AA 279 D – SPACECRAFT FORMATION-FLYING AND RENDEZVOUS: LECTURE 9

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Table of Contents

- Linear continuous control
- Nonlinear continuous control
- Simulations

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The mechanism of feedback

- <u>Open-loop control</u>
  - Control $\boldsymbol{u}$ is selected to produce $\boldsymbol{y}^{\text{ref}}$
  - Assumes perfect knowledge of process
  - The billiard game is an example

Open-loop control

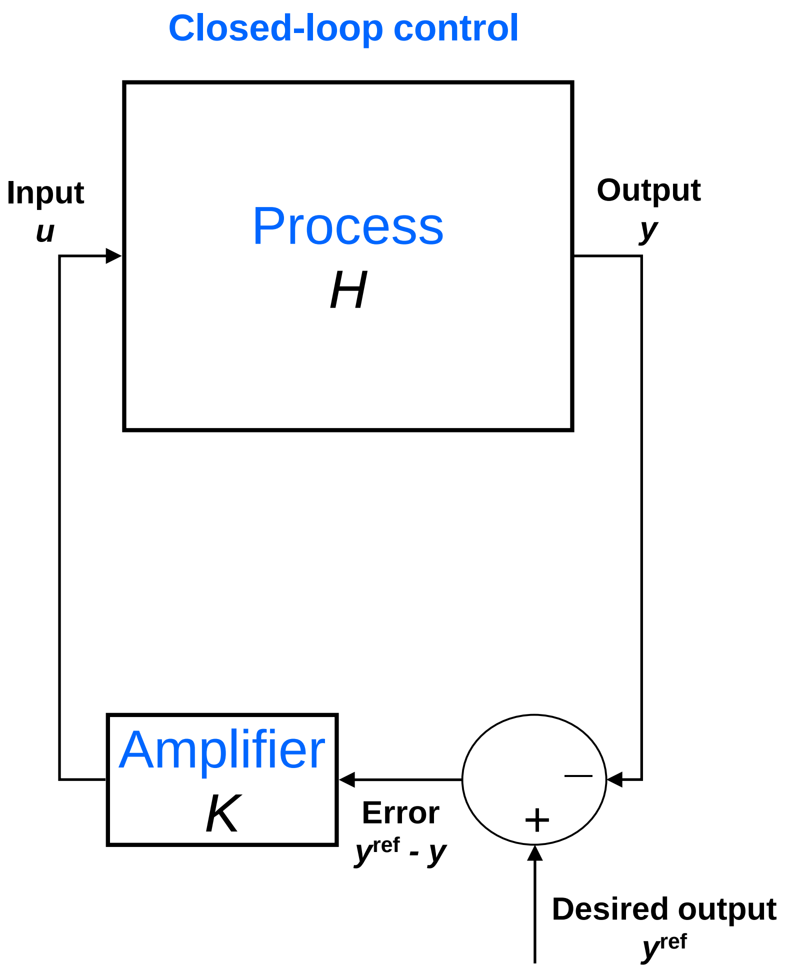
Input
u

Output
y

The mechanism of feedback

- Open-loop control
  - Control ***u*** is selected to produce y^{ref}
  - Assumes perfect knowledge of process
  - The billiard game is an example
- Closed-loop control
  - Control ***u*** is function of system error ***e***

$$u = Ke = K(y^{ref} - y)$$
$$y = Hu \Rightarrow y / y^{ref} = HK / (HK + 1)$$
$$K >> 1 \Rightarrow y \approx y^{ref}$$



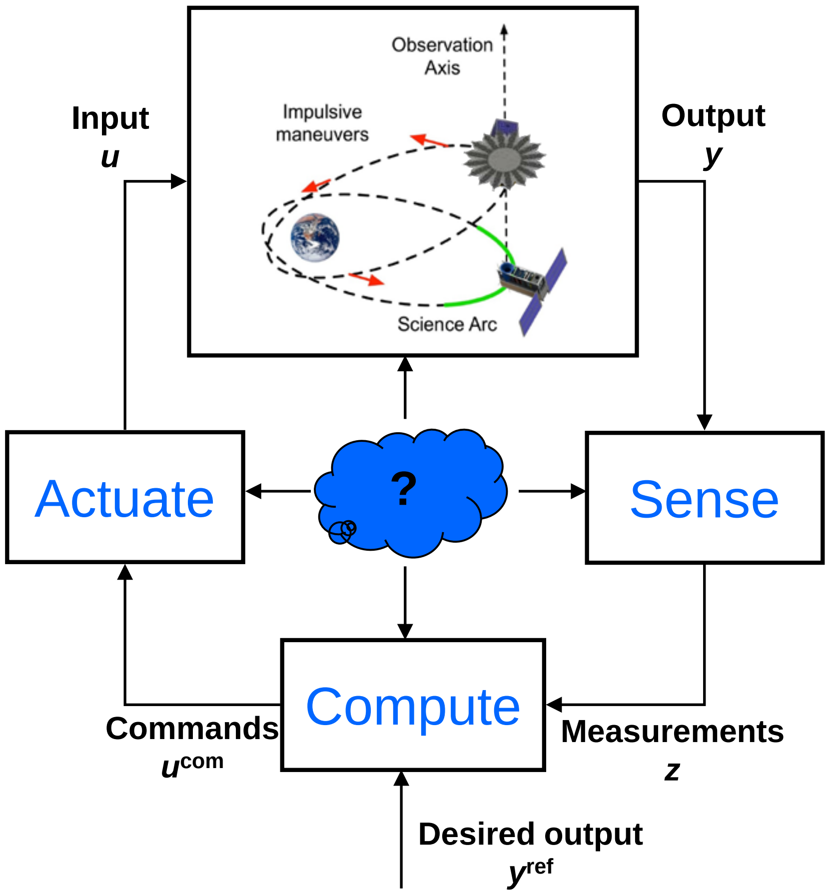
The mechanism of feedback

- <u>Open-loop control</u>
  - Control $\boldsymbol{u}$ is selected to produce $\boldsymbol{y}^{ref}$
  - Assumes perfect knowledge of process
  - The billiard game is an example
- <u>Closed-loop control</u>
  - Control $\boldsymbol{u}$ is function of system error $\boldsymbol{e}$

$$u = Ke = K(y^{ref} - y)$$
$$\cancel{y = Hu} \Rightarrow y / y^{ref} = HK / (HK + 1)$$
$$K >> 1 \Rightarrow y \approx y^{ref}$$

- <u>What is control engineering?</u>
  - Feedback to control dynamic process
  - Sensing + Computation + Actuation
  - Aims at stability, performance, robustness
  - Most important task: **process modeling**!

