AA 279 D – SPACECRAFT FORMATION-FLYING AND RENDEZVOUS: LECTURE 8

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D'Amico

Maneuver Planning in Near-Circular Orbit

$$\frac{da}{dt} = \frac{2a^2 v}{\mu} a_v$$

$$\frac{de}{dt} = \frac{1}{v}\left(\frac{r}{a}\sin f\, a_n + 2(e + \cos f) a_v\right)$$

$$\frac{di}{dt} = \frac{r\cos\theta}{h} a_h$$

$$\frac{d\Omega}{dt} = \frac{r\sin\theta}{h\sin i} a_h$$

$$\frac{d\omega}{dt} = \frac{1}{ev}\left(-\left(2e + \frac{r}{a}\right)\cos f\, a_n + 2\sin f\, a_v\right) - \frac{r\sin\theta\cos i}{h\sin i} a_h$$

$$\frac{dM}{dt} = n + \frac{b}{aev}\left(\frac{r}{a}\cos f\, a_n - 2\left(1 + e^2\frac{r}{p}\right)\sin f\, a_v\right)$$

GVE

- A relative orbit control system is necessary either to maintain the nominal formation geometry over the mission lifetime (i.e., formation keeping) or to acquire new formation geometries (i.e., formation reconfiguration)
- The inversion of the solution of the HCW equations expressed in terms of ROE provides the ideal framework to design closed-form deterministic impulsive maneuvering schemes

Current orbit location $u \geq u_M$

Maneuver location

Desired variation of ROE

$$\begin{aligned}
a\delta a &\approx && +2\delta v_t/n \\
a\delta\lambda &\approx -2\delta v_r/n && -3(u - u_M)\delta v_t/n \\
a\delta e_x &\approx +\delta v_r \sin u_M/n && +2\delta v_t \cos u_M/n \\
a\delta e_y &\approx -\delta v_r \cos u_M/n && +2\delta v_t \sin u_M/n \\
a\delta i_x &\approx && && +\delta v_n \cos u_M/n \\
a\delta i_y &\approx && && +\delta v_n \sin u_M/n
\end{aligned} \qquad (2.38)$$

Instantaneous variation of velocity

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Maneuver Planning: Out-Of-Plane

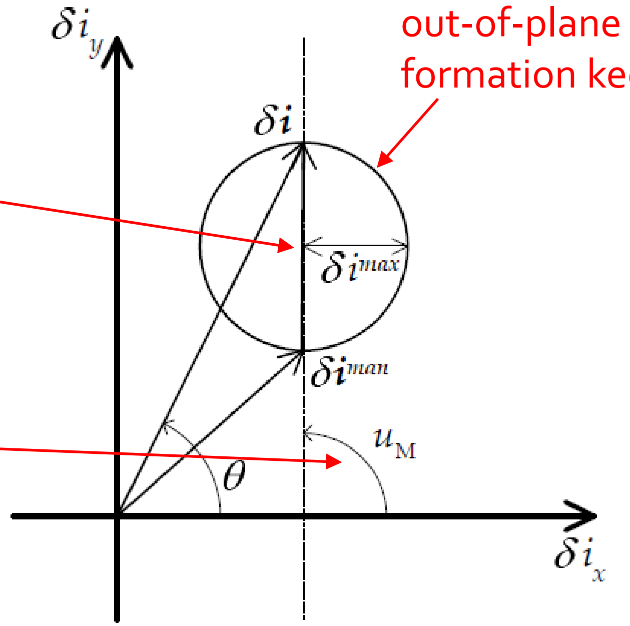
- The problem consists of 2 unknowns δv_n, u_M and 2 equations and can be solved through a single- or double-impulse

Magnitude of desired correction vector

$$
\begin{aligned}
\delta v_n &= na \left\| \delta \boldsymbol{i}^{\text{man}} - \delta \boldsymbol{i} \right\| \\
u_{\text{M}} &= \arctan \left[(\delta i_y^{\text{man}} - \delta i_y) / (\delta i_x^{\text{man}} - \delta i_x) \right]
\end{aligned} \tag{2.64}
$$

Phase of desired correction vector

Control window for out-of-plane formation keeping



Maneuver Planning: In-Plane (1)

- The problem consists of 3 unknowns δv_r, δv_t, u_M and 4 equations (over-determined) and can be solved exactly only through a double-impulse scheme which doubles the number of unknowns (under-determined)

$$
\begin{aligned}
\delta v_{t_1} &= \frac{na}{4}\left[\delta a + \delta e\cos(u_{M_1}-\xi)\right] & -\frac{na}{4}\chi\left[\frac{\delta\lambda}{2} + \delta e\sin(u_{M_1}-\xi)\right] \\
\delta v_{r1} &= \frac{na}{2}\left[-\frac{\delta\lambda}{2} + \delta e\sin(u_{M_1}-\xi)\right] & -\frac{na}{2}\chi\left[\delta a - \delta e\cos(u_{M_1}-\xi)\right] \\
\delta v_{t_2} &= \frac{na}{4}\left[\delta a - \delta e\cos(u_{M_1}-\xi)\right] & +\frac{na}{4}\chi\left[\frac{\delta\lambda}{2} + \delta e\sin(u_{M_1}-\xi)\right] \\
\delta v_{r2} &= \frac{na}{2}\left[-\frac{\delta\lambda}{2} - \delta e\sin(u_{M_1}-\xi)\right] & +\frac{na}{2}\chi\left[\delta a - \delta e\cos(u_{M_1}-\xi)\right]
\end{aligned}
\qquad (2.42)
$$

First maneuver location

Second maneuver location

$$\chi = \frac{\sin(\Delta u_M)}{\cos(\Delta u_M) - 1}$$

$$\xi = \arctan(\delta e_y / \delta e_x)$$

$$u_{M_2} - u_{M_1} \in]0, 2\pi[$$

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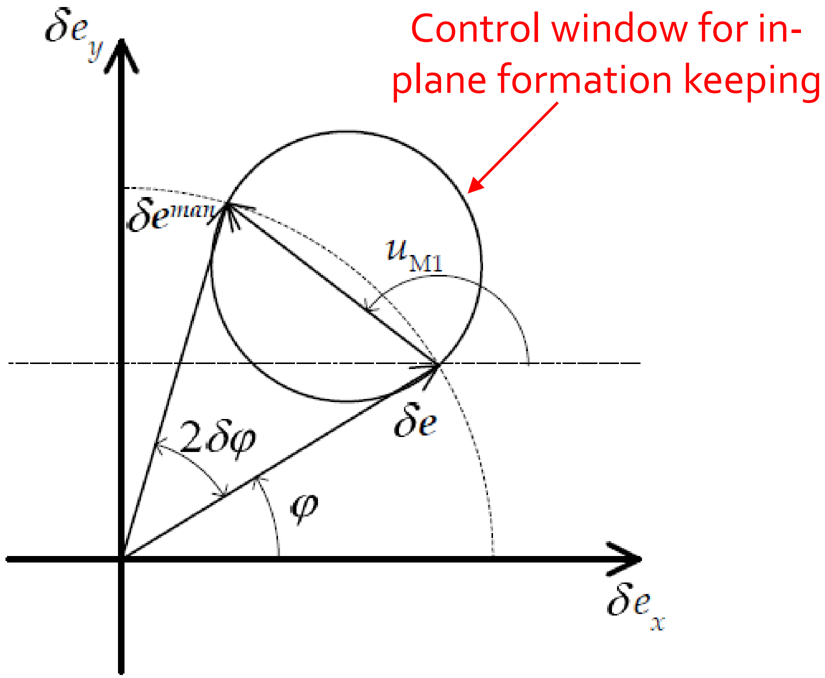
Maneuver Planning: In-Plane (2)

