Overall grade:Punctuality +2C

Crew Dragon: On-Orbit Operations

A Simulation of SpaceX's Crew Dragon and the mission of Rendezvous & Docking with the International Space Station

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Revision History

Table 1: Summary of project revisions.

Rev	Changes
PS1	- Created document - Added problem set 1 material

Contents

1 Abstract

This report outlines a detailed formation-flying and rendezvous simulation which summarizes the following fundamental Guidance, Navigation, and Control (GNC) requirements: Keplerian orbital mechanics and orbital perturbations; the general relative motion problem; linear formation-flying dynamics and control; impulsive station-keeping and reconfiguration; high-order relative motion equations; formulation of relative motion using orbital elements; perturbation-invariant formations; nonlinear formation control; low-thrust propulsion for formation flying; relative navigation using GNSS and optical navigation.

The generation of this report is supported in part by SpaceX's Paul Forquera (Director of GNC) and Justin Smith (Manager of Dragon GNC) to coincide with an Associate Engineering position to take place during the Summer of 2018 under the supervision of Dragon 2 GNC managers for the purpose of on-orbit operations with the International Space Station (ISS).

2 Mission Proposals

The following mission proposals represent two of SpaceX's leading satellite-based high-profile missions. Due to the lack of publicly available information on Crew Dragon mission specifications, analogies will be drawn from Dragon CRS-14 whenever possible to provide an accurate estimate of orbit planning based upon mission objectives and absolute/relative orbit parameters.

2.1 Primary Candidate: Dragon 2

The following information is taken from the launch of Dragon CRS-14 with the accompanying TLE in Appendix A:

- Mission Name and Operator
  - SpaceX CRS-14, USA
  - Contracted: NASA, Flown: SpaceX
- Mission Objectives
  - Commercial ISS Resupply Service Mission
    - Science investigations: 1,070 kg (2,359 lb)
    - Crew supplies: 344 kg (758 lb)
    - Vehicle hardware: 148 kg (326 lb)
    - Spacewalk equipment: 99 kg (218 lb)
    - Computer resources: 49 kg (108 lb)
    - Russian hardware: 11 kg (24 lb)
    - External payloads: 926 kg (2,041 lb)
- Number and Type of Satellites

  - One CRS Dragon C110.2 for ISS Resupply
- Absolute and Relative Orbit Parameters
  - Reference system: Geocentric
  - Regime: Low Earth
  - Launch date: 2 April 2018, 20:30:38 UTC
  - Perigee: 410.1 km
  - Apogee: 412.7 km
  - Period: 92.6 minutes
  - More information available in Section 3.1
- Basic Description of Functioning/Scientific Principle
  - The Dragon spacecraft rendezvoused with the ISS April 4th, 2018 in order to complete a scheduled resupply mission. After an r-bar rendezvous maneuver, it was captured by Canadarm2 at 10:40 UTC and was berthed to the Harmony module at 13:00 UTC. It is scheduled to remain there for approximately one month before de-orbiting and returning to Earth [2].
- Key GNC Requirements
  - For the purpose of on-orbit navigation, Dragon has a suite of Inertial Measurement Units (IMU), GPS Systems, Iridium Recovery Beacons, and Star Trackers. Both the IMUs and star trackers have an accuracy of $0.004°$ or smaller. The threshold of attitude control is also $0.012°$ on each axis in station-keeping Mode [4]. As a Dragon module, the following GNC requirements are required for successful rendezvous: GNC bay door deploys, exposing the GNC sensor suite; star tracker attitude initiation works as expected; TDRSS S-band telemetry and commanding works as expected; demonstrates precision R-bar arrival at 350 m below the ISS; initializes proximity sensors (LIDARs and thermal imagers) and converges a solution for range and range rate before proceeding; demonstrates hold and retreats commanded by the ISS crew; enters free drift at 10 m from the ISS with minimal vehicle body rates; successfully berths to Node 2 Nadir[5].
- Classification
  - Nadir Berthing Rendezvous

2.2 Secondary Candidate: Starlink Constellation

The following information is taken from the launch of Tintin A & B with the accompanying TLEs in Appendix A:

- Mission Name and Operator
  - SpaceX Starlink, USA
- Mission Objectives
  - Develop a low-cost, high-performance satellite bus and requisite customer ground transceivers to implement a new space-based Internet communication system. Secondary objectives include the

sale of satellites that use the same satellite bus that may be used for scientific or exploratory purposes in order to fund future Mars transport projects. [6]