Realistic dynamics system



Linearization

- <u>Choice of state variables</u>
  - Arbitrary but with physical meaning
  - Aims at first order differential equations

$$\dot{x} = f(x,u) \qquad x \in \Re^n, u \in \Re^p$$
$$y = h(x,u) \qquad y \in \Re^m$$

State equation of order n
Output equation of order m
Input: p control actions

- <u>Equilibrium state</u>
  - Represent stationary conditions for the dynamics

$$\begin{cases} \dot{x} = f(x,u) \\ \bar{x} : f(\bar{x},0) = 0 \end{cases}$$

Spontaneous motion and equilibria

- <u>Taylor expansion truncated to first order</u>
  - Linearization captures "tangent" dynamics
  - Provide local description of non linear systems

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$

State and output linear transfer equations

- <u>Linear controller design</u>
  - Many tools available for linear autonomous systems
    - Classical control theory (Nyquist, Bode)
    - Optimal control/estimator design (LQR, Kalman)
    - Robust control design (H_∞, μ)

$$\begin{cases} u = Kx \\ K = ? \qquad K \in \Re^{p,n} \end{cases}$$

How to obtain desired behavior?

Stability of linear systems (1)

$$\begin{cases} \dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \cancel{\boldsymbol{B}\boldsymbol{u}} \\ \cancel{\boldsymbol{y} = \boldsymbol{C}\boldsymbol{x} + \boldsymbol{D}\boldsymbol{u}} \end{cases}$$

Spontaneous motion

- Solution of linear system
  - Matrix exponential: series expansion
  - Simple case: diagonal system (***A***)

$$\boldsymbol{x}(t) = e^{\boldsymbol{A}t}\boldsymbol{x}(0) = \begin{bmatrix} e^{\lambda_1 t} & & 0 \\ & \ddots & \\ 0 & & e^{\lambda_n t} \end{bmatrix} \boldsymbol{x}(0)$$

- Transform to "Jordan" form
  - More general
  - Always possible
  - Jordan blocks $\boldsymbol{J}_i$ replace $e^{\lambda t}$ terms

$$\boldsymbol{A} = \boldsymbol{T}^{-1}\boldsymbol{J}\boldsymbol{T} \quad \boldsymbol{J} = \begin{bmatrix} \boldsymbol{J}_1 & & 0 \\ & \ddots & \\ 0 & & \boldsymbol{J}_k \end{bmatrix} \quad \boldsymbol{J}_i = \begin{bmatrix} \lambda_i & 1 & 0 \\ & \ddots & 1 \\ 0 & & \lambda_i \end{bmatrix}$$

η_i: Maximum dimension of Jordan blocks of λ_i

- Eigenvalues of ***A*** determine system behavior
  - Modes are associated to eigenvalues
  - Free motion is a combination of modes:
  - Stability is determined by eigenvalues

$$\begin{cases} t^{\eta_i - 1} e^{\lambda_i t} & \lambda_i \in \Re \\ t^{\eta_i - 1} e^{\sigma_i t} \sin(\omega_i t + \varphi_i) & \lambda_i = \sigma_i + j\omega_i \in C \end{cases}$$

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Stability of linear systems (2)

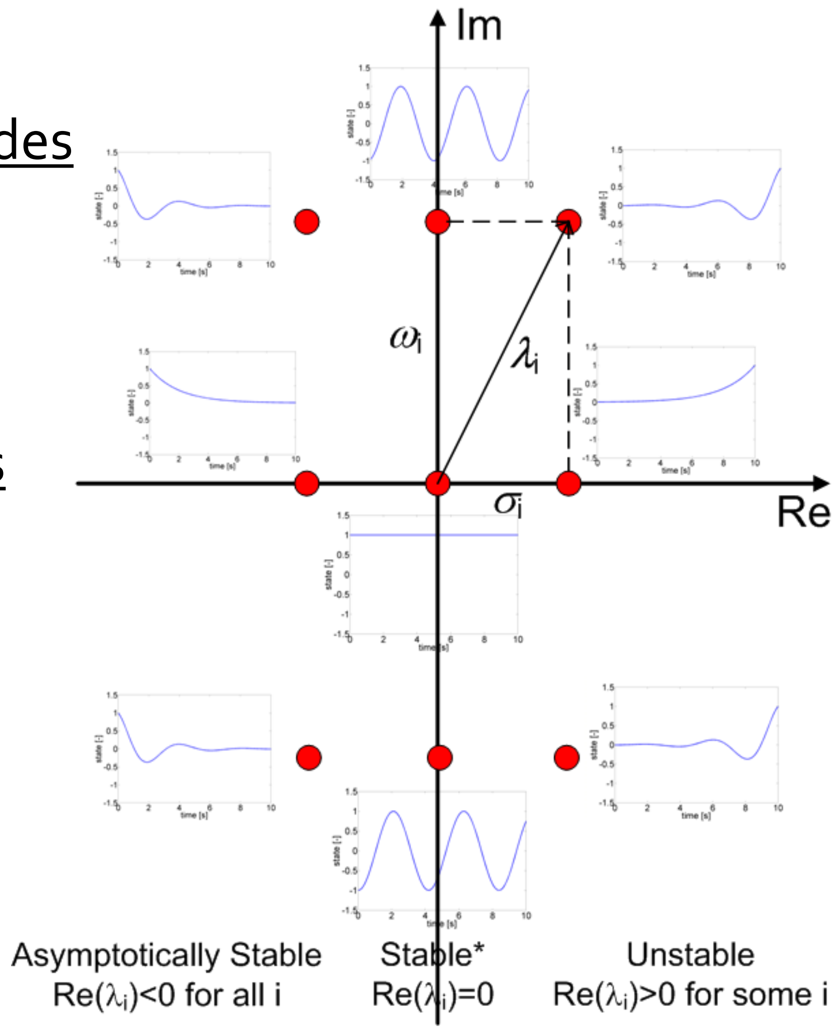
- <u>Free response is a combination of modes</u>

$$\begin{cases} t^{\eta_i - 1} e^{\lambda_i t} & \lambda_i \in \Re \\ t^{\eta_i - 1} e^{\sigma_i t} \sin(\omega_i t + \varphi_i) & \lambda_i = \sigma_i + j\omega_i \in C \end{cases}$$

- <u>Stability is determined by eigenvalues</u>
  - Asymptotically stable: response tends to zero
  - Stable: response is bounded
  - Unstable: response grows indefinitely

- <u>Eigenvalues and system behavior</u>
  - Frequency response: $\omega_i = \mathrm{Im}(\lambda_i)$
  - Time response (bandwidth): $\sigma_i = \mathrm{Re}(\lambda_i)$
  - Control law can modify eigenvalues!

*Multiple poles with zero real part implies instability since η>1



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Controller design for linear systems

- <u>Linear dynamics system</u>

$$\begin{cases} \dot{\boldsymbol{x}} = \boldsymbol{Ax} + \boldsymbol{Bu} & \boldsymbol{x} \in \Re^n, \boldsymbol{u} \in \Re^p \\ \boldsymbol{y} = \boldsymbol{Cx} + \boldsymbol{Du} & \boldsymbol{y} \in \Re^m \end{cases}$$

- <u>Goal</u>

  Find a linear control law $\boldsymbol{u} = \boldsymbol{Kx}$ such that the closed-loop system

$$\dot{\boldsymbol{x}} = \boldsymbol{Ax} + \boldsymbol{BKx} = (\boldsymbol{A} + \boldsymbol{BK})\boldsymbol{x}$$

  is stable around equilibrium

- <u>Remarks</u>
  - Stability determined by eigenvalues: use ***K*** to make eigenvalues of (***A***+***BK***) stable
  - The choice of the eigenvalues is linked to performance
  - Question: when can we place the eigenvalues anyplace we want?