- The most simple double-impulse scheme with $u_{\mathrm{M1}} = \xi$ and $u_{\mathrm{M2}} = u_{\mathrm{M1}} + \pi$ turns out to be the minimum cost (total delta-v) solution for formation keeping (evaluating Jacobian and bordered Hessian)

$$
\begin{aligned}
\delta v_{t_1} &= \frac{na}{4}\left[(\delta a^{\mathrm{man}} - \delta a) + ||\delta \boldsymbol{e}^{\mathrm{man}} - \delta \boldsymbol{e}||\right] \\
\delta v_{t_2} &= \frac{na}{4}\left[(\delta a^{\mathrm{man}} - \delta a) - ||\delta \boldsymbol{e}^{\mathrm{man}} - \delta \boldsymbol{e}||\right] \\
u_{\mathrm{M}_1} &= \arctan\left[(\delta e_y^{\mathrm{man}} - \delta e_y)/(\delta e_x^{\mathrm{man}} - \delta e_x)\right]
\end{aligned}
$$

Note: $\delta\lambda$ is controlled through δa^{man} achieved after maneuver pair



Eccentric Orbits and Delta-V Lower Bound

- Delta-v lower bound in eccentric orbits for tangential maneuvers:

$$\delta v_{\mathrm{LB}}/na\eta = \max\left(\frac{|\Delta\delta\bar{a}|}{2(1+e)}, \frac{|\Delta\delta\bar{\lambda}_e|}{3(1+e)\Delta M}, \frac{\|\Delta\delta\bar{e}\|}{\sqrt{3e^4-7e^2+4}}\right)$$

$$\delta\lambda_e = M_d - M_c + \eta[\omega_d - \omega_c + (\Omega_d - \Omega_c)\cos(i_c)]$$

- Delta-v lower bound in circular orbits for tangential maneuvers:

$$\delta v_{\mathrm{LB}}/na = \max\left(\frac{|\Delta\delta\bar{a}|}{2}, \frac{|\Delta\delta\bar{\lambda}_e|}{3\Delta M}, \frac{\|\Delta\delta\bar{e}\|}{2}\right)$$

- The task of minimizing fuel for formation control is equivalent to minimizing the length of the path taken from initial to final conditions in ROE space

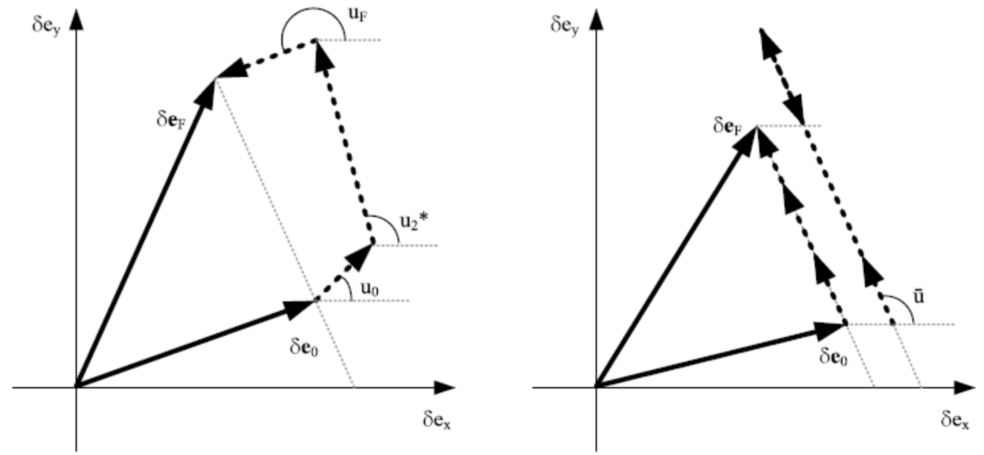


Fig. 3 Sketches in the $\boldsymbol{\delta e}$ plane for the three tangential maneuvers reconfiguration.

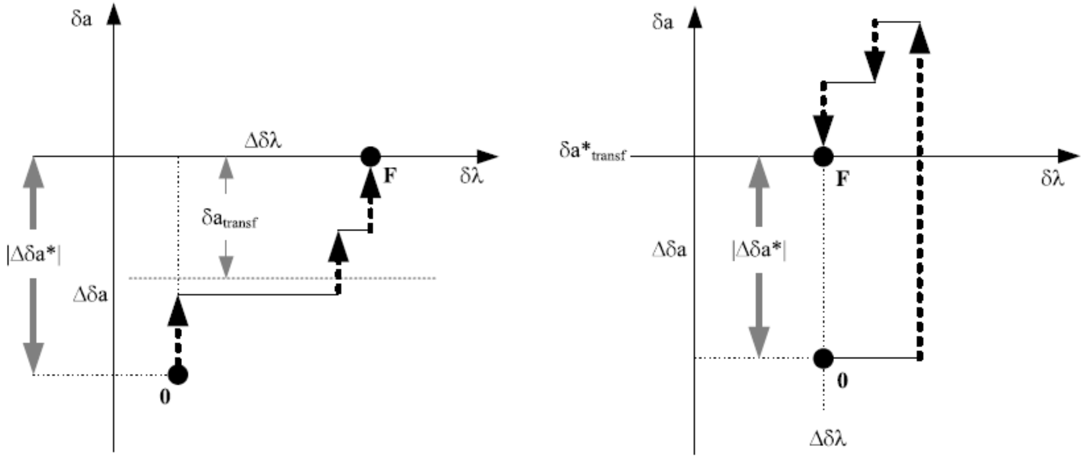
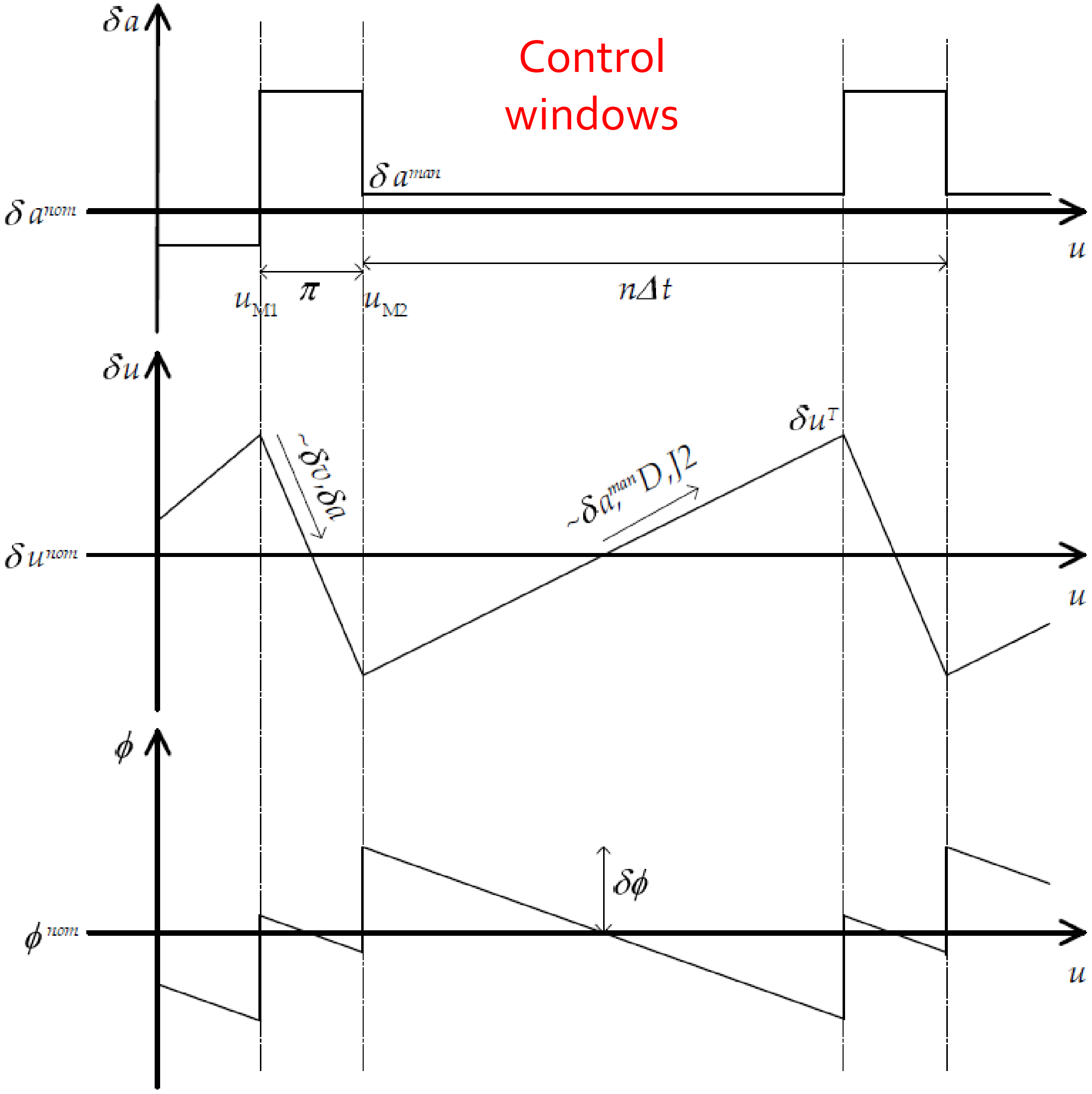


