AA 279 D – SPACECRAFT FORMATION-FLYING AND RENDEZVOUS: LECTURE 7

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Schaub

Numerical Simulations (1)

- Do equations (14.122), (14.127), and (14.131) for arbitrary, small, and near-zero eccentricity indeed predict the spacecraft formation geometry?
- Let the chief orbit elements and the orbit element differences be given by

Orbit elements	Value	Units
a	7555	km
e	0.03 or 0.13	
i	48.0	deg
Ω	20.0	deg
ω	10.0	deg
M_0	0.0	deg

Orbit elements	Value	Units
δa	0	km
δe	0.00095316	
δ_i	0.0060	deg
$\delta\Omega$	0.100	deg
$\delta\omega$	0.100	deg
δM_0	-0.100	deg

- The ratio $\rho/r \approx 0.003 << e$, thus we expect poor performance for small and near-zero eccentricity assumptions
- The relative motion is specified to be bounded (identical semi-major axis)

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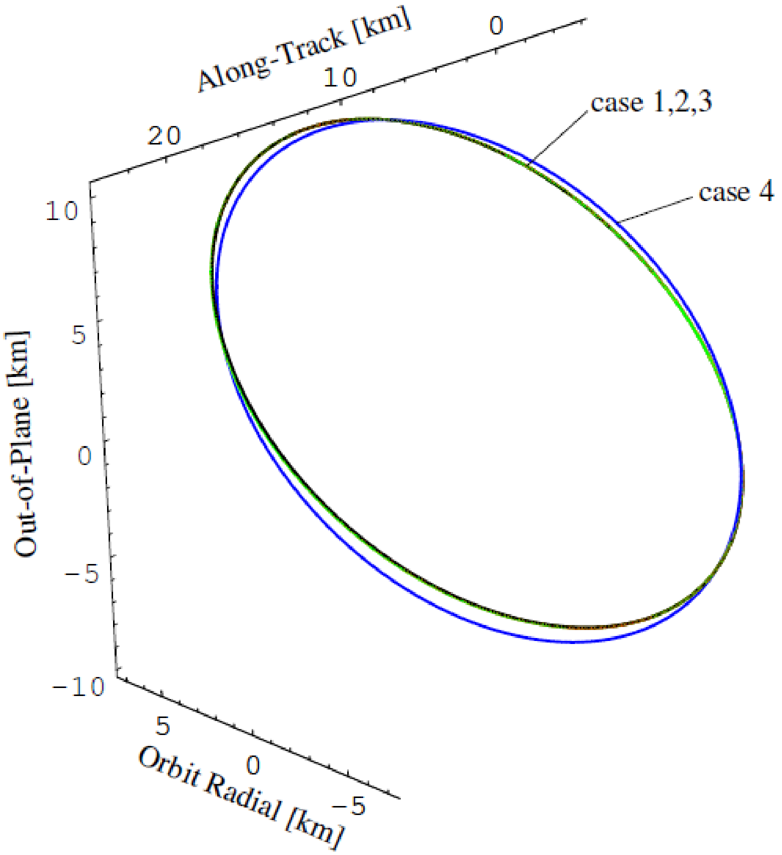
Numerical Simulations (2)

1) True nonlinear equations (numerical integration)

2) Linear solution for arbitrary *e* (analytical)

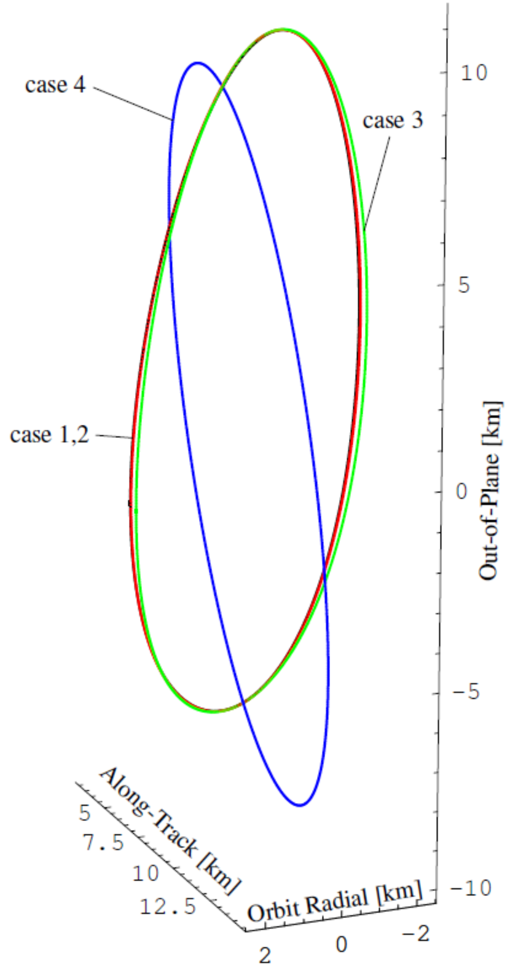
3) Linear solution for small *e* (analytical)

4) Linear solution for near-zero *e* (analytical)