- <u>Controllability</u>
  - The eigenvalues of the closed-loop system (***A***+***BK***) can be set to arbitrary values if and only if the pair (***A***,***B***) is controllable.

Controllability

$$\begin{cases} \dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} & \boldsymbol{x} \in \Re^n, \boldsymbol{u} \in \Re^p \\ \boldsymbol{y} = \boldsymbol{C}\boldsymbol{x} + \boldsymbol{D}\boldsymbol{u} & \boldsymbol{y} \in \Re^m \end{cases}$$

- <u>Practical definition</u>

  A system is said to be controllable if and only if it is possible, by means of the input, to transfer the system from any initial state to any other state in a finite time

- <u>Algebraic condition or test</u>
  - A linear system is controllable if and only if the [nxpn] controllability matrix

$$\boldsymbol{Q} = [\boldsymbol{B} \quad \boldsymbol{AB} \quad \boldsymbol{A}^2\boldsymbol{B} \quad \cdots \quad \boldsymbol{A}^{n-1}\boldsymbol{B}]$$

  is full rank, i.e. rank($\boldsymbol{Q}$) = max(n,np)

Pole placement

- <u>Problem</u>
  - Closed-loop pole assignment using state feedback

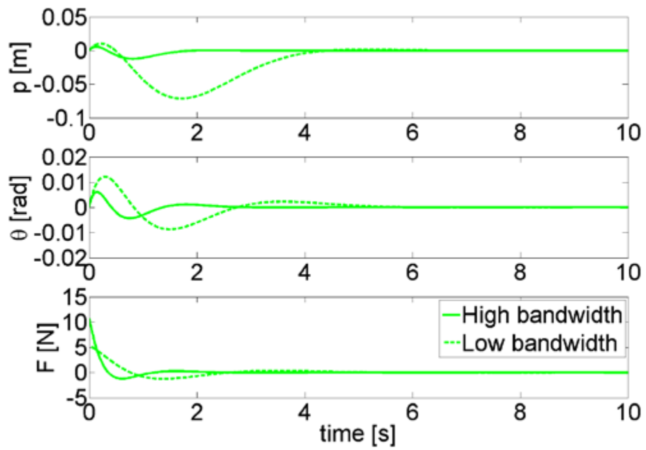
$$\begin{cases} \dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} & \boldsymbol{x} \in \Re^n, \boldsymbol{u} \in \Re^p \\ \boldsymbol{u} = \boldsymbol{K}\boldsymbol{x} & \boldsymbol{K} = ? \end{cases}$$

- <u>Solution</u>
  - Various methods exist
  - e.g. Ackerman formula:
  - $\boldsymbol{b}$ is the solution vector of a linear system with known coefficients
  - a_i are the coefficients of the characteristic equation in closed-loop

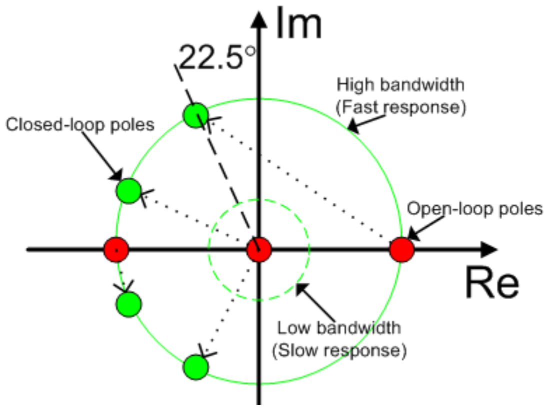
$$\begin{cases} \boldsymbol{K} = \boldsymbol{b}^T \alpha_c(\boldsymbol{A}), & \alpha_c(\boldsymbol{A}) = \boldsymbol{A}^n + \boldsymbol{a}_1\boldsymbol{A}^{n-1} + \ldots + \boldsymbol{a}_n\boldsymbol{A}^{n-1} \\ \boldsymbol{Q}^T\boldsymbol{b} = -\boldsymbol{e}_n, & \boldsymbol{e}_n = [0 \quad 0 \quad \ldots \quad 1]^T \end{cases}$$

- <u>Example: Inverted pendulum</u>

Proper selection of closed-loop poles makes system asymptotically stable and allows shaping of response



e.g. Butterworth polynomials



Shaping the dynamics response (1)

- How to relate performance requirements to gain matrix?
  - The closed-loop poles of a controllable system can be placed anywhere
  - Typical goal is minimization of both control error AND control effort
  - Broad guidelines for choice of closed-loop poles
    - Bandwidth high enough to achieve desired speed of response
    - Bandwidth low enough to avoid exciting un-modeled high-frequency effects, undesired response to noise, large control effort
    - Poles at approximately uniform distances from the origin for efficient use of control effort
- A systematic optimization method is the Linear Quadratic Regulator
  - Choose the feedback that minimizes a cost function:

$$J = \int_0^\infty (\boldsymbol{x}^T \boldsymbol{Q}\boldsymbol{x} + \boldsymbol{u}^T \boldsymbol{R}\boldsymbol{u})dt \Rightarrow \boldsymbol{u} = \boldsymbol{K}\boldsymbol{x}$$

  - Key advantage is the explicit trade-off between state error and input
  - Solution available in closed-form through Riccati equation $\boldsymbol{K} = \boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{P}$
  - Often hard to relate the weights ($\boldsymbol{Q}$, $\boldsymbol{R}$) to the desired system behavior