Fig. 4 Sketches in the $\boldsymbol{\delta\lambda/\delta a}$ plane for the three tangential maneuvers reconfiguration.

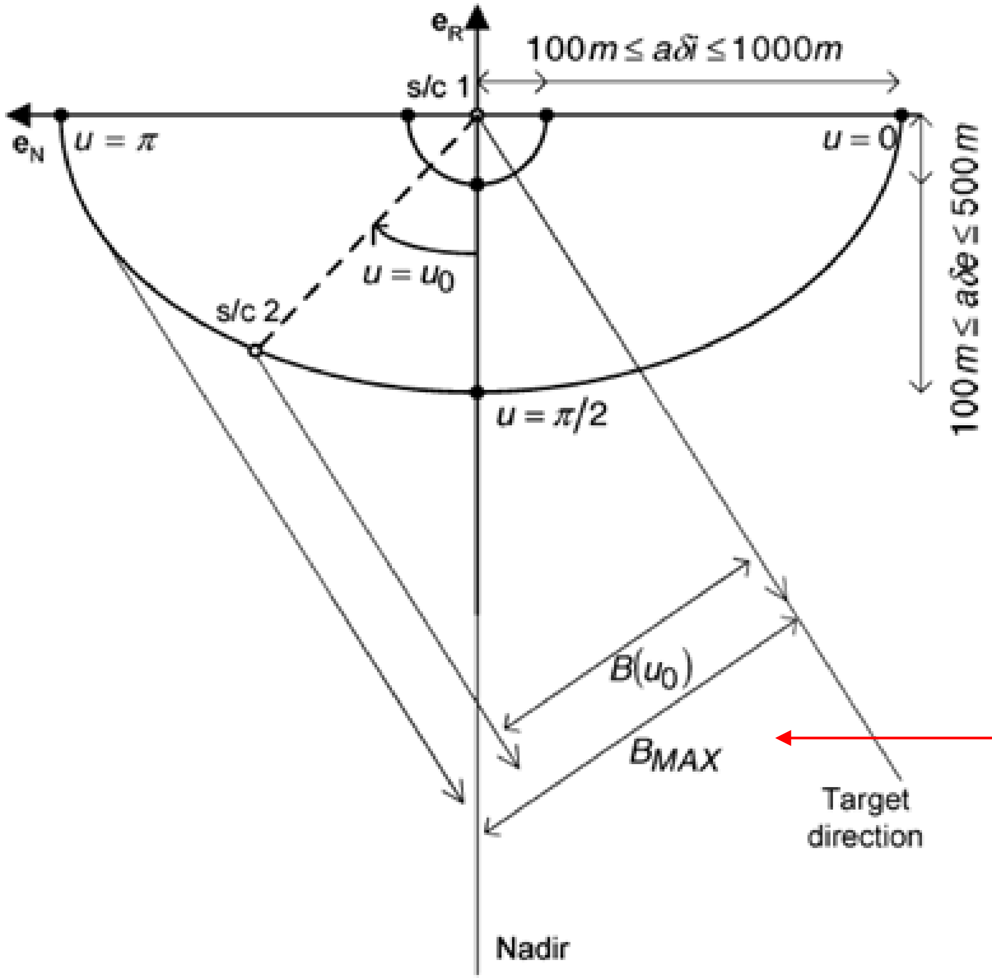
Maneuver Planning: In-Plane (3)



Goal

$$\delta a^{\mathrm{man}} = -\frac{2}{3}\frac{\delta u^{\mathrm{tot}}}{n\Delta t - \pi}$$

Practical Implementation (TanDEM-X concept)



Digital elevation modeling needs a so-called interferometric baseline

Practical Implementation (TanDEM-X plan)

Configuration	$a\delta e$ [m]	$a\delta i$ [m]	$\alpha = \theta - \varphi$ [°]	$a\Delta\lambda$ [km]	T [days]	$\Delta r_{\perp}^{MIN}$ [m]
Injection - A	-	-	-	3000…20	20	-
A	300	1000	0	20	70	300
B	300	300	-15…+30	0	165	≥ 212
C	300	400	-15…+30	0	132	≥ 236
D	300	500	-15…+30	0	121	≥ 246
E	500	300	-15…+30	0	44	≥ 246
E - F	-	-	-	0…20…0	11	-
F	300	300	150…195	0	165	≥ 212
G	300	400	150…195	0	132	≥ 236
H	300	500	150…195	0	121	≥ 246
I	500	300	150…195	0	44	≥ 246
L	500	500	180	0	11	500
M	500	1000	180	0	11	500
N	500	2000	180	0	11	500
O	500	4000	180	0	11	500
P	500	8000	180	0	11	500
Q	300	8000	180	130	11	300
Q - End	0	8000	180	130…7850	90	-

Injection and acquisition

Preliminary DEM

- small to large baseline
- large to small height of ambiguity
- mountainous to flat surfaces
- relative perigee: northern latitudes

Swap of relative e-vector

- through large along-track sep
- relative perigee southern latitudes

Final DEM

- larger baseline through RAAN
- ground-track repeatability
- along-track interferometry

Maneuver Budget – Relative e/i-vectors

Example for arithmetic evaluation

Orbit elements	Value	Relative orbit elements	Value
a [m]	7078135.0	$a\delta a^{\mathrm{nom}}$ [m]	0
u [°]	0.0	$a\delta\lambda^{\mathrm{nom}}$ [m]	0
e_x [-]	0.001	$a\delta e_x^{\mathrm{nom}}$ [m]	86.8241
e_y [-]	0.0	$a\delta e_y^{\mathrm{nom}}$ [m]	492.4039
i [°]	98.19	$a\delta i_x^{\mathrm{nom}}$ [m]	192.8363
Ω [°]	189.89086	$a\delta i_y^{\mathrm{nom}}$ [m]	229.8133