(i) Relative Orbits in Hill Frame for $e = 0.03$

Increase eccentricity



(ii) Relative Orbits in Hill Frame for $e = 0.13$

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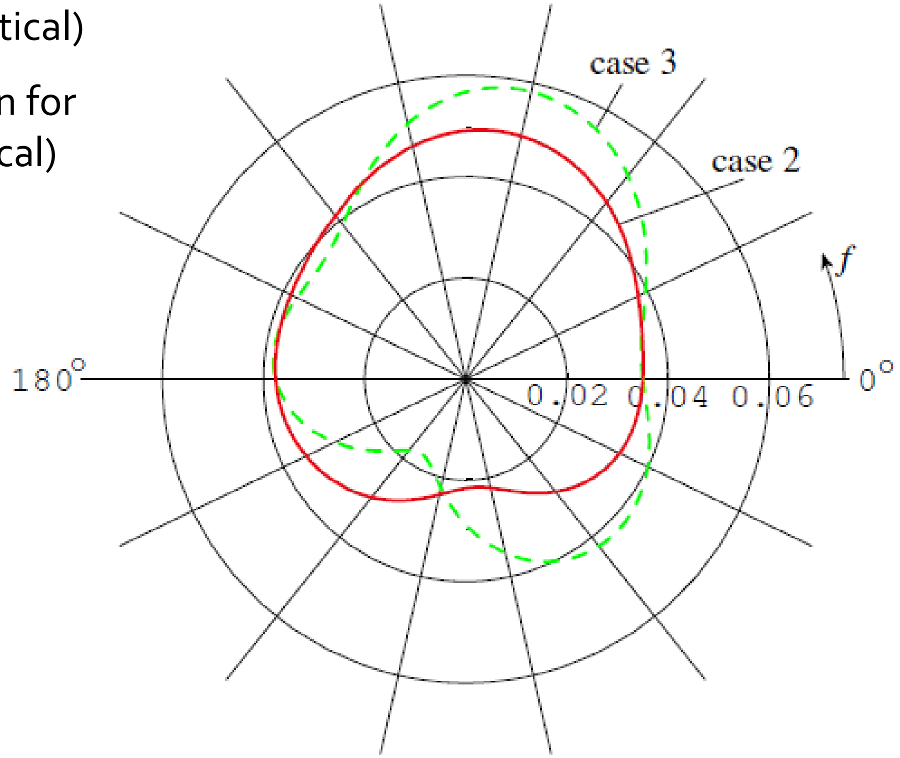
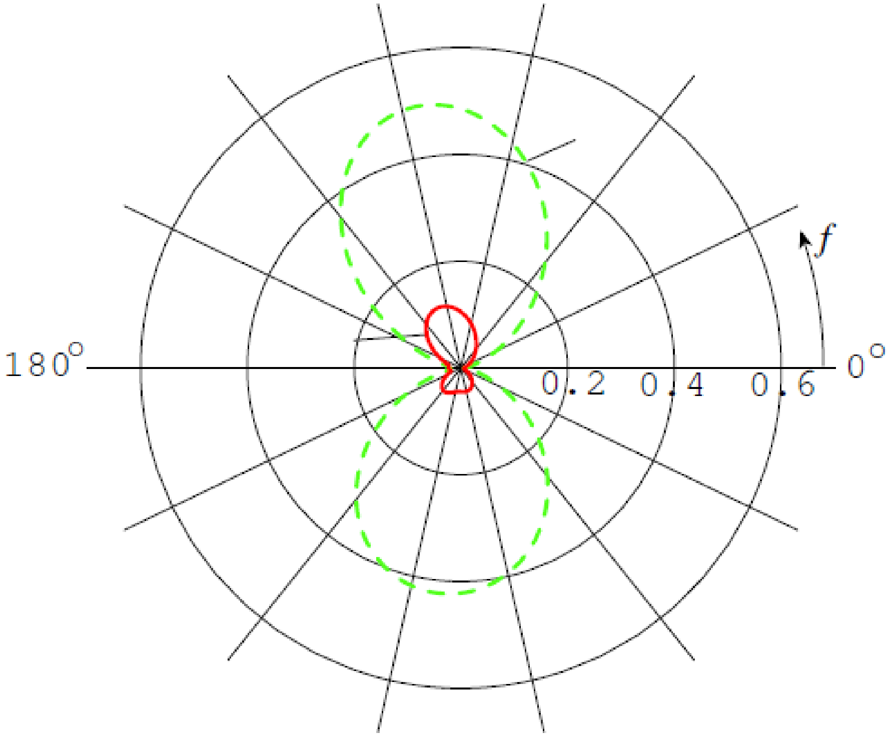
Numerical Simulations (3)

2) Linear solution for arbitrary *e* (analytical)

3) Linear solution for small *e* (analytical)



(iii) RMS Relative Orbit Error in kilometers vs. Chief True Anomaly Angle f for $e = 0.03$



(iv) RMS Relative Orbit Error in kilometers vs. Chief True Anomaly Angle f for $e = 0.13$

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D'Amico

Relative Eccentricity and Inclination Vectors (1)

- Usage of nearly non-singular orbital elements lead to an alternative representation of (14.131)

$$(2.1) \qquad \boldsymbol{\alpha} = \begin{pmatrix} a \\ u \\ e_x \\ e_y \\ i \\ \Omega \end{pmatrix} = \begin{pmatrix} a \\ \omega + M \\ e \cos\omega \\ e \sin\omega \\ i \\ \Omega \end{pmatrix}$$

Mean argument of latitude

u

Eccentricity vector

$$\boldsymbol{e} = (e_x, e_y)^T$$

- The orbit element differences are replaced by so called relative orbit elements

$$(2.2) \quad \delta\boldsymbol{\alpha} = \begin{pmatrix} \delta a \\ \delta\lambda \\ \delta e_x \\ \delta e_y \\ \delta i_x \\ \delta i_y \end{pmatrix} = \begin{pmatrix} (a_d - a)/a \\ (u_d - u) + (\Omega_d - \Omega)\cos i \\ e_{x_d} - e_x \\ e_{y_d} - e_y \\ i_d - i \\ (\Omega_d - \Omega)\sin i \end{pmatrix}$$

Relative semi-major axis

Relative mean longitude

Relative eccentricity vector

Relative inclination vector

Relative Eccentricity and Inclination Vectors (2)

- Cartesian or polar representations for relative ecc./incl. vectors can be used

$$(2.3) \qquad \delta\boldsymbol{e} = \begin{pmatrix} \delta e_x \\ \delta e_y \end{pmatrix} = \delta e \begin{pmatrix} \cos\varphi \\ \sin\varphi \end{pmatrix} \qquad \delta\boldsymbol{i} = \begin{pmatrix} \delta i_x \\ \delta i_y \end{pmatrix} = \delta i \begin{pmatrix} \cos\vartheta \\ \sin\vartheta \end{pmatrix} \qquad (2.4)$$

- The amplitudes of these vectors (*δ*) shall not be confused with the orbit element differences of Schaub (*Δ*) !!