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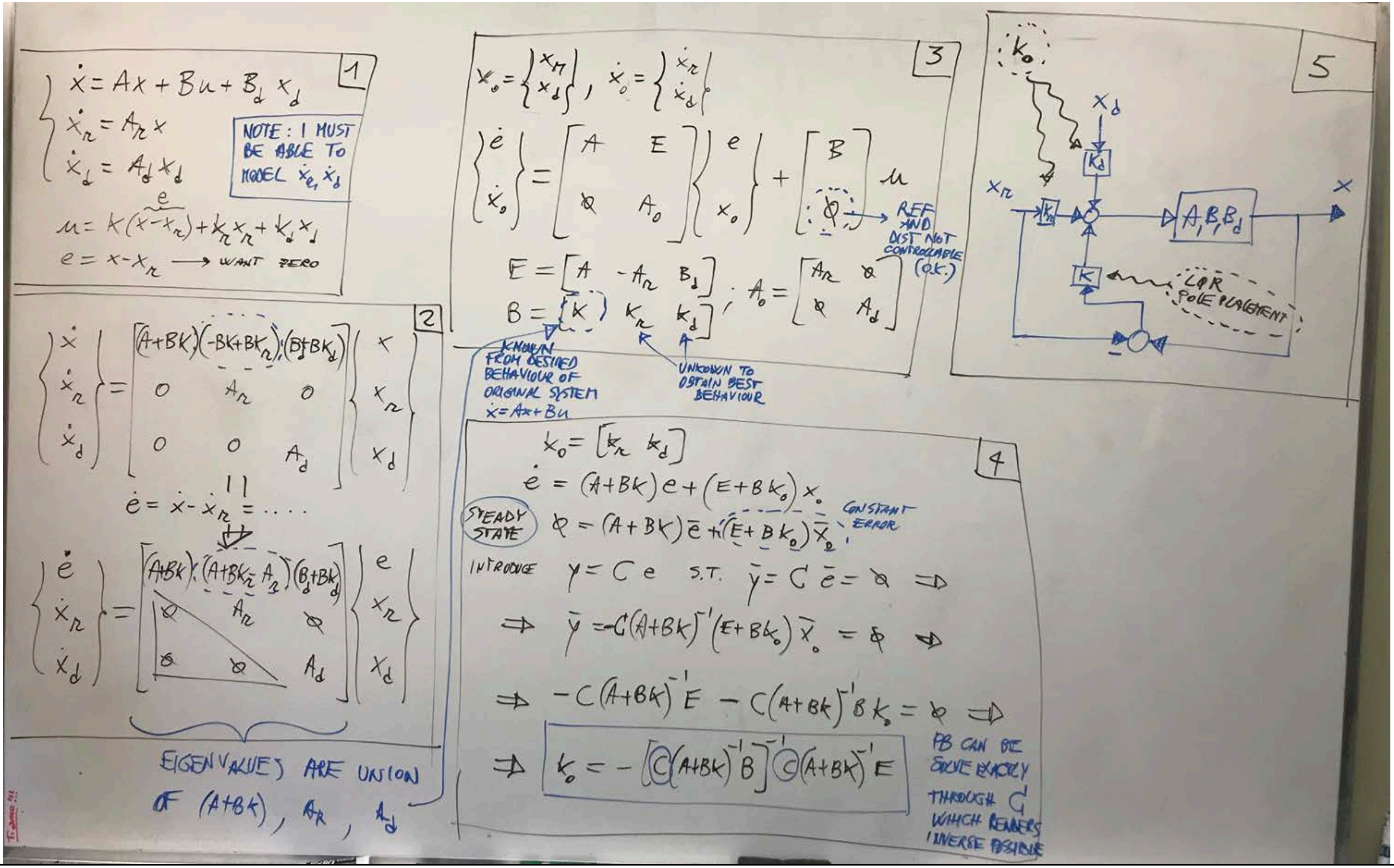
Shaping the dynamics response (2)

- <u>Gain matrix ***K*** for LQR control</u>
  - ***R***: Weight on control, small elements if control has little importance, which implies large gains
  - ***P***: Strictly related to ***Q*** weight on state error, small elements if state error has little importance, which implies small gains
  - Gain matrix is a trade-off between control input and state error, high gains are desired to reduce state error, while low gains are desired to reduce control effort
  - Choice of ***Q*** and ***R*** is thus fundamental to obtain desired system behavior in terms of performance, four possible approaches are available
    - *ITERATIVE*: guess ***Q*** and ***R***, verify performance, update ***Q*** and ***R***, repeat
    - *ENERGETIC*: express total energy of system in cost functional, derive ***Q*** and ***R***
    - *BOUNDED*: impose $|x| < x_{max}$ and $|u| < u_{max}$ by diag(***Q***) = $[1/ x^2_{max\ i}]$ and diag(***R***) = $k[1/ u^2_{max\ i}]$
    - *TRACKING*: introduce a fictitious dynamics system $\boldsymbol{A}_m,\boldsymbol{B}_m$ to follow and weights $\boldsymbol{Q}_m,\boldsymbol{R}_m$ to quantify the importance of following the reference, manipulating the cost functional provides

$$\boldsymbol{Q} = (\boldsymbol{A}-\boldsymbol{A}_m)^T \boldsymbol{Q}_m (\boldsymbol{A}-\boldsymbol{A}_m)$$
$$\boldsymbol{R} = \boldsymbol{R}_m + (\boldsymbol{B}-\boldsymbol{B}_m)\boldsymbol{Q}_m (\boldsymbol{B}-\boldsymbol{B}_m)$$

Optimal control with known disturbance and reference



Spacecraft Rendezvous (1)

- <u>Hill/Clohessy-Wiltshire Equations</u>
  - Pure Keplerian motion
  - Circular reference orbit (n: mean motion)
  - S/C separation small w.r.t. orbit radius

Equations of relative motion

$$\begin{cases} \ddot{R} = 2n\dot{T} + 3n^2 R \\ \ddot{T} = -2n\dot{R} \\ \ddot{N} = -n^2 N \end{cases}$$

Radial

Along-track

Cross-track

Out-of-plane:
harmonic oscillation

In-plane:
ellipse
radial offset
tangential drift

- <u>State-space representation</u>
  - e.g. Hill state coordinates
  - Reduce system to first order
  - Highlight system dynamics
  - Controllability test:

$$u = [a_R \quad a_T \quad 0]^T \Rightarrow rank(\mathbf{Q}) = 4 < 6$$

$$u = [0 \quad 0 \quad a_N]^T \Rightarrow rank(\mathbf{Q}) = 2 < 6$$

Tests failed!

$$u = [a_R \quad a_T \quad a_N]^T \Rightarrow rank(\mathbf{Q}) = 6 < 6$$

Test ok!

$$\begin{bmatrix} \dot{R} \\ \ddot{R} \\ \dot{T} \\ \ddot{T} \\ \dot{N} \\ \ddot{N} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 3n^2 & 0 & 0 & 2n & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -2n & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -n^2 & 0 \end{bmatrix} \begin{bmatrix} R \\ \dot{R} \\ T \\ \dot{T} \\ N \\ \dot{N} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_R \\ a_T \\ a_N \end{bmatrix}$$

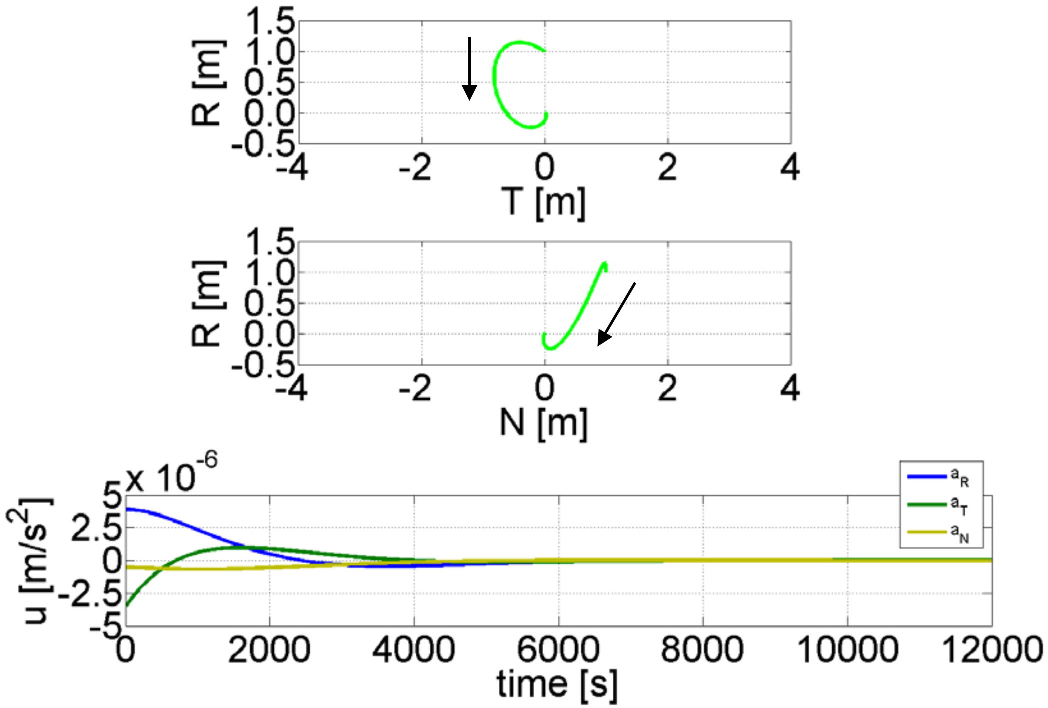
Spacecraft Rendezvous (2)