$a\delta u^{\mathrm{nom}} = 33.1\,\mathrm{m}$

$a\delta e^{\mathrm{nom}} = 500\,\mathrm{m}$
$\varphi^{\mathrm{nom}} = 80°$

$a\delta i^{\mathrm{nom}} = 300\,\mathrm{m}$
$\vartheta^{\mathrm{nom}} = 50°$

- Maneuver cycle Δt (N revolutions) dictated by control architecture and constraints from mission operations and propulsion system (orbits to days)
- Maximum allowed deviation of relative inclination vector and delta-v

$$a\delta i^{\mathrm{max}} = \left|\frac{3}{2} n\gamma a\delta i_x^{\mathrm{nom}} \Delta t \sin^2 i\right| \approx (0.7826 \cdot N)\,\mathrm{m} \quad \rightarrow \quad \delta v_n = 2na\delta i^{\mathrm{max}} \approx (1.7 \cdot N)\,\mathrm{mm/s}$$

- Maximum allowed deviation of relative eccentricity vector and delta-v

$$a\delta e^{\mathrm{max}} \approx a\delta e^{\mathrm{nom}} \sin\delta\varphi \approx \left|\frac{3}{4} n\gamma a\delta e^{\mathrm{nom}} \Delta t (5\cos^2 i - 1)\right| \approx (0.9306 \cdot N)\,\mathrm{m} \quad \rightarrow$$

$$\rightarrow \quad \delta v_{t_1} = -\delta v_{t_2} = na\delta e^{\mathrm{max}}/2 \approx (0.49 \cdot N)\,\mathrm{mm/s}$$

(2.72) - (2.76)

Maneuver Budget – Mean along-track sep

- Maneuver cycle Δt (N revolutions) dictated by control architecture and constraints from mission operations and propulsion system (orbits to days)
- Maximum allowed deviation of relative mean argument of latitude driven by baseline

$$a\delta u^{\max} = a\delta u_{\mathrm{T}} - a\delta u^{\mathrm{nom}} \approx \frac{3\pi}{4} a\delta e^{\max} = (2.1926 \cdot N)\,\mathrm{m}$$

- Required differential corrections of tangential maneuver sizes and accumulated along-track offsets caused by perturbations $\delta B/B$ = 136% (!)

Example for arithmetic evaluation

$a\delta u^{\max}$ [m]	N [rev]	$\delta v_{t_1} + \delta v_{t_2}$ [mm/s]	$a\delta u_{\mathrm{J}_2}$ [m]	$a\delta u_{\mathrm{D}}$ [m]
2.1926	1	-0.0209	1.8022	2.3037
4.3852	2	0.1301	3.6045	9.2148
6.5778	3	0.2656	5.4067	20.7332
8.7703	4	0.3979	7.2089	36.8591
10.9629	5	0.5290	9.0111	57.5923
13.1555	6	0.6596	10.8134	82.9329

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Basic Numerical Simulation – Setup

Nominal elements	Config. 1	Config. 2	Config. 3
$a\delta a^{\text{nom}}$ [m]	0	0	0
$a\delta\lambda^{\text{nom}}$ [m]	0	100	200
$a\delta e_x^{\text{nom}}$ [m]	86.8241	0	-52.0944
$a\delta e_y^{\text{nom}}$ [m]	492.4039	400	295.4423
φ^{nom} [°]	80	90	100
$a\delta i_x^{\text{nom}}$ [m]	192.8363	0	0
$a\delta i_y^{\text{nom}}$ [m]	229.8133	200	600
ϑ^{nom} [°]	50	90	90

First config. same as for delta-v budget analysis

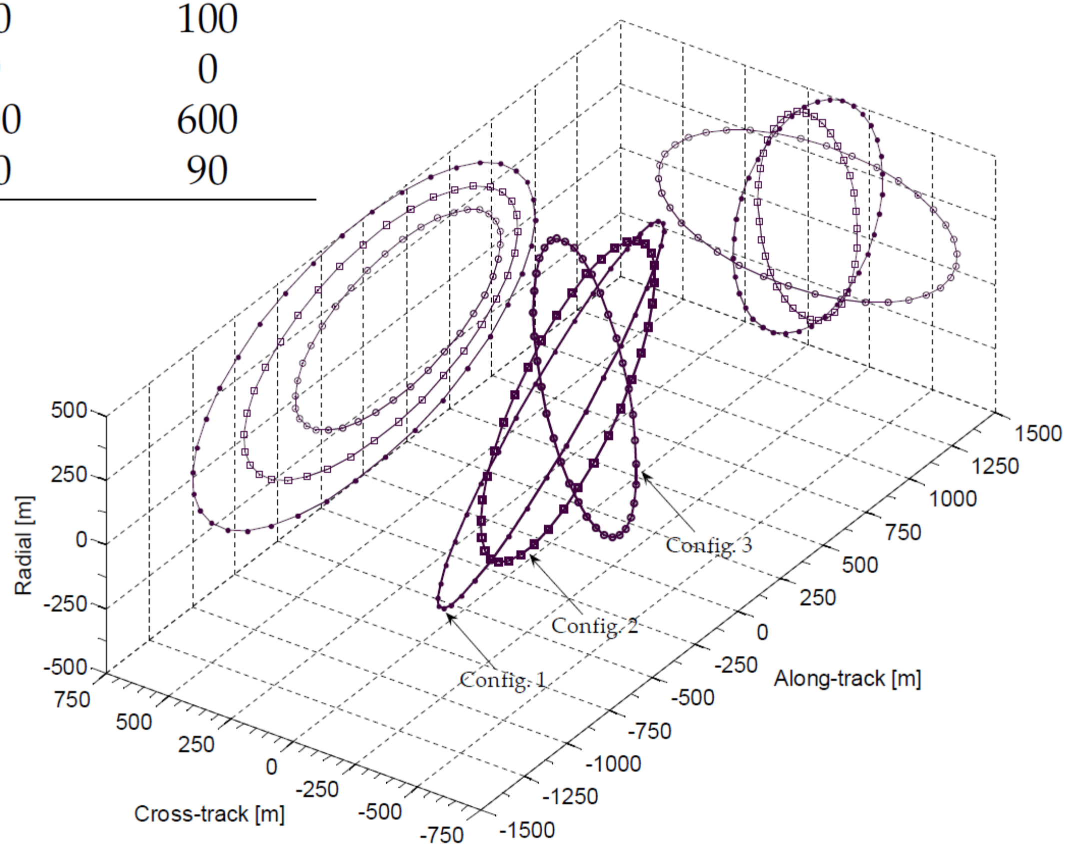
Only J_2 perturbations considered

Control windows

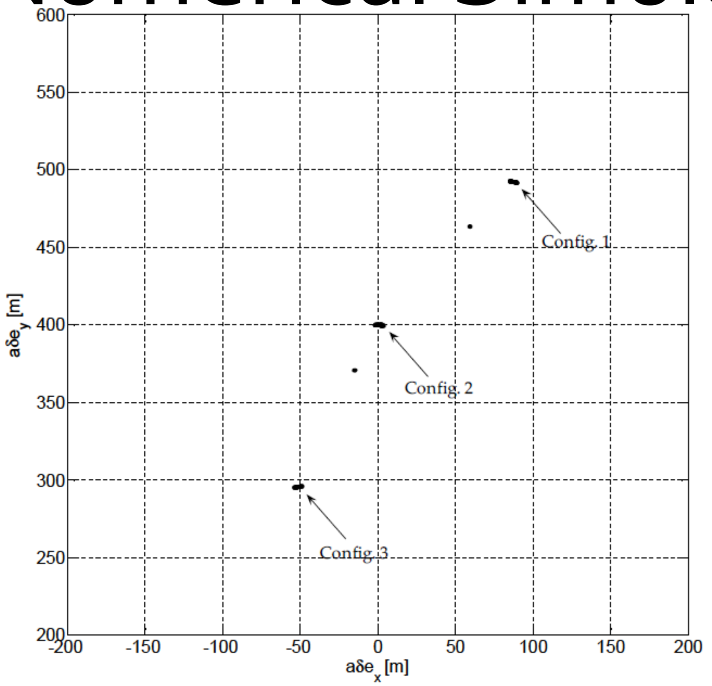
$$a\delta e^{\max} = a\delta i^{\max} = 2\,\text{m}$$
$$a\delta u^{\max} \approx 5\,\text{m}$$

