On Optimizing the Transmission Power of Multi-hop Underwater Acoustic Networks

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***Abstract*—This paper analyses how to optimize the transmission power for multi-hop underwater acoustic networks in a rigorous and theoretical way. In practical design, the transmission power is the function of two variables: internode distance *d* and signal frequency *f*, and for a prespecified *d*, there exists an optimal frequency which minimizes the transmission power. A close-form approximation for the optimal frequency as a function of *d* is obtained using curve-fitting, and the optimal transmission power as a function of only one variable *d* is deduced. A power-based cost function is proposed for a linear multi-hop network model to identify the minimum number of hops that optimizes the transmission power of the whole link. Besides, two communication strategies, namely direct-access strategy and relay strategy, are investigated in the linear multi-hop scenario to set forth how communication strategy affects the total power consumption. Simulation results show that the strategy which saves more power consumption is relaying, whose savings become more significant at longer distances.**

Keywords-transmission power, underwater acoustic, linear multi-hop network, optimal frequency, minimum number of hops

I. INTRODUCTION

Underwater exploration has been recognized as a key step towards a fuller understanding and sustaining of life on earth. However, communication in the underwater acoustic networks (UWANet) has been quite challenging [1, 2]. The most distinguishing property of acoustic channels is the fact that path loss depends on both the transmission distance and the signal frequency [3], and thus leads to much more transmission power when longer distance communication is needed. What's more, as the sensors are battery-powered, it could be difficult to replace or recharge batteries in underwater environments. Besides, in UWAN the power required for transmitting is typically about 100 times more than the power required for receiving [4, 5], so how to optimize transmission power as to prolong the lifetime of the network is a critical issue for UWANets.

Current researches focus on an information-theoretic approach to explore the correlations among capacity, bandwidth, band-edge frequency, transmission distance and power consumption, and on that basis seek ways to optimize transmission power. Ref. [6] assesses functional dependence of the system capacity on the transmission distance and defines the bandwidth corresponding to optimal signal energy allocation-one that maximizes the channel capacity subject to the constrained transmission power. Ref. [7] provides a closed-form approximate model for power consumption as functions of distance and capacity required for a data link, which is obtained by numerical evaluation of analytical results taking into account physical models of acoustic propagation loss and ambient noise. Ref. [8] proposes a network coding based lower bound for transmission power in UWAN, applying a tractable model for the underwater acoustic channel from Ref. [7] and compares this bound to the performance of several network layer schemes. Ref. [9] analytically optimizes the power spectral density of the signaling in a way analogous to waterfilling and examines the problem of determining the minimum number of hops to support a prespecified rate and reliability with and without a maximum coded packet length constraint.

In these researches, however, there is no close-form expression of the relationship between optimal frequency that maximums the narrow-band signal to noise ratio (*SNR*) and the transmission distance, and the transmission power is expressed from either a heuristic or an information-theoretic perspective, which is too complicated to analyse. So in this paper, we deal with this problem in a rigorous and theoretical way, aiming at revealing the optimal transmission power for multi-hop UWANets.

II. UNDERWATER ACOUSTICS FUNDAMENTALS

A. The passive sonar equation

A method for detecting acoustic signals in an underwater environment is passive sonar, the purpose of which is to detect signals embedded in noise. And a rough prediction of detection performance can be obtained from the passive sonar equation [10], which characterizes the *SNR* of an emitted underwater signal at the receiver as

$$SNR = SL - TL - NL + DI \tag{1}$$

where *SL* is the source level, *TL* is the transmission loss, *NL* is the noise level, and *DI* is the directivity index. All the quantities in eq. (1) are in dB re μPa, where the reference value of 1 μPa amounts to 0.67×10^{-22} Watts/cm^2. In the rest of the paper, the shorthand sign of dB is used to notate dB re μPa.