Initial position

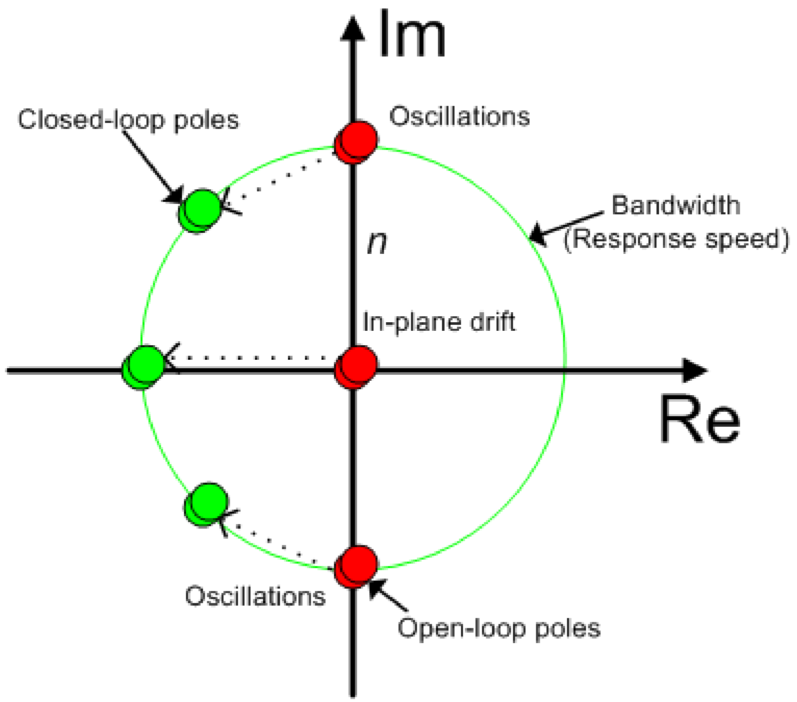
$$[R_0 \quad T_0 \quad N_0]^T = [1 \quad 0 \quad 1]^T \qquad [\mathrm{m}]$$

- Open-loop vs. closed-loop response

Relative motion in orbital frame



Pole placement



Schaub

Nonlinear Continuous Formation Control (1)

- Continuous feedback laws are necessary when the mission requirements call for higher control accuracy (forced motion)
- Instead of tracking errors in Hill coordinates, we can track errors in mean orbit element differences, e.g. considering only J_2 secular effects
- To this end, we re-write the Gauss Variational Equations in matrix form

(14.188)

(14.199)

Osculating orbit element vector

$$\boldsymbol{e}_{\text{osc}} = (a/r_e, e, i, \Omega, \omega, M)^T$$

Mean orbit element vector

$$\boldsymbol{e} = \boldsymbol{\xi}(\boldsymbol{e}_{\text{osc}})$$

Analytical transformation from Brouwer's theory

GVE

$$\dot{\boldsymbol{e}}_{\text{osc}} = (0, 0, 0, 0, 0, n)^T + [B(\boldsymbol{e}_{\text{osc}})]\boldsymbol{u}$$

Control input matrix [6x3]

Kepler-only

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Nonlinear Continuous Formation Control (2)

- Incorporating secular J_2 effects provides the following linear system for the mean orbit elements

Plant matrix [6x1]

$$\dot{\boldsymbol{e}} = [A(\boldsymbol{e})] + [\partial\boldsymbol{\xi}/\partial\boldsymbol{e}_{\text{osc}}][B(\boldsymbol{e}_{\text{osc}})]\boldsymbol{u}$$

- Studying the transformation between osculating and mean orbit elements, it is evident that the gradient of ξ is approximately the identity matrix
- Therefore, for control purposes, we can approximate the system as

$$\dot{\boldsymbol{e}} \approx [A(\boldsymbol{e})] + [B(\boldsymbol{e})]\boldsymbol{u}$$

$$[A(e)] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\frac{3}{2}J_2\left(\frac{r_{\text{eq}}}{p}\right)^2 n\cos i \\ \frac{3}{4}J_2\left(\frac{r_{\text{eq}}}{p}\right)^2 n(5\cos^2 i - 1) \\ n + \frac{3}{4}J_2\left(\frac{r_{\text{eq}}}{p}\right)^2 \eta n(3\cos^2 i - 1) \end{bmatrix}$$

$$[B(e)] = \begin{bmatrix} \frac{2a^2 e\sin f}{hr_e} & \frac{2a^2 p}{hr r_e} & 0 \\ \frac{p\sin f}{h} & \frac{(p+r)\cos f + re}{h} & 0 \\ 0 & 0 & \frac{r\cos\theta}{h} \\ 0 & 0 & \frac{r\sin\theta}{h\sin i} \\ -\frac{p\cos f}{he} & \frac{(p+r)\sin f}{he} & -\frac{r\sin\theta\cos i}{h\sin i} \\ \frac{\eta(p\cos f - 2re)}{he} & -\frac{\eta(p+r)\sin f}{he} & 0 \end{bmatrix}$$

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Nonlinear Continuous Formation Control (3)

- Given the true set of mean orbit elements of the deputy, the relative orbit tracking error is expressed in terms of mean orbit elements as

Tracking error $$\Delta \boldsymbol{e} = \hat{\boldsymbol{e}}_d - \boldsymbol{e}_d$$

True (from navigation)

Desired (e.g. natural passive orbit)

- Lyapunov control theory can be used to develop a feedback control law through the definition of a scalar function *V(Δ**e**)* which is **positive definite** and whose **time derivative is negative definite**
- We make the following arbitrary choice for the Lyapunov function

$$V(\Delta \boldsymbol{e}) = \frac{1}{2} \Delta \boldsymbol{e}^T \Delta \boldsymbol{e}$$ Positive definite

- whose time derivative is set to be negative definite

$$\dot{V} = \Delta \boldsymbol{e}^T \Delta \dot{\boldsymbol{e}} = \Delta \boldsymbol{e}^T ([A(\hat{\boldsymbol{e}}_d)] - [A(\boldsymbol{e}_d)] + [B(\hat{\boldsymbol{e}}_d)]\boldsymbol{u}) = -\Delta \boldsymbol{e}^T [P] \Delta \boldsymbol{e}$$

Negative definite through gain matrix P [6x6]

Nonlinear Continuous Formation Control (4)

