Quantifying Uncertainty

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Outline

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Motivation

Goal

Look out window - is it warm?

Rules

$\forall d\ Sunny(d) \Rightarrow Warm(d)$

Wrong; it can be sunny but cold

$\forall d\ Warm(d) \Rightarrow Sunny(d)$

Wrong; it can be warm and cloudy

Problem

Can't capture that sunny days are usually warm

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Probability Theory

Key Idea

Measure degrees of belief in propositions

Random Variable	An interesting part of the world e.g. *Sky* or *Temp*
Domain	Possible values of a random variable e.g. $domain(Temp) = \langle warm, cold, \ldots \rangle$
Proposition	A statement, like propositional logic e.g. $Sky = sunny \wedge Temp = warm$
Probability	The degree of belief in a proposition e.g. $P(Temp = warm) = 0.7$

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Aside: Fuzzy Logic

Probability Theory

- Propositions are believed to a certain degree
- Belief values range from [0, 1]

Fuzzy Logic

- Propositions are *true* to a certain degree
- *Truth* values range from [0, 1]

For example:

- $T(Tall(Steve)) = 0.5$
- $T(Fat(Steve)) = 0.1$

Better for describing indefinite classes than for reasoning

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Probability Rules

Range of Probabilities

$0 \leq P(a) \leq 1$

Propositions Known to be True or False

$$\begin{aligned} P(true) &= 1 \\ P(false) &= 0 \end{aligned}$$

Probability of Disjunctions

$P(a \vee b) = P(a) + P(b) - P(a \wedge b)$

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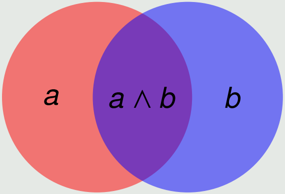

Random Variables

Boolean Random Variables

Have the domain $\langle true, false \rangle$, e.g.

- *IsSunny*

Discrete Random Variables

Have a countable domain, e.g.

- $domain(Sky) = \langle sunny, cloudy, \ldots \rangle$
- $domain(DieRoll) = \langle 1, 2, 3, 4, 5, 6 \rangle$

Continuous Random Variables

Have a real-valued domain, e.g.

- $domain(Temperature) = \langle \ldots, -40^\circ, \ldots, 98.6^\circ, \ldots \rangle$

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Definition

$P(a)$ The unconditional or prior probability of a proposition a is the degree of belief in that proposition *given no other information*

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Examples

$P(DieRoll = 5) \quad =$

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Examples

$$P(DieRoll = 5) = 1/6$$

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Examples

$$
\begin{aligned}
P(\mathit{DieRoll} = 5) &= 1/6 \\
P(\mathit{CardDrawn} = \mathrm{A}\spadesuit) &=
\end{aligned}
$$

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$P(a)$ The unconditional or prior probability of a proposition a is the degree of belief in that proposition *given no other information*

Examples

$$P(DieRoll = 5) = 1/6$$
$$P(CardDrawn = A\spadesuit) = 1/52$$

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Examples

$$
\begin{aligned}
P(\mathit{DieRoll} = 5) &= 1/6 \\
P(\mathit{CardDrawn} = \mathrm{A}\spadesuit) &= 1/52 \\
P(\mathit{SkyInBirmingham} = \mathit{sunny}) &=
\end{aligned}
$$

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$P(a)$ The unconditional or prior probability of a proposition a is the degree of belief in that proposition *given no other information*

Examples

$$
\begin{aligned}
P(\mathit{DieRoll} = 5) &= 1/6 \\
P(\mathit{CardDrawn} = \mathrm{A}\spadesuit) &= 1/52 \\
P(\mathit{SkyInBirmingham} = \mathit{sunny}) &= 210/365
\end{aligned}
$$

Joint Probability

Definition

$P(a_1, \ldots, a_n)$ The joint probability of propositions $a_1, \ldots, a_n$ is the degree of belief in the proposition $a_1 \wedge \ldots \wedge a_n$

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Example

$$
\begin{aligned}
& P(\mathit{DieRoll}_1 = 4, \mathit{DieRoll}_2 = 6) \\
& \quad = \quad P(\mathit{DieRoll}_1 = 4 \wedge \mathit{DieRoll}_2 = 6) \\
& \quad =
\end{aligned}
$$

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Example

$$
\begin{aligned}
& P(\mathit{DieRoll}_1 = 4, \mathit{DieRoll}_2 = 6) \\
& \quad = \quad P(\mathit{DieRoll}_1 = 4 \wedge \mathit{DieRoll}_2 = 6) \\
& \quad = \quad 1/36
\end{aligned}
$$

Conditional Probability

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$P(a|b)$ The posterior or conditional probability of a proposition a given a proposition b is the degree of belief in a, *given that we know only* b

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$$P(Card = \text{A}\spadesuit | CardSuit = \spadesuit) \quad =$$

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Examples

$$P(Card = \text{A}\spadesuit | CardSuit = \spadesuit) = 1/13$$

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Examples

$$
\begin{aligned}
P(Card = \mathrm{A}\spadesuit | CardSuit = \spadesuit) &= 1/13 \\
P(DieRoll_2 = 5 | DieRoll_1 = 5) &=
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Examples

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\begin{aligned}
P(\mathit{Card} = \text{A}\spadesuit | \mathit{CardSuit} = \spadesuit) &= 1/13 \\
P(\mathit{DieRoll}_2 = 5 | \mathit{DieRoll}_1 = 5) &= 1/6
\end{aligned}
$$

Prior, Joint or Conditional?

1. Probability of a having a cavity?
   Prior, $P(Cavity = true)$
2. Probability of it being warm and cloudy?
   Joint, $P(Temp = warm, Sky = cloudy)$
3. Probability of car being stolen?
   Prior, $P(CarStolen = true)$
4. Probability of car being stolen and being in Alabama?
   Joint, $P(CarStolen = true, InAlabama = true)$
5. The car is in Alabama. Probability of car being stolen?
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Relation between Joint and Conditional

Product Rule

$P(a \wedge b) = P(a|b)P(b)$ or $P(a|b) = P(a \wedge b)/P(b)$

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Example

$$P(\mathrm{A} \wedge \spadesuit) \quad =$$

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Example

$$P(\mathrm{A} \wedge \spadesuit) = 1/52$$

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\begin{aligned}
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P(\mathrm{A}|\spadesuit) &=
\end{aligned}
$$

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\end{aligned}
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P(\spadesuit) &=
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Example

$$
\begin{aligned}
P(\mathrm{A} \wedge \spadesuit) &= 1/52 \\
P(\mathrm{A}|\spadesuit) &= 1/13 \\
P(\spadesuit) &= 1/4
\end{aligned}
$$