B. Transmission loss and ambient noise

Underwater acoustic transmission loss is due to geometric spreading and absorption mechanisms. Urick [10] expresses the total transmission loss *TL* (in dB) for a signal of frequency *f* (in kHz) as

$$TL = k \times 10\log(d \times 10^3) + \alpha(f) \times d \tag{2}$$

where *k* is the spreading factor that describes the geometry of propagation (e.g. k = 2 corresponds to spherical spreading, k = 1 to cylindrical spreading, and k = 1.5 to the so-called practical spreading), *d* is the distance between source and receiver in kilometer, and $\alpha(f)$ is the frequency dependent absorption coefficient in dB/km, which can be expressed empirically, using [6]:

$$\alpha(f) = 0.11\frac{f^2}{1+f^2} + 0.011f^2 + 0.002 \tag{3}$$

For underwater acoustic channel, four basic sources that model the ambient noise *NL*, turbulence, shipping, waves and thermal noise, have to be considered, and the overall ambient noise in dB is given by [11]

$$NL(f) = NL_t(f) + NL_s(f) + NL_w(f) + NL_{th}(f) \tag{4}$$

where $NL_t(f)$, $NL_s(f)$, $NL_w(f)$ and $NL_{th}(f)$ represent the ambient noise caused by turbulence, shipping, waves and thermal noise, respectively. It may be useful to learn that the noise decays linearly in a certain frequency region as [6]

$$NL(f) = NL_a - \eta\log f \tag{5}$$

where NL_a = 50 dB and η =18 dB/decade.

C. Transmission power

We have shown how the source level *SL* relates to internode distance *d* and signal frequency *f* by passive sonar equation. *SL* also relates to the transmitted signal intensity I_t at 1 m from the source as

$$SL = 10\log\frac{I_t}{1\mu Pa} \tag{6}$$

where I_t is in μPa and can be attained through

$$I_t = 10^{SL/10} \times 0.67 \times 10^{-18} \tag{7}$$

in Watts/cm^2, where the constant has converted μPa into Watts/cm^2.

Therefore, the transmission power P_t needed to achieve an intensity I_t at a distance of 1 m from the source in the direction of the receiver can be expressed as [10]

$$P_t = 2\pi \times 1m \times H \times I_t \tag{8}$$

in Watts, where *H* is the water depth in meter.

III. Transmission Power Analysis

What has discussed in section II explicitly proposes a method to obtain the required transmission power P_t for signal transmissions at given *SNR*, water depth *H*, distance *d* and frequency *f*. First, we can compute the transmission loss *TL* in terms of *f* and *d* and ambient noise *NL* in terms of *f*. Then we can compute the source level *SL* in terms of a given *SNR*, which yields the source intensity I_t. Finally, we can obtain the corresponding transmission power P_t needed to achieve a source intensity of I_t at a given water depth *H*.

A. Properties of transmission power

As in practical design, *SNR* and *H* are predetermined as SNR_0 and H_0, so hereafter we only consider P_t as function of *d* and *f*, denoted by $P_t(d, f)$. But still the optimizing of $P_t(d, f)$ is rather complicated because it is a problem with two variables. So first, we address the following propositions.

Proposition 1 The transmission power $P_t(d,f)$ is a monotonically increasing function of internode distance *d*.

Proof Apparently, the monotony of $P_t(d,f)$ is consistent with that of *SL*. And we notice that

$$SL = 0.11d\frac{f^2}{1+f^2} + 0.011df^2 - 18\log f + 10\log d + 0.002d + SNR_0 + 80 \tag{9}$$

and further,

$$\frac{\partial SL}{\partial d} = 0.11\frac{f^2}{1+f^2} + 0.011f^2 + \frac{10}{d\ln 10} + 0.002 > 0 \tag{10}$$

Therefore, *SL* is a monotonically increasing function of *d*. So does $P_t(d,f)$. □

Remark Multi-hop approach is motivated for UWANets because shorter links require less transmission power.