- A positive definite feedback gain ***P*** needs to be chosen to guarantee the Lyapunov stability of the closed-loop dynamics system

$$[B]\overset{[3\text{x}1]}{\boldsymbol{u}} = -([A(\hat{\boldsymbol{e}}_d)] - [A(\boldsymbol{e}_d)]) - [P]\overset{[6\text{x}1]}{\Delta \boldsymbol{e}} \tag{14.200}$$

- Since the system of equations is over-determined, we could employ a least-square type inverse to solve for the control input ***u***

$$\boldsymbol{u} = -([B]^T[B])^{-1}[B]^T(([A(\hat{\boldsymbol{e}}_d)] - [A(\boldsymbol{e}_d)]) + [P]\Delta \boldsymbol{e}) \tag{14.202}$$

- Even if the resulting control law is no longer guaranteed to satisfy the stability constraint (14.200), numerical simulations indicate stability
- The key advantage of this method is that the matrix ***P*** can be chosen time- or location-dependent to selectively cancel particular orbit element errors and exploit our knowledge of the relative orbit dynamics

Nonlinear Continuous Formation Control (5)

- Studying the GVE gives indication on the effectiveness of the control vector to influence a particular orbit element
- A proposed solution is to give ***P*** the following diagonal form where *N* is an integer number
- This makes the various feedback gains maximum when the corresponding orbit elements are the most controllable
- *N* can be chosen such that the gain influence drops off and rises sufficiently fast
- Although there are infinite heuristic feedback gain logics, we could pose an optimization problem on ***P***(*t*) to extremize some performance measure

$$P_{11} = P_{a0} + P_{a1} \cos^N \frac{f}{2}$$

$$P_{22} = P_{e0} + P_{e1} \cos^N f$$

$$P_{33} = P_{i0} + P_{i1} \cos^N \theta$$

$$P_{44} = P_{\Omega 0} + P_{\Omega 1} \sin^N \theta$$

$$P_{55} = P_{\omega 0} + P_{\omega 1} \sin^N f$$

$$P_{66} = P_{M0} + P_{M1} \sin^N f$$

(14.201a)–(14.201f)

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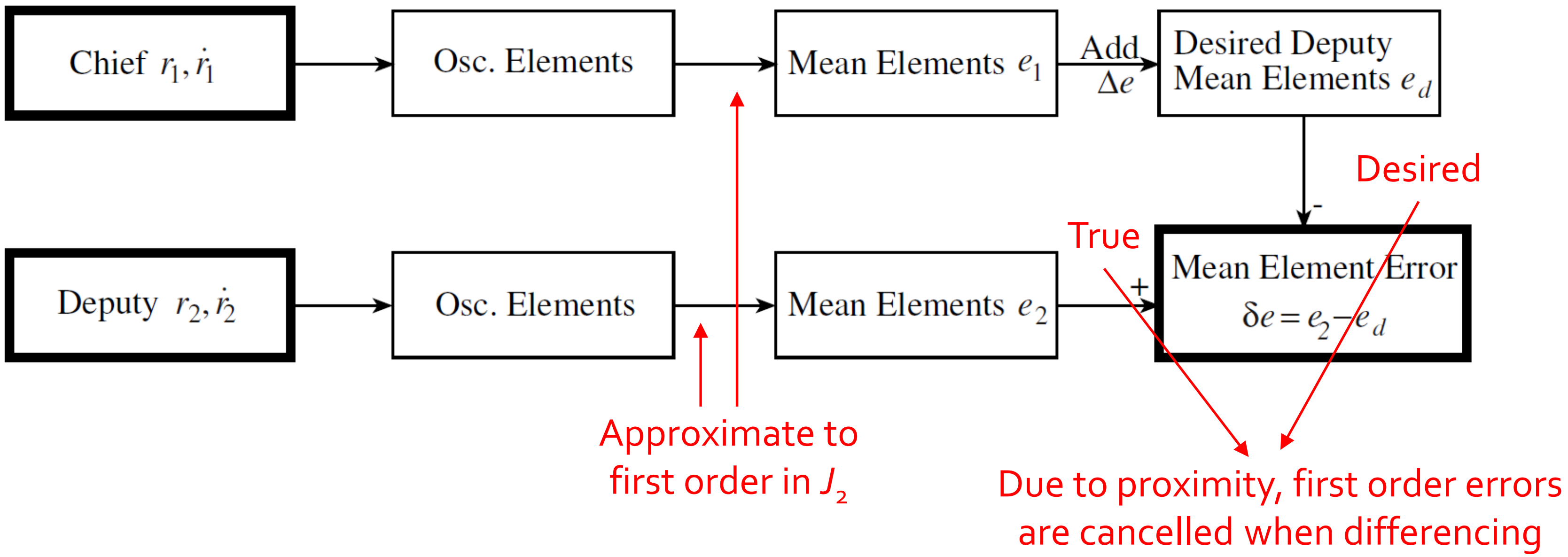
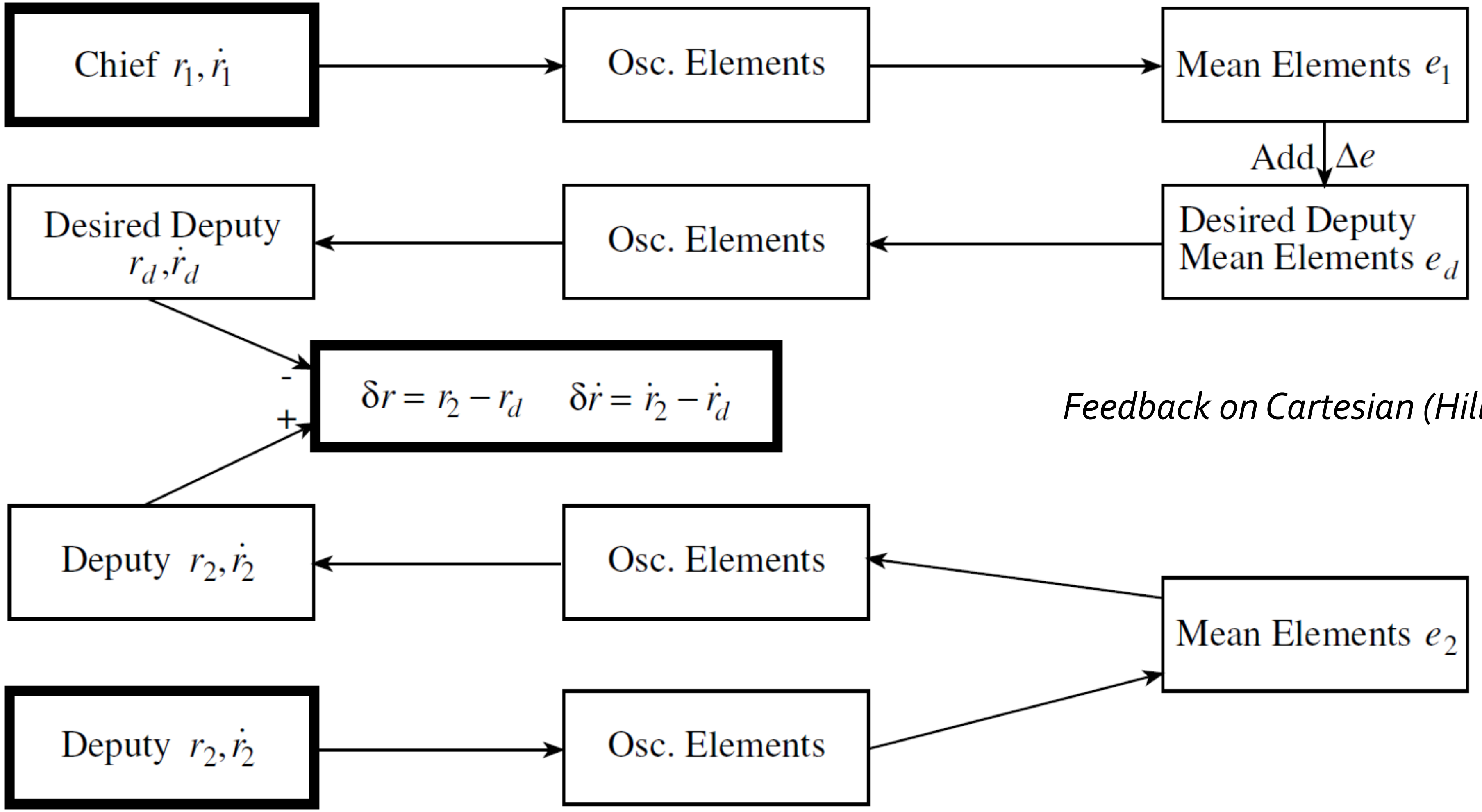
Mean Element Control Illustration (1)