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$$
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P(\spadesuit) &= 1/4 \\
P(\mathrm{A}|\spadesuit)P(\spadesuit) &=
\end{aligned}
$$

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Example

$$
\begin{aligned}
P(\mathrm{A} \wedge \spadesuit) &= 1/52 \\
P(\mathrm{A}|\spadesuit) &= 1/13 \\
P(\spadesuit) &= 1/4 \\
P(\mathrm{A}|\spadesuit)P(\spadesuit) &= 1/13 \cdot 1/4 = 1/52
\end{aligned}
$$

Chain Rule

Key Ideas

- Repeatedly apply product rule, $P(a, b) = P(a|b)P(b)$
- Joint probability $\rightarrow$ conditional probabilities

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Example

$P(sunny, dry, warm)$

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Key Ideas

- Repeatedly apply product rule, $P(a, b) = P(a|b)P(b)$
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Example

$$P(sunny, dry, warm) = P(sunny|dry, warm)P(dry, warm)$$

Chain Rule

Key Ideas

- Repeatedly apply product rule, $P(a, b) = P(a|b)P(b)$
- Joint probability $\rightarrow$ conditional probabilities

Example

$$\begin{aligned} & P(sunny, dry, warm) \\ &= P(sunny|dry, warm)P(dry, warm) \\ &= P(sunny|dry, warm)P(dry|warm)P(warm) \end{aligned}$$

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Probability Distributions

Definition

$\mathbf{P}(X)$ The probability distribution of a random variable X is a list of probabilities for each domain value

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Example

$\mathbf{P}(\mathit{SkyInBirmingham}) = \langle 210/365, 155/365 \rangle$ means

$$
\begin{aligned}
P(\mathit{SkyInBirmingham} = \mathit{sunny}) &= 210/365 \\
P(\mathit{SkyInBirmingham} = \mathit{cloudy}) &= 155/365
\end{aligned}
$$

Probability Distributions

Definition

$\mathbf{P}(X)$ The probability distribution of a random variable X is a list of probabilities for each domain value

Example

$\mathbf{P}(\mathit{SkyInBirmingham}) = \langle 210/365, 155/365 \rangle$ means

$$\begin{aligned} P(\mathit{SkyInBirmingham} = \mathit{sunny}) &= 210/365 \\ P(\mathit{SkyInBirmingham} = \mathit{cloudy}) &= 155/365 \end{aligned}$$

Notation Warning

- $P(a)$ or $P(X = a)$ means prior probability
- $\mathbf{P}(X)$ means probability distribution

Probability Distributions

Key Idea

$$\sum_{i}^{d} P(X = X_i) = 1$$

The sum of the probabilities for all possible value assignments of the random variable is always 1

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$$\sum_x P(Die = x)$$

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$$\begin{aligned}\sum_{x} P(Die = x) \\ = \quad & P(Die = 1) + P(Die = 2) + P(Die = 3) + \\ & P(Die = 4) + P(Die = 5) + P(Die = 6)\end{aligned}$$

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\sum_{x} P(Die = x) \\
&= P(Die = 1) + P(Die = 2) + P(Die = 3) + \\
&\quad P(Die = 4) + P(Die = 5) + P(Die = 6) \\
&= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}
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&= 1
\end{aligned}$$

Continuous Variable Probability Distributions

Definition

A probability density function is a probability distribution over a continuous variable

Key Idea

Function assigns a probability to all possible values

Example

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

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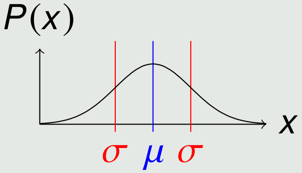

Joint Probability Distributions

Definition

$\mathbf{P}(X_1, \ldots, X_n)$

The joint probability distribution of random variables $X_1, \ldots, X_n$ is a table of probabilities for each combination of values in the variable domains

Example

$\mathbf{P}(Sex, Smoker) =$

	$Sex = male$	$Sex = female$
$Smoker = true$	0.113	0.107
$Smoker = false$	0.377	0.403

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& P(Sex = female, Smoker = false) \\
= \;& 0.377 + 0.107 + 0.403 = 0.887
\end{aligned}
$$

Marginalization

Key Idea

$$\mathbf{P}(Y) = \sum_z \mathbf{P}(Y, z)$$

Marginalization removes all variables but $\mathbf{Y}$ by summing over the values of the other variables

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$$\begin{aligned} P(Sex = female) & \\ = \; & P(Sex = female, Smoker = true) + \\ & P(Sex = female, Smoker = false) \\ = \; & 0.107 + 0.403 = 0.51 \end{aligned}$$

Normalization

Key Idea

$$\mathbf{P}(Y|z) = \frac{\mathbf{P}(Y,z)}{P(z)} = \frac{\mathbf{P}(Y,z)}{\sum_y P(y,z)} = \alpha\mathbf{P}(Y,z)$$

Normalization

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Calculating a Normalizing Constant

$$\begin{aligned} P(\spadesuit, \mathrm{A}) &= \frac{1}{52} \\ P(\clubsuit, \mathrm{A}) &= \frac{1}{52} \\ P(\diamondsuit, \mathrm{A}) &= \frac{1}{52} \\ P(\heartsuit, \mathrm{A}) &= \frac{1}{52} \end{aligned}$$

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$$\mathbf{P}(Suit|\mathrm{A}) = \langle \frac{13}{52}, \frac{13}{52}, \frac{13}{52}, \frac{13}{52} \rangle = \langle \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4} \rangle$$

Inference Exercises

$sunny$	$warm$	$snow$	0.01
$sunny$	$warm$	$\neg snow$	0.59
$sunny$	$cold$	$snow$	0.08
$sunny$	$cold$	$\neg snow$	0.14
$cloudy$	$warm$	$snow$	0.03
$cloudy$	$warm$	$\neg snow$	0.07
$cloudy$	$cold$	$snow$	0.06
$cloudy$	$cold$	$\neg snow$	0.02

1. $P(sunny) =$
2. $P(warm) =$
3. $P(snow) =$
4. $P(sunny \vee \neg snow) =$
5. $P(sunny | snow) =$
6. $P(snow | sunny, cold) =$

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1. $P(sunny) = 0.82$
2. $P(warm) = 0.70$
3. $P(snow) = 0.18$
4. $P(sunny \vee \neg snow) = 0.91$
5. $P(sunny|snow) = 0.50$
6. $P(snow|sunny, cold) = 0.36$

Joint Distribution Inference

Properties

Given n random variables with maximum domain size d:

Time Complexity? $O(d^n)$

Space Complexity? $O(d^n)$

Biggest Problem

How do you fill in a table of $O(d^n)$ probabilities?

