Quantum Computing Cheat Sheet

1 States

$$|0\rangle = \begin{bmatrix}1\\0\end{bmatrix} \quad |1\rangle = \begin{bmatrix}0\\1\end{bmatrix}$$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{bmatrix}\alpha\\\beta\end{bmatrix}, \quad |\alpha|^2 + |\beta|^2 = 1$$

$$|+\rangle = \frac{1}{\sqrt{2}}\big(|0\rangle + |1\rangle\big) = \begin{bmatrix}\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{bmatrix} \quad |-\rangle = \frac{1}{\sqrt{2}}\big(|0\rangle - |1\rangle\big) = \begin{bmatrix}\frac{1}{\sqrt{2}}\\-\frac{1}{\sqrt{2}}\end{bmatrix}$$

$$\left|\Phi^{\pm}\right\rangle = \frac{1}{\sqrt{2}}\big(|00\rangle \pm |11\rangle\big) \quad \left|\Psi^{\pm}\right\rangle = \frac{1}{\sqrt{2}}\big(|01\rangle \pm |10\rangle\big)$$

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle, \quad 0 \le \theta \le \pi,\ 0 \le \phi < 2\pi$$

2 Unitary Operators

$$I = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} \qquad \begin{matrix}|0\rangle \mapsto |0\rangle & |+\rangle \mapsto |+\rangle\\ |1\rangle \mapsto |1\rangle & |-\rangle \mapsto |-\rangle\end{matrix} \qquad -\!\!-\!\!-$$

$$X = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix} \qquad \begin{matrix}|0\rangle \mapsto |1\rangle & |+\rangle \mapsto |+\rangle\\ |1\rangle \mapsto |0\rangle & |-\rangle \mapsto -|-\rangle\end{matrix} \qquad -\boxed{X}-$$

$$Y = \begin{bmatrix}0 & -i\\i & 0\end{bmatrix} \qquad \begin{matrix}|0\rangle \mapsto i|1\rangle & |+\rangle \mapsto -i|-\rangle\\ |1\rangle \mapsto -i|0\rangle & |-\rangle \mapsto i|+\rangle\end{matrix} \qquad -\boxed{Y}-$$

$$Z = \begin{bmatrix}1 & 0\\0 & -1\end{bmatrix} \qquad \begin{matrix}|0\rangle \mapsto |0\rangle & |+\rangle \mapsto |-\rangle\\ |1\rangle \mapsto -|1\rangle & |-\rangle \mapsto |+\rangle\end{matrix} \qquad -\boxed{Z}-$$

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix} \qquad \begin{matrix}|0\rangle \mapsto |+\rangle & |+\rangle \mapsto |0\rangle\\ |1\rangle \mapsto |-\rangle & |-\rangle \mapsto |1\rangle\end{matrix} \qquad -\boxed{H}-$$

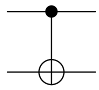
$$R_\theta = \begin{bmatrix}1 & 0\\0 & e^{i\theta}\end{bmatrix} \qquad \begin{matrix}|0\rangle \mapsto |0\rangle\\ |1\rangle \mapsto e^{i\theta}|1\rangle\end{matrix} \qquad -\boxed{R_\theta}-$$

$$S = \begin{bmatrix}1 & 0\\0 & i\end{bmatrix} \qquad \begin{matrix}|0\rangle \mapsto |0\rangle\\ |1\rangle \mapsto i|1\rangle\end{matrix} \qquad -\boxed{S}-$$

$$T = \begin{bmatrix}1 & 0\\0 & e^{i\pi/4}\end{bmatrix} \qquad \begin{matrix}|0\rangle \mapsto |0\rangle\\ |1\rangle \mapsto e^{i\pi/4}|1\rangle\end{matrix} \qquad -\boxed{T}-$$

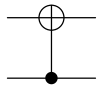
$$\mathrm{CNOT}_{0,1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{aligned} |00\rangle &\mapsto |00\rangle & \qquad |{+}{+}\rangle &\mapsto |{+}{+}\rangle \\ |01\rangle &\mapsto |01\rangle & |{+}{-}\rangle &\mapsto |{-}{-}\rangle \\ |10\rangle &\mapsto |11\rangle & |{-}{+}\rangle &\mapsto |{-}{+}\rangle \\ |11\rangle &\mapsto |10\rangle & |{-}{-}\rangle &\mapsto |{+}{-}\rangle \end{aligned}$$



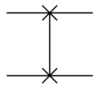
$$\mathrm{CNOT}_{1,0} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} |00\rangle &\mapsto |00\rangle & \qquad |{+}{+}\rangle &\mapsto |{+}{+}\rangle \\ |01\rangle &\mapsto |11\rangle & |{+}{-}\rangle &\mapsto |{+}{-}\rangle \\ |10\rangle &\mapsto |10\rangle & |{-}{+}\rangle &\mapsto |{-}{-}\rangle \\ |11\rangle &\mapsto |01\rangle & |{-}{-}\rangle &\mapsto |{-}{+}\rangle \end{aligned}$$



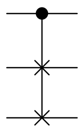
$$\mathrm{SWAP} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned} |00\rangle &\mapsto |00\rangle & \qquad |{+}{+}\rangle &\mapsto |{+}{+}\rangle \\ |01\rangle &\mapsto |10\rangle & |{+}{-}\rangle &\mapsto |{-}{+}\rangle \\ |10\rangle &\mapsto |01\rangle & |{-}{+}\rangle &\mapsto |{+}{-}\rangle \\ |11\rangle &\mapsto |11\rangle & |{-}{-}\rangle &\mapsto |{-}{-}\rangle \end{aligned}$$



