EFFECTIVE TWO-NUCLEON INTERACTION FROM AN ANALYSIS OF THE $^{16}O(t,p)^{18}O$ REACTION*

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Effective two-body matrix elements in the s-d shell are extracted from the ^{18}O wave function previously obtained. The matrix elements are tabulated and compared with results from shell model calculations.

In a recent analysis of the $^{16}O(t,p)^{18}O$ reaction by Kolltveit [1], the shell model wave functions of the three low-lying 0^+ states and the three 2^+ states of ^{18}O were obtained. The basic assumption was that the wave functions could be adequately described by the three basis vectors,

$|(d_{\frac{5}{2}})^2, J=0\rangle$, $|(S_{\frac{1}{2}})^2, J=0\rangle$, and $|\text{deformed}, J=0\rangle$

for the 0^+ states and

$|(d_{\frac{5}{2}})^2, J=2\rangle$, $|(d_{\frac{5}{2}}S_{\frac{1}{2}}), J=2\rangle$, and $|\text{deformed}, J=2\rangle$

for the 2^+ states.

The deformed states were assumed to be mainly 4 particle-2 hole states, though the only necessary restriction in the analysis was that they could not be reached in a direct two-particle transfer reaction with ^{16}O as target. If the ^{16}O ground state contains no core excited states, this would be true.

It was necessary to restrict the calculation to three basis vectors for the low-lying states. The number of equations needed to determine the amplitudes is $[\frac{1}{2}n(n-1)]$, where n is the number of basis vectors. For $n=3$, two linearly independent equations are given by the two-particle reaction cross sections. One equation was obtained from the electric quadrupole transition ratios. Fortunately, the amplitudes proved to be insensitive to the latter.

Due to the quadratic nature of the equations for the amplitudes of the wave functions, four different numerical solutions were obtained for both the $J=0$ and $J=2$ states. Of these solutions we find that solutions 4 (ref. 1) are the only acceptable ones. These are shown in table 1.

From these wave functions we can extract the energy matrix:

$$\langle n|H|m\rangle = \sum_{i=1}^{3} a_{in} E^i_{\text{exp}} a_{im}$$

where E^i_{exp} are the experimental energies and the a's are the amplitudes given in table 1. The ground state of ^{18}O is taken to be -3.9 MeV with the $d_{\frac{5}{2}}$ level at 0 MeV. Assuming the $S_{\frac{1}{2}}$ level to be at 0.87 MeV, we obtain the matrix elements shown in table 2. Since no model is assumed for the deformed states, only the total matrix elements (unperturbed energy plus interaction) are given for $\langle \text{def.}|H|\text{def.}\rangle$. In table 2 we also list matrix elements obtained from realistic forces

Table 1
Wave function of states of ^{18}O obtained from analysis of $^{16}O(t,p)^{18}O$ data [1].

J^π	E	Wave function			
		$d^2_{\frac{5}{2}}$	$S^2_{\frac{1}{2}}$	$d_{\frac{5}{2}}S_{\frac{1}{2}}$	def
0^+_1	0	0.75	0.43		0.51
0^+_2	3.63	0.65	-0.64		-0.42
0^+_3	5.33	0.15	0.64		-0.75
2^+_1	1.98	0.69		0.51	0.51
2^+_2	3.92	-0.66		0.74	0.16
2^+_3	5.25	-0.30		-0.45	0.84

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Table 2
Effective matrix elements in the 1s-0d shell.

	Present calculation	Federman and Talmi (case 2)	Kuo $V+V_{2p}$	Kuo $V+V_{2p}+V_{3p1h}+V_{2h}$
$\langle d_{\frac{5}{2}}^2 \lvert V \rvert d_{\frac{5}{2}}^2 \rangle_{J=0}$	-2.25	-3.24	-1.45	-2.44
$\langle S_{\frac{1}{2}}^2 \lvert V \rvert S_{\frac{1}{2}}^2 \rangle_{J=0}$	-1.97	-1.97	-1.18	-1.95
$\langle d_{\frac{5}{2}}^2 \lvert V \rvert S_{\frac{1}{2}}^2 \rangle_{J=0}$	-1.00	-0.77	-0.63	-0.97
$\langle d_{\frac{5}{2}}^2 \lvert V \rvert \mathrm{def} \rangle_{J=0}$	-1.59	-1.42		-1.31 *
$\langle S_{\frac{1}{2}}^2 \lvert V \rvert \mathrm{def} \rangle_{J=0}$	-1.58	-0.75		-0.96 *
$\langle \mathrm{def} \lvert V \rvert \mathrm{def} \rangle_{J=0} + E_{\mathrm{unper}}$	-0.26	0.65		
$\langle d_{\frac{5}{2}}^2 \lvert V \rvert d_{\frac{5}{2}}^2 \rangle_{J=2}$	-0.78	-1.59	-1.05	-1.04
$\langle d_{\frac{5}{2}} S_{\frac{1}{2}} \lvert V \rvert d_{\frac{5}{2}} S_{\frac{1}{2}} \rangle_{J=2}$	-1.05	-0.76	-1.24	-1.29
$\langle d_{\frac{5}{2}}^2 \lvert V \rvert d_{\frac{5}{2}} S_{\frac{1}{2}} \rangle_{J=2}$	-0.51	-0.48	-0.60	-0.85
$\langle d_{\frac{5}{2}}^2 \lvert V \rvert \mathrm{def} \rangle_{J=2}$	-1.04	-0.78		-0.75 *
$\langle d_{\frac{5}{2}} S_{\frac{1}{2}} \lvert V \rvert \mathrm{def} \rangle_{J=2}$	-1.01	-0.49		-1.12 *
$\langle \mathrm{def} \lvert V \rvert \mathrm{def} \rangle_{J=2} + E_{\mathrm{unper}}$	0.42	2.02		

* These are obtained from ref. 2.