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Outline

Independence

Key Ideas

- Sometimes no connection exists between variables
- Such independence determined by world knowledge

Formal Independence

$P(a, b) = P(a)P(b)$ or $P(a|b) = P(a)$

Examples

$$
\begin{aligned}
P(EyeColor, Gender) &= P(EyeColor)P(Gender) \\
P(Cavity, BlazersWon) &= P(Cavity)P(BlazersWon) \\
P(DieRoll_1, DieRoll_2) &= P(DieRoll_1)P(DieRoll_2)
\end{aligned}
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Independence

Using Independence

- No need to store entire joint probability table
- Can store several smaller independent tables

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Table Size

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Table Size

$\mathbf{P}(DieRoll_1, DieRoll_2)$

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Examples

	Table Size
$\mathbf{P}(DieRoll_1, DieRoll_2)$	$6 \cdot 6 = 36$

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$\mathbf{P}(DieRoll_1, DieRoll_2)$	$6 \cdot 6 = 36$
$\mathbf{P}(DieRoll_1)\mathbf{P}(DieRoll_2)$	

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	Table Size
$\mathbf{P}(DieRoll_1, DieRoll_2)$	$6 \cdot 6 = 36$
$\mathbf{P}(DieRoll_1)\mathbf{P}(DieRoll_2)$	$6 + 6 = 12$

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$\mathbf{P}(DieRoll_1, DieRoll_2)$	$6 \cdot 6 = 36$
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$\mathbf{P}(Age, Sex)\mathbf{P}(BlazersWon)$	$125 \cdot 2 + 2 = 252$

Conditional Independence

Key Idea

Two variables can sometimes become independent after the value of a third variable is observed

Definition

A random variable X is conditionally independent of random variable Y given the random variable Z if:

$$\mathbf{P}(X, Y|Z) = \mathbf{P}(X|Z)\mathbf{P}(Y|Z)$$

or

$$\mathbf{P}(X|Y, Z) = \mathbf{P}(X|Z)$$

Conditional Independence Example

Is *GrayHair* indpendent of *Bifocals*?

No, we expect the two to often come together, e.g.:

$P(\textit{gray-hair}, \textit{bifocals}) > P(\textit{gray-hair})P(\textit{bifocals})$

Is *GreyHair* independent of *Bifocals* given *Age*?

Yes, the bifocals add nothing if we know the age, e.g.:

$P(\textit{gray-hair}|\textit{bifocals}, \textit{Age}=x) = P(\textit{gray-hair}|\textit{Age}=x)$

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Conditional Independence Example

Noisy Phone

- Adam calls Betty and Charlie and says a number N_A
- Betty hears N_B and Charlie hears N_C

Are N_B and N_C independent?

No, we expect the numbers to be similar, e.g.:

$$P(N_B = 1, N_C = 1) > P(N_B = 1)P(N_C = 1)$$

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Bayes' Rule

Key Idea

Swap the conditioned and conditioning variables

Bayes' Rule

$$P(b|a) = \frac{P(a|b)P(b)}{P(a)}$$

Derivation

$$\begin{aligned} P(b|a) &= \frac{P(a \wedge b)}{P(a)} && \text{Definition} \\ &= \frac{P(a|b)P(b)}{P(a)} && \text{Product Rule} \end{aligned}$$

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Why Use Bayes' Rule?

Making Diagnoses Based on Causal Knowledge

$$P(Cause|Effect) = \frac{P(Effect|Cause)P(Cause)}{P(Effect)}$$

Example

$$P(meningitis|stiff\text{-}neck) = \frac{P(stiff\text{-}neck|meningitis)P(meningitis)}{P(stiff\text{-}neck)}$$

In an epidemic, where $P(meningitis)$ rises:

+Bayes $P(meningitis|stiff\text{-}neck)$ rises proportionally

-Bayes Collect data, re-estimate $P(meningitis|stiff\text{-}neck)$

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-Bayes Collect data, re-estimate $P(meningitis|stiff\text{-}neck)$

Bayes' Rule & Conditional Independence

Deriving More Manageable Models

$P(Cavity|toothache, catch)$

Bayes' Rule & Conditional Independence

Deriving More Manageable Models

$$\begin{aligned}
&\mathsf{P}(Cavity|toothache, catch) \\
&\quad = \alpha \mathsf{P}(toothache, catch|Cavity)\mathsf{P}(Cavity)
\end{aligned}$$

Bayes' Rule & Conditional Independence

Deriving More Manageable Models

$$\begin{aligned} &\mathsf{P}(Cavity|toothache, catch) \\ &\quad = \alpha \mathsf{P}(toothache, catch|Cavity)\mathsf{P}(Cavity) \\ &\quad = \alpha \mathsf{P}(toothache|Cavity)\mathsf{P}(catch|Cavity)\mathsf{P}(Cavity) \end{aligned}$$

Bayes' Rule & Conditional Independence

Deriving More Manageable Models

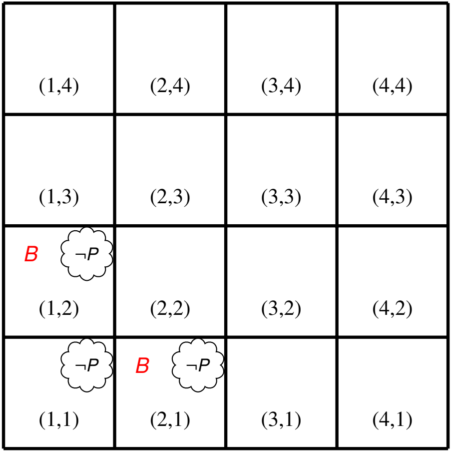
$$
\begin{aligned}
&P(Cavity|toothache, catch) \\
&\quad = \alpha P(toothache, catch|Cavity)P(Cavity) \\
&\quad = \alpha P(toothache|Cavity)P(catch|Cavity)P(Cavity)
\end{aligned}
$$

Naive Bayes Models

$P(Cause, Effect_1, \ldots, Effect_n) = P(Cause) \prod_i P(Effect_i|Cause)$

- All effects conditionally independent given cause
- Common class of machine learning models

Wumpus World Example



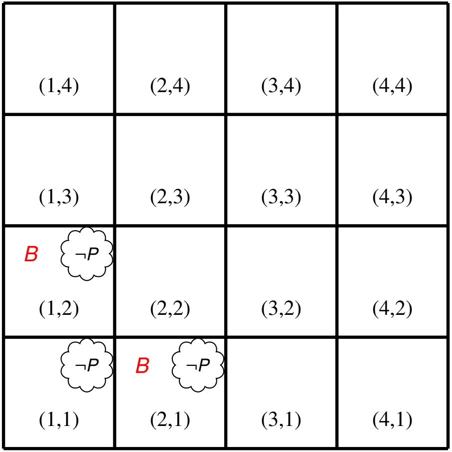
Query

$P_{1,3}$?

