Problem Set 4

AA 279D, Spring 2018

Due: May 9, 2018 (Wednesday) at 1:30 pm

Notes:

Submission Instructions

Please continue to document all tasks below in your project report. You should indicate any substantial changes in the change log and highlight them for review.
Please submit your report as a PDF file to the course Canvas website. You should use typesetting software like LaTeX or Microsoft Word to produce your documents. Do not submit extra files.

Topics

Lectures 5 and 6. Linearized equations of relative motion for eccentric orbits.

Problem 1

We would like to further our understanding of relative spacecraft motion according to the Tschauner-Hempel (TH) equations. These equations allow us to explore both a Cartesian relative state and orbit element differences. To this end, you are asked to perform the following mathematical analyses (no numbers needed).

(a) Consider the Yamanaka–Andersen (YA) solution of the TH equations (Eq. 1-3 in the Appendix below). Contrast these with the linear mapping offered by Schaub between the Cartesian relative state and the orbit element differences (Eq. 4-6 in the Appendix). In particular, provide approximate relationships between integration constants of the YA solution and Schaub's orbit element differences.
Hint: You may want to group like terms in both sets of equations to illustrate their parallel structures.

(b) Using the equations in the Appendix, develop or find expressions for the relative velocity of the YA solution and the orbit element difference approach. Manipulate the equations in such a way to demonstrate the correspondence between integration constants and orbit element differences as you did with the relative position equations.
Hint: Note that the time derivative of a parameter can be re-expressed with the chain rule as $\dfrac{d(\cdot)}{dt} = \dfrac{d(\cdot)}{df}\dfrac{df}{dt}$, where $\dfrac{df}{dt} = \sqrt{\dfrac{\mu}{a^3(1-e^2)^3}}\,[1+e\cos(f)]^2$

Problem 2

Now that we have developed some intuition via mathematical analysis, let us proceed with a simulation to further understand these dynamics models.

(a) Let the initial conditions for absolute and relative states be determined by Problem Set 3, however now you should increase the eccentricity of the reference orbit to a value of at least 0.1. Make sure that the initial conditions lie within the range of validity of the Tschauner-Hempel equations and that the resulting motion is bounded.
In other words, the separations between spacecraft should be small relative to the distance from Earth's center (e.g. $\|\vec{\rho}\| \approx 0.001\|\vec{r}_0\|$) and the chief and deputy spacecraft should have equal semi-major axes. Justify your final selection of initial conditions.

(b) Based on the initial conditions chosen in part (a), compute the exact values of the 6 integration constants of the YA solution (i.e. do not use the approximate relationships developed in Problem 1).
Hint: Sections 5.6.3 - 5.6.4 of Alfriend may offer some insight into this problem.

(c) Propagate the relative position and velocity using the YA solution over 15 orbits using true anomaly as the independent variable. Plot the resulting relative position and velocity in 3D and in the TR, NR, and TN planes (first letter indicates x-axis, second letter indicates y-axis).

(d) Are the trends obtained in part (c) according to expectation given the initial conditions and the integration constants? Is the relative motion bounded as expected from $\delta a = 0$ (energy matching condition). If not, why?

(e) Compute the orbit element differences as from Chapter 14 of Schaub.

(f) Propagate the relative position and velocity using the mapping offered by Schaub for arbitrary eccentricity. Plot the resulting relative position and velocity in the same plots as those in part (c).

(g) Are the results of the previous simulation as expected? How do the orbit element differences from part (e) and the integration constants from part (b) compare numerically?

(h) Produce the true relative position and velocity from analytical or numerical propagation and compare with the results from parts (c) and (f) on the same plots. Plot the propagation errors of the simulations from parts (c) and (f) in the RTN frame as usual. Which analytical solution is more accurate? Why?

(i) Repeat this exercise two more times, always starting with the definition of initial conditions and proceeding with comments on results and expectations. You should investigate the following cases:

   i. Difference in semi-major axis between deputy and chief. Justify your choice of δa by commenting on the expected along-track drift/orbit

   ii. Highly eccentric orbit of the chief spacecraft ($e > 0.5$)

Appendix

The Yamanaka–Andersen solution to the Tschauner–Hempel equations are provided in the subsequent equations (taken from Alfriend eq. 5.124):

$$\bar{x}(f) = c_1 k \sin(f) + c_2 k \cos(f) + c_3 \left[2 - 3ekI \sin(f)\right] \tag{1}$$

$$\bar{y}(f) = c_4 + c_1 k \left(1 + \frac{1}{k}\right) \cos(f) - c_2 k \left(1 + \frac{1}{k}\right) \sin(f) - 3c_3 k^2 I \tag{2}$$

$$\bar{z}(f) = c_5 \cos(f) + c_6 \sin(f) \tag{3}$$

Where $k = 1 + e\cos(f)$ and $I = \int_{f_0}^{f} \left[1 + e\cos(f)\right]^{-2} df$.

The linear mapping between the Cartesian relative state and Schaub's orbital element differences is provided in the following equations (taken from Schaub eq. 14.117):

$$\bar{x}(f) = \frac{\delta a}{a} + k\frac{e\delta M}{\eta^3} \sin(f) - \frac{k\delta e}{\eta^2} \cos(f) \tag{4}$$

$$\bar{y}(f) = k^2 \frac{\delta M}{\eta^3} + \delta\omega + (2 + e\cos(f)) \frac{\delta e}{\eta^2} \sin(f) + \cos(i)\delta\Omega \tag{5}$$

$$\bar{z}(f) = \sin(f + \omega)\delta i - \cos(f + \omega)\sin(i)\delta\Omega \tag{6}$$

Where $k = 1 + e\cos(f)$ and $\eta = \sqrt{1 - e^2}$.