- The phases of these vectors are called relative perigee and ascending node

- Reformulating the linear mapping for near-zero eccentricity provides

$$\begin{array}{lcllll} \delta r_r/a & \approx & \delta a & & -\delta e_x \cos u & -\delta e_y \sin u \\ \delta r_t/a & \approx & -\frac{3}{2}\delta a u & +\delta\lambda & +2\delta e_x \sin u & -2\delta e_y \cos u \\ \delta r_n/a & \approx & & & +\delta i_x \sin u & -\delta i_y \cos u \end{array} \qquad (2.17)$$

- or in amplitude/phase form

Compare with HCW !!

$$\boxed{\begin{array}{lclll} \delta r_r/a & = & \delta a & & -\delta e \cos(u-\varphi) \\ \delta r_t/a & = & \delta\lambda & -\frac{3}{2}\delta a u & +2\delta e \sin(u-\varphi) \\ \delta r_n/a & = & & & +\delta i \sin(u-\vartheta) \end{array}} \qquad (2.18)$$

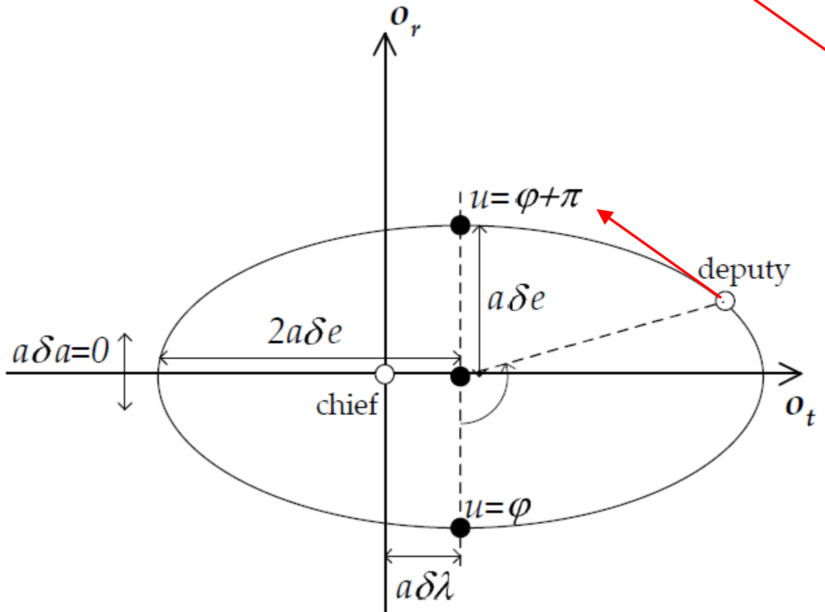
Revised Insight into Relative Orbit Geometry

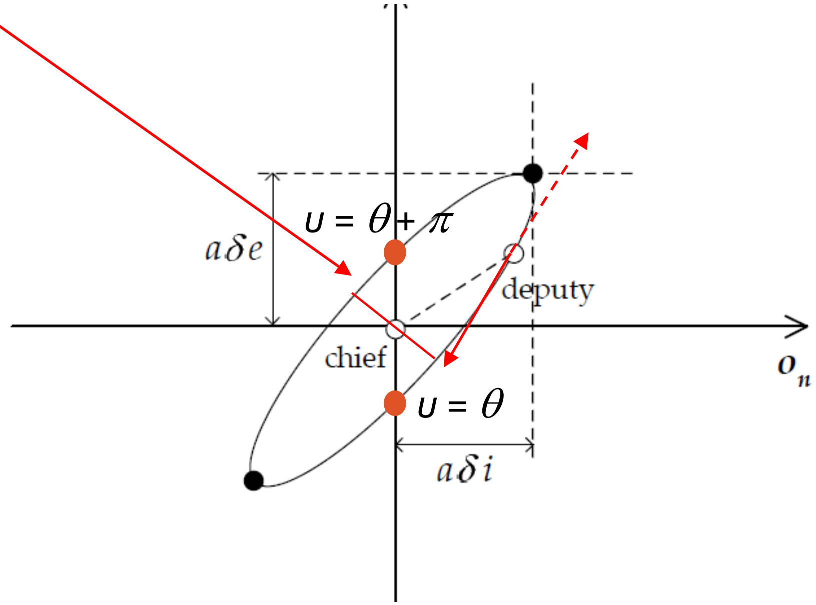
- Bounded relative motion
- Centered relative motion
- Minimum separation in RN

Arithmetic difference (Schaub)   Relative orbit element (D'Amico)

$$\begin{aligned}\delta a &= 0 \\ \delta\lambda &= 0 \;\Leftrightarrow\; \Delta u = -\Delta\Omega \cos i\end{aligned} \tag{2.19}$$

$$\delta r_{nr}^{\min} = \frac{\sqrt{2}a\,|\delta\boldsymbol{e}\cdot\delta\boldsymbol{i}|}{\left(\delta e^2 + \delta i^2 + |\delta\boldsymbol{e}+\delta\boldsymbol{i}|\cdot|\delta\boldsymbol{e}-\delta\boldsymbol{i}|\right)^{1/2}} \tag{2.22}$$