Brute Force Approach

- Calculate full joint distribution
- Sum all rows where: $p_{1,3}, \neg p_{1,1}, \neg p_{1,2}, \neg p_{2,1}, b_{1,2}, b_{2,1}$
- Result is $P(p_{1,3})$

Wumpus World Example



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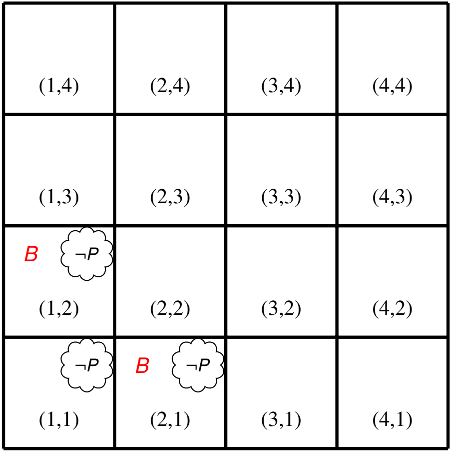

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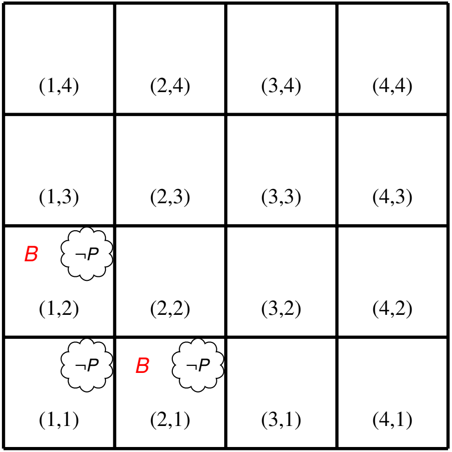

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Wumpus World Brute Force

(1,4)	(2,4)	(3,4)	(4,4)
(1,3)	(2,3)	(3,3)	(4,3)
(1,2)	(2,2)	(3,2)	(4,2)
(1,1)	(2,1)	(3,1)	(4,1)

Full Joint Distribution

$\mathbf{P}(P_{11}, \ldots, P_{44}, B_{11}, \ldots, B_{44})$

Total Rows?

$2^{16+16} = 2^{32}$

Rows Selected?

Assume:

$p_{13}, \neg p_{11}, \neg p_{12}, \neg p_{21}, b_{12}, b_{21}$

$2^{12+14} = 2^{28}$

There must be a better way!

Wumpus World Brute Force

(1,4)	(2,4)	(3,4)	(4,4)
(1,3)	(2,3)	(3,3)	(4,3)
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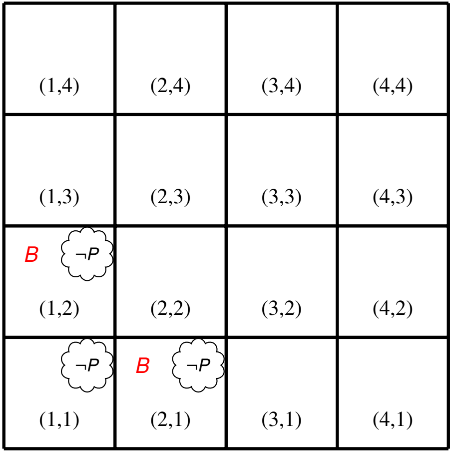
Wumpus World with Independence



Key Idea

Observations conditionally independent of other squares given neighbors

Goal

Manipulate equation until we can apply conditional independence formula

$$\mathbf{P}(b_{12}, b_{21} | \neg p_{11}, \neg p_{12}, \neg p_{21}, P_{13}, P_{fringe}, P_{other}) =$$
$$\mathbf{P}(b_{12}, b_{21} | \neg p_{11}, \neg p_{12}, \neg p_{21}, P_{13}, P_{fringe})$$

Wumpus World with Independence

Other (1,4)	Other (2,4)	Other (3,4)	Other (4,4)
Query (1,3)	Other (2,3)	Other (3,3)	Other (4,3)
B $\neg P$ (1,2)	Fringe (2,2)	Other (3,2)	Other (4,2)
$\neg P$ (1,1)	B $\neg P$ (2,1)	Fringe (3,1)	Other (4,1)

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Wumpus World Equation Manipulation

$P(p_{13}|\neg p_{11}, \neg p_{12}, \neg p_{21}, b_{12}, b_{21})$

Wumpus World Equation Manipulation

$$
\begin{aligned}
& P(p_{13} | \neg p_{11}, \neg p_{12}, \neg p_{21}, b_{12}, b_{21}) \\
& \quad = \quad \alpha P(p_{13}, \neg p_{11}, \neg p_{12}, \neg p_{21}, b_{12}, b_{21})
\end{aligned}
$$

Wumpus World Equation Manipulation

$$
\begin{aligned}
& P(p_{13} | \neg p_{11}, \neg p_{12}, \neg p_{21}, b_{12}, b_{21}) \\
& \quad = \quad \alpha P(p_{13}, \neg p_{11}, \neg p_{12}, \neg p_{21}, b_{12}, b_{21}) \\
& \quad = \quad \alpha P(p_{13}, p_{known}, b_{12}, b_{21})
\end{aligned}
$$

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$$\begin{aligned}
&P(p_{13}|\neg p_{11}, \neg p_{12}, \neg p_{21}, b_{12}, b_{21}) \\
&\quad = \alpha P(p_{13}, \neg p_{11}, \neg p_{12}, \neg p_{21}, b_{12}, b_{21}) \\
&\quad = \alpha P(p_{13}, p_{known}, b_{12}, b_{21}) \\
&\quad = \alpha \sum_{fringe+other} P(p_{13}, p_{known}, p_{fringe+other}, b_{12}, b_{21})
\end{aligned}$$

Wumpus World Equation Manipulation

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&P(p_{13} | \neg p_{11}, \neg p_{12}, \neg p_{21}, b_{12}, b_{21}) \\
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&\quad = \alpha \sum_{fringe} \sum_{other} P(p_{13}, p_{known}, p_{fringe}, p_{other}, b_{12}, b_{21})
\end{aligned}
$$