Proposition 2 For a prespecified internode distance *d*, there exists an optimal frequency $f_o(d)$ which minimizes the transmission power $P_t(d, f)$, i.e.

$$\min_f P_t(d,f) = P_t(d, f_o(d)) \tag{11}$$

Proof For *SL* with respect to a given *d*, we observer that

$$\lim_{f\to 0^+}\frac{\partial SL}{\partial f} < 0, \quad \text{and} \quad \lim_{f\to +\infty}\frac{\partial SL}{\partial f} > 0. \tag{12}$$

So, there exists an optimal frequency $f_o(d)$ which minimizes *SL* and such that

$$\left.\frac{\partial SL}{\partial f}\right|_{f = f_o(d)} = 0 \tag{13}$$

And it is of little difficulty to reach that $f_o(d)$ minimizes $P_t(d, f)$ as well. □

Remark Eq. (13) indicates that $f_o(d)$ can be a function of *d* so that $P_t(d, f)$ can be simplified to associate only with one variable *d*.

Fig. 1 shows the *SL* as a function of *f* and *d*. It is clear that for each *d*, there exists an optimal frequency for which the minimum *SL* is obtained, and it also shows that the optimal frequency decreases when *d* increases.

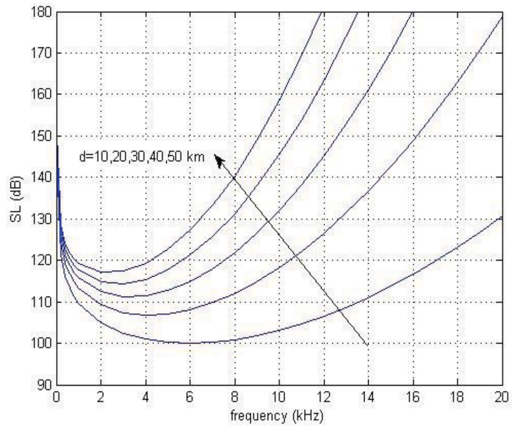

Figure 1. *SL* as a function of frequency and distance

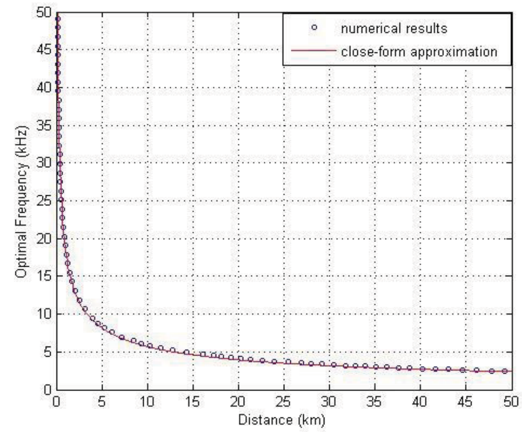

Figure 2. Distance vs. optimal frequency

B. Transmission power optimization

To obtain the relationship between d and $f_o(d)$, we solve eq. (13)

$$\left.\frac{\partial SL}{\partial f}\right|_{f=f_o(d)} = \frac{0.22df_o(d)}{(1+f_o^2(d))^2} + 0.022df_o(d) - \frac{18}{f_o(d)\ln 10} = 0 \quad (14)$$

and get

$$d = \frac{9000(1+f_o^{\,2}(d))^2}{11\ln 10(f_o^{\,6}(d)+2f_o^{\,4}(d)+11f_o^{\,2}(d))} \quad (15)$$

However, we can only attain a close-form solution for node distance as a function of optimal frequency from eq. (14), but not vice versa. A closer examination of the numerical results reveals that the optimal frequency decays almost linearly with internode distance on a logarithmic scale. Hence we use curve fitting methods to obtain such approximations of two general models, the parameters of which are shown in Tab. 1 with their goodness of fit in terms of the sum of squares due to error (SSE) and the root mean squared error (RMSE).

