Take Me Out to (Analyze) the Ballgame

Visualization and Analysis Techniques for Big Spatial Data

Chris Comiskey

Oregon State University

April 24, 2017

Baseball, Baseball, Baseball

- Chris, rookie year

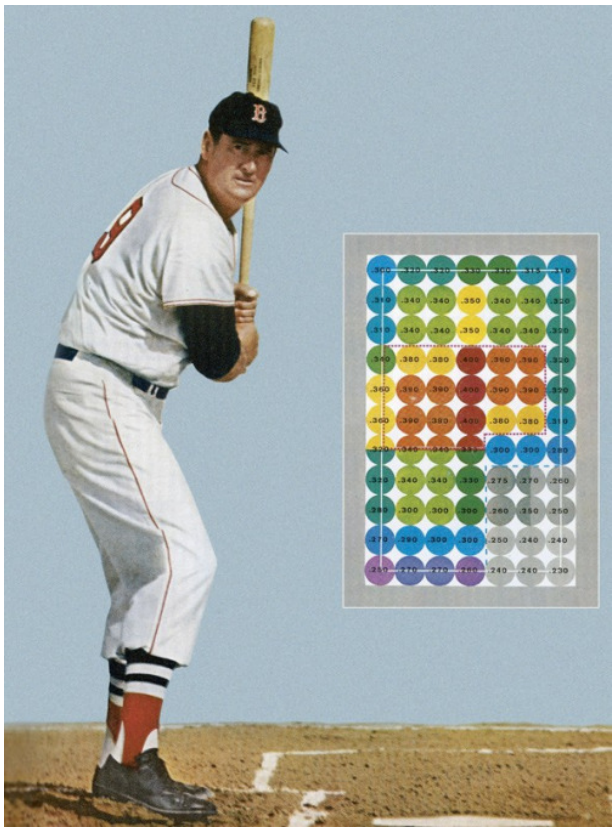
Baseball, Baseball, Baseball

- Chris, rookie year

- Chris, Boston Red Sox

Hitting Analytics, Then and Now

- "The Science of Hitting"$_{1970}$



- Conceptual breakthrough
- No data

Hitting Analytics, Then and Now

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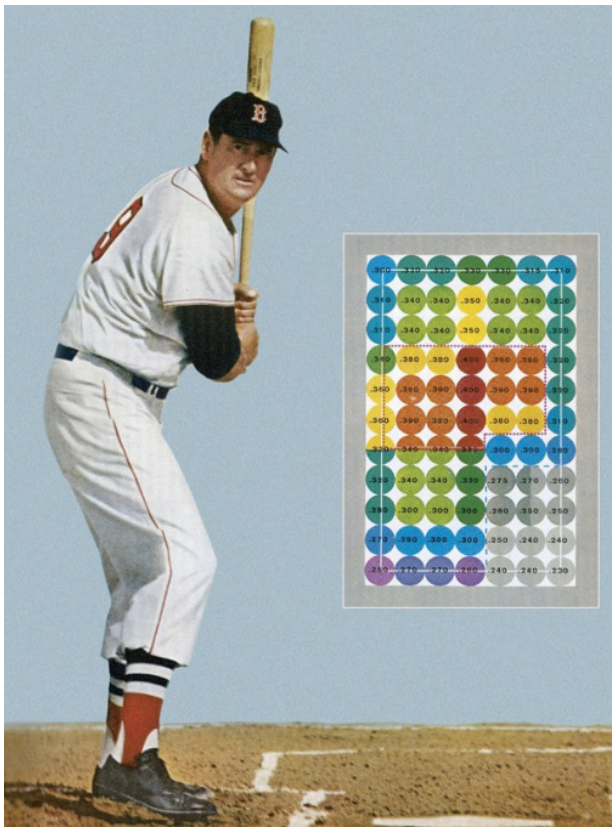


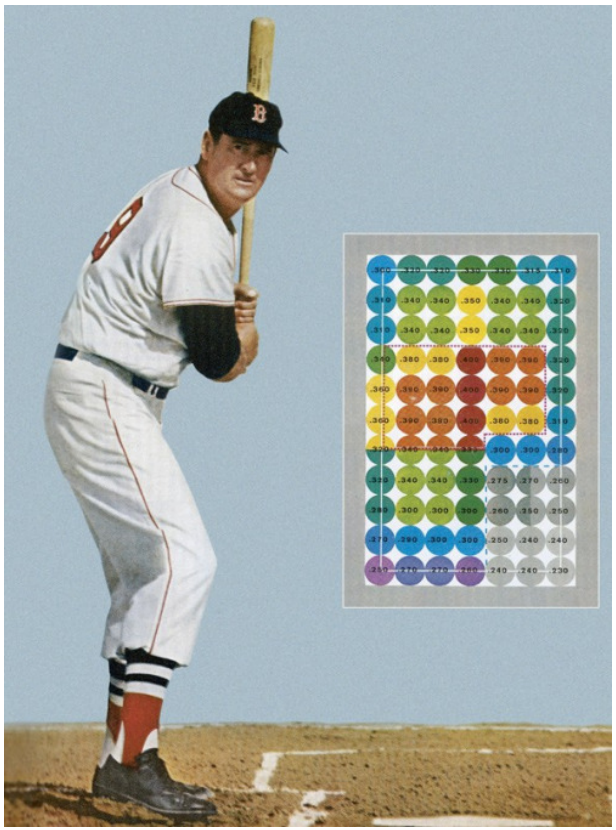
- Iconic breakthrough, hitter
- No data

- Hall of Fame exhibit

- Baseball fanatic statistician
- PITCHf/x data!

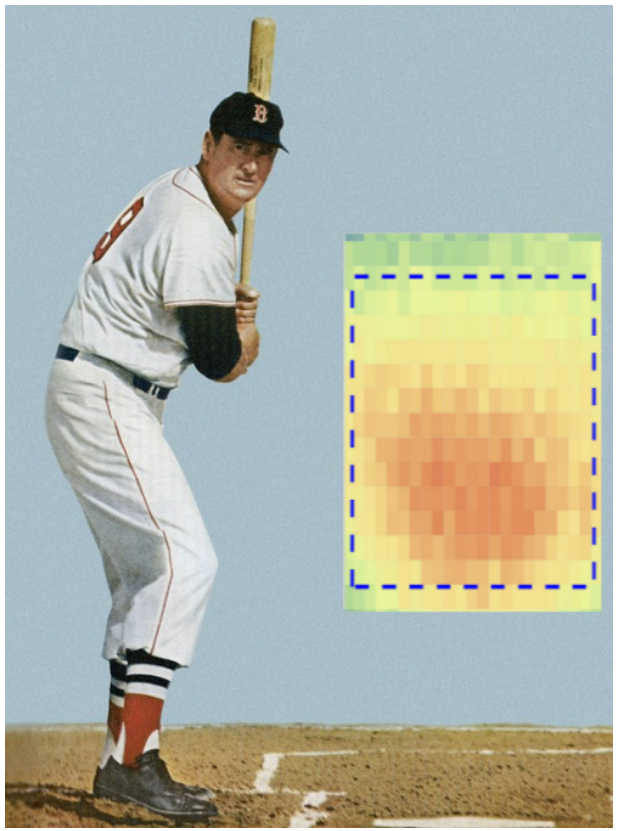
Hitting Analytics, Then and Now

- "The Science of Hitting"$_{1970}$



- Iconic breakthrough, hitter
- No data

- The Statistics of Hitting



- PITCHf/x data!
- Relevant?

Relevant Research

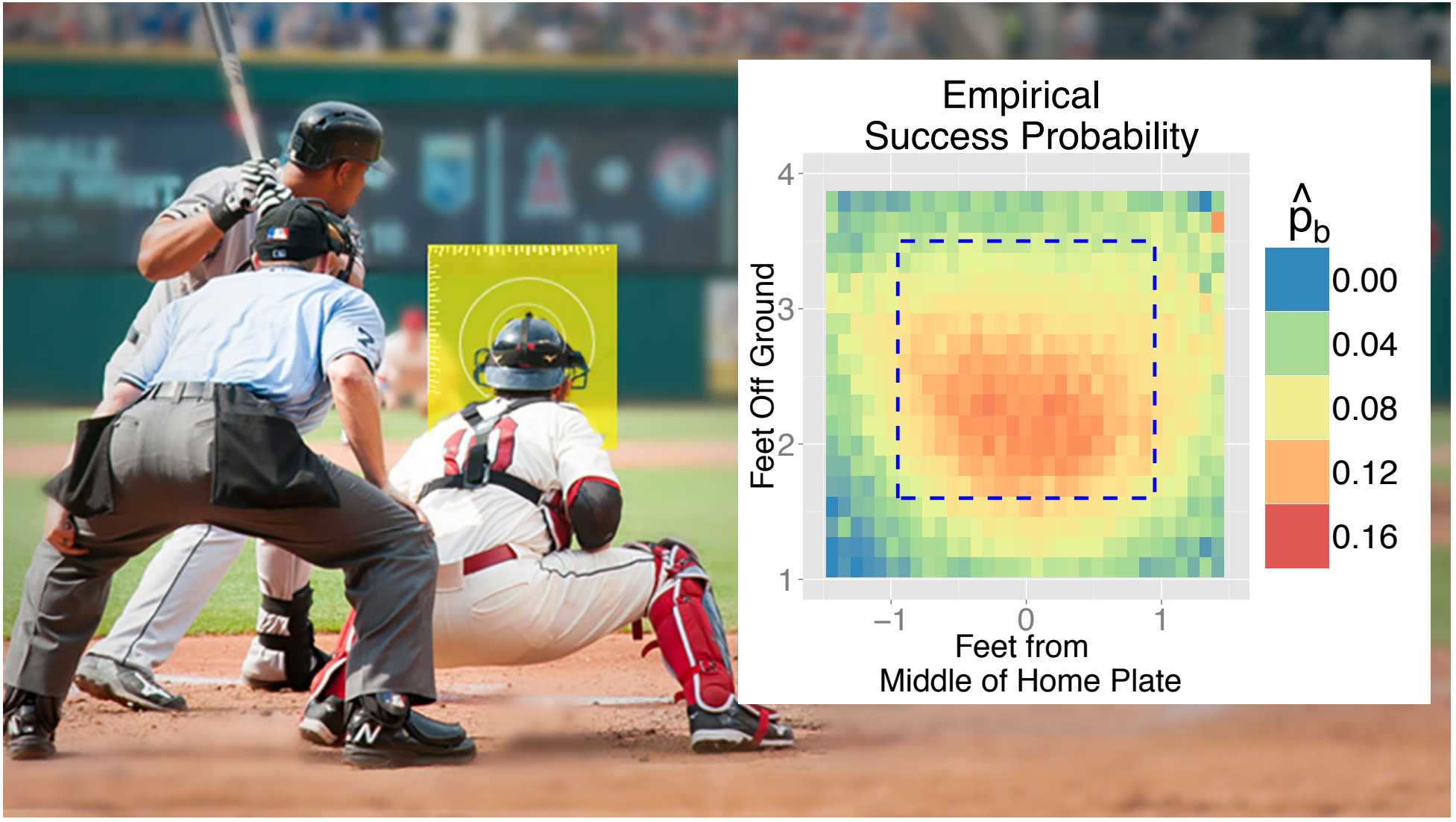
- Each MLB® team spends $\approx$ \$15 million/year
- Each MLB® team employs $\approx$ five quantitative analysts
- Heat maps popular on TV broadcasts
  - ESPN® and MLB® signed \$700 million/year contract
- Joey Votto (\$22.5 mil/year) packs dog-eared copy of "The Science of Hitting"
- "Big Data is Changing Baseball" - 1 TB/game [Delgado, 2014]

The Data

- PITCHf/x® - Sportsvision, high speed stereoscopic cameras, MLB Advanced Media, Gameday website, *open source*
  - http://www.sportvision.com/media/espn-k-zone
- MySQL - relational database management system, 2 GB, 'at bat' table 1,711,211 $\times$ 15, table joins
- Variables
  - **px** - horizontal location
  - **pz** - vertical location
  - **des** - pitch outcome
  - **ab_id** - ab bat ID number
  - **pitch_id** - pitch ID number
  - **pitch_type** - fastball, curve ball, etc.
  - **stand** - batter handedness
  - **batter** - batter ID number