Wumpus World Equation Manipulation

$$
\begin{aligned}
&P(p_{13} \mid \neg p_{11}, \neg p_{12}, \neg p_{21}, b_{12}, b_{21}) \\
&\quad = \alpha P(p_{13}, \neg p_{11}, \neg p_{12}, \neg p_{21}, b_{12}, b_{21}) \\
&\quad = \alpha P(p_{13}, p_{known}, b_{12}, b_{21}) \\
&\quad = \alpha \sum_{fringe+other} P(p_{13}, p_{known}, p_{fringe+other}, b_{12}, b_{21}) \\
&\quad = \alpha \sum_{fringe} \sum_{other} P(p_{13}, p_{known}, p_{fringe}, p_{other}, b_{12}, b_{21}) \\
&\quad = \alpha \sum_{fringe} \sum_{other} P(b_{12}, b_{21} \mid p_{13}, p_{known}, p_{fringe}, p_{other}) P(p_{13}, p_{known}, p_{fringe}, p_{other})
\end{aligned}
$$

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$$
\begin{aligned}
& P(p_{13} | \neg p_{11}, \neg p_{12}, \neg p_{21}, b_{12}, b_{21}) \\
& \quad = \alpha P(p_{13}, \neg p_{11}, \neg p_{12}, \neg p_{21}, b_{12}, b_{21}) \\
& \quad = \alpha P(p_{13}, p_{known}, b_{12}, b_{21}) \\
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\end{aligned}
$$

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& P(p_{13} | \neg p_{11}, \neg p_{12}, \neg p_{21}, b_{12}, b_{21}) \\
& \quad = \alpha P(p_{13}, \neg p_{11}, \neg p_{12}, \neg p_{21}, b_{12}, b_{21}) \\
& \quad = \alpha P(p_{13}, p_{known}, b_{12}, b_{21}) \\
& \quad = \alpha \sum_{fringe+other} P(p_{13}, p_{known}, p_{fringe+other}, b_{12}, b_{21}) \\
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& \quad = \alpha \sum_{fringe} \sum_{other} P(b_{12}, b_{21} | p_{13}, p_{known}, p_{fringe}) P(p_{13}) P(p_{known}) P(p_{fringe}) P(p_{other})
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& \quad = \quad \alpha P(p_{13}, \neg p_{11}, \neg p_{12}, \neg p_{21}, b_{12}, b_{21}) \\
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& \quad = \quad \alpha \sum_{fringe} \sum_{other} P(b_{12}, b_{21}|p_{13}, p_{known}, p_{fringe}) P(p_{13}, p_{known}, p_{fringe}, p_{other}) \\
& \quad = \quad \alpha \sum_{fringe} \sum_{other} P(b_{12}, b_{21}|p_{13}, p_{known}, p_{fringe}) P(p_{13}) P(p_{known}) P(p_{fringe}) P(p_{other}) \\
& \quad = \quad \alpha \sum_{fringe} P(b_{12}, b_{21}|p_{13}, p_{known}, p_{fringe}) P(p_{13}) P(p_{known}) P(p_{fringe}) \sum_{other} P(p_{other})
\end{aligned}
$$

Wumpus World Equation Manipulation

$$
\begin{aligned}
& P(p_{13}|\neg p_{11}, \neg p_{12}, \neg p_{21}, b_{12}, b_{21}) \\
& = \alpha P(p_{13}, \neg p_{11}, \neg p_{12}, \neg p_{21}, b_{12}, b_{21}) \\
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& \quad = \alpha P(p_{13}) P(p_{known}) \sum_{fringe} P(b_{12}, b_{21} | p_{13}, p_{known}, p_{fringe}) P(p_{fringe})
\end{aligned}
$$

Wumpus World Probability Calculation

Goal: Calculate the Summation

$$\sum_{p_{22},p_{31}} P(b_{12}, b_{21} | p_{13}, \neg p_{11}, \neg p_{12}, \neg p_{21}, P_{22}, P_{31}) P(P_{22}, P_{31})$$

Key Ideas

- First term is 1 if observed breezes are beside pits
- First term is 0 if observed breezes are not beside pits
- So find worlds consistent with observations
- And sum $P(P_{22}, P_{31})$ over these worlds

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Wumpus World Probability Calculation

Worlds Consistent with Observations

P		
B *¬P*	**P**	
¬P	*B* *¬P*	**P**

P		
B *¬P*	**P**	
¬P	*B* *¬P*	**¬P**

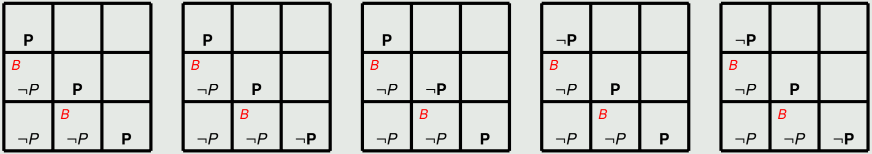
P		
B *¬P*	**¬P**	
¬P	*B* *¬P*	**P**

¬P		
B *¬P*	**P**	
¬P	*B* *¬P*	**P**

¬P		
B *¬P*	**P**	
¬P	*B* *¬P*	**¬P**

Wumpus World Probability Calculation

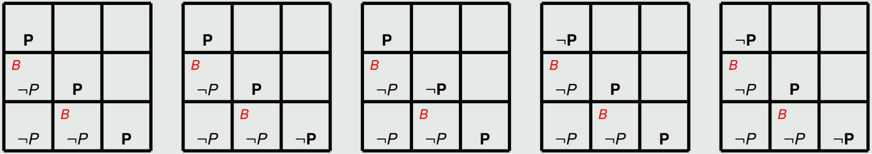
Worlds Consistent with Observations



Calculating $P(P_{22}, P_{31})$, assuming pit probability is 0.2

Wumpus World Probability Calculation

Worlds Consistent with Observations



$.2 \times .2 = .04$

Calculating $P(P_{22}, P_{31})$, assuming pit probability is 0.2

Wumpus World Probability Calculation

Worlds Consistent with Observations

P		
B ¬*P*	**P**	
¬*P*	*B* ¬*P*	**P**

$.2 \times .2 = .04$

P		
B ¬*P*	**P**	
¬*P*	*B* ¬*P*	¬**P**

$.2 \times .8 = .16$

P		
B ¬*P*	¬**P**	
¬*P*	*B* ¬*P*	**P**

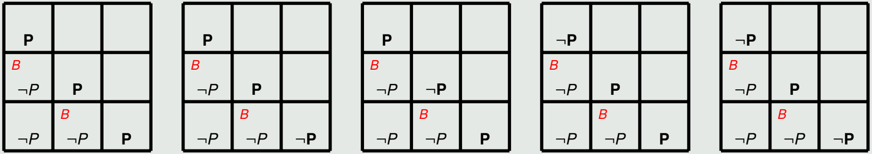
¬**P**		
B ¬*P*	**P**	
¬*P*	*B* ¬*P*	**P**