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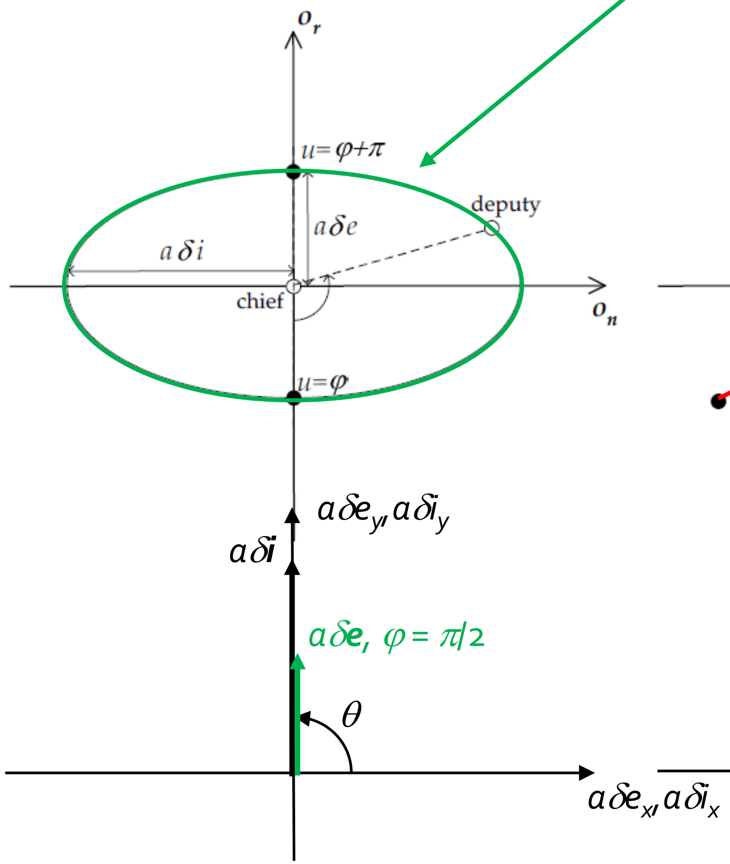
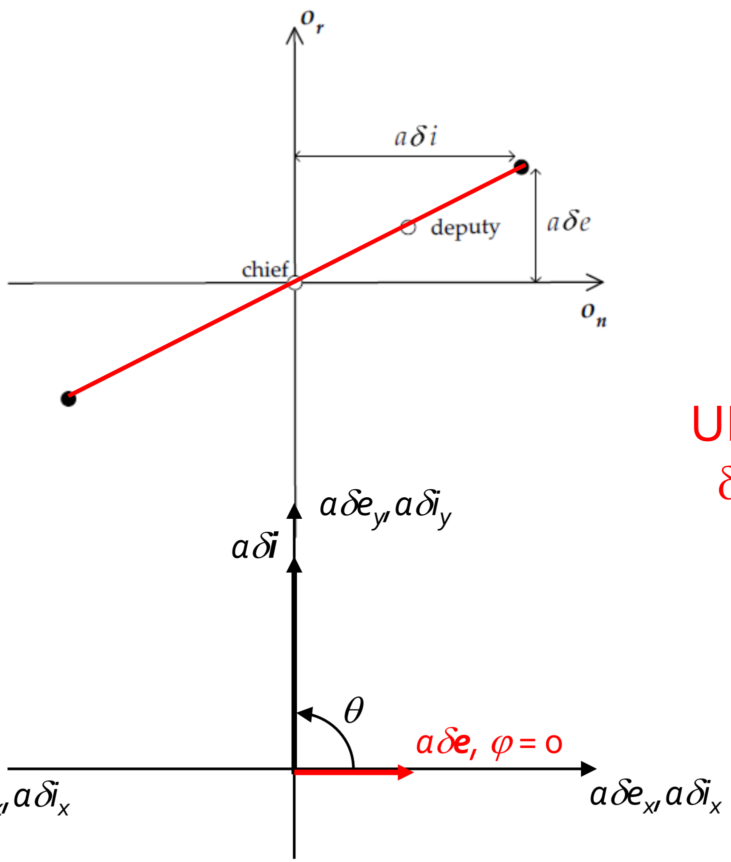
Relative Eccentricity/Inclination Vector Separation

$$\delta r_{nr}^{\min}(u = \varphi, \varphi + k\pi) = a \min\{\delta e, \delta i\}$$