Empirical Success Probability Heat Map

Empirical Success Probability Heat Map

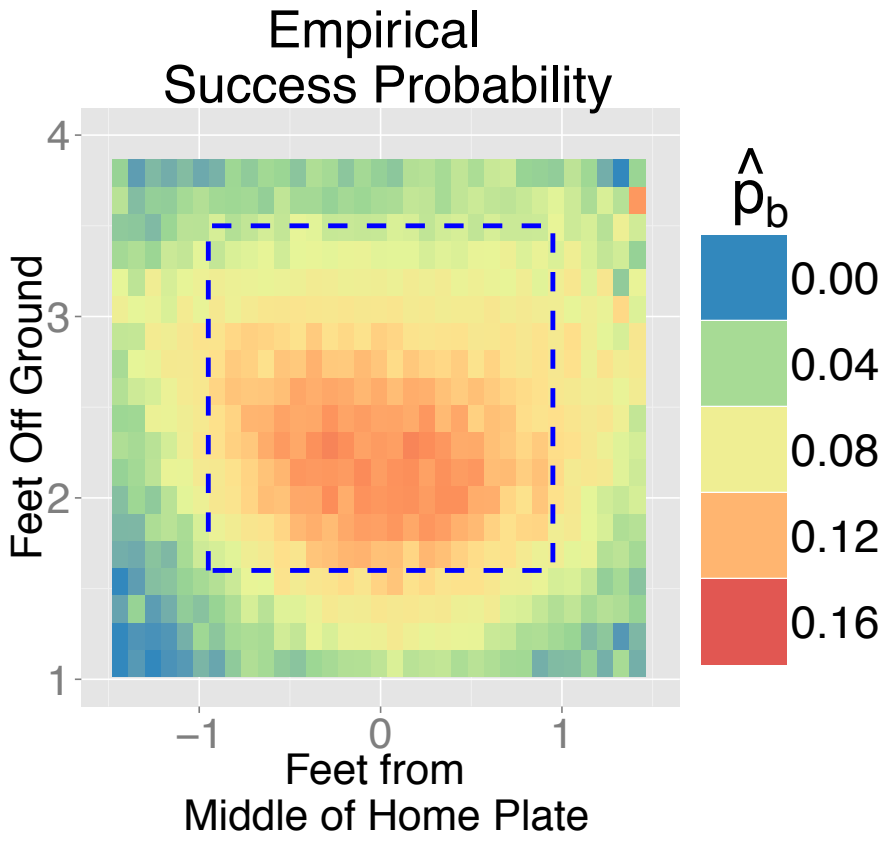


Empirical Success Probability Heat Map

- Grid vertical strike zone face
- Swing i, grid box b
- **Every swing - Bernoulli trial**
  - $Y_i = 1$, for swing success
- n_b = number of box b swings
- box b success probability:

$$\hat{p}_b = \frac{1}{n_b} \sum_{i \in b} Y_i$$

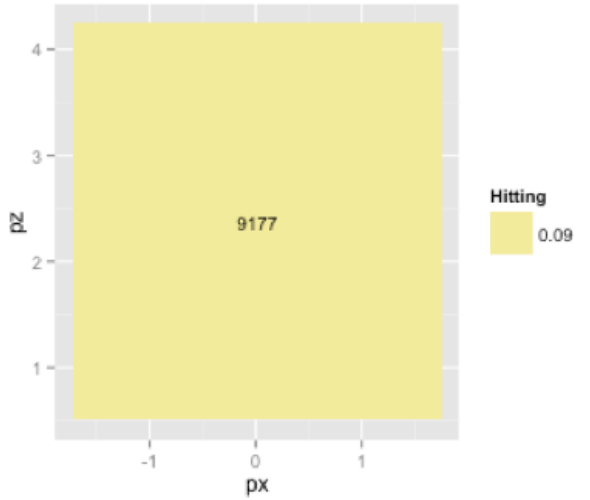
- Map $\hat{p}_b$ to box color

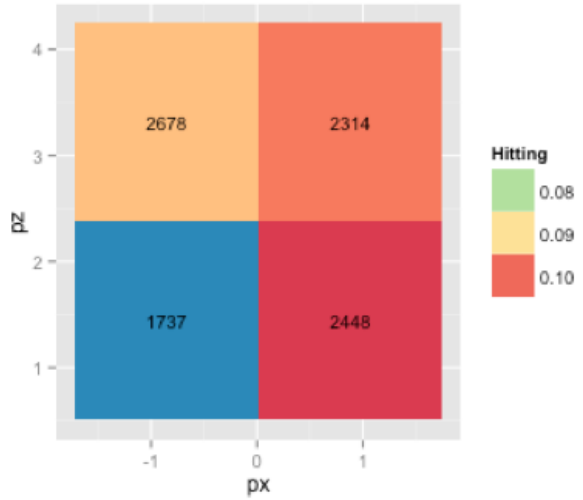


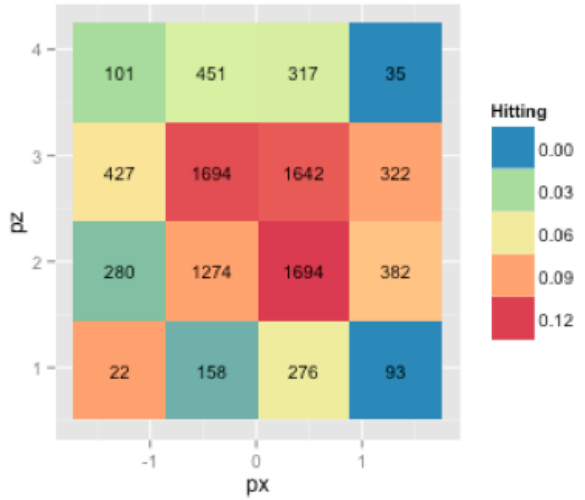
Note: $\hat{p}_b$ = **success probability**, not batting average

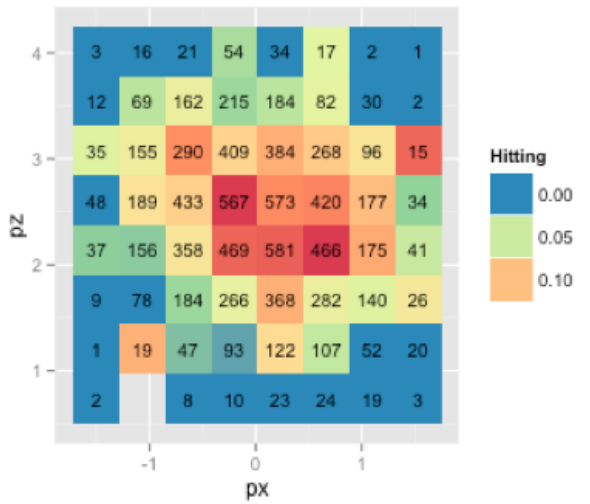
Heat Map Resolution, Jhonny Peralta

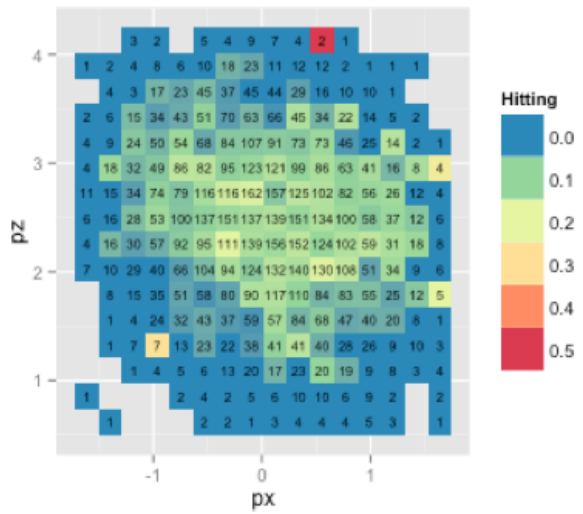
Decisions, Decisions...