- Number and Type of Satellites
  - Two Smallsat-class communication satellites Tintin A & B
- Absolute and Relative Orbit Parameters
  - Both:
    - Reference system: Geocentric
    - Regime: Low Earth
    - Orbit Type: Sun-Synchronous
    - Launch date: 22 February 2018
  - Tintin A
    - Perigee: 504.2 km
    - Apogee: 526.5 km
    - Period: 94.8 minutes
    - Epoch: 15 April 2018, 15:18:56 UTC.
    - $a = 6886.14$ km
    - $e = 0.0016$
    - $i = 97.4630°$
    - $\Omega = 114.1702°$
    - $\omega = 97.6203°$
    - $M_0 = 262.6859°$
  - Tintin B
    - Perigee: 504.1 km
    - Apogee: 526.2 km
    - Period: 94.8 minutes
    - Epoch: 15 April 2018, 5:12:45 UTC.
    - $a = 6886.35$ km
    - $e = 0.0016$
    - $i = 97.4562°$
    - $\Omega = 114.6986°$
    - $\omega = 91.5260°$
    - $M_0 = 268.7829°$

- Basic Description of Functioning/Scientific Principle
  - The satellites will employ optical inter-satellite links and phased array beam forming and digital processing technologies in the Ku- and Ka band. Ka- and Ku-band satellites will orbit at an altitude of 1,200 km (750 miles), and V-band satellites will orbit lower at 210 miles (340 km.). The proposed Starlink constellation includes 4,425 higher orbit satellites and 7,518 lower orbit satellites using a peer-to-peer protocol simpler than IPv6. Since each satellite would orbit at 1/30 of geostationary orbits, they offer practical latencies around 25 to 35 ms, comparable to currently existing cable or fiber networks [7].
- Key GNC Requirements
  - Not much is known about the Tintin A & B satellites or the Starlink Constellation project due to the nature of these demonstrations as technical proofs of concept in a market that is growing to be increasingly more competitive. Therefore, information regarding the GNC of these satellites is not publicly available.
- Classification
  - Communication Satellite Constellation

3 Orbit Simulation

The following orbit simulation and data analysis was conducted on Dragon CRS-14 for a berthing maneuver to the ISS. Whereas Crew Dragon will forgo berthing for a docking approach, CRS-14 provides an accurate measure of orbit parameters that Crew Dragon will be designed to emulate. All initial conditions were derived from the TLE in Appendix A.

3.1 Keplerian Initial Conditions

The initial conditions of the simulation are provided as a set of Keplerian orbital elements and an initial epoch date and time. Since a TLE for this mission is available, initial launch time is replaced with an active epoch time.

Therefore, although the mission has a launch date of 2 April 2018, 20:30:38 UTC, all initial conditions are taken from the TLE at Epoch: 15 April 2018, 12:27:43 UTC.

$a = 6782.42$ km
$e = 0.0002.42$
$i = 51.6438°$
$\Omega = 331.1221°$
$\omega = 355.8915°$
$M_0 = 76.9789°$

3.2 Inertial Position and Velocity I.C.'s

By treating these initial Keplerian elements as osculating quantities, computing the corresponding initial position and velocity in the appropriate inertial frame (Earth- Centered Inertial for an Earth-orbiting mission), leads to the following vectors:

$r_0{}^{ECI} = [\,3689.667,\, 2558.674,\, 5082.996\,]'$ [km]
$v_0{}^{ECI} = [\,-5.739,\, 4.764,\, 1.769\,]'$ [km/s]

3.3 Unperturbed Orbit Propagation

For an unperturbed system, vehicle dynamics follow a simple second-order non-linear ODE with zero disturbance forces present. This ODE represents the gravitational acceleration felt on the vehicle in ideal conditions. Using the initial inertial states as provided in section 3.2, the following equation can be numerically integrated [1]:

$$\ddot{\mathbf{r}} + \frac{\mu \mathbf{r}}{r^3} = \mathbf{0}$$

By neglecting any perturbations beyond the two-body spherical gravity force acting on the orbit, the following orbit path was produced over a span of 100 complete orbits with a numerical integration step size of 1 second.