SAFE

$\delta \mathbf{e} \,//\, \delta \mathbf{i}$



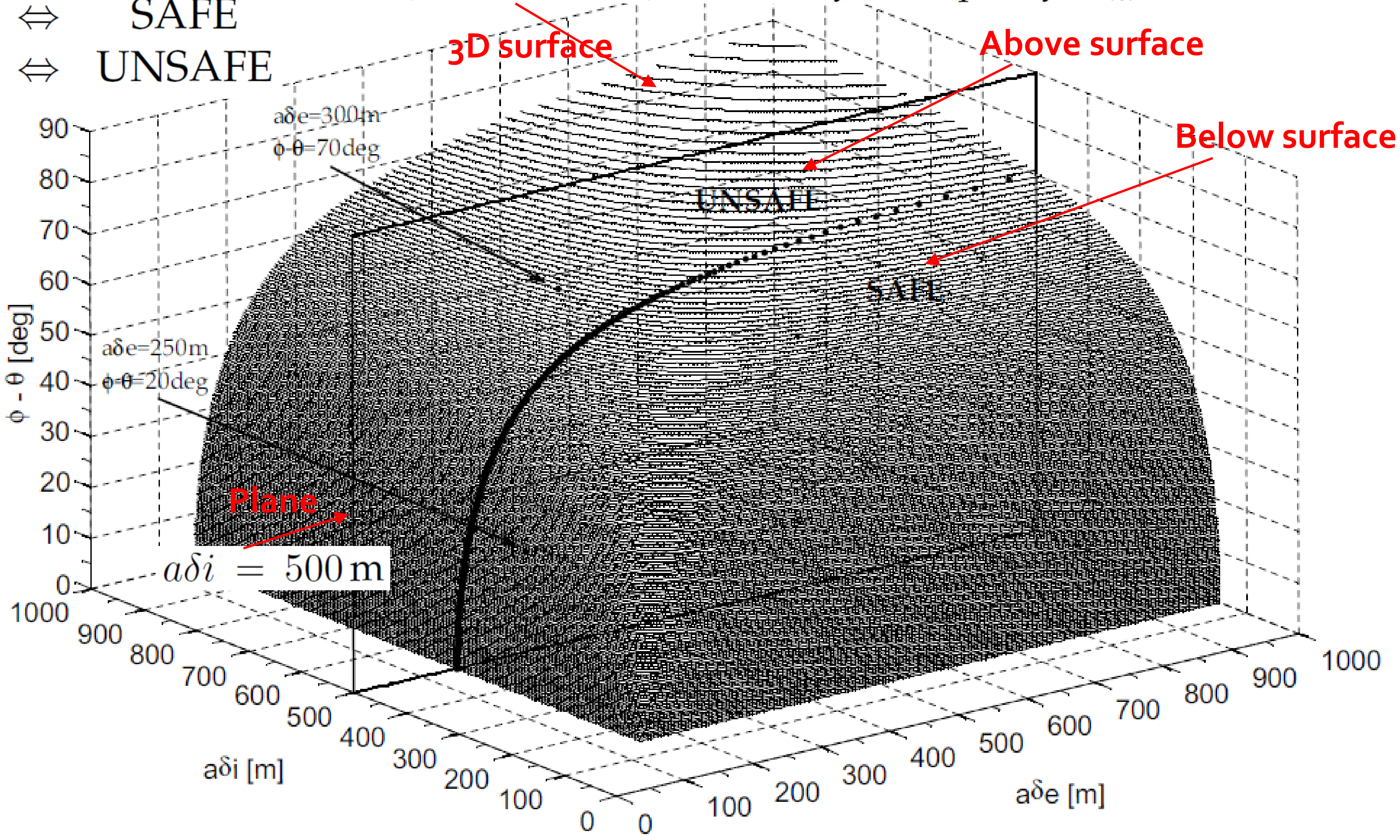
UNSAFE

$\delta \mathbf{e} \perp \delta \mathbf{i}$

Collision-Free Formation Flying Configurations

$$\delta r_{nr}^{\min} \geq d_{\min} \Leftrightarrow \text{SAFE}$$
$$\delta r_{nr}^{\min} < d_{\min} \Leftrightarrow \text{UNSAFE}$$

$(a\delta e, a\delta i, \varphi - \vartheta)$ that satisfy the equality $\delta r_{nr}^{\min} = 150\,\text{m}$



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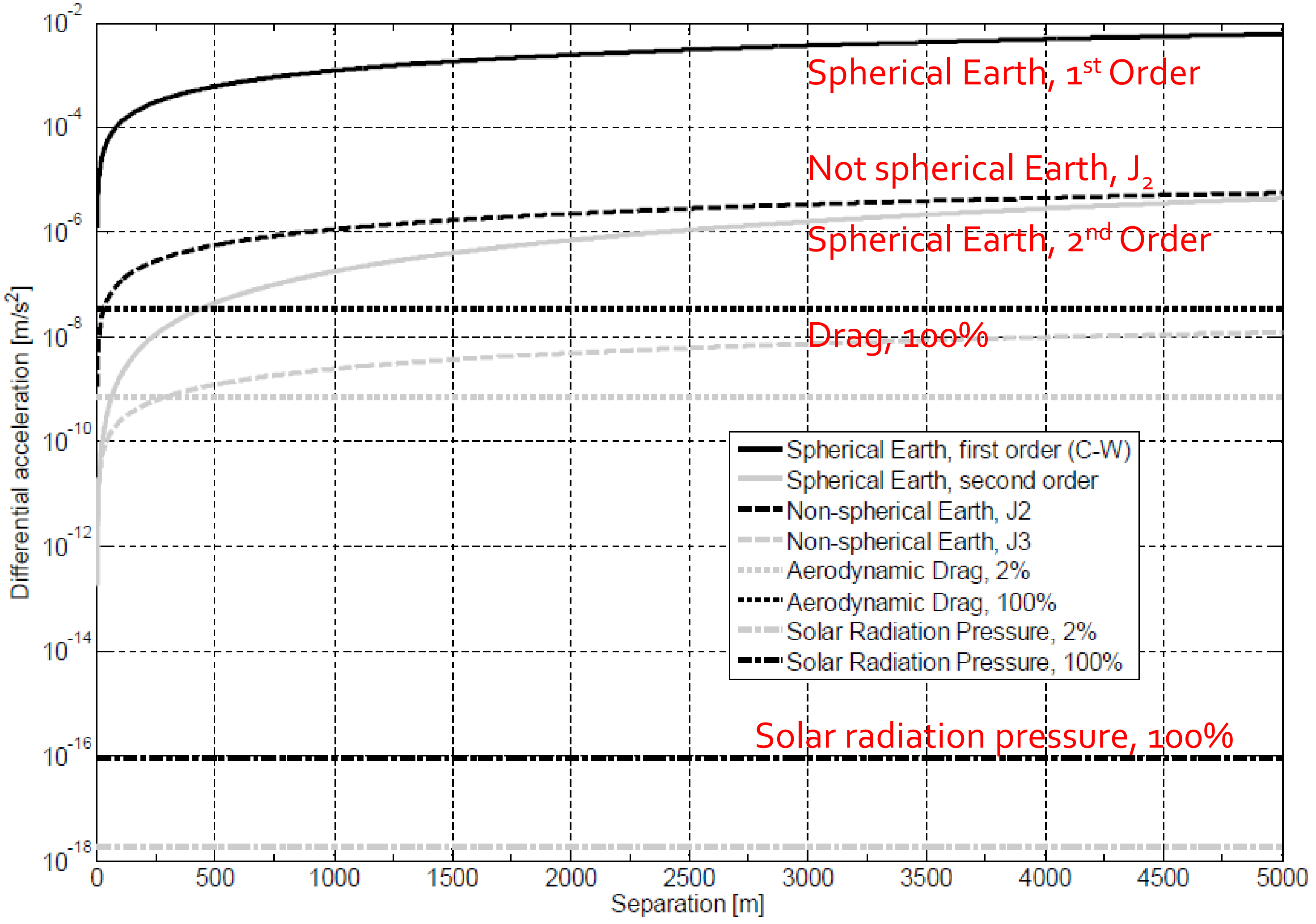
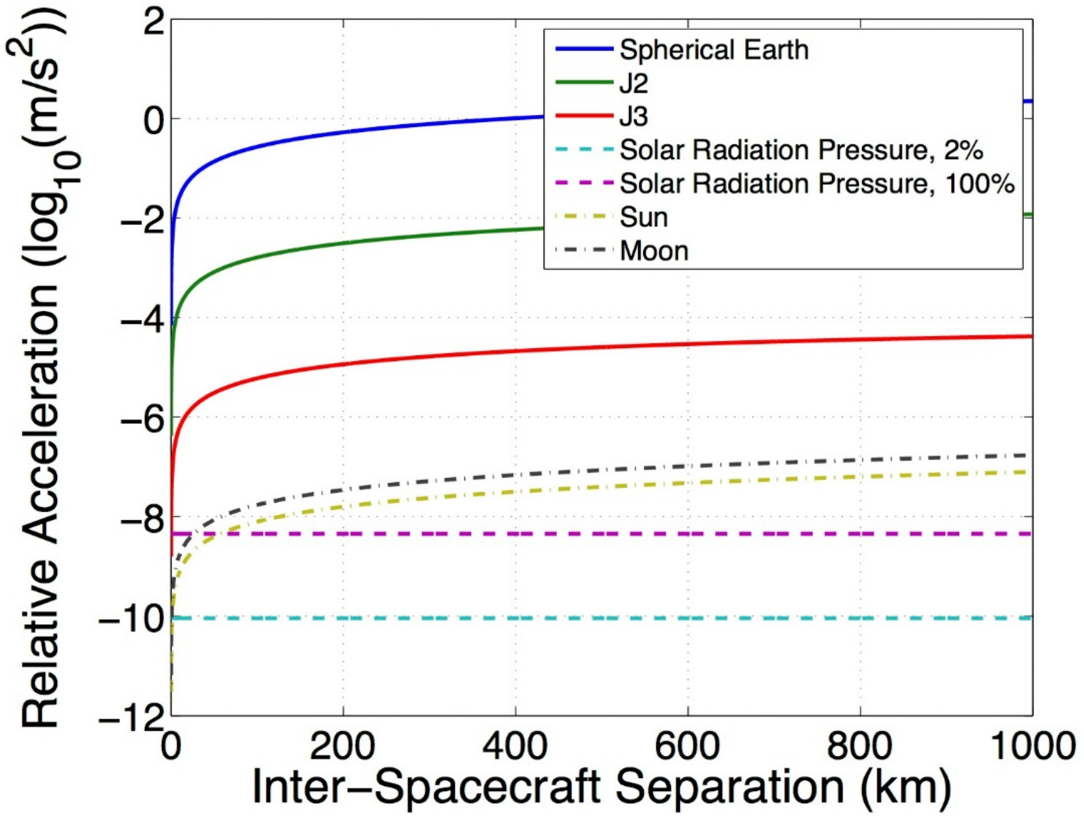
Perturbed Relative Motion (LEO <1500km)

