Quantum Computing Cheat Sheet

1 States

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle, \quad |\alpha|^2 + |\beta|^2 = 1$$

$$|+\rangle = \frac{1}{\sqrt{2}}\big(|0\rangle + |1\rangle\big) = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} \quad |-\rangle = \frac{1}{\sqrt{2}}\big(|0\rangle - |1\rangle\big) = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\left|\Phi^\pm\right\rangle = \frac{1}{\sqrt{2}}\big(|00\rangle \pm |11\rangle\big) \quad \left|\Psi^\pm\right\rangle = \frac{1}{\sqrt{2}}\big(|01\rangle \pm |10\rangle\big)$$

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle, \quad 0 \le \theta \le \pi,\ 0 \le \phi < 2\pi$$

2 Unitary Operators

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \begin{matrix} |0\rangle \mapsto |0\rangle & |+\rangle \mapsto |+\rangle \\ |1\rangle \mapsto |1\rangle & |-\rangle \mapsto |-\rangle \end{matrix} \qquad \text{———}$$

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad \begin{matrix} |0\rangle \mapsto |1\rangle & |+\rangle \mapsto |+\rangle \\ |1\rangle \mapsto |0\rangle & |-\rangle \mapsto -|-\rangle \end{matrix} \qquad -\boxed{X}-$$

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \qquad \begin{matrix} |0\rangle \mapsto i|1\rangle & |+\rangle \mapsto -i|-\rangle \\ |1\rangle \mapsto -i|0\rangle & |-\rangle \mapsto i|+\rangle \end{matrix} \qquad -\boxed{Y}-$$

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \qquad \begin{matrix} |0\rangle \mapsto |0\rangle & |+\rangle \mapsto |-\rangle \\ |1\rangle \mapsto -|1\rangle & |-\rangle \mapsto |+\rangle \end{matrix} \qquad -\boxed{Z}-$$

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad \begin{matrix} |0\rangle \mapsto |+\rangle & |+\rangle \mapsto |0\rangle \\ |1\rangle \mapsto |-\rangle & |-\rangle \mapsto |1\rangle \end{matrix} \qquad -\boxed{H}-$$

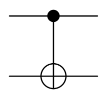
$$R_\theta = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{bmatrix} \qquad \begin{matrix} |0\rangle \mapsto |0\rangle \\ |1\rangle \mapsto e^{i\theta}|1\rangle \end{matrix} \qquad -\boxed{R_\theta}-$$

$$S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \qquad \begin{matrix} |0\rangle \mapsto |0\rangle \\ |1\rangle \mapsto i|1\rangle \end{matrix} \qquad -\boxed{S}-$$

$$T = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix} \qquad \begin{matrix} |0\rangle \mapsto |0\rangle \\ |1\rangle \mapsto e^{i\pi/4}|1\rangle \end{matrix} \qquad -\boxed{T}-$$

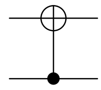
$$\mathrm{CNOT}_{0,1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{aligned} |00\rangle &\mapsto |00\rangle & \qquad |{+}{+}\rangle &\mapsto |{+}{+}\rangle \\ |01\rangle &\mapsto |01\rangle & |{+}{-}\rangle &\mapsto |{-}{-}\rangle \\ |10\rangle &\mapsto |11\rangle & |{-}{+}\rangle &\mapsto |{-}{+}\rangle \\ |11\rangle &\mapsto |10\rangle & |{-}{-}\rangle &\mapsto |{+}{-}\rangle \end{aligned}$$



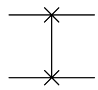
$$\mathrm{CNOT}_{1,0} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} |00\rangle &\mapsto |00\rangle & \qquad |{+}{+}\rangle &\mapsto |{+}{+}\rangle \\ |01\rangle &\mapsto |11\rangle & |{+}{-}\rangle &\mapsto |{+}{-}\rangle \\ |10\rangle &\mapsto |10\rangle & |{-}{+}\rangle &\mapsto |{-}{-}\rangle \\ |11\rangle &\mapsto |01\rangle & |{-}{-}\rangle &\mapsto |{-}{+}\rangle \end{aligned}$$