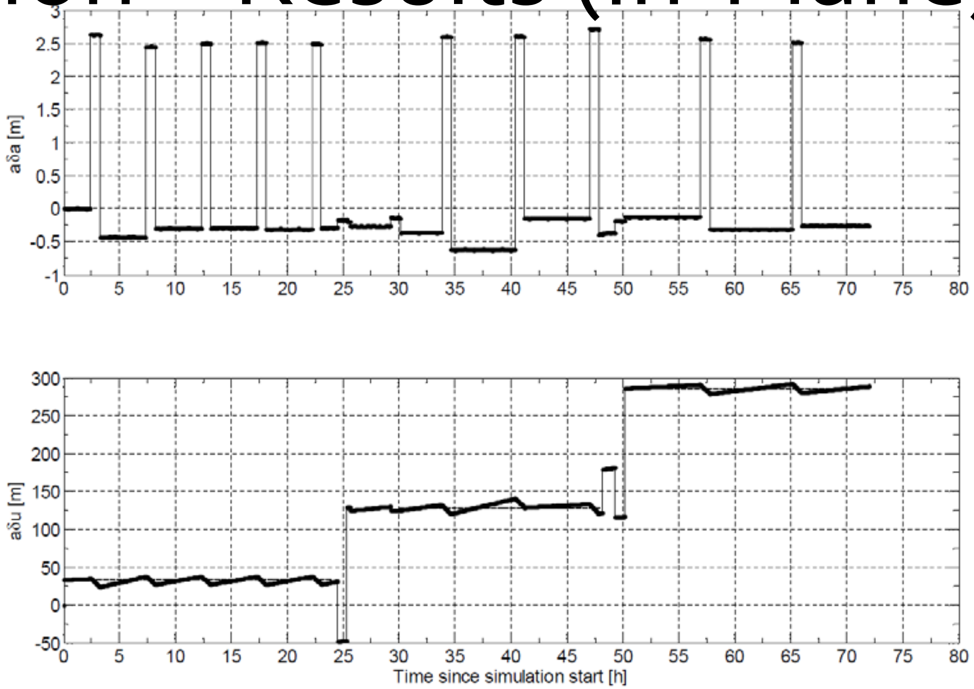
Formation **keeping**: 1 day/config., pairs of along-track delta-vs

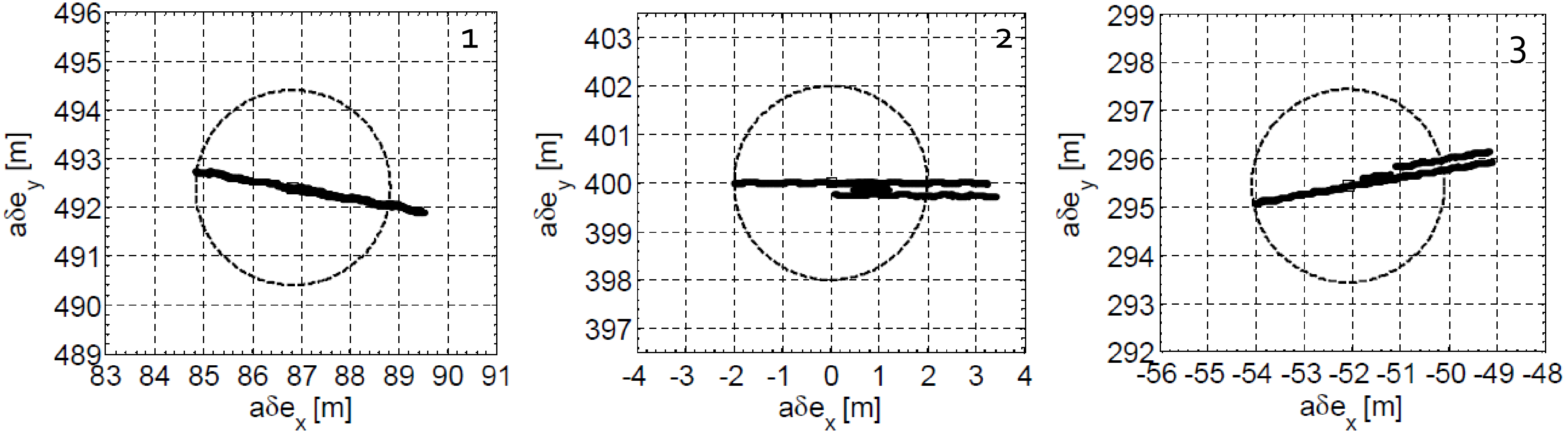
Formation **reconfiguration**: pairs of radial/along-track delta-vs



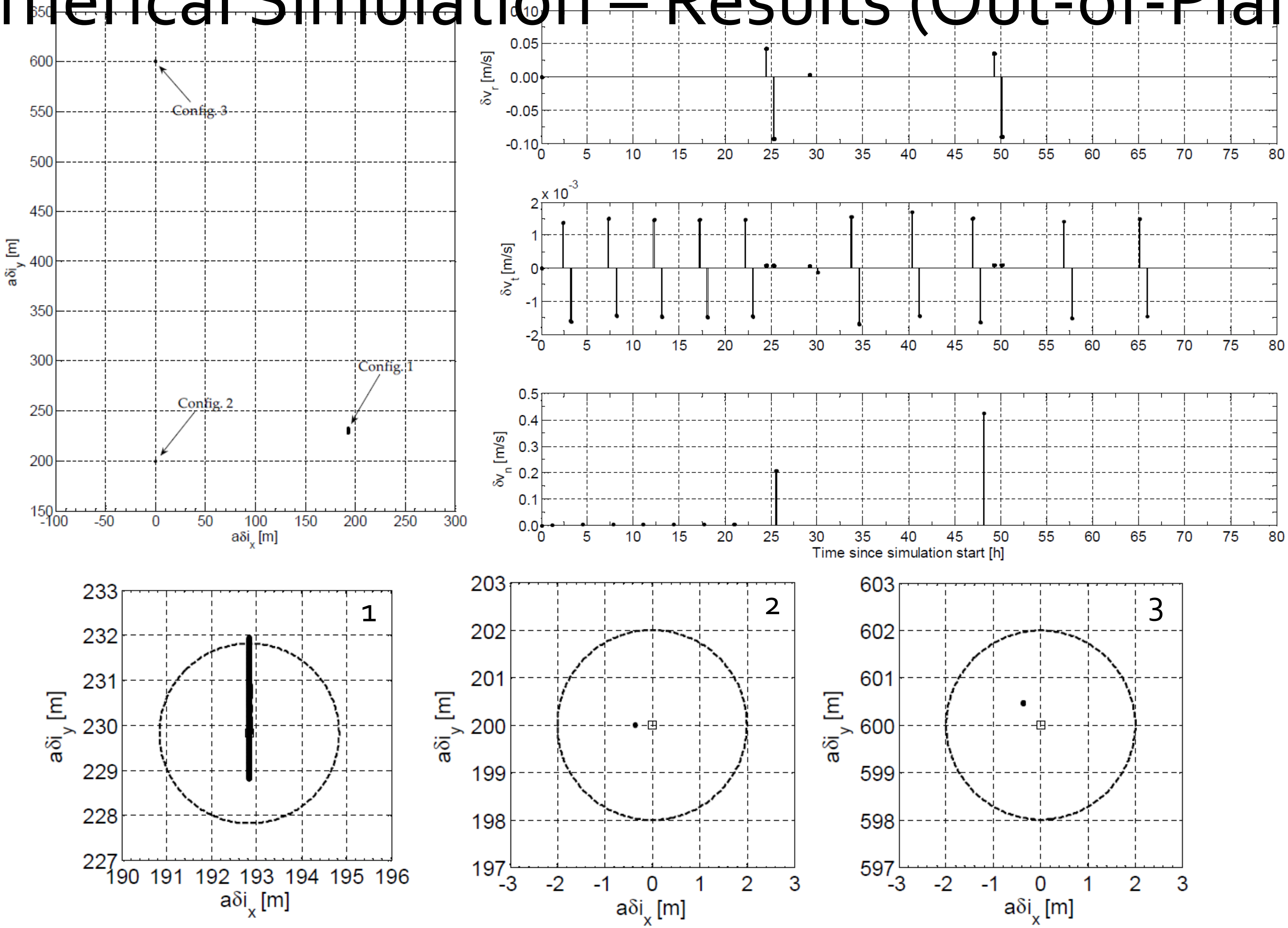
Numerical Simulation – Results (In-Plane)