¬**P**		
B ¬*P*	**P**	
¬*P*	*B* ¬*P*	¬**P**

Calculating $P(P_{22}, P_{31})$, assuming pit probability is 0.2

Wumpus World Probability Calculation

Worlds Consistent with Observations



$.2 \times .2 = .04$ $.2 \times .8 = .16$ $.8 \times .2 = .16$

Calculating $P(P_{22}, P_{31})$, assuming pit probability is 0.2

Wumpus World Probability Calculation

Worlds Consistent with Observations

P		
B $\neg P$	**P**	
$\neg P$	*B* $\neg P$	**P**

P		
B $\neg P$	**P**	
$\neg P$	*B* $\neg P$	**¬P**

P		
B $\neg P$	**¬P**	
$\neg P$	*B* $\neg P$	**P**

¬P		
B $\neg P$	**P**	
$\neg P$	*B* $\neg P$	**P**

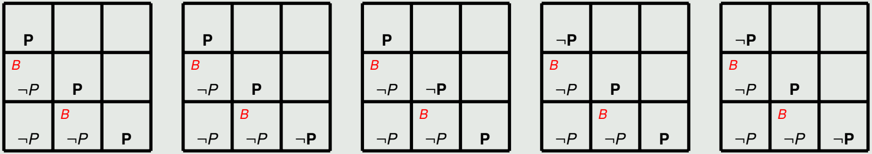
¬P		
B $\neg P$	**P**	
$\neg P$	*B* $\neg P$	**¬P**

$.2 \times .2 = .04$ $.2 \times .8 = .16$ $.8 \times .2 = .16$ $.2 \times .2 = .04$

Calculating $P(P_{22}, P_{31})$, assuming pit probability is 0.2

Wumpus World Probability Calculation

Worlds Consistent with Observations

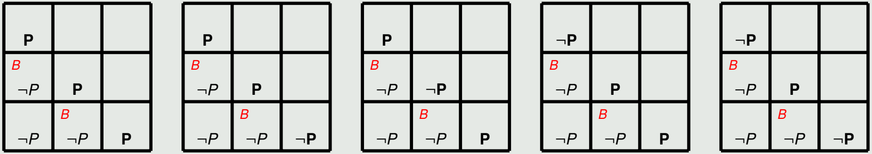


$.2 \times .2 = .04$ $.2 \times .8 = .16$ $.8 \times .2 = .16$ $.2 \times .2 = .04$ $.2 \times .8 = .16$

Calculating $P(P_{22}, P_{31})$, assuming pit probability is 0.2

Wumpus World Probability Calculation

Worlds Consistent with Observations



$.2 \times .2 = .04$ $.2 \times .8 = .16$ $.8 \times .2 = .16$ $.2 \times .2 = .04$ $.2 \times .8 = .16$

Calculating $P(P_{22}, P_{31})$, assuming pit probability is 0.2

Final Calculation

$$\sum_{p_{22},p_{31}} P(b_{12}, b_{21} | p_{13}, \ldots) P(P_{22}, P_{31}) = 0.04 + 0.16 + 0.16$$

$$\sum_{p_{22},p_{31}} P(b_{12}, b_{21} | \neg p_{13}, \ldots) P(P_{22}, P_{31}) = 0.04 + 0.16$$

Wumpus World Probability Calculation

$P(p_{13}|\neg p_{11}, \neg p_{12}, \neg p_{21}, b_{12}, b_{21})$

Wumpus World Probability Calculation

$$
\begin{aligned}
& P(p_{13}|\neg p_{11}, \neg p_{12}, \neg p_{21}, b_{12}, b_{21}) \\
& \quad = \ \alpha P(p_{13}) P(\neg p_{11}, \neg p_{12}, \neg p_{21}) \sum \ldots p_{13} \ldots
\end{aligned}
$$

Wumpus World Probability Calculation

$$
\begin{aligned}
& P(p_{13}|\neg p_{11}, \neg p_{12}, \neg p_{21}, b_{12}, b_{21}) \\
& \quad = \alpha P(p_{13}) P(\neg p_{11}, \neg p_{12}, \neg p_{21}) \sum \ldots p_{13} \ldots \\
& \quad = \alpha \times 0.2 \times P(\neg p_{11}, \neg p_{12}, \neg p_{21}) \sum \ldots p_{13} \ldots
\end{aligned}
$$

Wumpus World Probability Calculation

$$
\begin{aligned}
&P(p_{13}|\neg p_{11}, \neg p_{12}, \neg p_{21}, b_{12}, b_{21}) \\
&\quad = \alpha P(p_{13}) P(\neg p_{11}, \neg p_{12}, \neg p_{21}) \sum \ldots p_{13} \ldots \\
&\quad = \alpha \times 0.2 \times P(\neg p_{11}, \neg p_{12}, \neg p_{21}) \sum \ldots p_{13} \ldots \\
&\quad = \alpha \times 0.2 \times 0.8 \times 0.8 \times 0.8 \times \sum \ldots p_{13} \ldots
\end{aligned}
$$

Wumpus World Probability Calculation

$$
\begin{aligned}
& P(p_{13} | \neg p_{11}, \neg p_{12}, \neg p_{21}, b_{12}, b_{21}) \\
& \quad = \alpha P(p_{13}) P(\neg p_{11}, \neg p_{12}, \neg p_{21}) \sum \ldots p_{13} \ldots \\
& \quad = \alpha \times 0.2 \times P(\neg p_{11}, \neg p_{12}, \neg p_{21}) \sum \ldots p_{13} \ldots \\
& \quad = \alpha \times 0.2 \times 0.8 \times 0.8 \times 0.8 \times \sum \ldots p_{13} \ldots \\
& \quad = \alpha \times 0.2 \times 0.8 \times 0.8 \times 0.8 \times (0.04 + 0.16 + 0.16)
\end{aligned}
$$

Wumpus World Probability Calculation

$$
\begin{aligned}
& P(p_{13}|\neg p_{11}, \neg p_{12}, \neg p_{21}, b_{12}, b_{21}) \\
&= \alpha P(p_{13}) P(\neg p_{11}, \neg p_{12}, \neg p_{21}) \sum \ldots p_{13} \ldots \\
&= \alpha \times 0.2 \times P(\neg p_{11}, \neg p_{12}, \neg p_{21}) \sum \ldots p_{13} \ldots \\
&= \alpha \times 0.2 \times 0.8 \times 0.8 \times 0.8 \times \sum \ldots p_{13} \ldots \\
&= \alpha \times 0.2 \times 0.8 \times 0.8 \times 0.8 \times (0.04 + 0.16 + 0.16) \\
&= 0.036864\alpha
\end{aligned}
$$