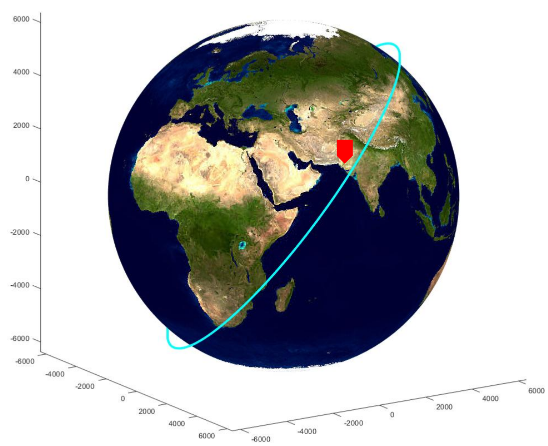

Figure 1: Unperturbed Orbit Path

3.4 Orbit Propagation with J_2 Effects

By considering J_2 effects from Earth oblateness, small disturbances can be introduced into the numerical integration to produce a non-constant orbit path. The governing dynamics then introduce non-zero forces proportional to J_2 as prescribed below [1]:

$$\ddot{X} = -\frac{\mu X}{r^3}\left[1 - \frac{3}{2}J_2\left(\frac{R_e}{r}\right)^2\left(5\frac{Z^2}{r^2} - 1\right)\right]$$
$$\ddot{Y} = -\frac{\mu Y}{r^3}\left[1 - \frac{3}{2}J_2\left(\frac{R_e}{r}\right)^2\left(5\frac{Z^2}{r^2} - 1\right)\right]$$
$$\ddot{Z} = -\frac{\mu Z}{r^3}\left[1 - \frac{3}{2}J_2\left(\frac{R_e}{r}\right)^2\left(5\frac{Z^2}{r^2} - 3\right)\right]$$

The following orbit path was run over the same parameters as Figure 1 with the addition of J_2 effects.

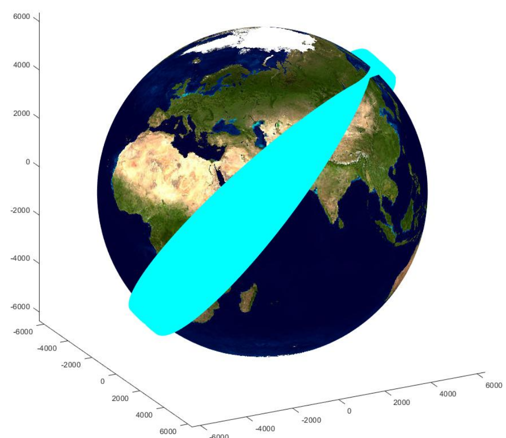

Figure 2: Undirected graph composed of four nodes

3.5 Error Propagation

By comparing the state output from the unperturbed numerical integration against an analytical Keplerian propagation, the accuracy of the integrator can be demonstrated. The error in absolute position and inertial velocity expressed in the local vertical, local horizontal (RTN) coordinate system can be seen below for a simulation over a span of 10 complete orbits with a numerical integration step size of 0.01 seconds.

As the error in the system propagates over the length of the simulation, error accumulated over the span of 10 orbits was shown to accuracy drift and potential divergence.

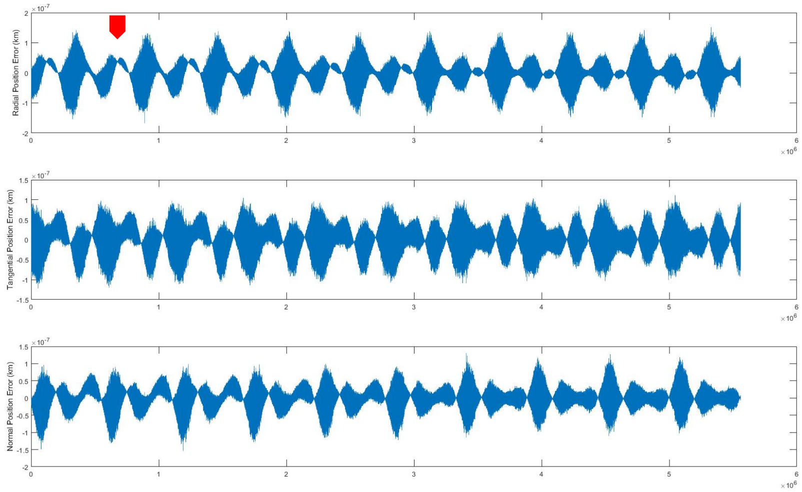

Figure 3: RTN Position Error [km]

As seen in the vertical axis of this plot, the error over 10 orbits is scaled to $10^{-}7$ km, or 0.1 mm. Therefore, the numerically integrated position vector, r^{RTN}, is accurate and has negligible error propagation over 10 orbits.