Numerical Simulation – Results (Out-of-Plane)



Operational Considerations

- Propulsion constraints
  - Minimum impulse bit of propulsion system drives accuracy
  - Control window or maneuver cycle shall be selected accordingly
  - Analytical relationships are straightforward from delta-v budget analysis
- Selection of maneuvering scheme
  - Minimum delta-v demands along-track maneuvers
  - Minimum number of pulses drives selection of pairs or triplets
  - Radial maneuvers might be necessary for precision (no change of semi-major axis) or to cope with minimum impulse bit (larger delta-v)
- Control architecture
  - **Autonomy** decreases operational costs and increases accuracy, but requires on-board software development and careful verification
  - **Ground-in-the-loop** is subject to delays due to ground-contact limitations, but can be automated to reduce manpower
- Closed-form solutions are compatible with control accuracies at the meter level and do not demand ultimate precision in on-board navigation

Alfriend

Impulsive Formation Control – Eccentric Orbit (1)

- The energy matching condition (4.37) is a necessary and sufficient condition for 1:1 commensurable relative motion
- In practice, due to initialization errors, this constraint cannot be satisfied exactly
- The violation of this constraint results in a "drift" in the relative position which must be satisfied through control, typically through impulsive maneuvers
- We can develop an impulsive formation-keeping maneuver strategy aimed at correcting the relative position drift with minimum fuel consumption
- Methodology valid for orbits of arbitrary eccentricity, here we proceed in Cartesian space

Initial condition

Initialization error

$$\begin{aligned} \boldsymbol{\rho}^{\delta}(0) &= \boldsymbol{\rho}(0) + \delta\boldsymbol{\rho}(0) \\ \mathbf{v}^{\delta}(0) &= \mathbf{v}(0) + \delta\mathbf{v}(0) \end{aligned} \tag{4.39}$$

Energy matching condition

Impulsive maneuver by deputy

$$[\Delta \mathbf{v}]_{\mathscr{L}} = [\Delta v_x, \Delta v_y, \Delta v_z]^T$$

sought to satisfy energy matching at t_i

Chief-fixed RTN

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Impulsive Formation Control – Eccentric Orbit (2)

- Since the impulsive maneuver does not change the position, the energy matching condition to be satisfied by the maneuver is

$$\mathcal{E}_1^+ = \frac{1}{2}\left[(v_x^- + \Delta v_x)^2 + (v_y^- + \Delta v_y)^2 + (v_z^- + \Delta v_z)^2\right] - \frac{\mu}{r_1}$$
$$= -\frac{\mu}{2a_0} \qquad (4.40)$$

Deputy

Chief

3 unknowns/1 equation

- where - indicates quantities prior to the impulsive maneuver at t_i

$$v_x^- = \dot{x}^-(t_i) - \dot{\theta}_0^-(t_i)y(t_i) + \dot{r_0}^-(t_i) \qquad (4.41)$$

$$v_y^- = \dot{y}^-(t_i) + \dot{\theta}_0^-(t_i)[x(t_i) + r_0(t_i)] \qquad (4.42)$$

$$v_z^- = \dot{z}^-(t_i) \qquad (4.43)$$

$$r_1 = \sqrt{[r_0(t_i) + x(t_i)]^2 + y^2(t_i) + z^2(t_i)} \qquad (4.44)$$

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Impulsive Formation Control – Eccentric Orbit (3)

- The extra degrees of freedom can be used to minimize fuel consumption through a static optimization problem (e.g., t_i fixed by operational constraints):

Problem statement

$$\Delta\mathbf{v}^* = \arg\min_{\Delta\mathbf{v}} \|\Delta\mathbf{v}\|^2$$
s. t.
$$\mathcal{E}_1^+ = -\frac{\mu}{2a_0}$$

Lagrangian and Lagrange multiplier

$$\mathcal{L} = \|\Delta\mathbf{v}\|^2 + \lambda\left(\mathcal{E}_1^+ + \frac{\mu}{2a_0}\right)$$

- Evaluating the Jacobian (w.r.t. $\Delta\boldsymbol{v}$) and bordered Hessian of the Lagrangian we found an optimum solution

$$\frac{\Delta v_x^*}{v_x^-} = \frac{\Delta v_y^*}{v_y^-} = \frac{\Delta v_z^*}{v_z^-} = -1 + \frac{1}{v_1^-}\sqrt{\frac{\mu(2a_0 - r_1)}{a_0 r_1}} \quad (4.55)$$

$$\Delta v^* = v_1^- - \sqrt{\frac{\mu(2a_0 - r_1)}{a_0 r_1}} \quad (4.56)$$

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Impulsive Formation Control – Eccentric Orbit (4)

- The delta-v solution was derived in the chief-fixed RTN/LVLH frame
- Using the Gauss Variational Equations (GVE), we can reformulate the solution in the deputy-fixed RTN/LVLH frame
- We integrate GVE (2.107a) over the impulsive maneuver to derive

$$\Delta \mathbf{v}_1 = [\Delta v_r,\, \Delta v_\theta,\, \Delta v_h]^T \quad \Rightarrow \quad \text{Desired} \quad \Delta a_1 = \frac{2a_1^2}{h_1}\left(\Delta v_r e_1 \sin f_1 + \frac{p_1}{r_1}\Delta v_\theta\right) \qquad (4.63)$$

- The optimization problem can be reformulated as

Problem statement

$$\Delta \mathbf{v}_1^* = \arg \min_{\Delta v_r, \Delta v_\theta} \Delta v_r^2 + \Delta v_\theta^2$$

s. t.

$$\Delta a_1 = \frac{2a_1^2}{h_1}\left(\Delta v_r e_1 \sin f_1 + \frac{p_1}{r_1}\Delta v_\theta\right)$$

(4.64)