[2,3] and by empirical fitting [4]. The $(V+V_{2p})$ and $(V+V_{2p}+V_{3p\text{-}1h}+V_{2h})$ matrix elements [3] are listed separately.

Strictly speaking, the shell model expansion is an infinite, linear combination of shell model states. Most two-particle shell model configurations give strong contributions to the transfer reaction. Since we have restricted our analysis to two two-particle states, this implies that the excluded two-particle states are mixed with our states.

If our assumption that ^{16}O has no core excited configurations were true, the core excited configurations in ^{18}O would give no contribution to the transfer reaction. Consequently, from our only assumption about the deformed state, that the deformed state gives no contribution to the transfer reaction, one might hope that *all* the core excited configurations in ^{18}O are embedded in the deformed state. We should therefore compare our matrix elements with $(V+V_{2p})$ in table 2. However, the existence of core excited configurations in ^{16}O causes transitions to core excited configurations in ^{18}O and the above argument is consequently no longer true. Hence, it seems most reasonable to compare our matrix elements with the $(V+V_{2p}+V_{3p\text{-}1h}+V_{2h})$ matrix elements [5].

One should remember, however, that particle transfer reactions favor certain states and that transitions involving some configurations are forbidden. The spectroscopic factors therefore describe the "transition allowed" configurations in ^{18}O. By normalizing our states to unity, the states that were excluded in the transfer reaction analysis will be arbitrarily included in our amplitudes. The comparison with shell model calculations should therefore be made with care.

In table 2 we see that our $J=0$ matrix elements agree fairly well with $(V+V_{2p}+V_{3p\text{-}1h}+V_{2h})$, while $(V+V_{2p})$ generally is too small in magnitude. However, Kuo has explicitly included the $d_{\frac{3}{2}}$ configuration in his calculation, so the matrix elements may not be directly comparable. A crude estimate of the effect of the $d_{\frac{3}{2}}$ configuration for $J=0$ matrix elements may be obtained by adding a perturbation "correction" of the form

$$\Delta E_{\mathrm{nm}}^{J=0} \approx -\left[\langle n, J=0\ V_{\mathrm{Kuo}}\ (d_{\frac{3}{2}})^2,\ J=0\rangle \times \langle (d_{\frac{3}{2}})^2,\ J=0\ V_{\mathrm{kuo}}\ m, J=0\rangle\right]/10\ \mathrm{MeV}$$

to Kuo's matrix elements. When this is done, the agreement with the $(V+V_{2p})$ matrix elements is very good, while the $(V+V_{2p}+V_{3p\text{-}1h}+V_{2h})$ matrix elements now become too big in magnitude.

For the $J=2$ matrix elements we find that the agreement with Kuo's matrix elements is not so

good. Presumably this is due to the exclusion of the $d_{\frac{3}{2}}$ configuration in the two-particle transfer analysis. The $d_{\frac{3}{2}}$ configuration in the $J = 2$ case adds two more states against only one in the $J = 0$ case, and one of the states has an unperturbed energy of only 5 MeV above the $(d_{\frac{5}{2}})^2$ configuration. The value of $B(E_2; (2^+)_2 \rightarrow (0^+)_1)$ used in the analysis of the $J = 2$ levels, was also very uncertain.

From our listed matrix elements, $\langle d_{\frac{3}{2}}^2, J = 0 \,|V|\, \text{def.} \rangle$ may be estimated. This, however, requires a model for the deformed state. If we assume SU_3 to be a good approximation, only $S = 0$ will contribute, and an estimate may be given by:

$$\langle (d_{\frac{5}{2}})^2, J = 0 \,|V|\, \text{deformed}, J = 0 \rangle =$$

$$= \frac{A\begin{pmatrix} 2 & \frac{1}{2} & \frac{3}{2} \\ 2 & \frac{1}{2} & \frac{3}{2} \\ 0 & 0 & 0 \end{pmatrix} \langle (d_{\frac{5}{2}})^2, J = 0 \,|V|\, \text{deformed}, J = 0 \rangle}{A\begin{pmatrix} 2 & \frac{1}{2} & \frac{5}{2} \\ 2 & \frac{1}{2} & \frac{5}{2} \\ 0 & 0 & 0 \end{pmatrix}}$$

where $A\begin{pmatrix} l & s_1 & j_1 \\ l_2 & s_2 & j_2 \\ L & S & J \end{pmatrix}$ are Kennedy-Cliff coefficients [6]. Due to the exclusion of the $d_{\frac{3}{2}}$ configuration, our deformed matrix elements may be overestimated, and a proper reduction ought to be made first.

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