

Quantum Computing Cheat Sheet

1 States

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}, \quad |\alpha|^2 + |\beta|^2 = 1$$

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} \quad |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$|\Phi^\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle) \quad |\Psi^\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$$

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle, \quad 0 \leq \theta \leq \pi, \quad 0 \leq \phi < 2\pi$$

2 Unitary Operators

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \begin{array}{ll} |0\rangle \mapsto |0\rangle & |+\rangle \mapsto |+\rangle \\ |1\rangle \mapsto |1\rangle & |-\rangle \mapsto |-\rangle \end{array} \quad \text{---}$$

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \begin{array}{ll} |0\rangle \mapsto |1\rangle & |+\rangle \mapsto |+\rangle \\ |1\rangle \mapsto |0\rangle & |-\rangle \mapsto -|-\rangle \end{array} \quad \text{---} \boxed{X} \text{---}$$

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad \begin{array}{ll} |0\rangle \mapsto i|1\rangle & |+\rangle \mapsto -i|-\rangle \\ |1\rangle \mapsto -i|0\rangle & |-\rangle \mapsto i|+\rangle \end{array} \quad \text{---} \boxed{Y} \text{---}$$

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad \begin{array}{ll} |0\rangle \mapsto |0\rangle & |+\rangle \mapsto |-\rangle \\ |1\rangle \mapsto -|1\rangle & |-\rangle \mapsto |+\rangle \end{array} \quad \text{---} \boxed{Z} \text{---}$$

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad \begin{array}{ll} |0\rangle \mapsto |+\rangle & |+\rangle \mapsto |0\rangle \\ |1\rangle \mapsto |-\rangle & |-\rangle \mapsto |1\rangle \end{array} \quad \text{---} \boxed{H} \text{---}$$

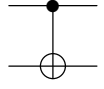
$$R_\theta = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{bmatrix} \quad \begin{array}{ll} |0\rangle \mapsto |0\rangle & \\ |1\rangle \mapsto e^{i\theta}|1\rangle & \end{array} \quad \text{---} \boxed{R_\theta} \text{---}$$

$$S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \quad \begin{array}{ll} |0\rangle \mapsto |0\rangle & \\ |1\rangle \mapsto i|1\rangle & \end{array} \quad \text{---} \boxed{S} \text{---}$$

$$T = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix} \quad \begin{array}{ll} |0\rangle \mapsto |0\rangle & \\ |1\rangle \mapsto e^{i\pi/4}|1\rangle & \end{array} \quad \text{---} \boxed{T} \text{---}$$

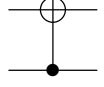
$$\text{CNOT}_{0,1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{array}{ll} |00\rangle \mapsto |00\rangle & |++\rangle \mapsto |++\rangle \\ |01\rangle \mapsto |01\rangle & |+-\rangle \mapsto |--\rangle \\ |10\rangle \mapsto |11\rangle & |-+\rangle \mapsto |-+\rangle \\ |11\rangle \mapsto |10\rangle & |--\rangle \mapsto |+-\rangle \end{array}$$



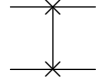
$$\text{CNOT}_{1,0} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\begin{array}{ll} |00\rangle \mapsto |00\rangle & |++\rangle \mapsto |++\rangle \\ |01\rangle \mapsto |11\rangle & |+-\rangle \mapsto |+-\rangle \\ |10\rangle \mapsto |10\rangle & |-+\rangle \mapsto |--\rangle \\ |11\rangle \mapsto |01\rangle & |--\rangle \mapsto |-+\rangle \end{array}$$



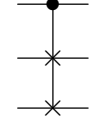
$$\text{SWAP} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{array}{ll} |00\rangle \mapsto |00\rangle & |++\rangle \mapsto |++\rangle \\ |01\rangle \mapsto |10\rangle & |+-\rangle \mapsto |-+\rangle \\ |10\rangle \mapsto |01\rangle & |-+\rangle \mapsto |+-\rangle \\ |11\rangle \mapsto |11\rangle & |--\rangle \mapsto |--\rangle \end{array}$$