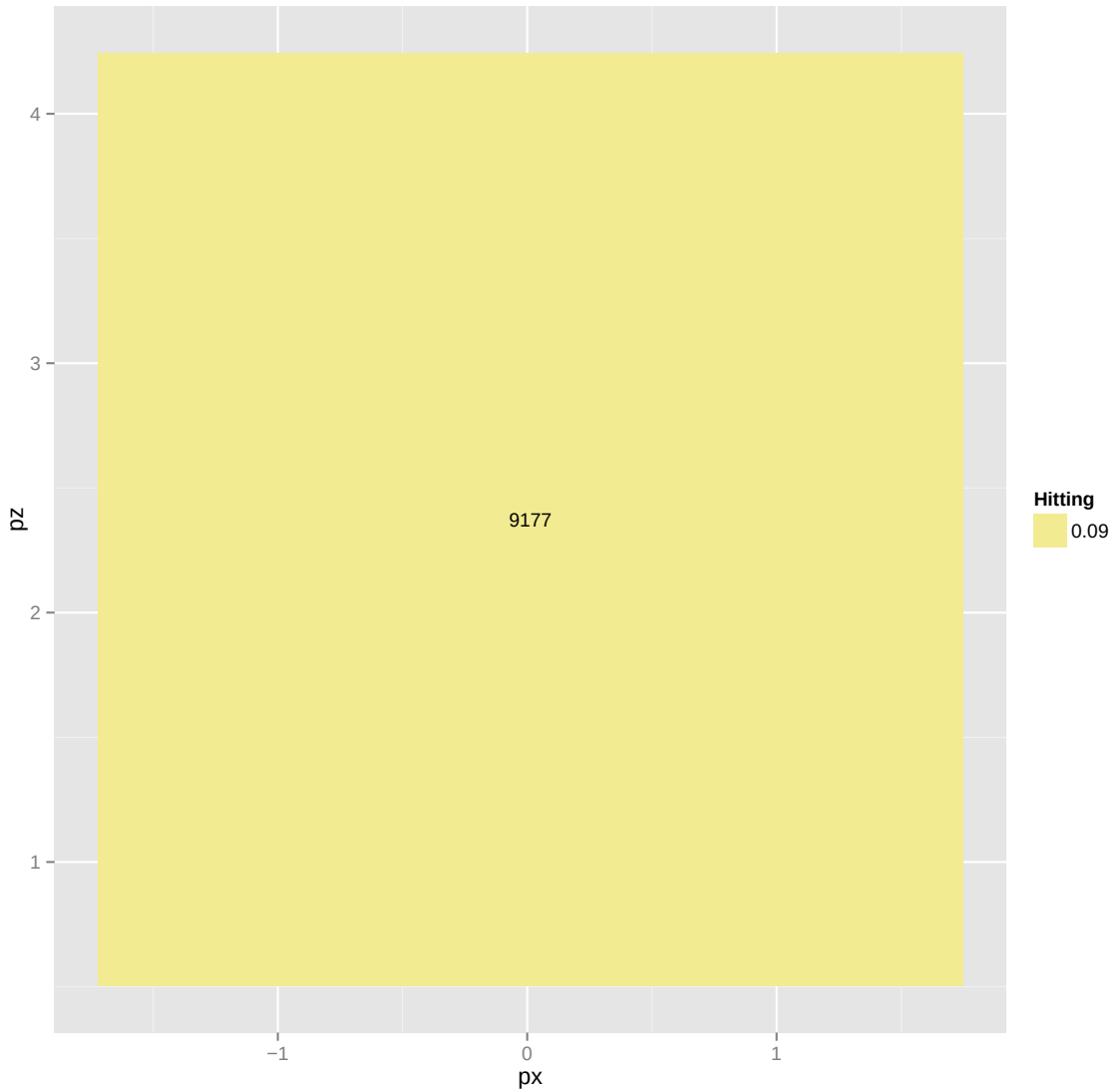






Heat Map Resolution, Jhonny Peralta

Starting from Scratch



Heat Map Resolution, Jhonny Peralta

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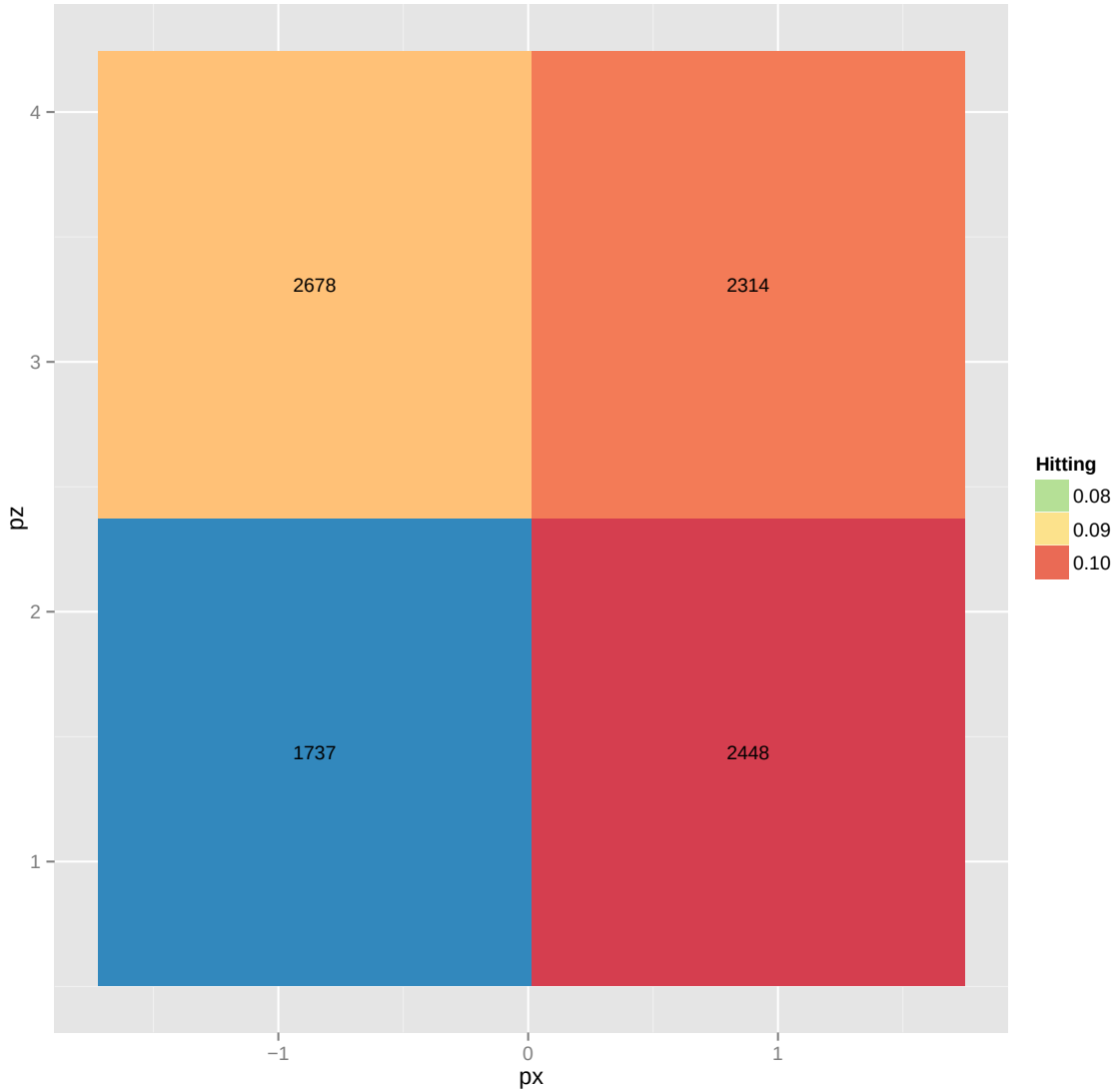

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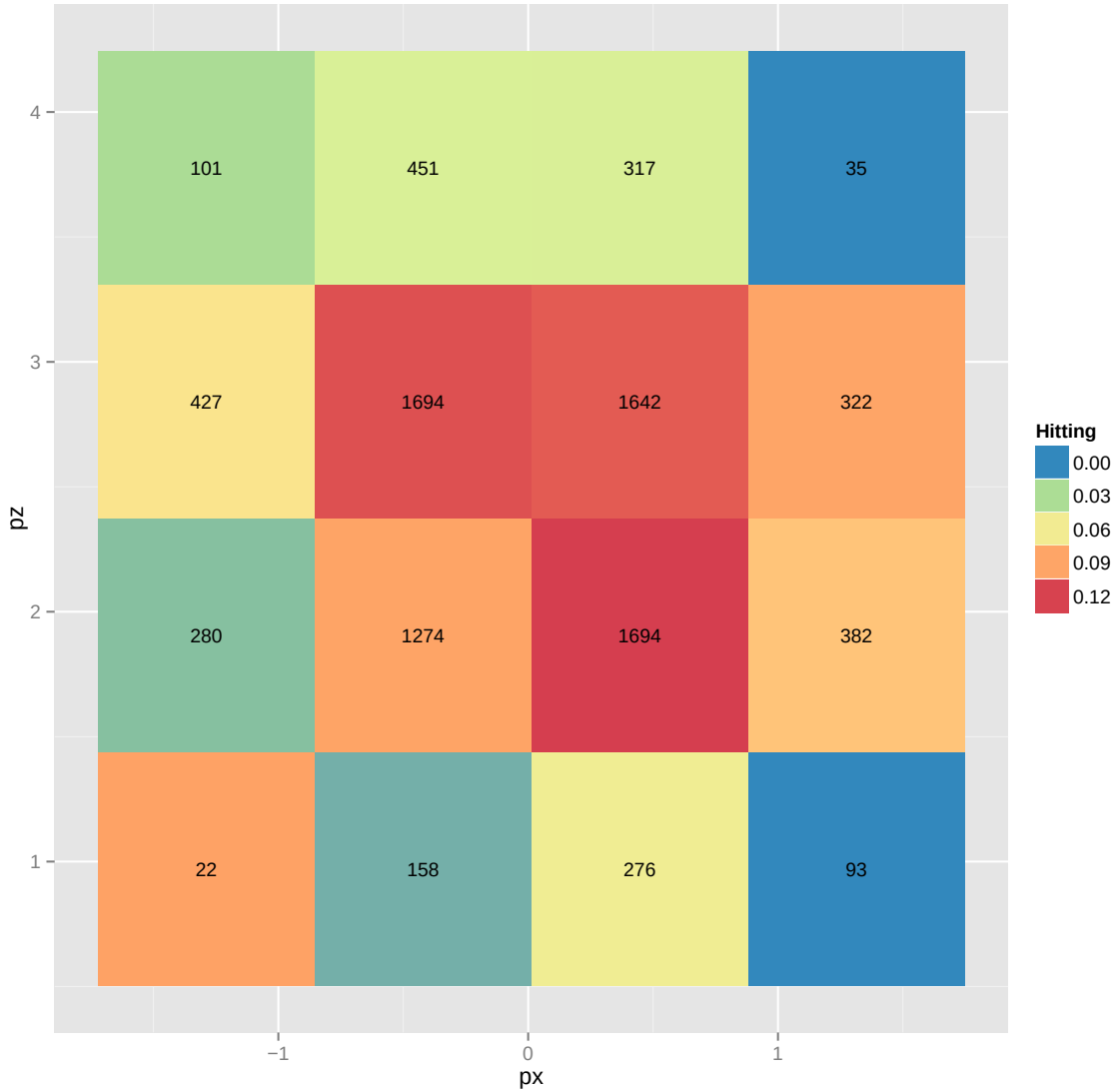

Heat Map Resolution, Jhonny Peralta

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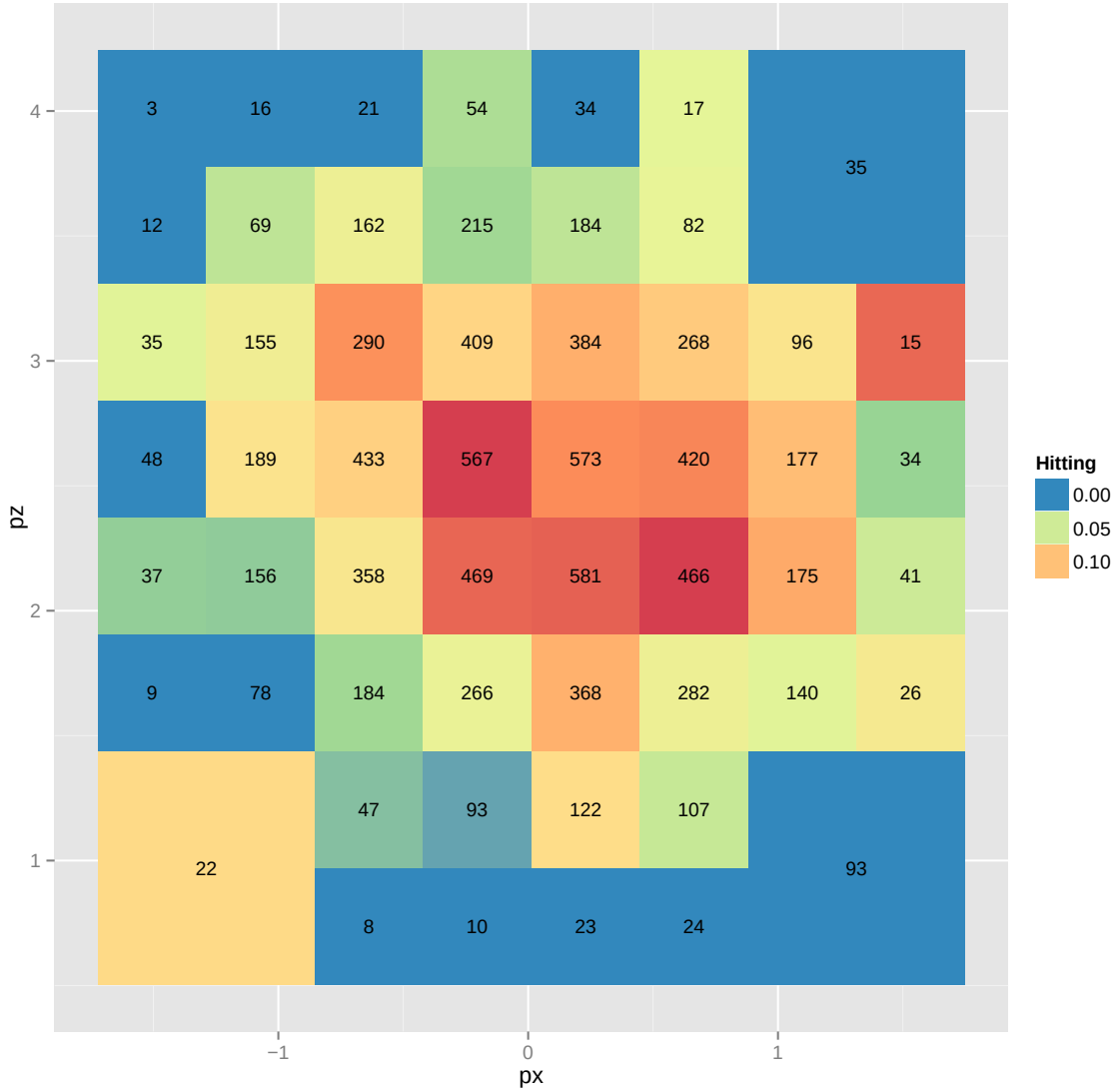