Feedback on mean elements error



Mean Element Control Illustration (2)



Feedback on Cartesian (Hill) error

Numerical Simulations – Setup

- Numerical propagation includes up to 5 order and degree in geopotential
- Formation control is done by feeding back errors in orbit element differences and Hill coordinates, alternatively
- The initial control tracking errors are

$$\left\{\begin{aligned} \delta a &= -100 \text{ m} \\ \delta i &= 0.05 \text{ deg} \\ \delta\Omega &= \text{-}0.01 \text{ deg} \end{aligned}\right.$$

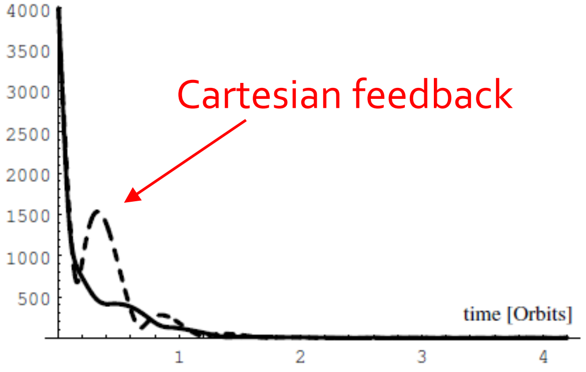
Mean Chief Orbit Elements	Value	Units	Desired Mean Deputy Orbit Element Differences	Value	Units
a	7555	km	Δa	-0.00192995	km
e	0.05		Δe	0.000576727	
i	48	deg	Δi	0.006	deg
Ω	0.0	deg	$\Delta\Omega$	0.0	deg
ω	10.0	deg	$\Delta\omega$	0.0	deg
M	120.0	deg	ΔM	0.0	deg

Numerical Simulations – Results

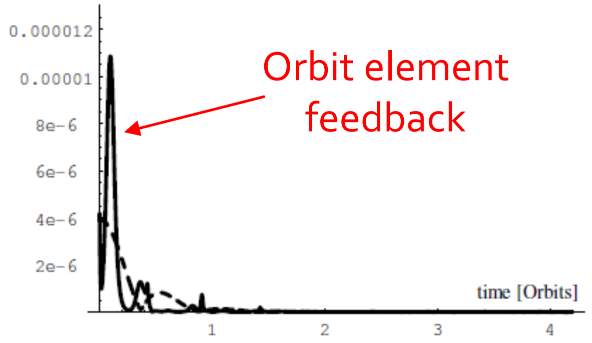
$[P]$

Parameter	Value	Parameter	Value
P_{a0}	0.024	P_{a1}	0.024
P_{e0}	.020	P_{e1}	0.020
P_{i0}	0.00004	P_{i1}	0.005
$P_{\Omega 0}$	0.00004	$P_{\Omega 1}$	0.005
$P_{\omega 0}$	0.0002	$P_{\omega 1}$	0.040
P_{M0}	0.000001	P_{M1}	0.010

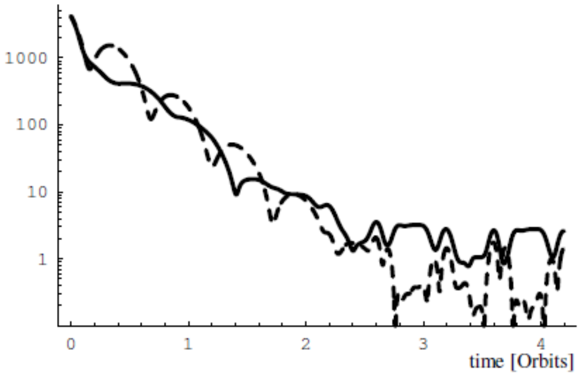
- Pulse-like control
- Total delta-v 7.5 m/s
- Convergence: order of orbits
- Accuracy: meter level
- Residual errors: J2
- Control laws for comparison achieves similar performance



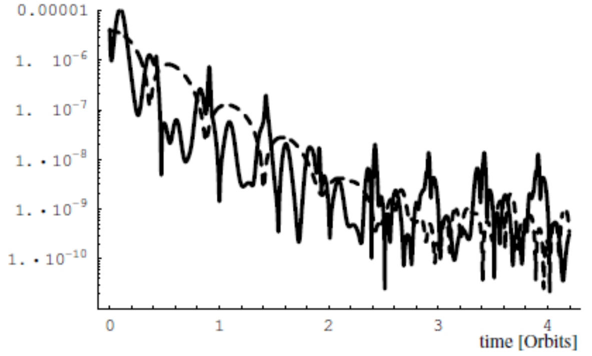
(a) Tracking Error Magnitude $|\delta r|$ (m)



(b) Control Magnitude (km/s^2)



(c) Tracking Error Magnitude $|\delta r|$ (m)



(d) Control Magnitude (km/s^2)

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Overcoming Limitations of Lyapunov Controller

- Lyapunov control theory provides only a guarantee of asymptotic stability, however a reference governor can be designed to prescribe an efficient control policy and enforce constraints on state and control input

Applied by reference governor (guidance)

$$\boldsymbol{u} = -\boldsymbol{B}^{\star}\left[\boldsymbol{A}\delta\boldsymbol{\alpha} + \boldsymbol{P}\Delta\delta\boldsymbol{\alpha}\right] \qquad \Delta\delta\boldsymbol{\alpha} = \delta\boldsymbol{\alpha} - \delta\boldsymbol{\alpha}_a$$

  - Central equation of reference governor

Minimum threshold

w.r.t. reference

Conservative force field and potential (attractive or repulsive)

$$\delta\dot{\boldsymbol{\alpha}}_a = \xi\left[\Gamma - V\right]\cdot\boldsymbol{\rho} \qquad V = 0.5\Delta\delta\boldsymbol{\alpha}_a^{\mathrm{T}}\Delta\delta\boldsymbol{\alpha}_a \qquad \boldsymbol{\rho} = \boldsymbol{\nabla}\phi$$