Wumpus World Probability Calculation

$$
\begin{aligned}
& P(p_{13}|\neg p_{11}, \neg p_{12}, \neg p_{21}, b_{12}, b_{21}) \\
& \quad = \alpha P(p_{13}) P(\neg p_{11}, \neg p_{12}, \neg p_{21}) \sum \ldots p_{13} \ldots \\
& \quad = \alpha \times 0.2 \times P(\neg p_{11}, \neg p_{12}, \neg p_{21}) \sum \ldots p_{13} \ldots \\
& \quad = \alpha \times 0.2 \times 0.8 \times 0.8 \times 0.8 \times \sum \ldots p_{13} \ldots \\
& \quad = \alpha \times 0.2 \times 0.8 \times 0.8 \times 0.8 \times (0.04 + 0.16 + 0.16) \\
& \quad = 0.036864\alpha
\end{aligned}
$$

$$P(\neg p_{13}|\neg p_{11}, \neg p_{12}, \neg p_{21}, b_{12}, b_{21})$$

Wumpus World Probability Calculation

$$
\begin{aligned}
&P(p_{13}|\neg p_{11}, \neg p_{12}, \neg p_{21}, b_{12}, b_{21}) \\
&\quad = \alpha P(p_{13}) P(\neg p_{11}, \neg p_{12}, \neg p_{21}) \textstyle\sum \ldots p_{13} \ldots \\
&\quad = \alpha \times 0.2 \times P(\neg p_{11}, \neg p_{12}, \neg p_{21}) \textstyle\sum \ldots p_{13} \ldots \\
&\quad = \alpha \times 0.2 \times 0.8 \times 0.8 \times 0.8 \times \textstyle\sum \ldots p_{13} \ldots \\
&\quad = \alpha \times 0.2 \times 0.8 \times 0.8 \times 0.8 \times (0.04 + 0.16 + 0.16) \\
&\quad = 0.036864\alpha
\end{aligned}
$$

$$
\begin{aligned}
&P(\neg p_{13}|\neg p_{11}, \neg p_{12}, \neg p_{21}, b_{12}, b_{21}) \\
&\quad = \alpha P(\neg p_{13}) P(\neg p_{11}, \neg p_{12}, \neg p_{21}) \textstyle\sum \ldots \neg p_{13} \ldots
\end{aligned}
$$

Wumpus World Probability Calculation

$$
\begin{aligned}
&P(p_{13}|\neg p_{11}, \neg p_{12}, \neg p_{21}, b_{12}, b_{21}) \\
&\quad = \alpha P(p_{13}) P(\neg p_{11}, \neg p_{12}, \neg p_{21}) \sum \ldots p_{13} \ldots \\
&\quad = \alpha \times 0.2 \times P(\neg p_{11}, \neg p_{12}, \neg p_{21}) \sum \ldots p_{13} \ldots \\
&\quad = \alpha \times 0.2 \times 0.8 \times 0.8 \times 0.8 \times \sum \ldots p_{13} \ldots \\
&\quad = \alpha \times 0.2 \times 0.8 \times 0.8 \times 0.8 \times (0.04 + 0.16 + 0.16) \\
&\quad = 0.036864\alpha
\end{aligned}
$$

$$
\begin{aligned}
&P(\neg p_{13}|\neg p_{11}, \neg p_{12}, \neg p_{21}, b_{12}, b_{21}) \\
&\quad = \alpha P(\neg p_{13}) P(\neg p_{11}, \neg p_{12}, \neg p_{21}) \sum \ldots \neg p_{13} \ldots \\
&\quad = \alpha \times 0.8 \times 0.8 \times 0.8 \times 0.8 \times (0.04 + 0.16)
\end{aligned}
$$

Wumpus World Probability Calculation

$$
\begin{aligned}
&P(p_{13}|\neg p_{11}, \neg p_{12}, \neg p_{21}, b_{12}, b_{21}) \\
&\quad = \alpha P(p_{13}) P(\neg p_{11}, \neg p_{12}, \neg p_{21}) \textstyle\sum \ldots p_{13} \ldots \\
&\quad = \alpha \times 0.2 \times P(\neg p_{11}, \neg p_{12}, \neg p_{21}) \textstyle\sum \ldots p_{13} \ldots \\
&\quad = \alpha \times 0.2 \times 0.8 \times 0.8 \times 0.8 \times \textstyle\sum \ldots p_{13} \ldots \\
&\quad = \alpha \times 0.2 \times 0.8 \times 0.8 \times 0.8 \times (0.04 + 0.16 + 0.16) \\
&\quad = 0.036864\alpha
\end{aligned}
$$

$$
\begin{aligned}
&P(\neg p_{13}|\neg p_{11}, \neg p_{12}, \neg p_{21}, b_{12}, b_{21}) \\
&\quad = \alpha P(\neg p_{13}) P(\neg p_{11}, \neg p_{12}, \neg p_{21}) \textstyle\sum \ldots \neg p_{13} \ldots \\
&\quad = \alpha \times 0.8 \times 0.8 \times 0.8 \times 0.8 \times (0.04 + 0.16) \\
&\quad = 0.08192\alpha
\end{aligned}
$$

Wumpus World Probability Calculation

$$
\begin{aligned}
&P(p_{13}|\neg p_{11}, \neg p_{12}, \neg p_{21}, b_{12}, b_{21}) \\
&\quad = \alpha P(p_{13}) P(\neg p_{11}, \neg p_{12}, \neg p_{21}) \sum \ldots p_{13} \ldots \\
&\quad = \alpha \times 0.2 \times P(\neg p_{11}, \neg p_{12}, \neg p_{21}) \sum \ldots p_{13} \ldots \\
&\quad = \alpha \times 0.2 \times 0.8 \times 0.8 \times 0.8 \times \sum \ldots p_{13} \ldots \\
&\quad = \alpha \times 0.2 \times 0.8 \times 0.8 \times 0.8 \times (0.04 + 0.16 + 0.16) \\
&\quad = 0.036864\alpha
\end{aligned}
$$

$$
\begin{aligned}
&P(\neg p_{13}|\neg p_{11}, \neg p_{12}, \neg p_{21}, b_{12}, b_{21}) \\
&\quad = \alpha P(\neg p_{13}) P(\neg p_{11}, \neg p_{12}, \neg p_{21}) \sum \ldots \neg p_{13} \ldots \\
&\quad = \alpha \times 0.8 \times 0.8 \times 0.8 \times 0.8 \times (0.04 + 0.16) \\
&\quad = 0.08192\alpha
\end{aligned}
$$