Heat Map Resolution, Jhonny Peralta

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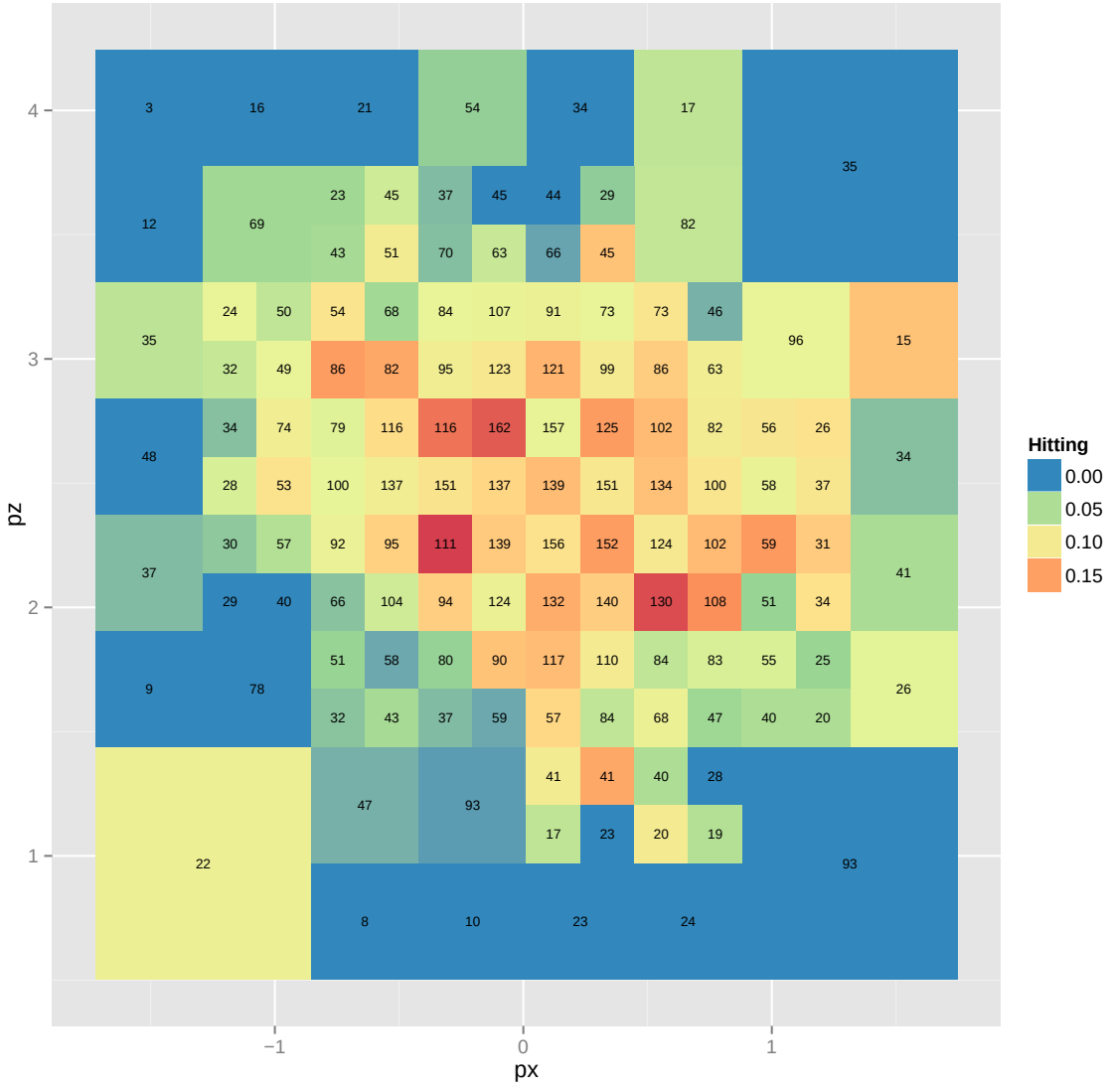

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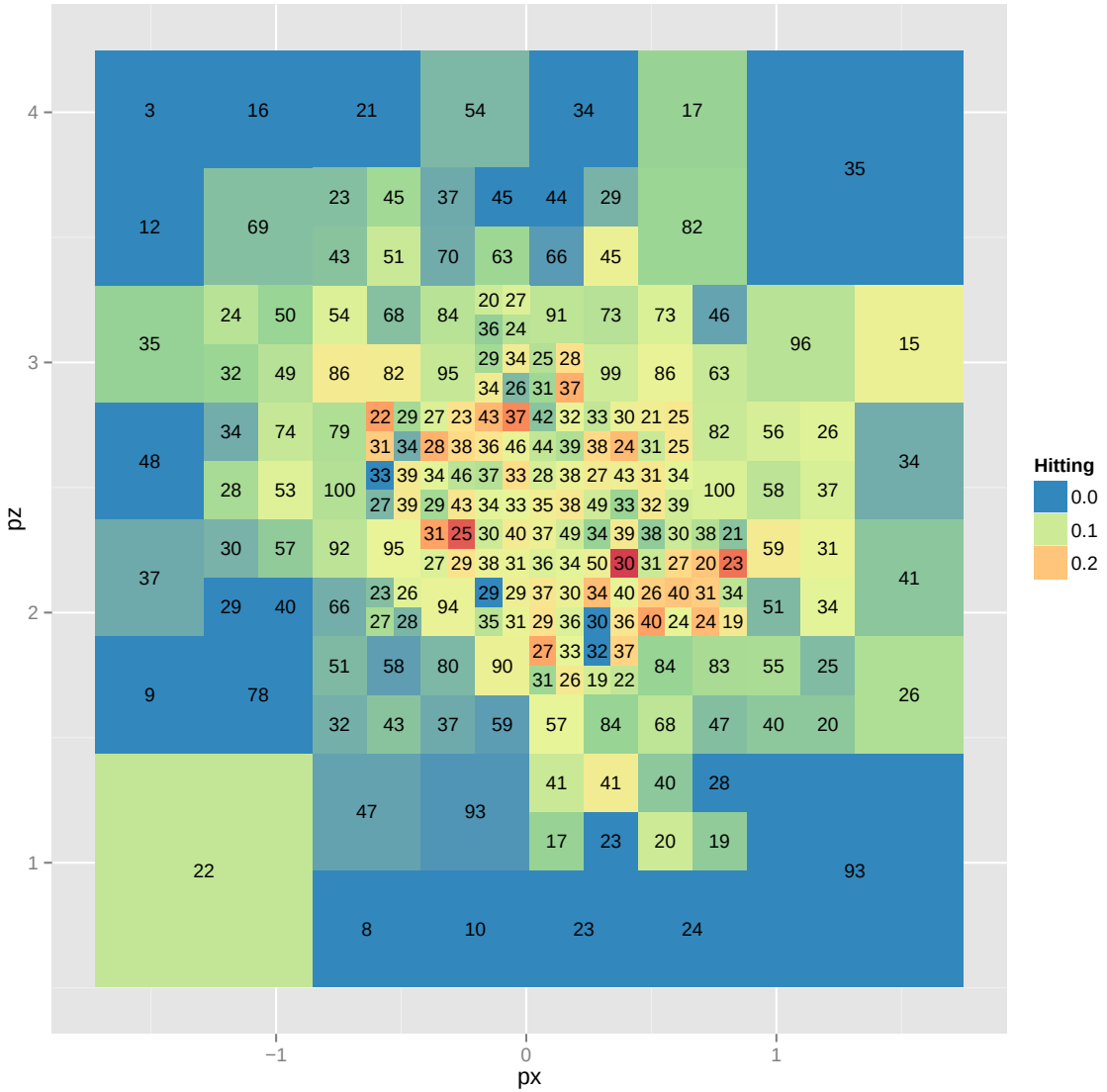


Best of Both (Spatial) Worlds

- Location specific resolution, rather than one global resolution
- Stopping rule $\iff$ **sample size** threshold
- Local resolution conveys data density
  - Big box $\iff$ low density
  - Bigger box, bigger variance*
- Choose subdivision algorithm

Empirical $\rightarrow$ Model

- Seek smooth(er) surface
- Models show relationships
- Models help understanding
- Modeling is fun!

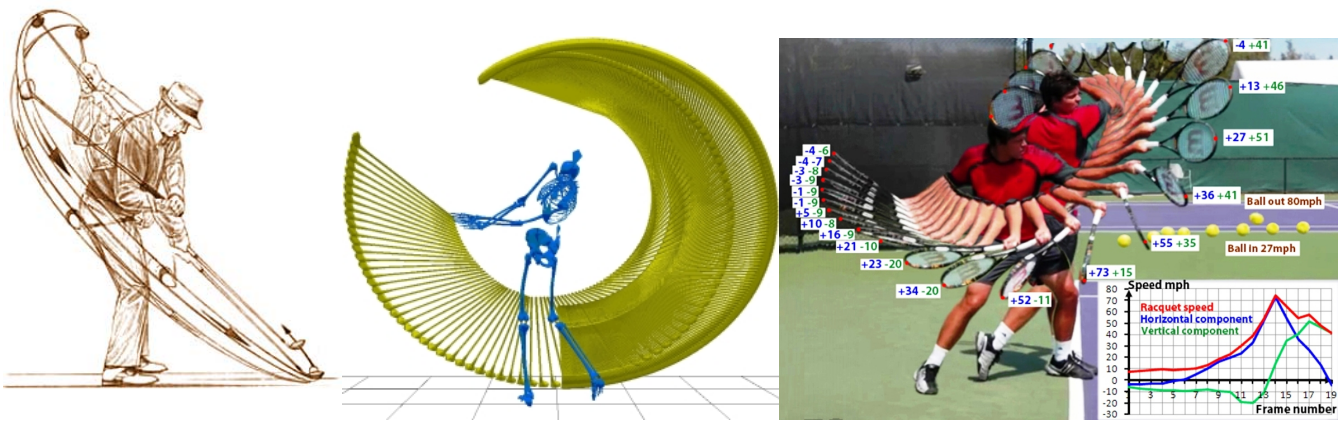
Let's Model!

- Hitter, pitcher, umpire
- Location, pitch type, previous pitch type/location
- Game state; count, baserunners
- Ballpark, day/night, weather

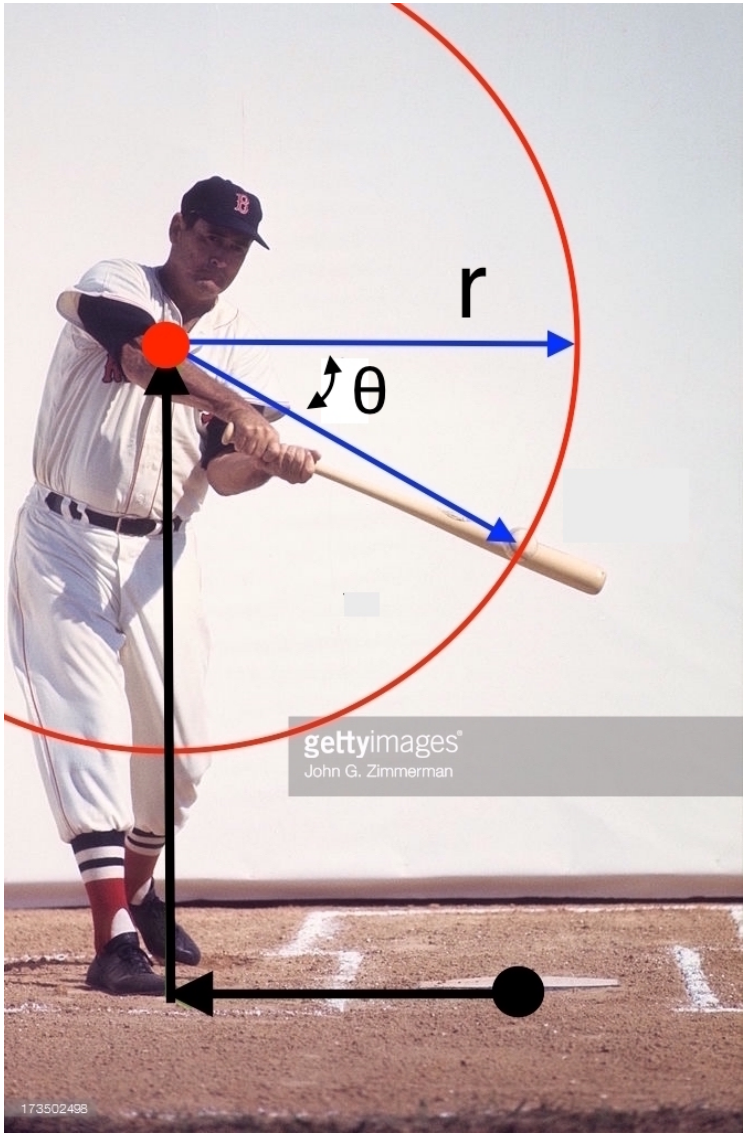
Simplifying assumption: success probabilities depend only on location (i) and hitter (j):

- $Y_{ij} \sim \text{Bernoulli}(p_{ij})$
- $\text{E}[Y_{ij}] = p_{ij}$
- $\text{logit}(p_{ij}) = \boldsymbol{X}_{ij}\beta$
  - covariate vector $\boldsymbol{X}_{ij}$, parameter vector β
- Covariates? Biomechanics.