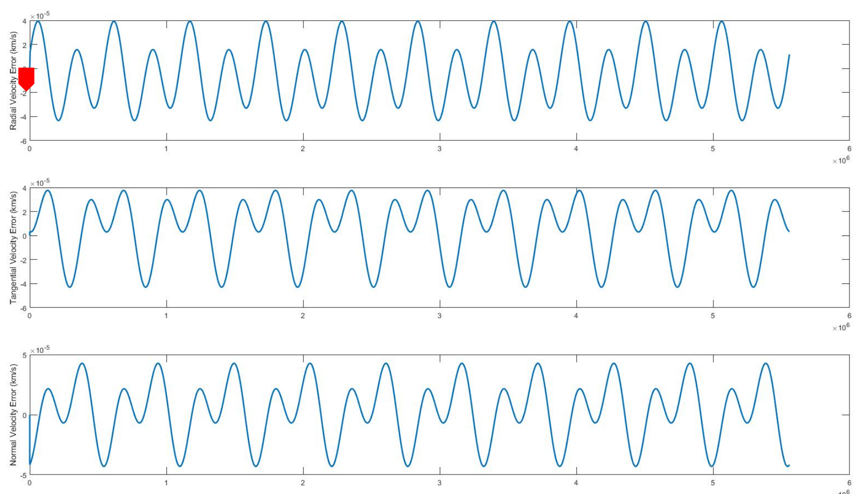

Figure 4: RTN Velocity Error [km/s]

In this plot, the error over 10 orbits is scaled to $10^{-}5$ km, or 1 cm. Therefore, although larger than the positional error, the numerically integrated velocity vector, v^{RTN}, is also accurate and has negligible error propagation over 10 orbits.

3.6 Osculation Disturbances

When excluding J_2 effects, it can be seen that (disregarding negligible integration noise), that the disturbance-free time history for the Keplerian orbital elements (a, e, i, Ω, ω), the magnitude of the angular momentum, and the specific mechanical energy are all constant. The only non-constant value is the true anomaly, which is monotonically increasing (modulo 360°).

The following was run over a span of 1 complete orbit with a numerical integration step size of 0.01 seconds.

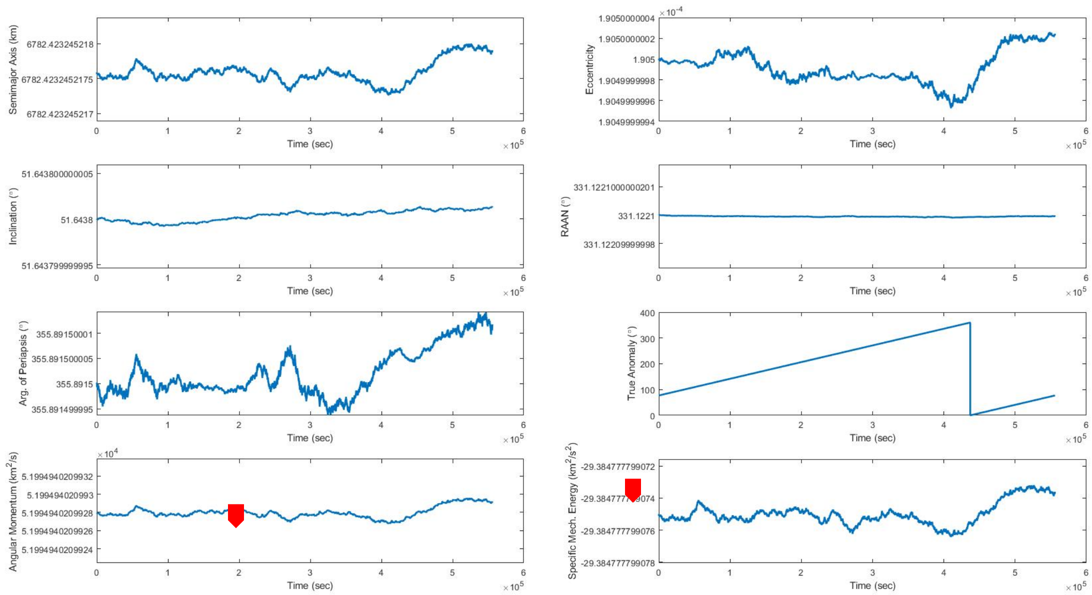

Figure 5: Unperturbed Osculating Time Histories

With the addition of J_2 effects, the oblateness of the Earth provides a small disturbance in the gravitational acceleration of the vehicle that is no longer constant, but has a low frequency mode relating to its orbit and relative position to the equator.