Lagrangian (now time-dependent)

$$\mathcal{L}_1 = \Delta v_r^2 + \Delta v_\theta^2 + \lambda_1\left[\frac{2a_1^2}{h_1}\left(\Delta v_r e \sin f_1 + \frac{p_1}{r_1}\Delta v_\theta\right) - \Delta a_1\right]$$

(4.66)

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Impulsive Formation Control – Eccentric Orbit (5)

- Evaluating the Jacobian (now w.r.t. $\Delta \boldsymbol{v}_1$ and f_1) and bordered Hessian of the Lagrangian we found an optimum solution

$$\Delta v_r = 0,\ \Delta v_\theta = \frac{h_1 \Delta a_1}{2a_1^2(1+e_1)},\ \lambda_1 = -\frac{h_1^2 \Delta a_1}{2a_1^4(e_1+1)^2},\ f_1 = 0 \qquad (4.71)$$

$$\Delta \mathbf{v}_1^* = [0,\ \Delta v_\theta(f_1 = 0),\ 0]^T \qquad (4.73)$$

- This method tells us that the minimum-fuel maneuver should be performed at periapsis ($f = 0$) and in (anti-)along-track (θ or T) direction
- The magnitude of the delta-v must be equal in the chief-fixed (4.56) and deputy-fixed (4.71) co-rotating frames (rotations do not change magnitude)

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Impulsive Formation Control – Example (1)

Example 4.2. *Consider a chief spacecraft on an elliptic orbit. Normalize positions by a_0 and angular velocities by $\sqrt{\mu/a_0^3}$ so that $a_0 = \mu = 1$. Let the nominal normalized initial conditions be as in Eq. (4.38) with $\bar{x}(0) = -0.01127$. Assume that the initialization errors are*

$$\delta\bar{x}(0) = 0.001,\ \delta\bar{y}(0) = 0.001,\ \delta\bar{z}(0) = 0.01$$
$$\delta\bar{x}'(0) = 0,\ \delta\bar{y}'(0) = 0,\ \delta\bar{z}'(0) = 0 \tag{4.57}$$

Compute the minimum-fuel maneuver required to obtain a 1 : 1 bounded relative motion assuming that the maneuver is to be applied after one orbital period of the chief.

- (4.38)+(4.57) gives new normalized initial conditions, and integration of (4.14-4.16) gives relative motion after 1 period (t_i = 1)

$$\begin{gathered}\bar{x} = -0.015374,\ \bar{y} = -0.084596,\ \bar{z} = 0.109547 \\ (\bar{x}')^- = 0.00994,\ (\bar{y}')^- = 0.021792,\ (\bar{z}')^- = 0.011765 \\ (\theta_0')^- = 1.22838,\ \bar{r}_0 = 0.9,\ (\bar{r}_0')^- = 0\end{gathered} \tag{4.58}$$

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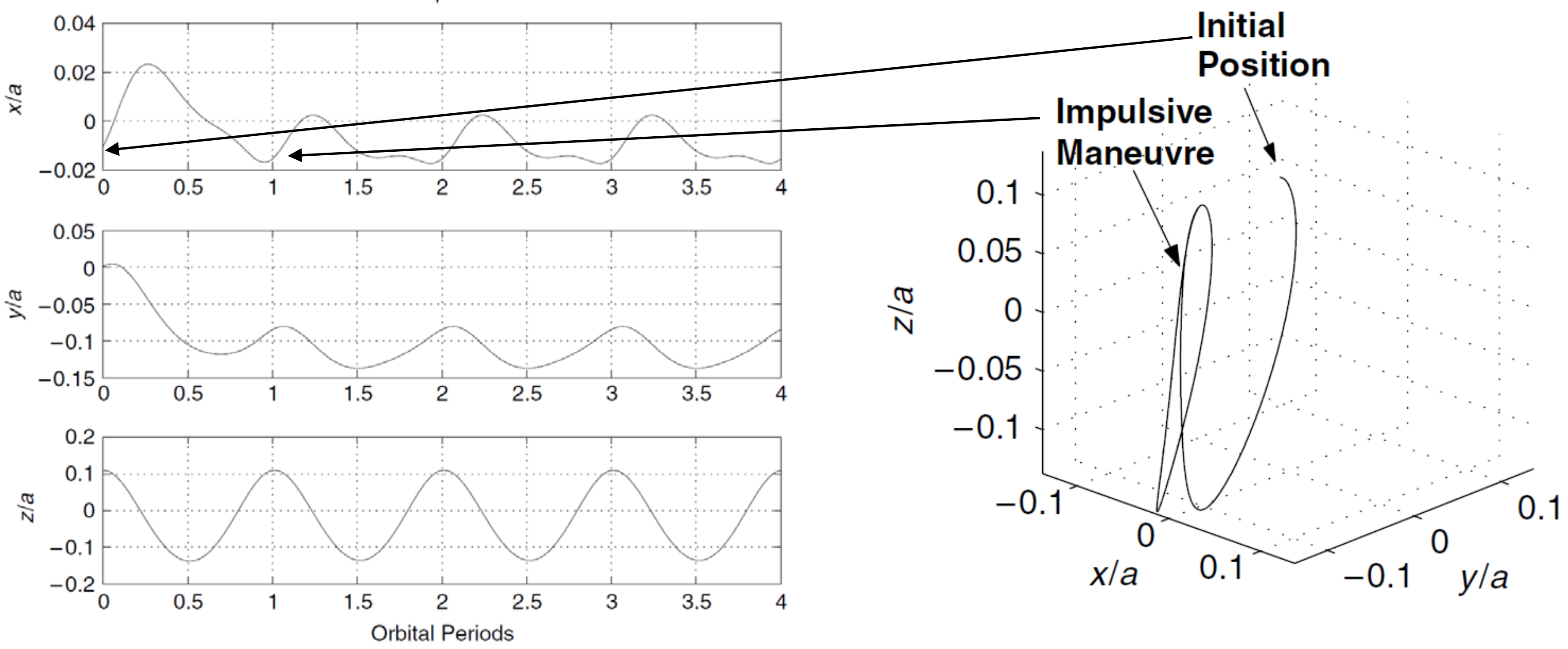
Impulsive Formation Control – Example (2)

- Using (4.41-4.44) and substituting in (4.55) yields the optimal formation-keeping maneuver components (in normalized units)