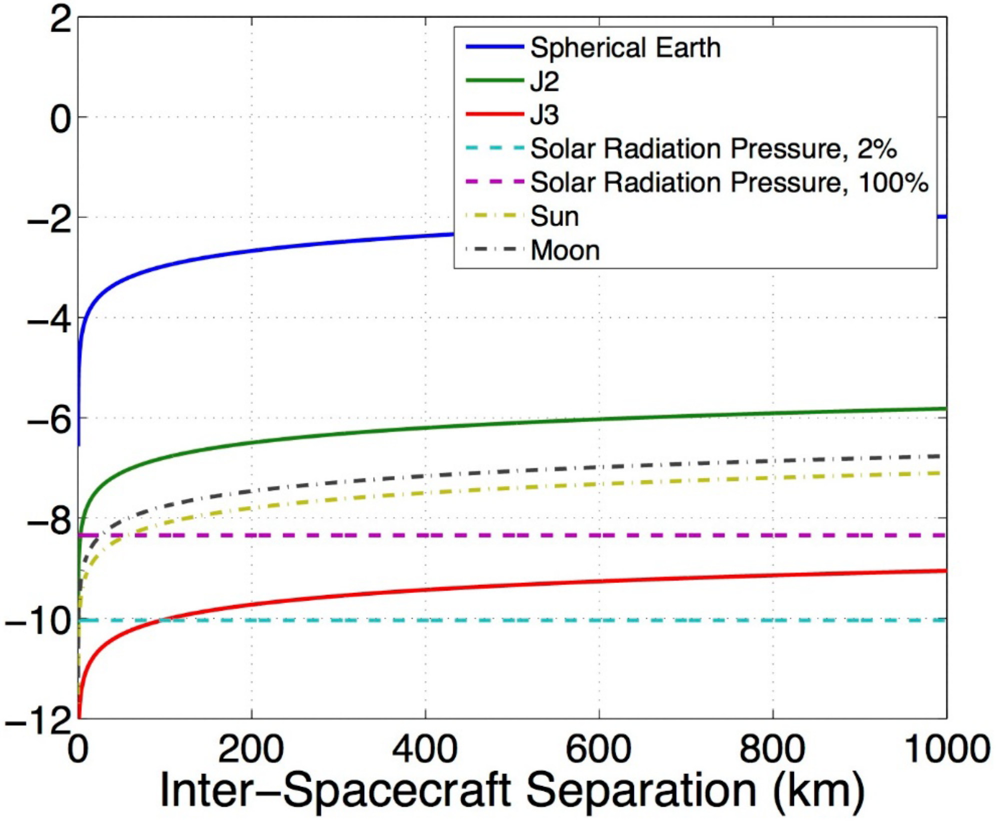


Perturbed Relative Motion at Different Altitudes

LEO Perturbations



GEO Perturbations



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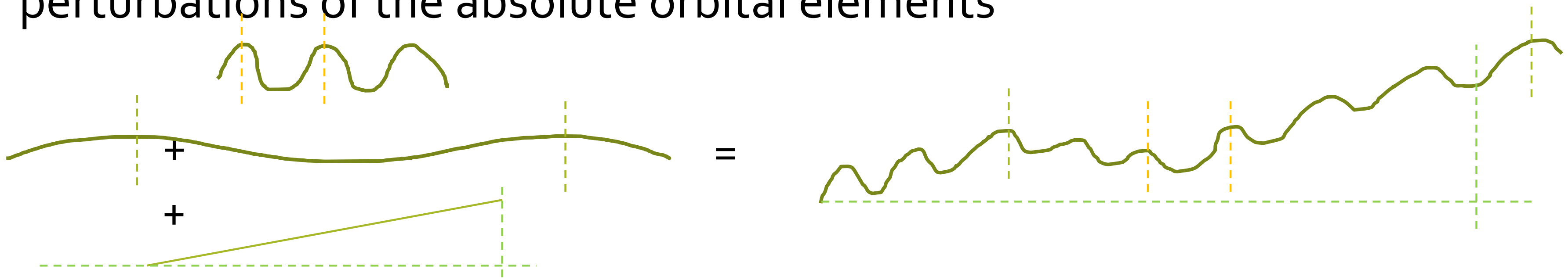
Earth's Oblateness J_2 Effects (Absolute, 1)