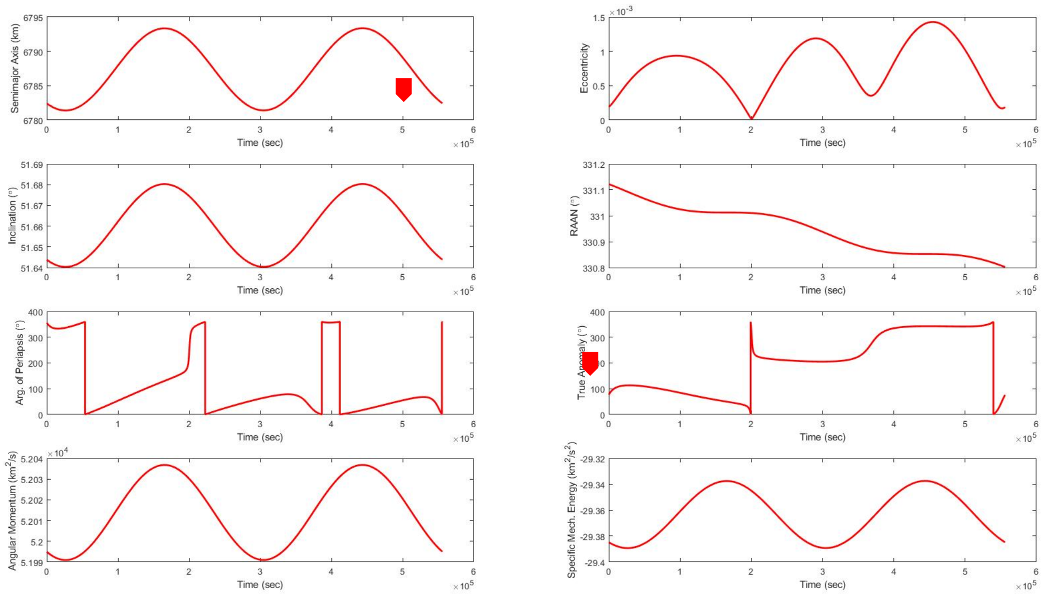

Figure 6: J_2-Affected Osculating Time Histories

Although difficult to see on such a timescale, these results are as expected from the results of averaging theory (discussed in the following section), where the semimajor axis, eccentricity, and inclination only experience periodic changes, while the RAAN, argument of periapsis, and true anomaly undergo both periodic and secular effects.

3.7 Mean Keplerian Orbital Elements

By using Averaging Theory as a comparison to the osculating orbital elements from Figure 6, the following simplified first-order linear ODE's can be used to propagate the mean Keplerian orbital elements with J2 effects [1]:

$$
\begin{aligned}
\frac{\mathrm{d}\bar{a}}{\mathrm{d}t} &= 0 \\
\frac{\mathrm{d}\bar{e}}{\mathrm{d}t} &= 0 \\
\frac{\mathrm{d}\bar{i}}{\mathrm{d}t} &= 0 \\
\frac{\mathrm{d}\bar{\Omega}}{\mathrm{d}t} &= -\frac{3}{2} J_2 \left(\frac{R_e}{\bar{p}}\right)^2 \bar{n} \cos \bar{i} \\
\frac{\mathrm{d}\bar{\omega}}{\mathrm{d}t} &= \frac{3}{4} J_2 \left(\frac{R_e}{\bar{p}}\right)^2 \bar{n} (5 \cos^2 \bar{i} - 1) \\
\frac{\mathrm{d}\bar{M}_0}{\mathrm{d}t} &= \frac{3}{4} J_2 \left(\frac{R_e}{\bar{p}}\right)^2 \bar{n} \bar{\eta} (3 \cos^2 \bar{i} - 1)
\end{aligned}
$$

It should be noted that the inputs for this system are mean elements, not instantaneous ones. The following plot compares the osculating orbital elements (solid, red) to the mean Keplerian orbital elements (dotted, black).