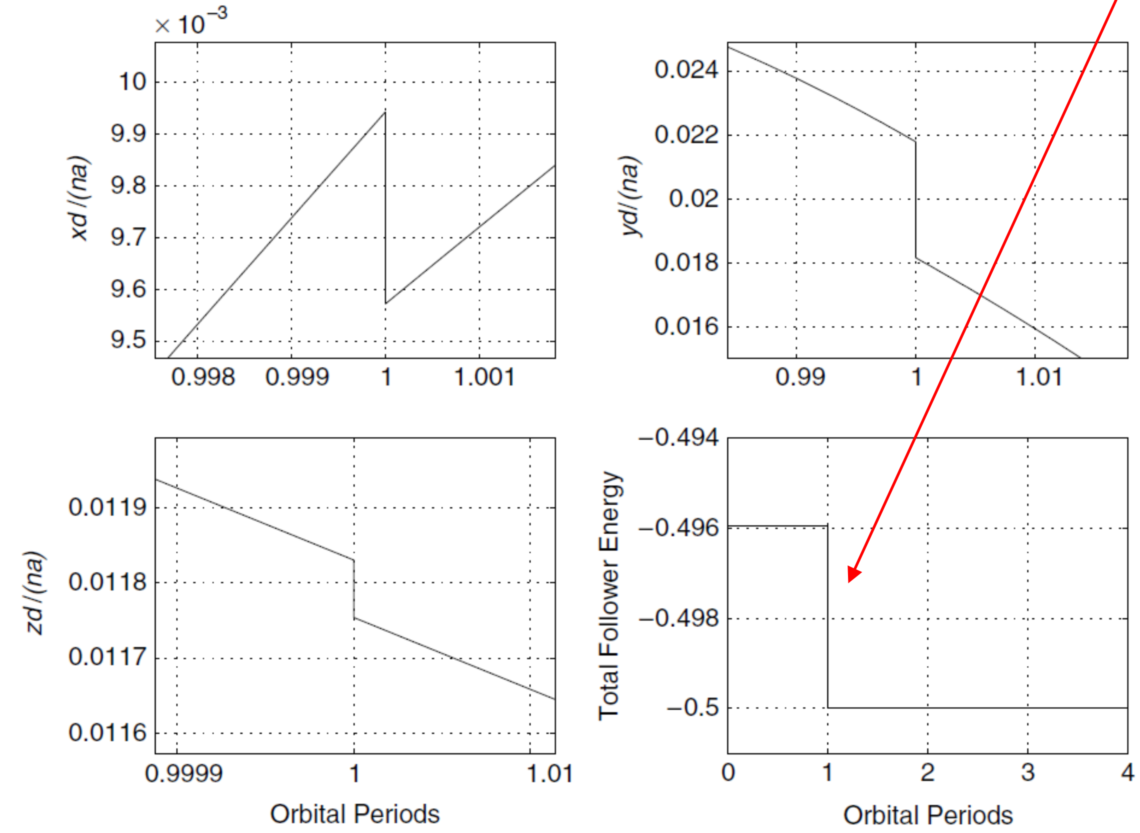
$$\Delta\bar{v}_x^* = -0.00037144,\ \Delta\bar{v}_y^* = -0.00361606, \qquad (4.60)$$
$$\Delta\bar{v}_z^* = -0.00003838$$

$$\Delta\bar{v}^* = \sqrt{(\Delta\bar{v}_x^*)^2 + (\Delta\bar{v}_y^*)^2 + (\Delta\bar{v}_z^*)^2} = 0.0036353 \qquad (4.61)$$



Impulsive Formation Control – Example (3)

- Magnification of the relative velocity components at the vicinity of the maneuver shows the applied velocity impulse, decreasing the total energy of the deputy to match the total energy of the chief



Schaub

Impulsive control using GVE in eccentric orbits (1)

- Mission requirements call for formation keeping and reconfiguration
- Keeping aims at the maintenance of a nominal configuration
- Reconfiguration aims at the acquisition of a nominal configuration
- Efficient feedback control laws are based on the Gauss Variational Eqs.
- These relate the effects of a control acceleration vector ***u*** to the osculating orbit element time derivatives

$$\boldsymbol{u} = \left(u_r, u_\theta, u_h\right)^T$$

(14.186a) (14.186f)

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \frac{2a^2}{h}\left(e \sin f\, u_r + \frac{p}{r} u_\theta\right)$$

$$\frac{\mathrm{d}e}{\mathrm{d}t} = \frac{1}{h}\left(p \sin f\, u_r + ((p+r)\cos f + re)u_\theta\right)$$

$$\frac{\mathrm{d}i}{\mathrm{d}t} = \frac{r\cos\theta}{h} u_h$$

$$\frac{\mathrm{d}\Omega}{\mathrm{d}t} = \frac{r\sin\theta}{h \sin i} u_h$$

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{1}{he}\left[-p\cos f\, u_r + (p+r)\sin f\, u_\theta\right] - \frac{r\sin\theta\cos i}{h\sin i} u_h$$

$$\frac{\mathrm{d}M}{\mathrm{d}t} = n + \frac{\eta}{he}\left[(p\cos f - 2re)u_r - (p+r)\sin f\, u_\theta\right]$$

Notes:
r-radial, θ-along-track, *h*-cross-track
Singularities for zero *e* and *i*

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Impulsive control using GVE in eccentric orbits (2)

- Most formation-flying missions avoid continuous thrust actuation to minimize interference with the payload activities and reduce propellant consumption
- Gauss variational equations can be used to derive impulsive control laws in orbits of arbitrary eccentricity
- Schaub proposes the following scheme by correcting 3 pairs of orbit elements "independently" at three stages: i and Ω, ω and M, a and e

p: perigee
a: apogee

1 cross-track impulse

$$\Delta i = \frac{r\cos\theta}{h}\Delta v_h$$

$$\Delta\Omega = \frac{r\sin\theta}{h\sin i}\Delta v_h$$

(14.238)
|
(14.257)

$$\Delta\omega = \frac{1}{he}\left(-p(\Delta v_{r_p} - \Delta v_{r_a}\right) - \Delta\Omega\cos i$$

$$\Delta M = \frac{\eta}{he}\left((p - 2r_p e)\Delta v_{r_p} - (p + 2r_a e)\Delta v_{r_a}\right)$$

2 radial impulses

$$\Delta a = \frac{2a^2}{h}\left(\frac{p}{r_p}\Delta v_{\theta_p} + \frac{p}{r_a}\Delta v_{\theta_a}\right)$$

$$\Delta e = \frac{1}{h}\left((p + r_p + r_p e)\Delta v_{\theta_p} + (-p - r_a + r_a e)\Delta v_{\theta_a}\right)$$

2 along-track impulses

Impulsive control using GVE in eccentric orbits (3)

- The resulting expressions represent 3 pairs of 2 equations each in 2 unknowns which can be solved independently to provide the individual delta-v's
- Single cross-track maneuver

$$\Delta v_h = \frac{h}{r}\sqrt{\Delta i^2 + \Delta\Omega^2 \sin i^2}$$

$$\theta_c = \arctan\frac{\Delta\Omega \sin i}{\Delta i}$$

Solution corresponding to positive Δv_h taken

- Two radial and two along-track maneuvers over one orbit (sequence relevant!)

$$\Delta v_{r_p} = -\frac{na}{4}\left(\frac{(1+e)^2}{\eta}(\Delta\omega + \Delta\Omega\cos i) + \Delta M\right)$$

$$\Delta v_{r_a} = -\frac{na}{4}\left(\frac{(1-e)^2}{\eta}(\Delta\omega + \Delta\Omega\cos i) + \Delta M\right)$$

$$\Delta v_{\theta_p} = \frac{na\eta}{4}\left(\frac{\Delta a}{a} + \frac{\Delta e}{1+e}\right)$$

$$\Delta v_{\theta_a} = \frac{na\eta}{4}\left(\frac{\Delta a}{a} - \frac{\Delta e}{1-e}\right)$$

Attention with coupling of orbit element differences due to Kepler

Impulsive control using GVE in eccentric orbits (4)

- In order to implement this control scheme, the mean orbit element errors are established at some arbitrary point in the orbit (ignoring short-period effects)
- They are held constant during the orbit to compute the appropriate delta-v's
- Any neglected error evolution over the current orbit (e.g., due to J_2) can be corrected during the next orbit (or next maneuver cycle)
- Exception is made for the coupling between semi-major axis and mean anomaly differences, since different orbit periods will cause substantial relative drift over one orbit
- In order to circumvent this problem and reduce propellant consumption, the correction of (ω, M) could begin at the second orbit, only after the semi-major axis difference is nullified
- This method is shown to be *efficient* for arbitrary correction of orbit elements (few percent increase over multi-impulse optimal solution)

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AA 279 D – SPACECRAFT FORMATION-FLYING AND RENDEZVOUS: LECTURE 8

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