- The Earth's equatorial bulge causes short-, long-period and secular perturbations of the absolute orbital elements



- The variation of parameters method (see Brouwer and Lyddane [1959-1963]) provides the analytical tool to capture these effects for the absolute orbit

$$\frac{d}{dt}\begin{pmatrix} a \\ e \\ i \\ \Omega \\ \omega \\ M \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -3\gamma n \cos i \\ \frac{3}{2}\gamma n(5\cos^2 i - 1) \\ \frac{3}{2}\gamma\eta n(3\cos^2 i - 1) \end{pmatrix} \tag{2.26}$$

Secular variations of Keplerian orbital elements caused by J_2

$$\gamma = \frac{J_2}{2}\left(\frac{R_E}{a}\right)^2 \frac{1}{\eta^4} \tag{2.25}$$

Earth's Oblateness J_2 Effects (Relative, 2)

- We can substitute the long-period and secular effects into our definition of ROE and neglect 2nd order effects (in *e* and *δ*) as done previously to obtain

Secular variations of relative orbital elements caused by J_2

$$\delta\dot{\boldsymbol{\alpha}} = \begin{pmatrix} 0 \\ -\frac{21}{2}\gamma n \sin(2i)\delta i_x \\ -\frac{3}{2}\gamma n(5\cos^2 i - 1)\delta e_y \\ \frac{3}{2}\gamma n(5\cos^2 i - 1)\delta e_x \\ 0 \\ 3\gamma n \sin^2 i \delta i_x \end{pmatrix} \quad (2.28)$$

- Using the mean argument of latitude *u* as independent variable, after integration over u-u_0, we obtain

$$\delta\boldsymbol{\alpha}(t) = \begin{pmatrix} \delta a \\ \delta\lambda - \frac{21}{2}(\gamma\sin(2i)\delta i_x + \frac{1}{7}\delta a)(u(t) - u_0) \\ \delta e\cos(\varphi + \varphi'(u(t) - u_0)) \\ \delta e\sin(\varphi + \varphi'(u(t) - u_0)) \\ \delta i_x \\ \delta i_y + 3\gamma\sin^2 i\delta i_x(u(t) - u_0) \end{pmatrix} \quad (2.29)$$

$$\varphi' = \frac{d\varphi}{du} = \frac{3}{2}\gamma(5\cos^2 i - 1) \quad (2.30)$$

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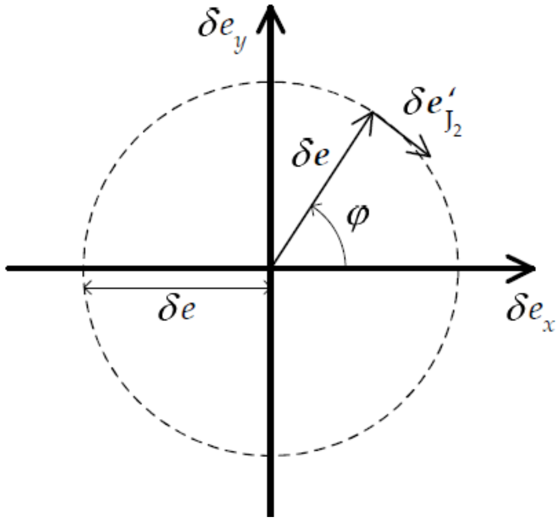
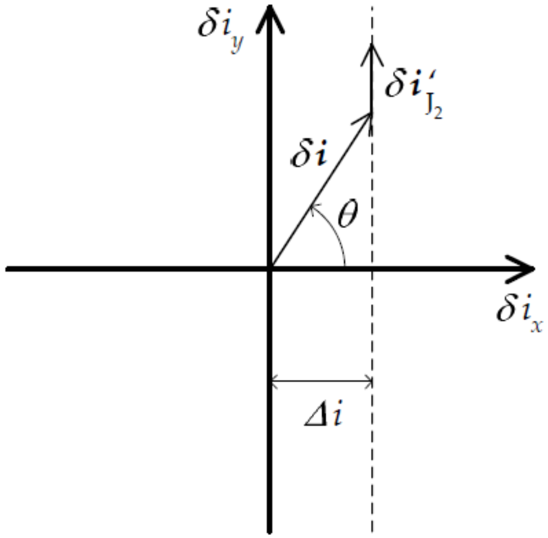
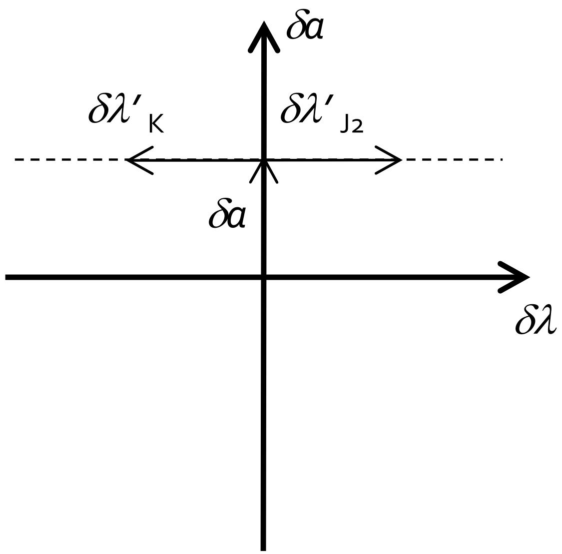
Earth’s Oblateness J_2 Effects (Relative, 3)



$$\varphi' = \frac{d\varphi}{du} = \frac{3}{2}\gamma(5\cos^2 i - 1)$$



$$\delta i'_{J2} = 3\gamma \sin^2 i\,\delta i_x$$



$$\delta\lambda' = -\frac{21}{2}\left(\gamma\sin(2i)\delta i_x + \frac{1}{7}\delta a\right)$$

Clockwise for sun-synchronous orbits with period of about 100-200 days or about 1000 times the orbital period. A critical inclination exists!