- Rotational movement
- Change impact point $\Longrightarrow$ biomechanical adjustments
- Adjustments affect impact conditions, outcome probabilities



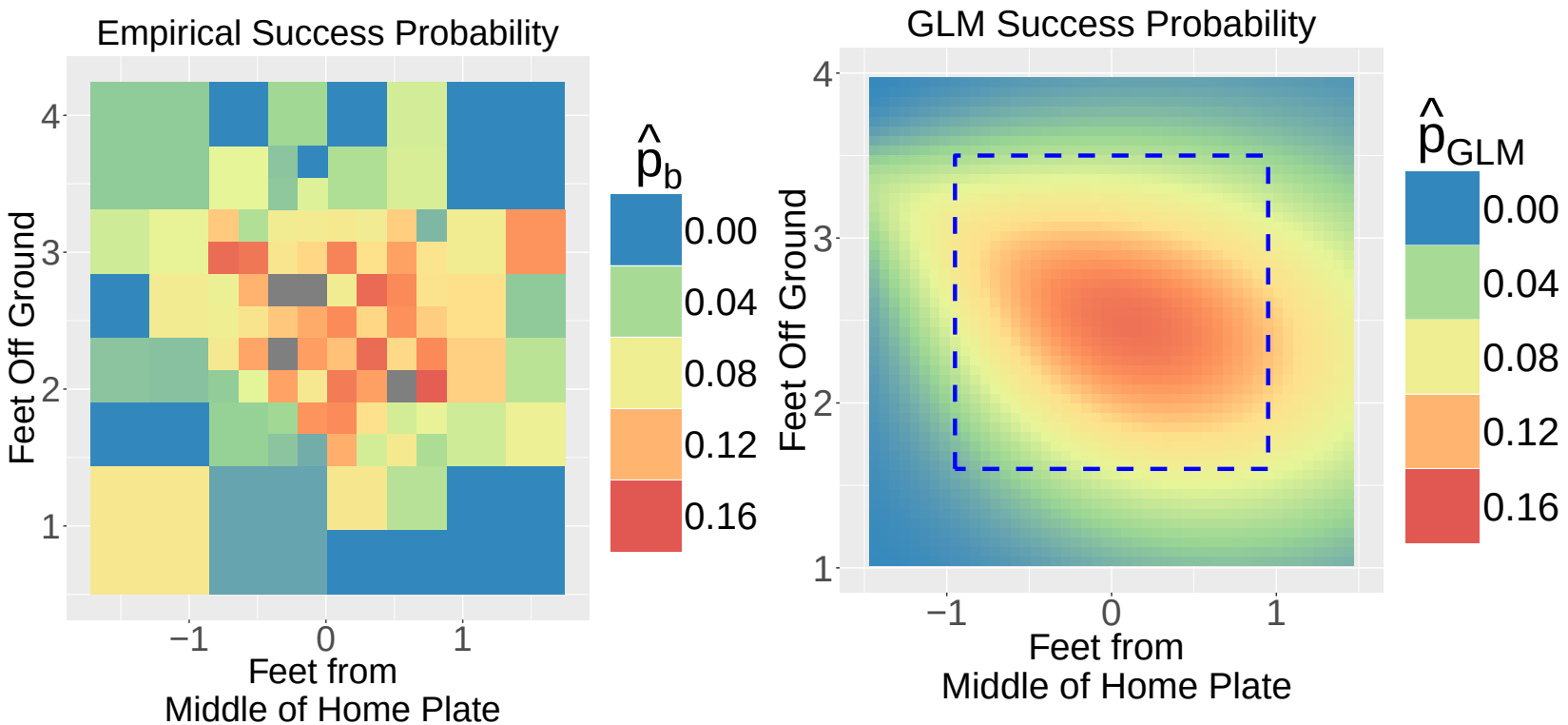
Baseball Biomechanics



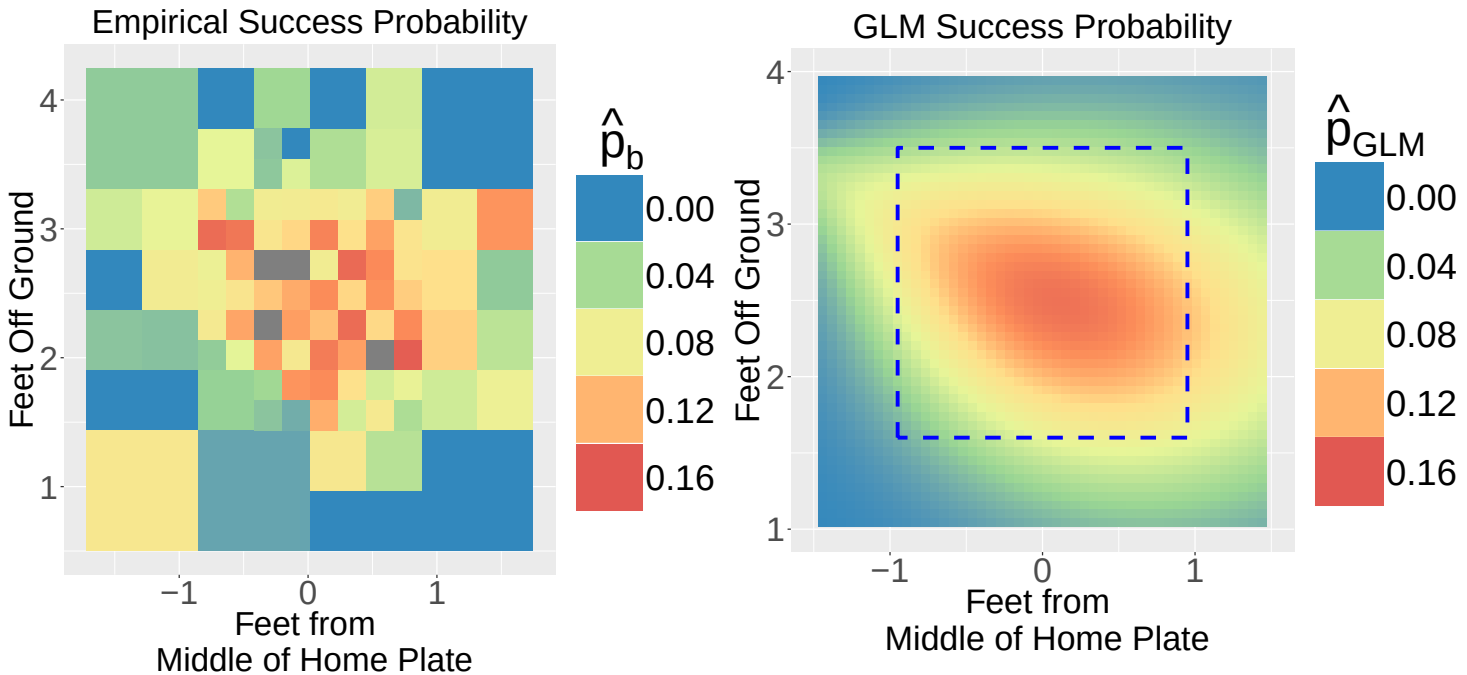
- Polar coordinates
- Translate origin
- Radius and angle biomechanically meaningful
- Big challenge: new origin location
  - Fleisig, ASMI
  - Dowling, Motus
- Let's model Jhonny Peralta, Mr. 9172

Logistic Regression Model, Jhonny Peralta

- $\text{logit}(p_{ij}) = \boldsymbol{X}_{ij}\beta$
  - swing i, hitter j = Jhonny Peralta



Logistic Regression Model, Jhonny Peralta

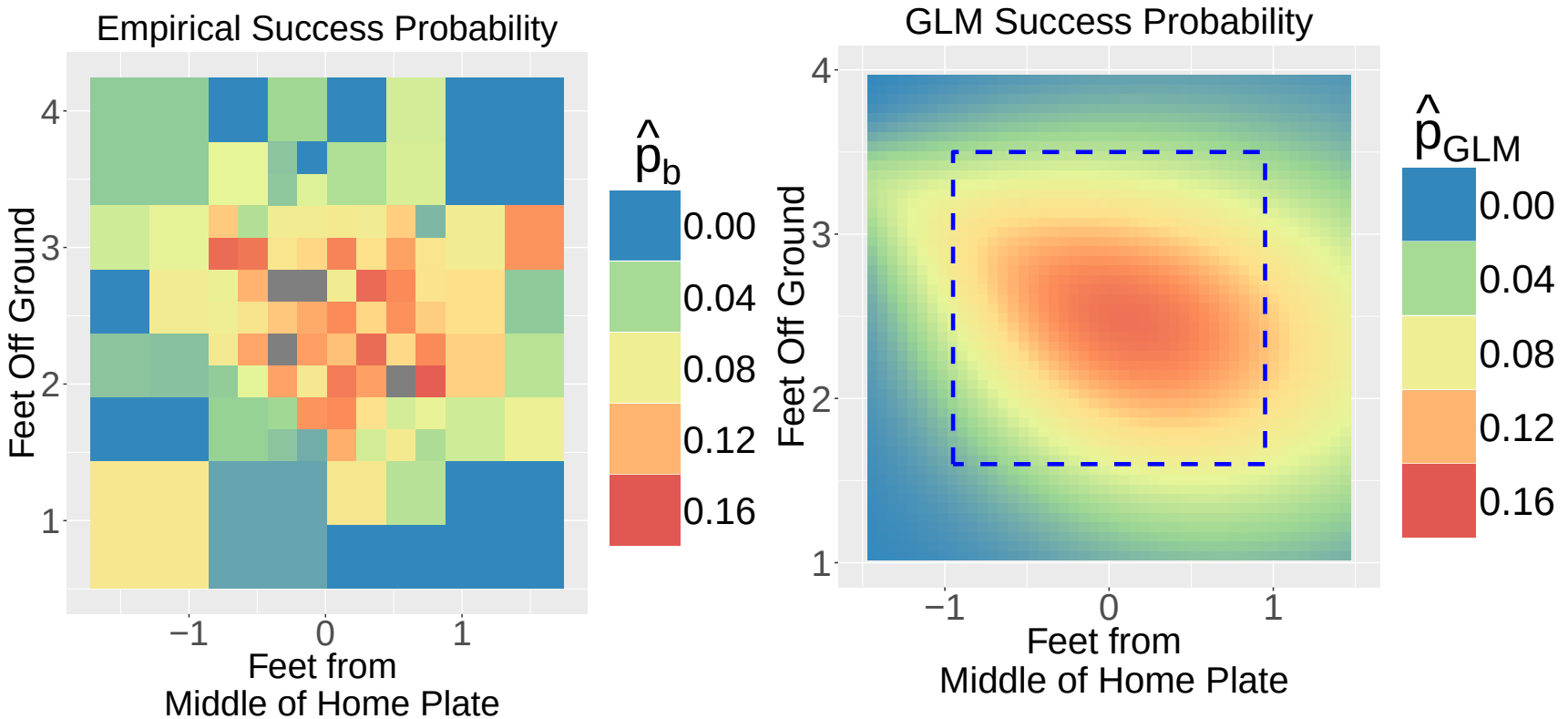


- Hosmer-Lemeshow (logistic regression) GOF test
  - Like Pearson χ^2 test, but cont. covariates (no categories)
  - H_0: Well fit
    H_A: Lack of fit
  - p-value = 0.8217

Logistic Regression Model

What about a confidence interval?

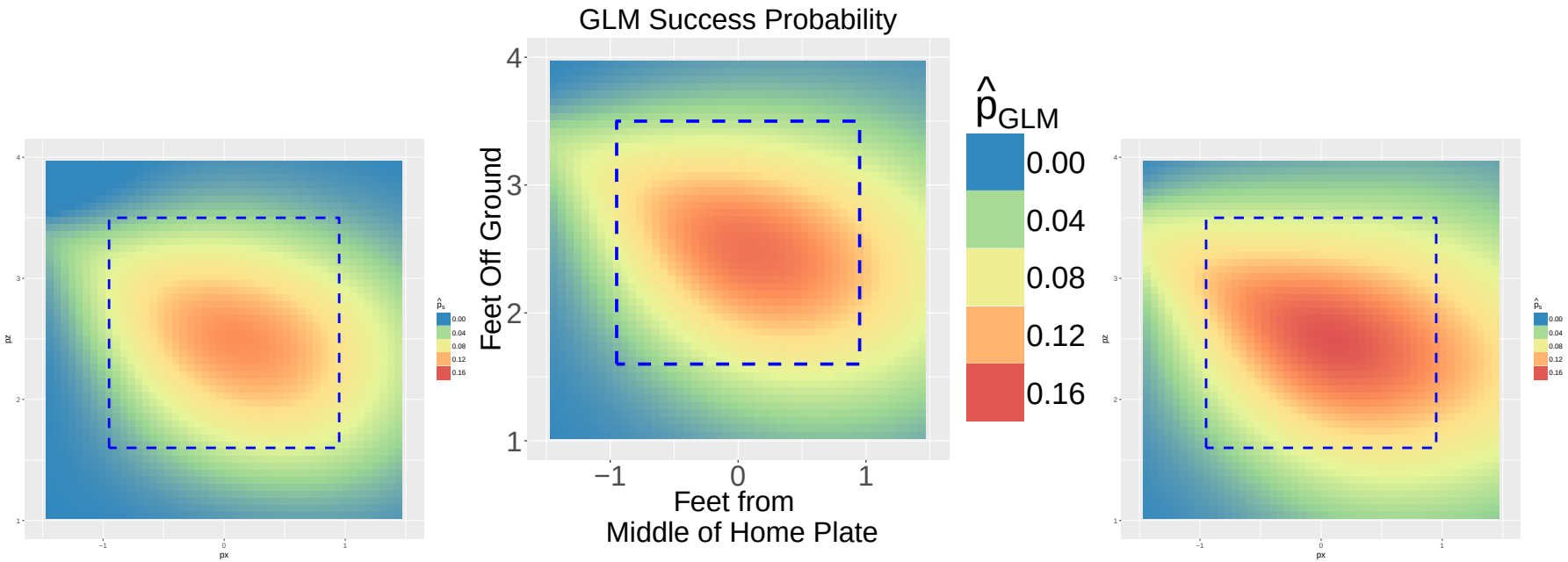
- $\text{logit}(p_{ij}) = \boldsymbol{X}_{ij}\beta$
  - swing i, hitter j



Logistic Regression Model

The State of the Art

- $\text{logit}(p_{ij}) = \boldsymbol{X}_{ij}\beta$



- Think about 1D and 2D CIs

Meet SHINY!!!