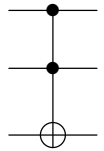
$$\text{CSWAP} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{array}{l} |000\rangle \mapsto |000\rangle \\ |001\rangle \mapsto |001\rangle \\ |010\rangle \mapsto |010\rangle \\ |011\rangle \mapsto |011\rangle \\ |100\rangle \mapsto |100\rangle \\ |101\rangle \mapsto |110\rangle \\ |110\rangle \mapsto |101\rangle \\ |111\rangle \mapsto |111\rangle \end{array}$$



$$\text{TOFFOLI} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{array}{l} |000\rangle \mapsto |000\rangle \\ |001\rangle \mapsto |001\rangle \\ |010\rangle \mapsto |010\rangle \\ |011\rangle \mapsto |011\rangle \\ |100\rangle \mapsto |100\rangle \\ |101\rangle \mapsto |101\rangle \\ |110\rangle \mapsto |111\rangle \\ |111\rangle \mapsto |110\rangle \end{array}$$



3 Operator identities

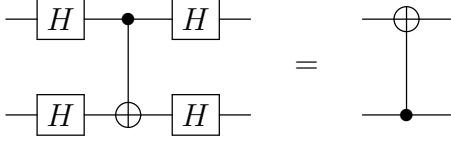
$$X^2 = Y^2 = Z^2 = H^2 = I$$

$$T^2 = S \quad S^2 = Z$$

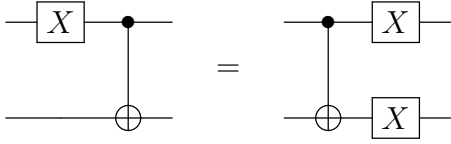
$$\begin{array}{lll} XY = iZ & YX = -iZ & ZX = iY \\ XZ = -iY & YZ = iX & ZY = -iX \end{array}$$

$$\begin{aligned}
HX &= ZH & SX &= XZS \\
HZ &= XH & SZ &= ZS
\end{aligned}$$

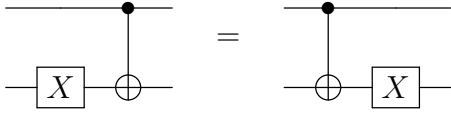
$$\begin{aligned}
HXH &= Z & SXS^\dagger &= Y \\
HYH &= -Y & SY S^\dagger &= -X \\
HZH &= X & SZS^\dagger &= Z
\end{aligned}$$



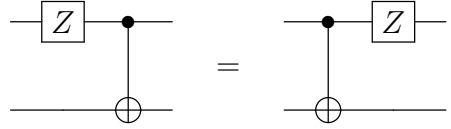
$$(H \otimes H)\text{CNOT}_{0,1}(H \otimes H) = \text{CNOT}_{1,0}$$



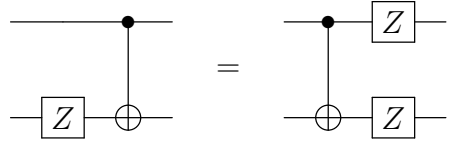
$$(X \otimes I)\text{CNOT}_{0,1} = \text{CNOT}_{0,1}(X \otimes X)$$



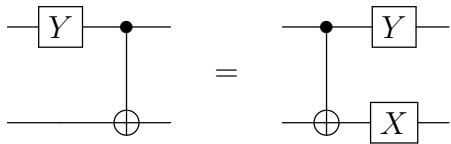
$$(I \otimes X)\text{CNOT}_{0,1} = \text{CNOT}_{0,1}(I \otimes X)$$



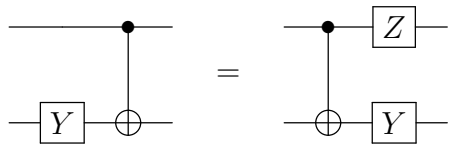
$$(Z \otimes I)\text{CNOT}_{0,1} = \text{CNOT}_{0,1}(Z \otimes I)$$



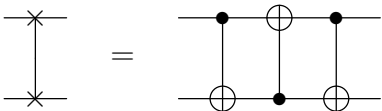
$$(I \otimes Z)\text{CNOT}_{0,1} = \text{CNOT}_{0,1}(Z \otimes Z)$$



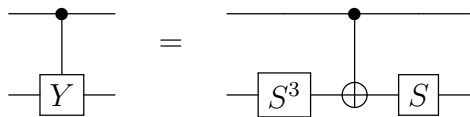
$$(Y \otimes I)\text{CNOT}_{0,1} = \text{CNOT}_{0,1}(Y \otimes X)$$



$$(I \otimes Y)\text{CNOT}_{0,1} = \text{CNOT}_{0,1}(Z \otimes Y)$$



$$\text{SWAP} = \text{CNOT}_{0,1}\text{CNOT}_{1,0}\text{CNOT}_{0,1}$$



$$CY_{0,1} = (I \otimes S) \text{CNOT}_{0,1} (I \otimes S^3)$$