Proportional to Δi and J_2. Note that $\sin(2i)$ is negative for sun-synchronous orbits and closed relative orbits are given by

$$\Delta a \approx 1000\Delta i$$

Differential Drag Effects (1)

GVE

$$\frac{\mathrm{d}a}{\mathrm{d}t} = -\left(\frac{A}{m}\right) C_d \rho \frac{v^3}{an^2}$$

$$\frac{\mathrm{d}e}{\mathrm{d}t} = -\left(\frac{A}{m}\right) C_d \rho (e + \cos f) v$$

$$\frac{\mathrm{d}i}{\mathrm{d}t} = 0$$

$$\frac{\mathrm{d}\Omega}{\mathrm{d}t} = 0$$

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = -\left(\frac{A}{m}\right) C_d \rho \frac{\sin f}{e} v$$

$$\frac{\mathrm{d}M}{\mathrm{d}t} = n + \frac{b}{ae}\left(\frac{A}{m}\right) C_d \rho \left(1 + e^2 \frac{r}{p}\right) \sin f v$$

- The interaction of the upper atmosphere with the satellite's surface produces the dominant non-conservative disturbance for LEO spacecraft

$$|\ddot{r}_t| = \frac{1}{2}\rho v^2 C_{\mathrm{D}} \frac{A}{m}$$

Along-track acceleration

$$B = C_{\mathrm{D}} \frac{A}{m}$$

Ballistic coefficient

- If we neglect density variations over distances of less than a few kilometers, the relative along-track acceleration for two formation-flying spacecraft is driven by the differential ballistic coefficient ΔB

$$\delta r_t = \frac{1}{2} a \Delta \ddot{u} (t - t_0)^2 = \frac{3}{4n^2} \Delta B \rho v^2 (u(t) - u_0)^2$$

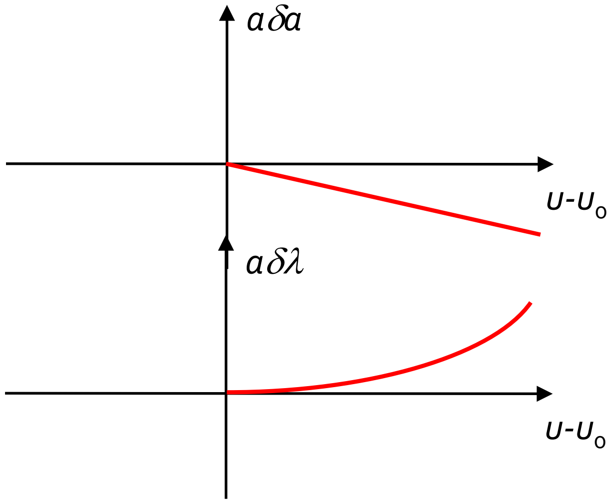
- The first-order relative motion model can be extended to include this accumulated along-track offset, either using Cartesian or ROE parameters

Differential Drag Effects (2)

Only due to drag

$$a\delta\lambda(t) = \frac{3}{8n^2}\Delta B\rho v^2 \left(u(t) - u_0\right)^2$$

$$a\delta a(t) = -\frac{1}{2n^2}\Delta B\rho v^2 \left(u(t) - u_0\right)$$



- Impact of differential drag can be minimized by employing identically designed spacecraft. The ballistic coefficients can be matched to roughly 1% at launch.
- Mass variations during lifetime can cause an additional difference of 1%
- Considering typical atmospheric density values in LEO, differential accelerations of <10^1 nm/s^2 are encountered which require negligible delta-vs
- This conclusion is no longer valid during safe modes (10^2 nm/s^2) or for non-cooperative spacecraft where differential drag can match absolute drag

New State Transition Matrix based on ROEs

$$\begin{pmatrix} \delta\dot{a} \\ \delta\boldsymbol{\alpha} \end{pmatrix}_t = \boldsymbol{\Phi}(t - t_0) \begin{pmatrix} \delta\dot{a} \\ \delta\boldsymbol{\alpha} \end{pmatrix}_{t_0}$$

$$\boldsymbol{\Phi}_{F,0} = \begin{array}{c} \begin{array}{ccccccc} \text{DRAG} & \text{KEPLER} & & & \text{SECULAR J2} & & \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ (t_F - t_0) & 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{\nu}{2}(t_F - t_0)^2 & \nu(t_F - t_0) & 1 & 0 & 0 & \mu(t_F - t_0) & 0 \\ 0 & 0 & 0 & 1 & -\dot{\varphi}(t_F - t_0) & 0 & 0 \\ 0 & 0 & 0 & \dot{\varphi}(t_F - t_0) & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda(t_F - t_0) & 1 \end{bmatrix} \end{array}$$

$$\nu = -\frac{3}{2}n \qquad \dot{\varphi} = \frac{3}{2}n\gamma(5\cos^2 i - 1) \qquad \mu = -\frac{21}{2}n\gamma\sin 2i \qquad \lambda = 3n\gamma\sin^2 i$$

Osculating effects on relative motion

From numerical integration

J_2 in LEO/HEO

→

SRP in GEO

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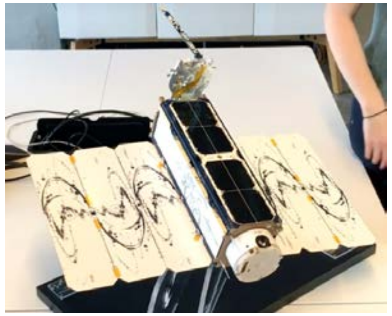
Relative eccentricity/inclination vector separation for swarming

Planet's Dove



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