Spatial Logistic Regression Model

What about a spatial random effect?

- $\text{logit}(p_{ij}) = \boldsymbol{X}_{ij}\beta + w_{ij}$
- Missing covariates
- Tobler's First Law of Geography
- Unexplained **spatial** variation in the mean

$\rightarrow$ Spatial Generalized Linear Mixed Model (SGLMM)

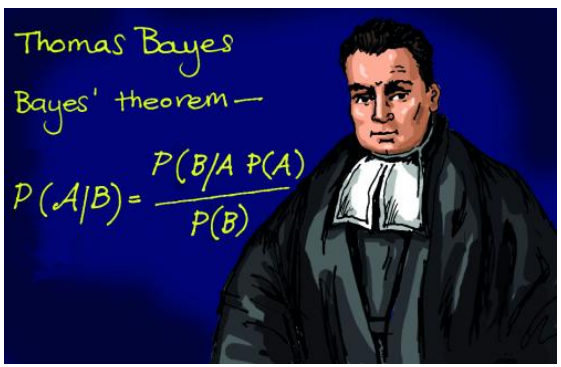
Spatial Generalized Linear Mixed Models

- SGLMM: $\text{logit}(p_j) = \boldsymbol{X}_{ij}\beta + w_{ij}$
- Gaussian Random Field (GRF)
  - $\boldsymbol{w}|\boldsymbol{\theta} \sim MVN(\mathbf{0}, \Sigma(\boldsymbol{\theta}))$
- Covariance function
  - $\Sigma(\phi, \sigma^2)_{i,j} = \sigma^2 exp(-||\boldsymbol{s}_i - \boldsymbol{s}_j||/\phi)$
  - $||s_i - s_j||$ - Euclidean distance between $\boldsymbol{s}_i$ and $\boldsymbol{s}_j$
  - σ^2 - scale parameter
  - ϕ - range parameter.

Challenges

$$\text{logit}(p_{ij}) = \boldsymbol{X}_{ij}\beta + w_{ij}$$

- We do not observe p_{ij}, instead Bernoulli(p_{ij}) trial outcome
- We do not observe n latent w_{ij}
- w_{ij} has a correlation structure, unknown correlation parameters $\rightarrow$ p_{ij} has complicated correlation structure
- Classical methods infeasible
  - Imagine writing, maximizing the likelihood, partial derivatives



Bayesian Hierarchical Model

$$f(\theta|Y) \propto f(Y|\theta)f(\theta)$$

- $Y_{ij}|p_{ij} \sim \text{Bernoulli}(p_{ij})$
- $\text{logit}(p_{ij}) = \boldsymbol{X}_{ij}\beta + w_{ij}$
  - $\beta \sim f_\beta(\beta)$
  - $\boldsymbol{w}_{ij}|(\sigma^2, \phi) \sim \text{MVN}(\mathbf{0}, \Sigma_{\sigma^2,\phi})$
  - $\sigma \sim f_\sigma$, $\phi \sim f_\phi$
- $f(\beta_j, \sigma, \phi|Y_{ij}) \propto f(Y_{ij}|\beta_j, w) \cdot f_\beta(\beta_j) \cdot f(w|\sigma, \phi) \cdot f_\sigma(\sigma) \cdot f_\phi(\phi)$

Great structure, now implementation.

Markov Chain Magic

Markov Chain Monte Carlo (MCMC)

- Most popular Bayesian approach, by far
- Target and update parameters (or groups of) in sequence, procedurally reduced dimension
- Produce **samples from posterior**; no closed form
- It works!
- Can take time, converges asymptotically, correlated draws

There's just one more thing.

One More Big (N) Problem

- GRF $\boldsymbol{w}_j$, recall $n \times n$ correlation matrix
- Recall multivariate normal pdf
  - $n \times n$ correlation matrix $\rightarrow$ invert $n \times n$ matrix
  - Jhony Peralta: 9172×9172!
- Iterative MCMC approach, invert lots of high dimension matrices
- Known as “Big N” problem

How bad can it be? Plus, we have physics, and Stan!!

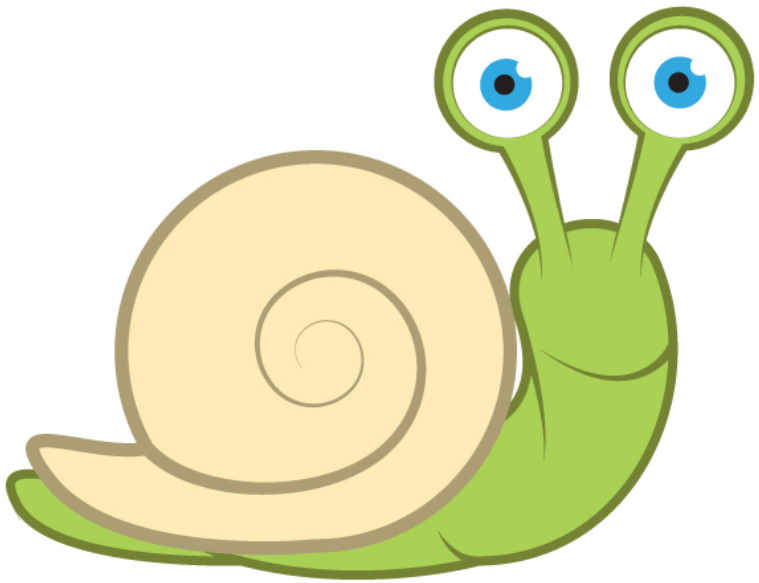
Hamiltonian Markov Chain (HMC) in Stan

- Proposal distribution - key component of MCMC algorithm
- Stan proposal machinery: HMC, from physics (molecular motion)
  - $H(q(t), p(t)) = U(q(t)) + K(p(t))$
  - Total energy of a system
  - t = time
  - **q = position (variables of interest)**, U = potential energy
  - p = momentum (auxiliary), K = kinetic energy
- $H(q, p) = -\log f_q(q) + p^T \boldsymbol{M}^{-1} p/2$
- Tidy partial derivatives
- **Short version**: randomly sample momentum p (auxiliary), calculate new position q — that's your Metropolis proposal.

Stan will be unstoppable!

Slow, Slow, Slow

Meet Stan the snail.



Slow, Slow, Slow

Meet Stan the snail.

How slow? Prohibitively slow. Think geologic time scales; earth is 4.54 billion years old.

Optimization

Give Stan a turbo-boost!

Stan Optimization

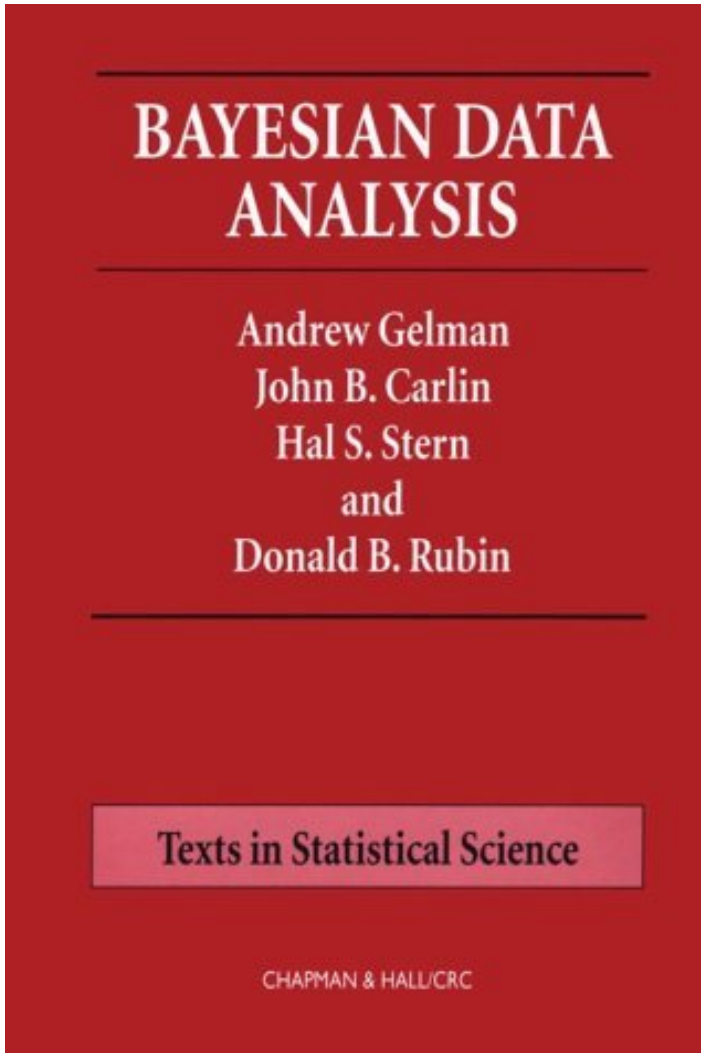
Stan Modeling Language

User's Guide and Reference Manual

Stan Development Team

Stan Version 2.12.0

Tuesday 6th September, 2016



- Stan Development Team Members
  - Andrew Gelman
  - Bob Carpenter
  - Rob Trangucci

Stan Optimization

- Informative (sharp tailed) prior on exponential covariance length-scale parameter ϕ for model identifiability (normal or log-normal)
- Computing time/convergence - “Complex models” such as spatial hierarchical models require proper priors for all β coefficients.
- QR factorization on covariate matrix X; reparameterize, adjust priors

$$
\begin{aligned}
\boldsymbol{X} &= \boldsymbol{QR} \\
\boldsymbol{X}\beta &= \boldsymbol{QR}\beta \rightarrow \text{Let } \boldsymbol{\theta} = \boldsymbol{R}\beta \\
\boldsymbol{X}\beta &= \boldsymbol{Q}\boldsymbol{\theta} \\
\beta &= \boldsymbol{R}^{-1}\boldsymbol{\theta}
\end{aligned}
$$

Stan Optimization

- Add noise to the covariance matrix diagonal, ensure numerical positive-definiteness
- Cholesky decomposition and tactical reparameterization

```
L = cholesky_decompose(Sigma);
Z ~ normal(0, 1);
Z_mod = L * Z;
Y ~ bernoulli_logit(Q*theta + Z_mod);
```