TABLE I. PARAMETERS OF THE CLOSE-FORM APPROXIMATION

General model	Coefficients	Goodness of fit
$f_o(d) = \alpha d^{\beta}$	$a = 18.89$, $\beta = -0.5012$	SSE: 1.2946, RMSE: 0.1642
$f_o(d) = \alpha d^{\beta}+\chi$	$a = 19.12$, $\beta = -0.4962$, $\chi = -0.2796$	SSE: 0.8086, RMSE: 0.1274

As shown in eq. (15), $f_o(d)$ is in the form of even power (such as square, biquadrate) while the parameter β of the close-form approximations nearly equals to -0.5, which enables us to simplify our calculations. For this reason, we finally choose the close-form approximation as

$$f_o(d) = \frac{19}{\sqrt{d}} \quad (16)$$

with its SSE 2.6207 and RMSE 0.2196, which is illustrated in Fig. 2 (solid curve) and exhibits good agreement with numerical results (circles).

The transmission power $P_t(d, f)$ thus can be simplified by substituting $f_o(d)$ for f as

$$P_t(d) = 2\pi H_0 \times 10^{1.9\log d + \frac{3.971d}{d+361} + 0.0002d + 0.1SNR_0 + 6.095} \times 0.67 \times 10^{-18} \quad (17)$$

which reflects the relationship between optimal transmission power and internode distance.

IV. OPTIMIZING TRANSMISSION POWER FOR NETWORKS

Section III analyses the optimal transmission power for a point-to-point communication model. On account of the fact that shorter links require less transmission power, multi-hop approach is motivated. And the question spontaneously arises as to whether there exists an optimal number of hops so as to optimize the link power consumption. So in this section we take a linear multi-hop network model to identify the optimal number of hops under the power-based cost function.

A. Linear multi-hop network model

The linear multi-hop network model, which can be used for nearshore environment monitoring, is illustrated in Fig. 3. Nodes are denoted by N_k, $k = 0, 1, 2, \ldots, K$, among which N_0 is the source, N_K is the destination, and the remaining nodes are relays located between N_0 and N_K. The relay nodes receive the incoming signal, regenerate it, and pass it on to the next hop until the final destination is reached. All the nodes are uniformly distributed along a straight line so that the distance between every two adjacent nodes is d/K. The transmission power of each node thus is

$$P_t(\frac{d}{K}) = 2\pi H_0 \times 10^{1.9\log(d/K) + \frac{3.971d/K}{d/K+361} + 0.0002d/K + 0.1SNR_0 + 6.095} \times 0.67 \times 10^{-18} \quad (18)$$

Without loss of generality as well as for simplicity, we assume $SNR_0 = 19.05$ and $H_0 = 237.5$ so that $0.1SNR_0 + 6.095 = 8$ and $2\pi H_0 \times 0.67 = 1000$. Actually, for any H_0 we can turn it

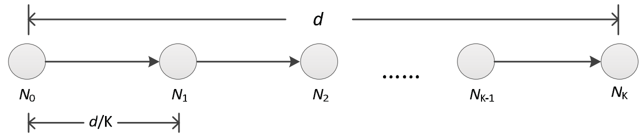

Figure 3. Linear multi-hop network model

into the form of 10^x and thus simplify the calculation. Therefore, eq. (18) can be simplified as

$$P_t(\frac{d}{K}) = 10^{1.9\log(d/K)+\frac{3.971d/K}{d/K+361}+0.0002d/K-7} \tag{19}$$

And the transmission power of the whole link is

$$P_T(d) = KP_t(\frac{d}{K}) = K10^{1.9\log(d/K)+\frac{3.971d/K}{d/K+361}+0.0002d/K-7} \tag{20}$$

which trails off with the increase of the number of hops as is demonstrated in Fig. 4.

B. Minimum number of hops

Since the more number of hops the link bears, the less transmission power it may cost, it seems that more relays should be used to gain the least power consumption. In practice, however, more relays mean more additional costs, such as hardware and deployment costs, or the costs of total delay [12]. So there is a tradeoff between transmission power and practical costs, i.e. a minimum number of hops. Note that the end-to-end coding delay is beyond the scope of this paper, so we only consider the fixed deployment costs, expressed by C_0 in Watts.