Arbitrary scalars >0

>1

  - Potential field map

$$\boldsymbol{\rho} = -\boldsymbol{\nabla}\bar{\phi} - \sum_{i=1}^{M}\boldsymbol{\nabla}\phi_i$$

Influence distance

$$\bar{\phi} = \begin{cases} \|\delta\boldsymbol{\alpha}_a - \delta\boldsymbol{\alpha}_r\|, & \text{if } \|\delta\boldsymbol{\alpha}_a - \delta\boldsymbol{\alpha}_r\| \geq \eta \\ \frac{1}{2}\frac{\|\delta\boldsymbol{\alpha}_a - \delta\boldsymbol{\alpha}_r\|^2}{\eta} + \frac{1}{2}\eta, & \text{otherwise} \end{cases} \qquad \phi_i = \begin{cases} \frac{-\Upsilon_i^2(\zeta_i - C_i)^2}{(\zeta_i^2 - \Upsilon_i^2)C_i}, & \text{if } C_i \leq \zeta_i \\ 0, & \text{otherwise} \end{cases}$$

$$\boldsymbol{\nabla}\bar{\phi} = \frac{\delta\boldsymbol{\alpha}_a - \delta\boldsymbol{\alpha}_r}{\max\{\|\delta\boldsymbol{\alpha}_a - \delta\boldsymbol{\alpha}_r\|, \eta\}}$$

Current distance to the boundary and Lyapunov threshold

$$\Gamma_i = \frac{1}{2}C_i^2 = \frac{1}{2}(\boldsymbol{c}_i^{\mathrm{T}}\delta\boldsymbol{\alpha}_a - d_i)^2$$

$$C_i = |\boldsymbol{c}_i^{\mathrm{T}}\delta\boldsymbol{\alpha}_a - d_i|$$

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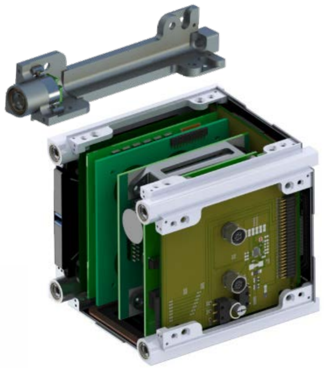
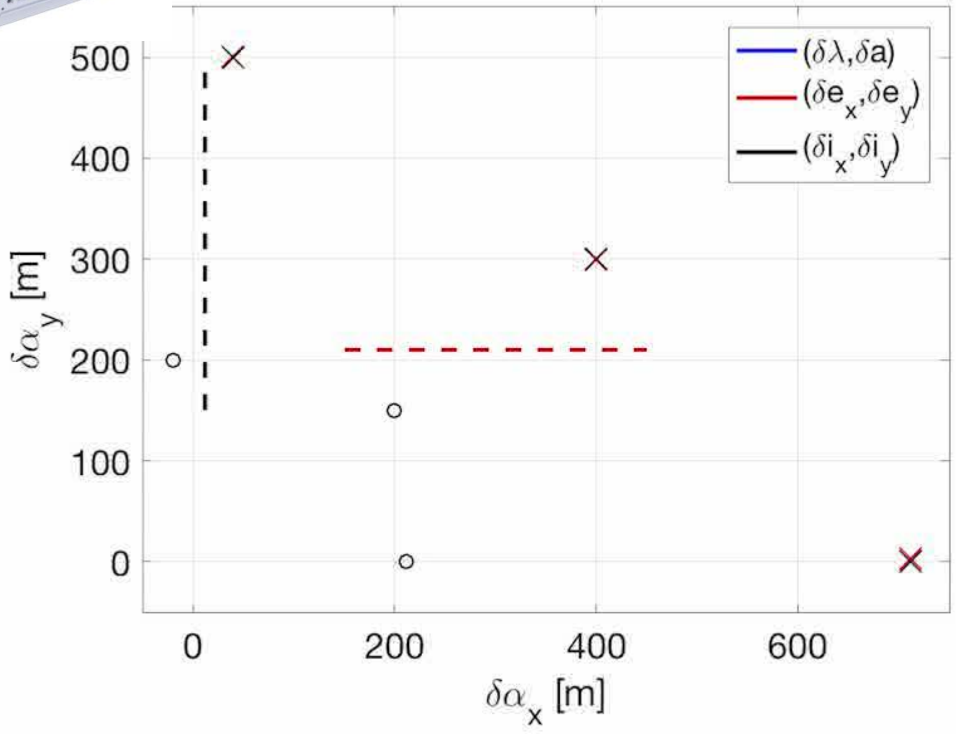
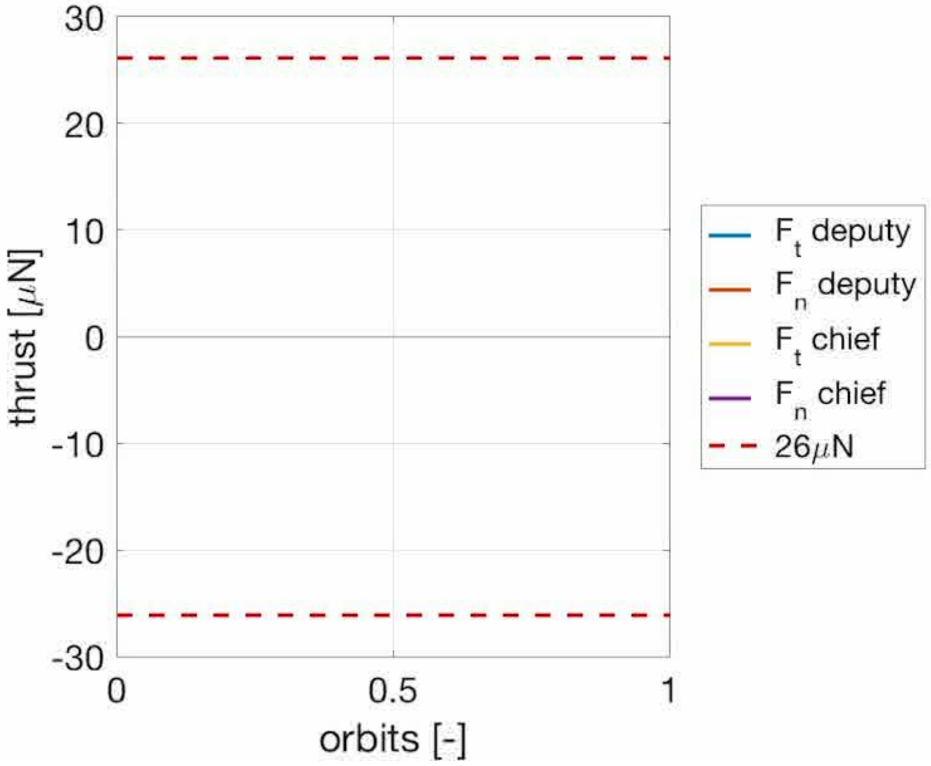
Near-Optimal Reconfiguration using Lyapunov Controller with arbitrary constraints







AA 279 D – SPACECRAFT FORMATION-FLYING AND RENDEZVOUS: LECTURE 9

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