- Matrix algebra and vectors, over 'for loops' and scalars

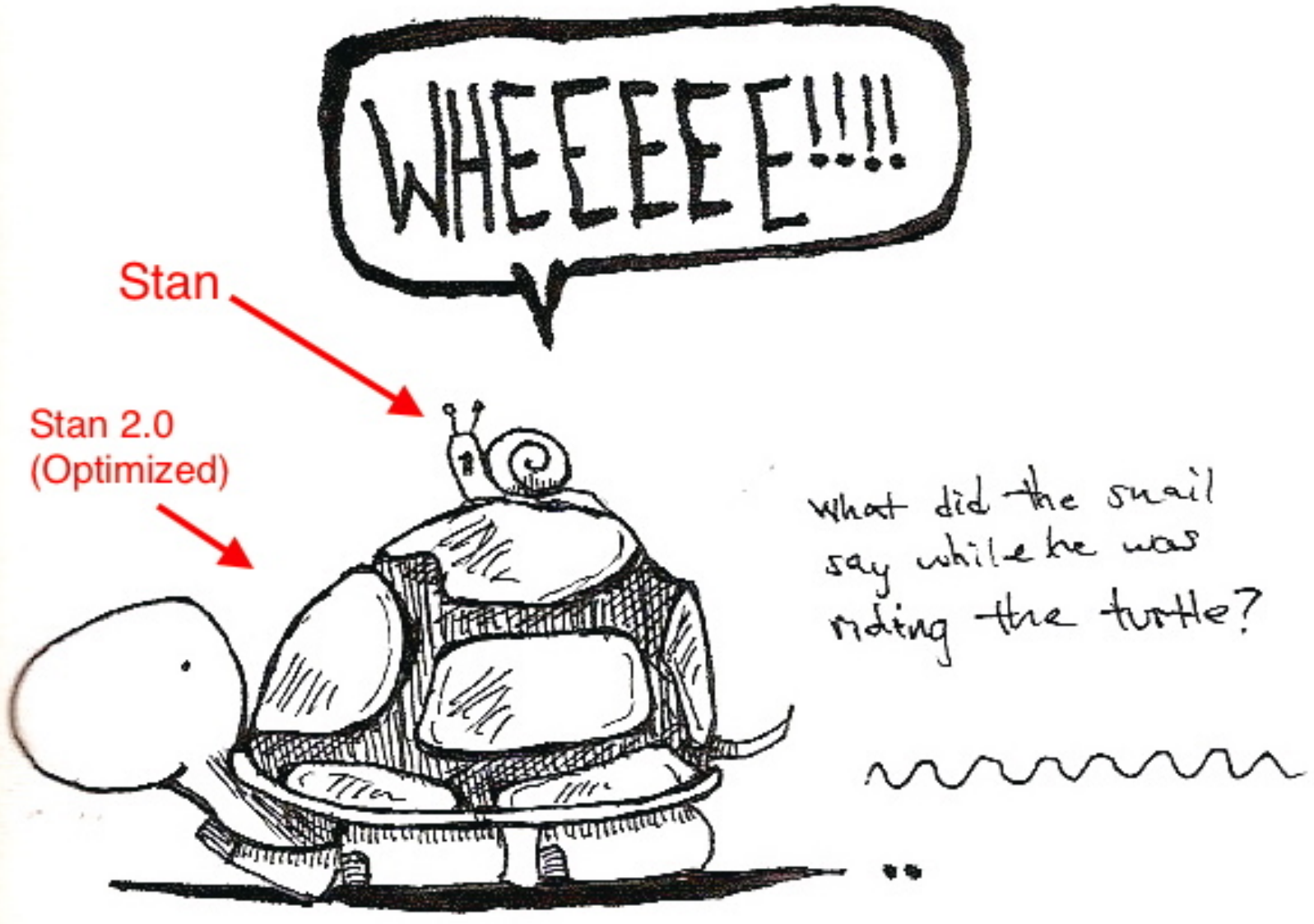
```
hit ~ bernoulli_logit(X*beta + Z)
```

is faster than

```
for (n in 1:N)
    hit[n] = bernoulli_logit(X[n]*beta[n] + Z[n]);
```

On your mark. Get set...

Go Stan 2.0!


WHEEEEE!!!!
Stan
Stan 2.0
(Optimized)
what did the snail
say while he was
riding the turtle?

How Slow is Stan 2.0?

- N = 1000: 7 hours 45 mins for 1500 draws
  - Note: w/o spatial effect: 6 seconds.
- N = 2000 overnight, 350 draws

The “Big N” Problem Revisited

- SGLMM computational costs
  - Increase at a rate of $\mathcal{O}(n^3)$:

$$t(n) \leq M \cdot n^3 \text{ as } n \to \infty$$

  - Spatial random effect covariance matrix Σ is $n \times n$
  - Every iteration requires Σ^{-1}
  - Determinant of Σ
- Computational bottleneck

Persistence

Secret 7. You can do it. A PhD is 10% intelligence and 90% persistence.

Predictive Process Models (PPMs)

Dimension Reduction

NCAR workshop, Andrew Finley

- MSU Deptartments of Forresty and Geography

“Gaussian predictive process models for large spatial data sets” [Banerjee et al., 2008]

- Competitive approach with computational advantages
- Project original process onto lower dimensioned subspace; generate parent process realizations at knots
- “Spatial sampling design for parameter estimation of the covariance function” [Zhu and Stein, 2005]

PPM Procedure

Nuts and Bolts

$$\text{logit}(p_{ij}|\boldsymbol{s}_{ij}) = \boldsymbol{X}_{ij}(\boldsymbol{s}_{ij})\beta_j + w(\boldsymbol{s}_{ij})$$
$$\boldsymbol{w}(\boldsymbol{s})|\boldsymbol{\theta} \sim \text{GRF}(\boldsymbol{0}, \boldsymbol{C}(\boldsymbol{\theta}))$$

- $\boldsymbol{w}(\boldsymbol{s})$ - $n \times 1$ vector of random effects, at locations $\boldsymbol{s}$
- $\boldsymbol{0}$ - $n \times 1$ zero vector
- $\boldsymbol{C}(\boldsymbol{\theta})$ - $n \times n$, symmetric, positive-definite covariance matrix
  - $\boldsymbol{\theta}$ - covariance parameters
- $C(\boldsymbol{s}_i, \boldsymbol{s}_j; \boldsymbol{\theta})$ - covariance of random effects at locations $\boldsymbol{s}_i$ and $\boldsymbol{s}_j$
  - $C(\boldsymbol{\theta}) = [C(\boldsymbol{s}_i, \boldsymbol{s}_j; \boldsymbol{\theta})]_{i,j=1}^{n}$

PPM Knots

- $\boldsymbol{S}^* = \{\boldsymbol{s}_1^*, \ldots, \boldsymbol{s}_m^*\}$ — m knot locations
  - $m << n$
- $\boldsymbol{w}^* = \left[w(\boldsymbol{s}_i^*)\right]_{i=1}^m$ – knot location random effects
- $\boldsymbol{C}^*(\boldsymbol{\theta}) = \left[C(\boldsymbol{s}_i^*, \boldsymbol{s}_j^*)\right]_{i,j=1}^m$ – knot covariance matrix
  - $m \times m$ — much smaller dimension
- $\boldsymbol{w}^* | \boldsymbol{\theta} \sim \text{GRF}\{\boldsymbol{0}, \boldsymbol{C}^*(\boldsymbol{\theta})\}$ – m-dimensional GRF

PPM, Krig $w(\boldsymbol{s}_0)$

- Interpolate $w(\boldsymbol{s}_0)$ using: (i) m knots, (ii) parent process covariance structure, (iii) kriging procedure
- $\tilde{w}(\boldsymbol{s}_0)$ - interpolated random effect

$$\begin{aligned}\tilde{w}(\boldsymbol{s}_0) &= E[w(\boldsymbol{s}_0)|\boldsymbol{w}^*] \\ &= \boldsymbol{c}^T(\boldsymbol{s}_0;\boldsymbol{\theta}) \cdot \boldsymbol{C}^{*-1}(\boldsymbol{\theta}) \cdot \boldsymbol{w}^*\end{aligned}$$

- $\boldsymbol{c}(\boldsymbol{s}_0;\boldsymbol{\theta}) = \left[C(\boldsymbol{s}_0, \boldsymbol{s}_j^*;\boldsymbol{\theta})\right]_{j=1}^{m}$ — $m \times 1$ vector, Cov$(w(\boldsymbol{s}_0),$ knots$)$
- Spatially varying linear combination
- Minimize squared error loss function (linear predictors, GRFs)

Now replace $w(\boldsymbol{s})$ with $\tilde{w}(\boldsymbol{s})$.

The Predictive Process! $\tilde{w}(\boldsymbol{s})$

Another Gaussian Random Field

$$\tilde{\boldsymbol{w}}(\boldsymbol{s}) \sim \text{GRF}\{0, \tilde{C}(\cdot)\}$$
$$\tilde{C}(\boldsymbol{s}, \boldsymbol{s}'; \boldsymbol{\theta}) = \boldsymbol{c}^T(\boldsymbol{s}; \boldsymbol{\theta}) \cdot \boldsymbol{C}^{*-1}(\boldsymbol{\theta}) \cdot \boldsymbol{c}(\boldsymbol{s}'; \boldsymbol{\theta})$$

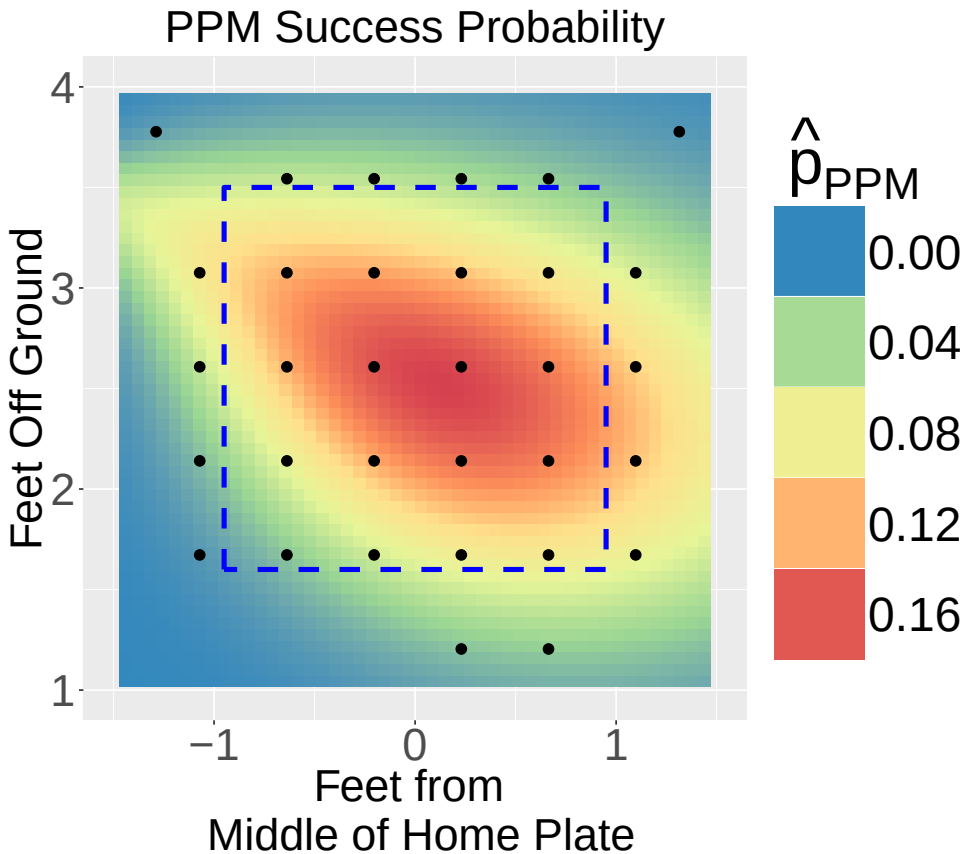
And a predictive process model!

$$\text{logit}(p_{ij}|\boldsymbol{s}_{ij}) = \boldsymbol{X}_{ij}(\boldsymbol{s}_{ij})\beta_j + \tilde{w}(\boldsymbol{s}) \tag{1}$$

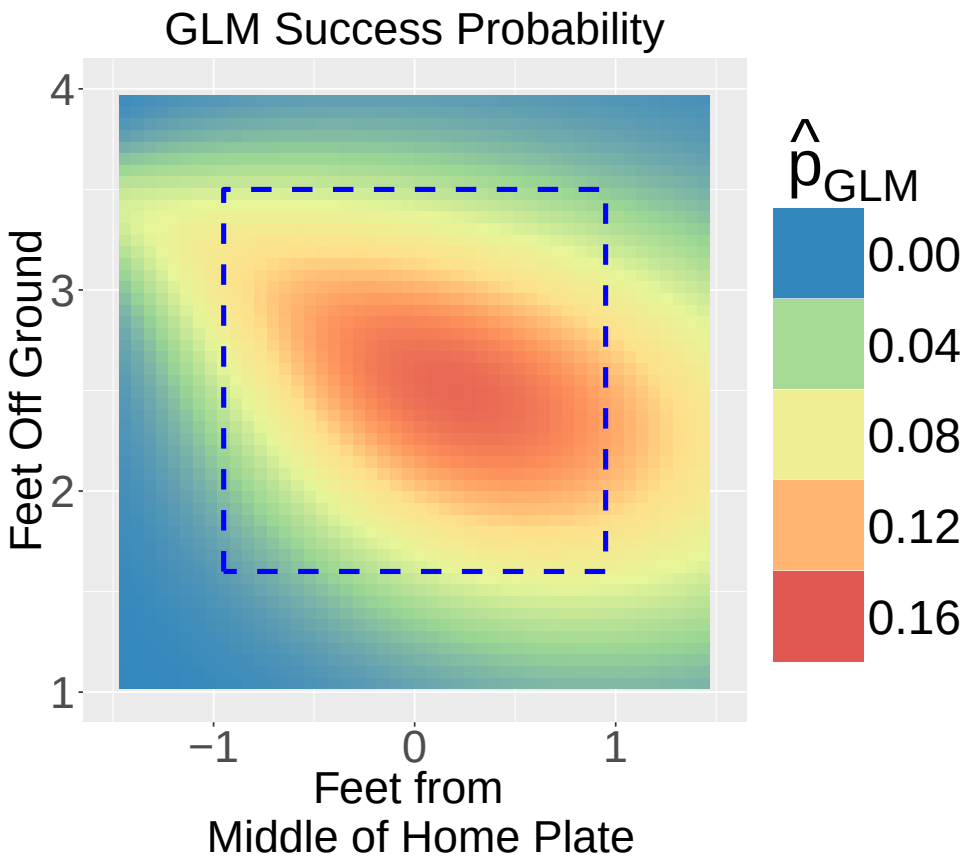
- Note dimension of covariance matrix.
- Implement in `spBayes` [Finley et al., 2007].
- Faster?