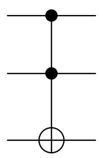
$$\mathrm{CSWAP} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned} |000\rangle &\mapsto |000\rangle \\ |001\rangle &\mapsto |001\rangle \\ |010\rangle &\mapsto |010\rangle \\ |011\rangle &\mapsto |011\rangle \\ |100\rangle &\mapsto |100\rangle \\ |101\rangle &\mapsto |110\rangle \\ |110\rangle &\mapsto |101\rangle \\ |111\rangle &\mapsto |111\rangle \end{aligned}$$



$$\mathrm{TOFFOLI} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{aligned} |000\rangle &\mapsto |000\rangle \\ |001\rangle &\mapsto |001\rangle \\ |010\rangle &\mapsto |010\rangle \\ |011\rangle &\mapsto |011\rangle \\ |100\rangle &\mapsto |100\rangle \\ |101\rangle &\mapsto |101\rangle \\ |110\rangle &\mapsto |111\rangle \\ |111\rangle &\mapsto |110\rangle \end{aligned}$$



3 Operator identities

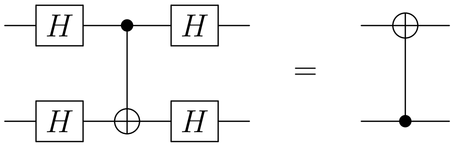
$$X^2 = Y^2 = Z^2 = H^2 = I$$

$$T^2 = S \qquad S^2 = Z$$

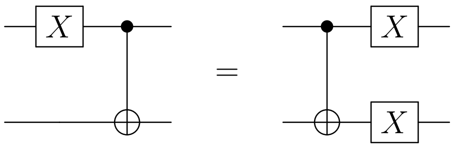
$$\begin{aligned} XY &= iZ & \qquad YX &= -iZ & \qquad ZX &= iY \\ XZ &= -iY & YZ &= iX & ZY &= -iX \end{aligned}$$

$$
\begin{aligned}
HX &= ZH & \qquad SX &= XZS \\
HZ &= XH & \qquad SZ &= ZS
\end{aligned}
$$

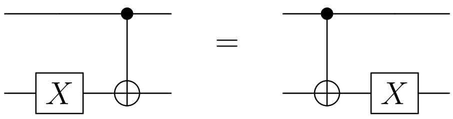
$$
\begin{aligned}
HXH &= Z & \qquad SXS^\dagger &= Y \\
HYH &= -Y & \qquad SYS^\dagger &= -X \\
HZH &= X & \qquad SZS^\dagger &= Z
\end{aligned}
$$



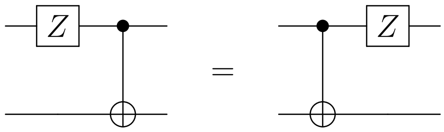
$$(H \otimes H)\mathrm{CNOT}_{0,1}(H \otimes H) = \mathrm{CNOT}_{1,0}$$



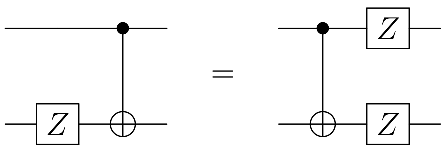
$$(X \otimes I)\mathrm{CNOT}_{0,1} = \mathrm{CNOT}_{0,1}(X \otimes X)$$



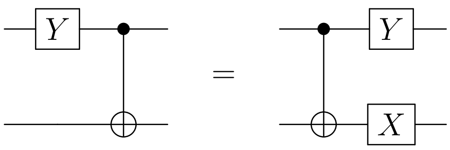
$$(I \otimes X)\mathrm{CNOT}_{0,1} = \mathrm{CNOT}_{0,1}(I \otimes X)$$



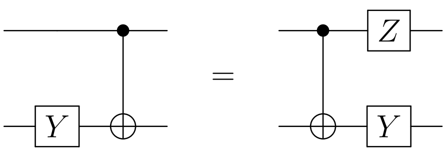
$$(Z \otimes I)\mathrm{CNOT}_{0,1} = \mathrm{CNOT}_{0,1}(Z \otimes I)$$



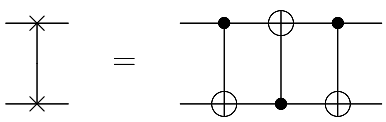
$$(I \otimes Z)\mathrm{CNOT}_{0,1} = \mathrm{CNOT}_{0,1}(Z \otimes Z)$$



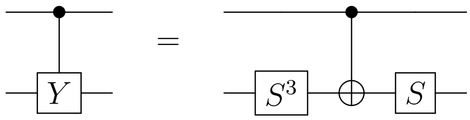
$$(Y \otimes I)\mathrm{CNOT}_{0,1} = \mathrm{CNOT}_{0,1}(Y \otimes X)$$



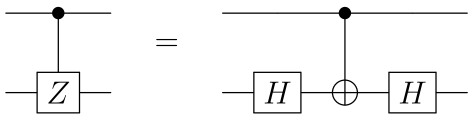
$$(I \otimes Y)\mathrm{CNOT}_{0,1} = \mathrm{CNOT}_{0,1}(Z \otimes Y)$$



$$\mathrm{SWAP} = \mathrm{CNOT}_{0,1}\mathrm{CNOT}_{1,0}\mathrm{CNOT}_{0,1}$$



$$\mathrm{CY}_{0,1} = (I \otimes S)\mathrm{CNOT}_{0,1}(I \otimes S^3)$$



$$\mathrm{CZ}_{0,1} = (I \otimes H)\mathrm{CNOT}_{0,1}(I \otimes H)$$