Define the cost function in respect to optimal transmission power as

$$C_T(d,K) = P_T(d) + C_0(K-1) = KP_t(\frac{d}{K}) + C_0(K-1) \tag{21}$$

In that $K \geq 2$, we can see

$$\lim_{K\to 2}\frac{\partial C_T(d,K)}{\partial K} < 0 \quad \text{and} \quad \lim_{K\to\infty}\frac{\partial C_T(d,K)}{\partial K} > 0 \tag{22}$$

so the minimum number of hops K_o can be determined analytically by $\frac{\partial C_T(d,K)}{\partial K} = 0$, treating K as a continuous variable. Though theoretically feasible, the calculation of K_o is vastly more complicated, so we evaluate the cost function numerically. The power based cost function versus the number of hops in different distances is illustrated in Fig. 5. It clearly shows the existence of an optimal number of hops to use over a given link distance d. Fig. 6 presents the minimum number of hops as the prespecified distance is changed from 5 to 50 km, and the K_o is piecewise integer valued from 1 to 11 as the distance increases. It should be pointed out at last that how to set the value of the fixed per-node deployment cost remains open.

C. Effect of communication strategy

We further consider two communication strategies in the linear multi-hop scenario, which is not exactly the same as in section IV-B since the nodes N_k, $k = 1, 2, \ldots, K$-1 in this section are not just treated as relays but ones that can also transmit signal. In the first strategy, each node (N_k, $k = 0, 1, 2, \ldots, K$-1) has a direct access to the destination node, and in the second strategy, each node transmits only to its nearest neighbour, which then relays the signal toward the destination node. The total transmission power for both cases are explored.

For the direct access strategy, the total transmission power is

$$P_{direct} = P_t(\frac{d}{K}) + P_t(\frac{2d}{K}) + \ldots + P_t(\frac{Kd}{K}) = \sum_{i=1}^{K} P_t(\frac{id}{K}) \tag{23}$$

For the relay strategy, the total transmission power is

$$P_{relay} = P_t(\frac{d}{K}) + 2P_t(\frac{d}{K}) + \ldots + KP_t(\frac{d}{K}) = \frac{K(K+1)}{2}P_t(\frac{d}{K}) \tag{24}$$

The total consumed power for both strategies is illustrated in Fig. 7. Obviously the case of relay saves more transmission power, and the savings are more notable for a larger number of relay hops and become more significant at longer distances. The price of the relay strategy to pay, as mentioned before, is hardware and deployment costs, the costs of total delay as well as more sophisticated communication protocols, and the tradeoff between the number of hops and the cost therefore can be determined with the method discussed in section IV-B.

V. CONCLUSION

In this paper, we offered an insight into optimizing transmission power for multi-hop underwater acoustic networks in a rigorous and theoretical way. We first addressed two properties of the transmission power: 1) The transmission power is a monotonically increasing function of internode distance d; 2) For a prespecified d, there exists an optimal frequency which minimizes the transmission power. Then we used curve fitting methods to obtain a close-form approximation for the optimal frequency as a function of d, and finally deduced the optimal transmission power as a function of only one variable d. We further took a linear multi-hop network model, proposed a power-based cost function, and identified the minimum number of hops that optimizes the transmission power of the whole link. Besides, we investigate two communication strategies, namely direct-access strategy and relay strategy, in the linear multi-hop scenario, and find that the strategy which saves more power consumption is relaying, whose savings become more remarkable at longer distances. Our further works will be devoted to the analysis of the minimum number of hops for more complex network scenarios, taking into account the cost function in terms of capacity, link delay, reliability and available bandwidth.