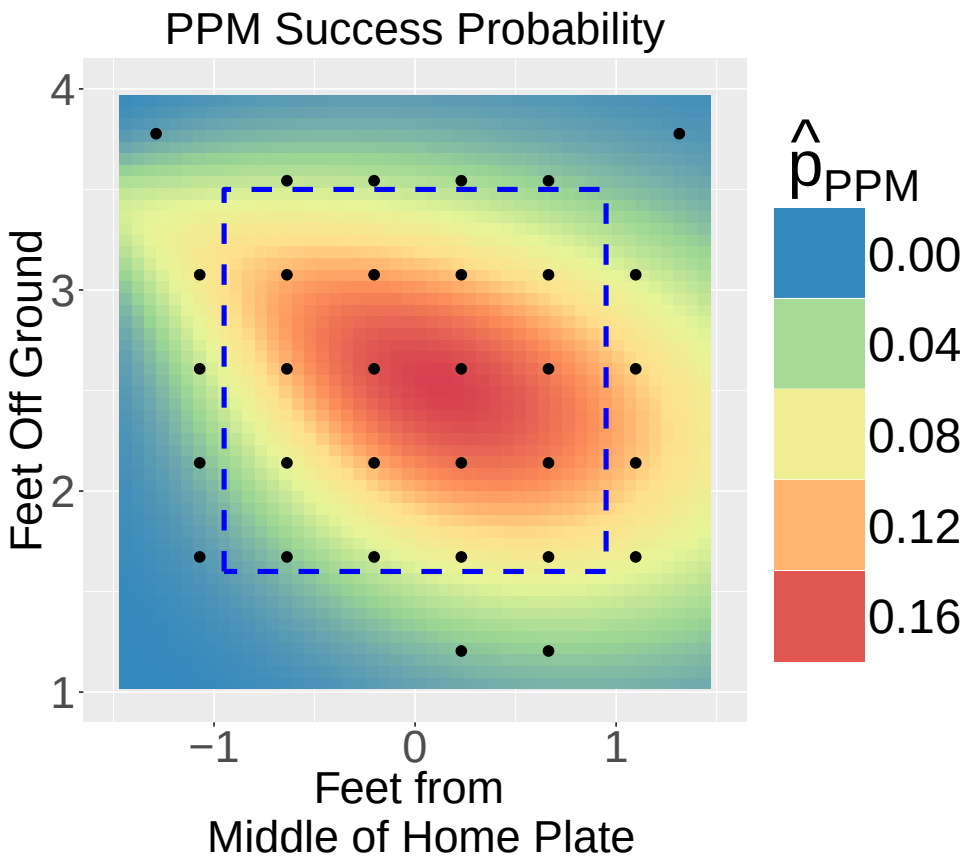
PPMs, Plot and Results

- n = 500, knots = 97, 10K samples, $\approx$ 4 mins
- n = 1000, knots = 97, 10K samples, $\approx$ 6.7 mins
- n = 1000, knots = 49, 30K samples, $\approx$ 7 mins
- n = 3000, knots = 49, 80K samples, $\approx$ 54 mins



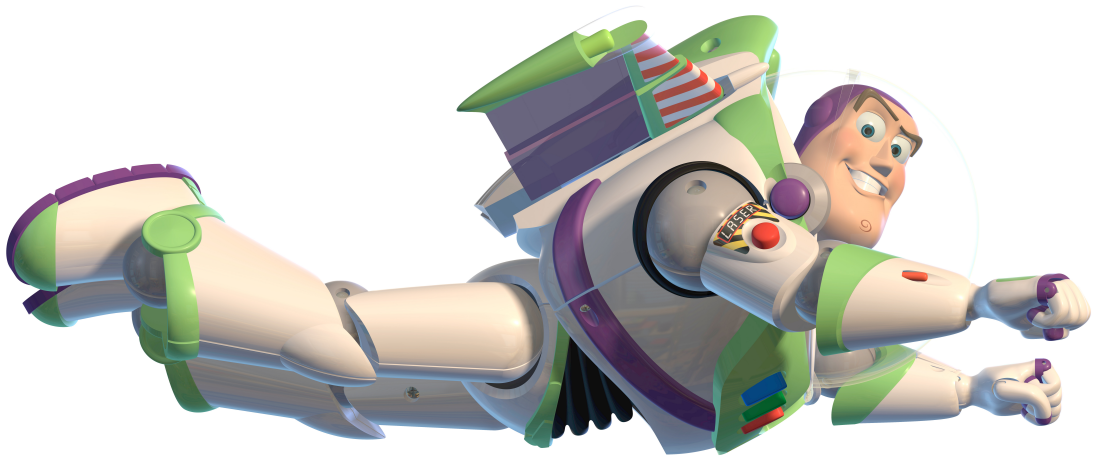
PPM, Plot and Results





Current Work, SPDE-INLA

- What about n = 9172?
- Approximation method, works well for Bayesian hierarchical models with latent GRF
- Essentially a two part process for continuous domain
  - Part 1: SPDE
  - Part 2: INLA
- To n = 9172... and beyond!!



Approximation Part 1, SPDE

SPDE - Stochastic partial differential equation [Lindgren et al., 2011]

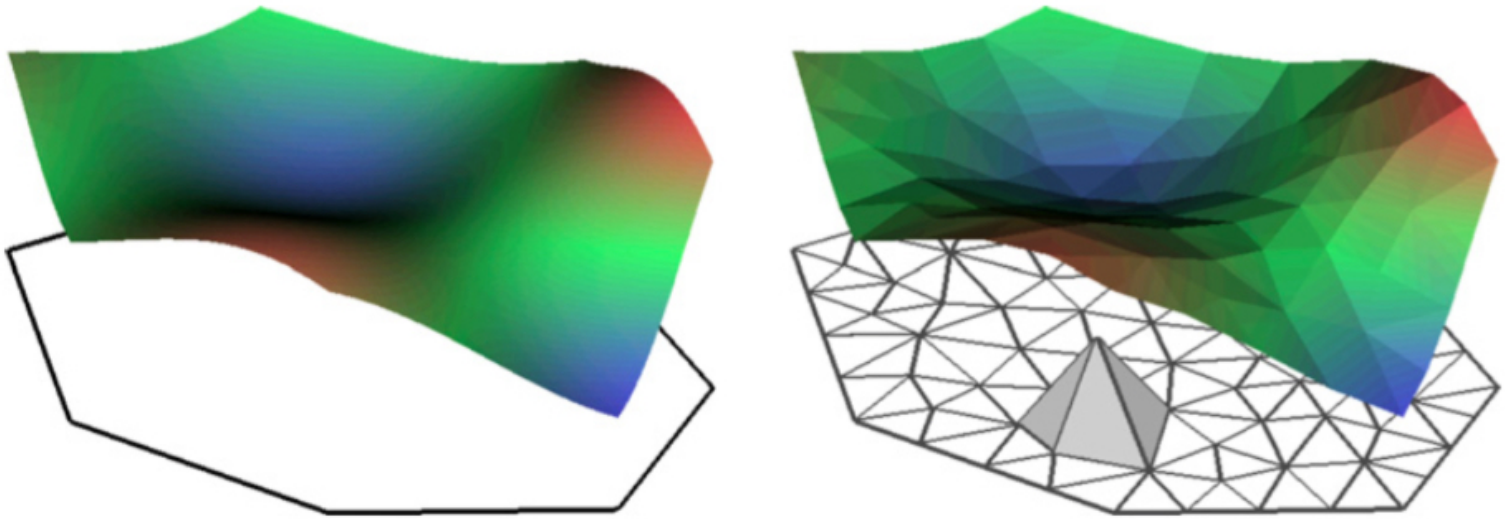
$$(\kappa^2 - \Delta)^{\alpha/2} x(\boldsymbol{s}) = \mathcal{W}(\boldsymbol{s})$$

- Convert GRF to Gaussian **Markov** Random Field (GMRF)
  - Sparse precision matrix
- Finite Element Method: project SPDE onto basis representation
- Triangular mesh, defines piecewise linear basis representation

$$\tilde{x}(\boldsymbol{s}) = \sum_k \psi_k(\boldsymbol{s}) x_k$$

  - $\psi_k(\boldsymbol{s})$ — deterministic basis functions
  - x_k — weights, explicit and sparse precision matrix
  - $\tilde{x}(\boldsymbol{s})$ approximates SPDE solution

SPDE, A Picture Worth 1000 Equations



(a) A continuous function.

(b) A piecewise linear approximation.

[Simpson et al., 2012]

Approximation Part 2, INLA

INLA - Integrated Nested Laplace Approximation [Rue et al., 2009]

1. Gaussian approximation: $\tilde{p}(\boldsymbol{w}|\boldsymbol{\theta}, \boldsymbol{y})$
2. Laplace approximation:

$$p(\boldsymbol{\theta}|\boldsymbol{y}) \propto \frac{p(\boldsymbol{\theta}, \boldsymbol{y}, \boldsymbol{w})}{p(\boldsymbol{w}|\boldsymbol{\theta}, \boldsymbol{y})}\Big|_{\boldsymbol{w}=\boldsymbol{w}^*(\boldsymbol{\theta})} \approx \frac{p(\boldsymbol{\theta}, \boldsymbol{y}, \boldsymbol{w})}{\tilde{p}(\boldsymbol{w}|\boldsymbol{\theta}, \boldsymbol{y})}\Big|_{\boldsymbol{w}=\boldsymbol{w}^*(\boldsymbol{\theta})}$$

   - $\boldsymbol{w}^*(\boldsymbol{\theta}) = \text{argmax}_w \tilde{p}(\boldsymbol{w}|\boldsymbol{\theta}, \boldsymbol{y})$

3. Numerical integration: $p(\theta_k|\boldsymbol{y}) \approx \int \tilde{p}(\boldsymbol{\theta}|\boldsymbol{y}) d\boldsymbol{\theta}_{-k}$
4. Numerical integration: $p(w_j|\boldsymbol{y}) \approx \int \tilde{p}(w_j|\boldsymbol{\theta}, \boldsymbol{y}) \tilde{p}(\boldsymbol{\theta}|\boldsymbol{y}) d\boldsymbol{\theta}$

Current Work, SPDE-INLA

- Implement in R-INLA [Lindgren and Rue, 2015]
- Computational costs - increase at a rate of $\mathcal{O}(n^{3/2})$
- Trade-offs - speed for bias (Binomial data)

Next Steps

- Cool SPDE-INLA pictures
- Model evaluation, scoring rules [Bickel, 2007]
- Cross validation study
- Variable-resolution heat map R package?
- Dynamic Heat Map CIs – R package?

Thanks for listening. Questions?

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