}$ , and therefore we can obtain

$$P_{k} = e^{j2\pi k \Delta \tau/N} \left| H_{k,1} \right|^{2} + \Omega \tag{20}$$

where  $\Omega$  is a Gaussian distribution whose average value is 0. Now, if we take the mathematical expectation of  $P_k$  as

$$E[P_k] = e^{j2\pi k \Delta \tau/N} \left| H_{k,1} \right|^2 \tag{21}$$

we can see that  $\Delta \tau$  only depends on the compound angle of  $E[P_{\iota}]$ . Therefore,

$$\Delta \tau = N/2\pi k \times \arg[E[P_k]] \tag{22}$$

Now that  $\Delta \tau$  can be achieved, the revised channel frequency response can be achieved using (19).

The proposed algorithm can also be extended to get revised time after time as

$$H_{k,m}^{\Theta} = \alpha_1 \hat{H}_{k,m} + \alpha_2 H_{k,m-1}^{\circ (n-1)} + \dots + \alpha_n H_{k,m-n+1}^{\circ}$$
 (23)

where  $\alpha_1 + \alpha_2 + ... + \alpha_n = 1$ , and  $H_{k,m-1}^{\circ (n-1)}$  is an intermediate variable rephrased for n-1 times.

# IV. SIMULATIONS

The simulations are focused on the simulated bit error rate (BER) and mean square error (MSE) performance comparison between the conventional LS channel estimation algorithm and the improved LS algorithm proposed in this paper. The considered UWA OFDM system parameters are listed in Table I.

Fig. 2 and Fig. 3 respectively show the BER and MSE performance of the two algorithms. From Fig. 2, we can see that when  $\alpha = 0.3$ , the proposed algorithm needs about 27.5dB to reach  $10^{-3}$  BER performance and when  $\alpha = 0.6$ , it needs 25dB, while the conventional algorithm needs nearly 30dB. Similar results can be seen from Fig. 3, and they both show the performance superiority of the proposed algorithm.

TABLE I. SYSTEM PARAMETERS

Parameters	Specifications
Bandwidth	1MHz
Carrier frequency	1GHz
Number Of Subcarriers	128
Subcarrier Interval	7.8kHz
Guard Interval	25.6ns
Pilot ratio	3
Modulation	QPSK
Channel model	Rayleigh channel

They also show that the BER and MSE performances are improved when the weighting coefficient get closer to 0.5 (actually we can prove that when the weighting coefficient equals to 0.5 the system has the best performance).

#### V. CONCLUSION

Channel estimation based on LS algorithm for UWA OFDM systems is very sensitive to noise and frequency synchronization errors. To improve its performance, this paper proposes a modified PSAM based LS channel estimation algorithm, which utilizes the channel frequency response of one pilot symbol to revise that of the following symbol with weighted average. Since this method uses part of the channel frequency response of the former symbol, it can effectively exercise its restrains on the error diffusion that results from the former error symbol. Simulation results verify that the proposed algorithm greatly outperforms the conventional LS algorithm.

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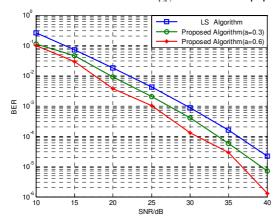


Figure 2. BER performance according to SNR

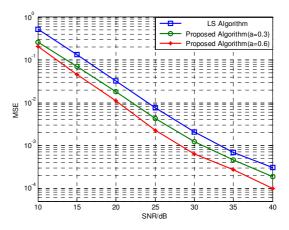


Figure 3. MSE performance according to SNR

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