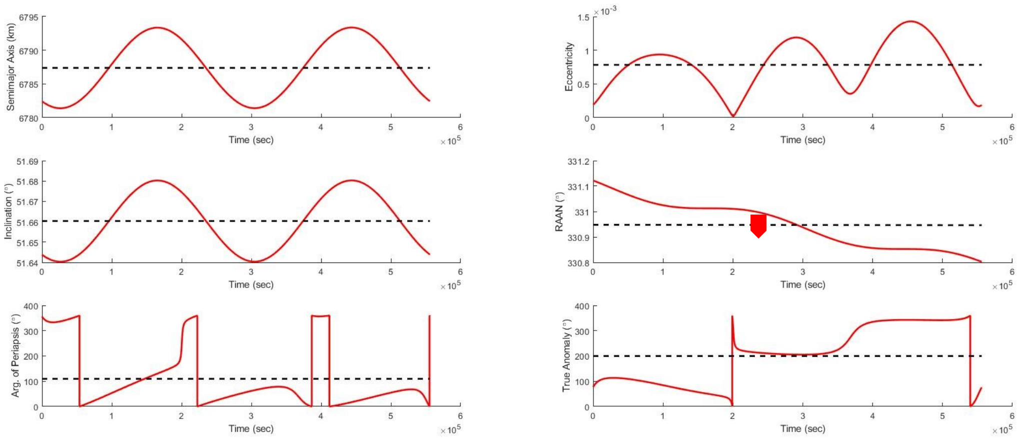

Figure 7: Mean and Osculating Keplerian Orbital Elements

Clearly, the mean orbital elements represent the mean of the osculating orbital elements as expected from Averaging Theory.

3.8 Inconsistencies in Mean vs. Osculating Element Methods

It is possible to encounter inconsistencies when comparing osculating and mean orbital elements due the initialization procedure. In fact, one uses osculating states as inputs and provides osculating states as outputs, whereas the other uses mean states as inputs and provides mean states as outputs.

Since the mean states cannot be calculated without the osculating orbit, it is possible to run a simulation of a single orbit to provide enough information to initialize the mean states. Therefore, if computational costs are a factor, the calculation of the osculating orbital elements must only be computed for one orbit and the mean Keplerian orbital elements may be used for the remainder of the calculation. It should be noted, however, that the stronger the J_2 effects are on the orbit, the greater the standard deviation in the osculating terms from the mean terms (which increasing instantaneous error).

In order to convert between osculating and mean orbital elements, a few methods are available: Brouwer's Satellite Theory, Short-Period Kozai-Izsak theory, and Eckstein-Ustinov theory to name a few (variations on PLEs and GVEs).

Appendices

A Two Line Element Sets (TLE)

```
DRAGON CRS14
1 43267U 18032A   18105.51925310 +.00002699 +00000-0 +47756-4 0  9996
2 43267 051.6438 331.1221 0001905 355.8915 076.9789 15.54265438108777

TINTIN A
1 43216U 18020B   18106.53254881  .00002260  00000-0  11029-3 0  9995
2 43216  97.4562 114.6986 0016194  91.5260 268.7829 15.19214506  8037

TINTIN B
1 43217U 18020C   18105.93439396 -.00000056 +00000-0 +60291-6 0  9995
2 43217 097.4630 114.1702 0016101 097.6203 262.6859 15.19282153007945
```

References

[1] K. Alfriend, S. Vadali, P. Gurfil, J. How, and L. Breger, *Spacecraft Formation Flying: Dynamics, Control, and Navigation*. Elsevier Astrodynamics Series, 2010.

[2] C. Stephen (2 April 2018). "Launch Log". Spaceflight Now. Archived from the original on 5 April 2018.

[3] C. Stephen (4 April 2018). "Dragon cargo capsule reaches space station for second time". Spaceflight Now. Retrieved 4 April 2018.

[4] Spaceflight101 (n.d.). Dragon - Cargo Version. Retrieved April 14, 2018, from http://spaceflight101.com/spacecraft/dragon/

[5] "Overview: SpaceX CRS-14 Mission" (PDF). NASA. Retrieved 4 April 2018.

[6] Grush, Loren (15 February 2018). "SpaceX is about to launch two of its space Internet satellites: the first of nearly 12,000". The Verge. Retrieved 16 February 2018.

[7] de Selding, Peter B. (2016-10-05). "SpaceX's Shotwell on Falcon 9 inquiry, discounts for reused rockets and Silicon Valley's test-and-fail ethos". SpaceNews. Retrieved 2016-10-08.

[8] Gates, Dominic (16 January 2015). "Elon Musk touts launch of 'SpaceX Seattle'". Seattle Times. Retrieved 19 January 2015.

[9] Messier, Doug (2017-03-03). "SpaceX Wants to Launch 12,000 Satellites". Parabolic Arc. Retrieved 2018-01-22.

[10] Musk, Elon (22 February 2018). "First two Starlink demo satellites, called Tintin A & B, deployed and communicating to Earth stations (video)". Twitter. Retrieved 22 February 2018.