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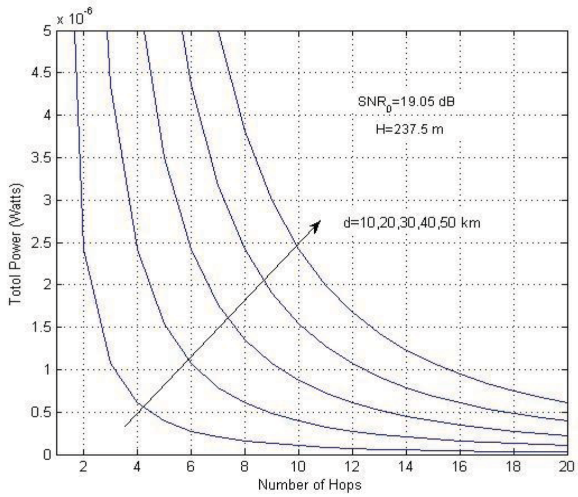

Figure 4 Total power as a function of number of hops and distance

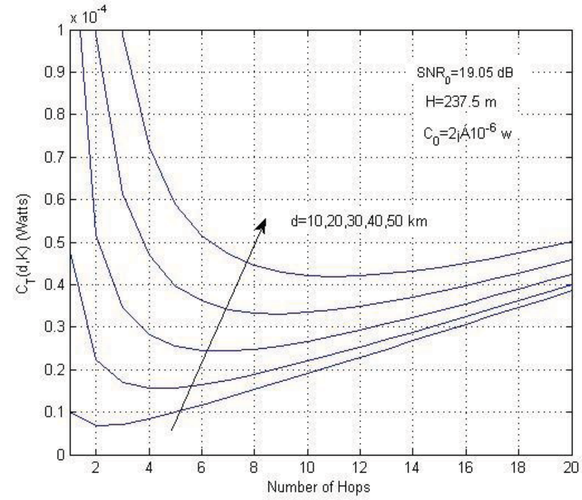

Figure 5 Power cost as a function of number of hops and distance

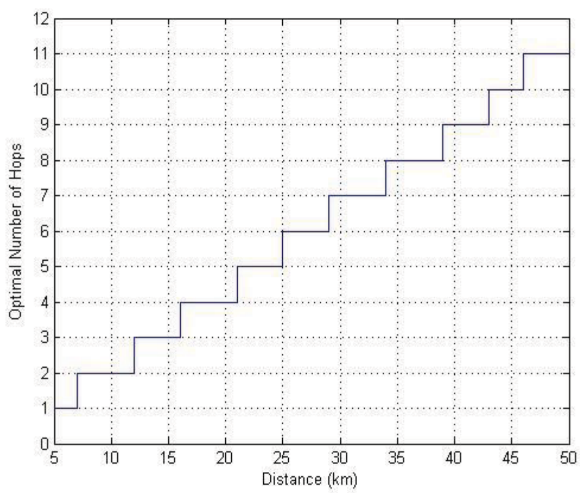

Figure 6 Minimum number of hops vs. distance

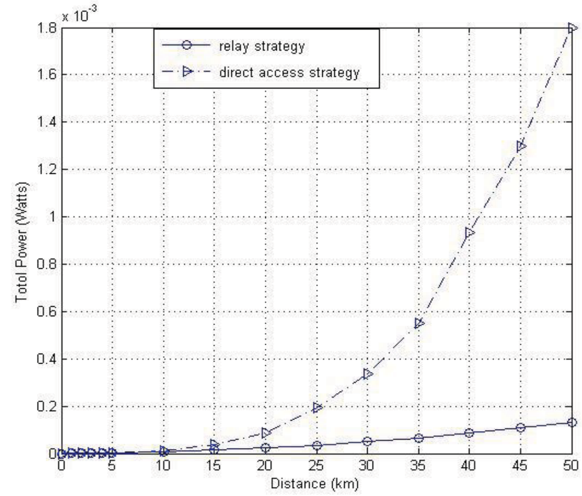

Figure 7 Total power of both strategies

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