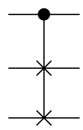
$$\mathrm{SWAP} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned} |00\rangle &\mapsto |00\rangle & \qquad |{+}{+}\rangle &\mapsto |{+}{+}\rangle \\ |01\rangle &\mapsto |10\rangle & |{+}{-}\rangle &\mapsto |{-}{+}\rangle \\ |10\rangle &\mapsto |01\rangle & |{-}{+}\rangle &\mapsto |{+}{-}\rangle \\ |11\rangle &\mapsto |11\rangle & |{-}{-}\rangle &\mapsto |{-}{-}\rangle \end{aligned}$$



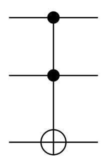
$$\mathrm{CSWAP} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned} |000\rangle &\mapsto |000\rangle \\ |001\rangle &\mapsto |001\rangle \\ |010\rangle &\mapsto |010\rangle \\ |011\rangle &\mapsto |011\rangle \\ |100\rangle &\mapsto |100\rangle \\ |101\rangle &\mapsto |110\rangle \\ |110\rangle &\mapsto |101\rangle \\ |111\rangle &\mapsto |111\rangle \end{aligned}$$



$$\mathrm{TOFFOLI} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{aligned} |000\rangle &\mapsto |000\rangle \\ |001\rangle &\mapsto |001\rangle \\ |010\rangle &\mapsto |010\rangle \\ |011\rangle &\mapsto |011\rangle \\ |100\rangle &\mapsto |100\rangle \\ |101\rangle &\mapsto |101\rangle \\ |110\rangle &\mapsto |111\rangle \\ |111\rangle &\mapsto |110\rangle \end{aligned}$$



3 Operator identities

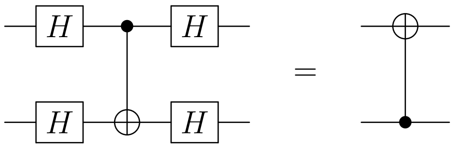
$$X^2 = Y^2 = Z^2 = H^2 = I$$

$$T^2 = S \qquad S^2 = Z$$

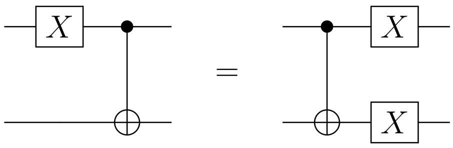
$$\begin{aligned} XY &= iZ & \qquad YX &= -iZ & \qquad ZX &= iY \\ XZ &= -iY & YZ &= iX & ZY &= -iX \end{aligned}$$

$$HX = ZH \qquad SX = XZS$$
$$HZ = XH \qquad SZ = ZS$$

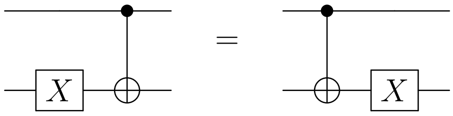
$$HXH = Z \qquad SXS^\dagger = Y$$
$$HYH = -Y \qquad SYS^\dagger = -X$$
$$HZH = X \qquad SZS^\dagger = Z$$



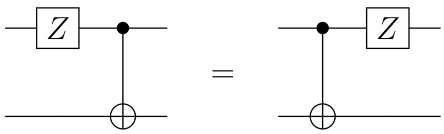
$$(H \otimes H)\mathrm{CNOT}_{0,1}(H \otimes H) = \mathrm{CNOT}_{1,0}$$



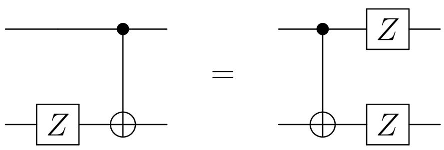
$$(X \otimes I)\mathrm{CNOT}_{0,1} = \mathrm{CNOT}_{0,1}(X \otimes X)$$



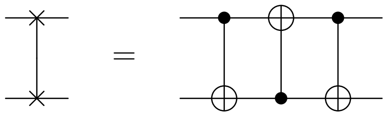
$$(I \otimes X)\mathrm{CNOT}_{0,1} = \mathrm{CNOT}_{0,1}(I \otimes X)$$



$$(Z \otimes I)\mathrm{CNOT}_{0,1} = \mathrm{CNOT}_{0,1}(Z \otimes I)$$



$$(I \otimes Z)\mathrm{CNOT}_{0,1} = \mathrm{CNOT}_{0,1}(Z \otimes Z)$$



$$\mathrm{SWAP} = \mathrm{CNOT}_{0,1}\mathrm{CNOT}_{1,0}\mathrm{CNOT}_{0,1}$$