Problem Set 5
AA 279D, Spring 2018
Due: May 16, 2018 (Wednesday) at 1:30 pm

Notes:

Submission Instructions

Please continue to document all tasks below in your project report. You should indicate any substantial changes in the change log and highlight them for review.
Please submit your report as a PDF file to the course Canvas website. You should use typesetting software like LaTeX or Microsoft Word to produce your documents. Do not submit extra files.

Topics

Lectures 6 and 7. Relative orbit elements. J2 perturbed dynamics.

Problem 1

We would like to explore the properties of perturbed J2 relative orbit motion in close near-circular orbits using analytical solutions and numerical integration. To this end, you are asked to

(a) Let the osculating initial conditions for absolute (chief) states be determined by Problem Set 3. Make sure that the initial conditions lie within the range of validity of the HCW equations, i.e. $e << 1$. Justify your selection of initial conditions.

(b) Let a second initial relative state of deputy w.r.t. chief be defined by the following osculating (quasi-non-singular) relative orbit elements: $a_c(\delta a, \delta\lambda, \delta e_x, \delta e_y, \delta i_x, \delta i_y) =$ (0, 100, 50, 100, 30, 200) m.

(c) Perform a numerical integration of the equations of motion for chief AND deputy using position and velocity as state variables, for at least 15 orbits. You should repeat the simulation four times: using initial conditions from (a) and (b), and including/excluding J2 effects. Note: this is a copy of what you did in Problem Set 2, where the consistency of your propagator has been verified. Compute the following auxiliary quantities along the simulation and plot them over time:

   i. Osculating quasi-non-singular orbital elements (i.e. based on mean argument of latitude and eccentricity vector)

ii. Osculating relative quasi-non-singular orbital elements (i.e. based on relative eccentricity and inclination vectors)

iii. Mean quasi-non-singular orbital elements (i.e. applying an osculating to mean mapping)

iv. Mean relative quasi-non-singular orbital elements

It is advised that you give each of the six absolute/relative orbit elements its own time history plot for each of the four cases. However, if you wish to compare results (e.g. J2 on vs. J2 off, or mean vs. osculating), feel free to plot these results on the same figures.

(d) Plot relative position in RTN as usual (3D, TR, NR, and TN planes) throughout the simulation. Use axis equal and [m] as length units. Show unperturbed and J2 perturbed case on the same plots. Are the results according to expectation given your initial relative orbital elements?

(e) Plot the osculating and mean relative quasi-non-singular orbital elements on the following plots (always axis equal, with units of [m]):

i. x-y axis for relative eccentricity vector

ii. x-y axis for relative inclination vector

iii. $\delta\lambda$-δa axis for relative mean longitude and relative semi-major axis

Compare with part (d). Are the results according to expectation given your relative position in RTN and your initial conditions?

(f) What is the minimum change you can make on the initial relative orbital elements given in part (b) to remove J2 secular effects on the formation? What would be size, direction, and location (u_M) of the necessary maneuver to accomplish this?

(g) Rerun the simulation from (c) with your new initial conditions (no need to plot all of the orbit elements this time). Are the secular effects removed?

(h) Now consider the linearized analytical solution for the secular evolution of the relative orbital elements under J2 effects (Lecture 7). Plot the resulting motion on the same plots generated in part (e) for the two sets of initial conditions in (b) and (f). Are the results consistent with the numerical integration?