$$P(p_{13}|\neg p_{11}, \neg p_{12}, \neg p_{21}, b_{12}, b_{21})$$

Wumpus World Probability Calculation

$$
\begin{aligned}
&P(p_{13}|\neg p_{11}, \neg p_{12}, \neg p_{21}, b_{12}, b_{21}) \\
&\quad = \alpha P(p_{13}) P(\neg p_{11}, \neg p_{12}, \neg p_{21}) \sum \ldots p_{13} \ldots \\
&\quad = \alpha \times 0.2 \times P(\neg p_{11}, \neg p_{12}, \neg p_{21}) \sum \ldots p_{13} \ldots \\
&\quad = \alpha \times 0.2 \times 0.8 \times 0.8 \times 0.8 \times \sum \ldots p_{13} \ldots \\
&\quad = \alpha \times 0.2 \times 0.8 \times 0.8 \times 0.8 \times (0.04 + 0.16 + 0.16) \\
&\quad = 0.036864\alpha
\end{aligned}
$$

$$
\begin{aligned}
&P(\neg p_{13}|\neg p_{11}, \neg p_{12}, \neg p_{21}, b_{12}, b_{21}) \\
&\quad = \alpha P(\neg p_{13}) P(\neg p_{11}, \neg p_{12}, \neg p_{21}) \sum \ldots \neg p_{13} \ldots \\
&\quad = \alpha \times 0.8 \times 0.8 \times 0.8 \times 0.8 \times (0.04 + 0.16) \\
&\quad = 0.08192\alpha
\end{aligned}
$$

$$
\begin{aligned}
&P(p_{13}|\neg p_{11}, \neg p_{12}, \neg p_{21}, b_{12}, b_{21}) \\
&\quad = 0.036864/(0.036864 + 0.08192) \approx 0.31
\end{aligned}
$$

Probability Rules

Product Rule

$$P(a, b) = P(a|b)P(b)$$

Independence

$$P(a, b) = P(a)P(b)$$

Conditional Independence

$$P(a, b|c) = P(a|c)P(b|c)$$
$$P(a|b, c) = P(a|c)$$

Probability Rules

Product Rule

$$P(a, b) = P(a|b)P(b)$$

Independence

$$P(a, b) = P(a)P(b)$$

Conditional Independence

$$P(a, b|c) = P(a|c)P(b|c)$$
$$P(a|b, c) = P(a|c)$$

Given: w indep. x, y, z
x indep. z given y

Show:

$$P(w, x, y, z) = P(w)P(x|y)P(z, y)$$

Probability Rules

Product Rule

$P(a, b) = P(a|b)P(b)$

Independence

$P(a, b) = P(a)P(b)$

Conditional Independence

$P(a, b|c) = P(a|c)P(b|c)$

$P(a|b, c) = P(a|c)$

Given: w indep. x, y, z

x indep. z given y

Show:

$P(w, x, y, z) = P(w)P(x|y)P(z, y)$

Proof:

$P(w, x, y, z)$

Probability Rules

Product Rule

$P(a, b) = P(a|b)P(b)$

Independence

$P(a, b) = P(a)P(b)$

Conditional Independence

$P(a, b|c) = P(a|c)P(b|c)$
$P(a|b, c) = P(a|c)$

Given: w indep. x, y, z
x indep. z given y

Show:

$P(w, x, y, z) = P(w)P(x|y)P(z, y)$

Proof:

$P(w, x, y, z)$
$= P(w)P(x, y, z)$

Probability Rules

Product Rule

$$P(a, b) = P(a|b)P(b)$$

Independence

$$P(a, b) = P(a)P(b)$$

Conditional Independence

$$P(a, b|c) = P(a|c)P(b|c)$$
$$P(a|b, c) = P(a|c)$$

Given: w indep. x, y, z
x indep. z given y

Show:

$$P(w, x, y, z) = P(w)P(x|y)P(z, y)$$

Proof:

$$P(w, x, y, z)$$
$$= P(w)P(x, y, z)$$
$$= P(w)P(x, z|y)P(y)$$

Probability Rules

Product Rule

$P(a,b) = P(a|b)P(b)$

Independence

$P(a,b) = P(a)P(b)$

Conditional Independence

$P(a,b|c) = P(a|c)P(b|c)$
$P(a|b,c) = P(a|c)$

Given: w indep. x, y, z
x indep. z given y

Show:

$P(w,x,y,z) = P(w)P(x|y)P(z,y)$

Proof:

$$
\begin{aligned}
&P(w,x,y,z) \\
&= P(w)P(x,y,z) \\
&= P(w)P(x,z|y)P(y) \\
&= P(w)P(x|y)P(z|y)P(y)
\end{aligned}
$$

Probability Rules

Product Rule

$P(a, b) = P(a|b)P(b)$

Independence

$P(a, b) = P(a)P(b)$

Conditional Independence

$P(a, b|c) = P(a|c)P(b|c)$

$P(a|b, c) = P(a|c)$

Given: w indep. x, y, z

x indep. z given y

Show:

$$P(w, x, y, z) = P(w)P(x|y)P(z, y)$$

Proof:

$$\begin{aligned} & P(w, x, y, z) \\ &= P(w)P(x, y, z) \\ &= P(w)P(x, z|y)P(y) \\ &= P(w)P(x|y)P(z|y)P(y) \\ &= P(w)P(x|y)P(z, y) \end{aligned}$$

Probability Rule Exercises

Product Rule

$P(a, b) = P(a|b)P(b)$

Independence

$P(a, b) = P(a)P(b)$

Conditional Independence

$P(a, b|c) = P(a|c)P(b|c)$
$P(a|b, c) = P(a|c)$

Given:
z indep. w, x, y
w indep. x given y

Show:

$$P(w, z|x, y) = P(w|y)P(z)$$

Given:
w indep. y, z given x
x indep. z given y
y indep. z

Show:

$$P(w, x, y, z) = P(w|x)P(x|y)P(y)P(z)$$

Normalization Rules

Complements

$P(\neg a) + P(a) = 1$

Summing Out

$P(a, b) + P(\neg a, b) = P(b)$

Distributions

$$\sum_{i}^{d} P(X = x_i) = 1$$

Normalization Rules

Complements

$$P(\neg a) + P(a) = 1$$

Summing Out

$$P(a, b) + P(\neg a, b) = P(b)$$

Distributions

$$\sum_{i}^{d} P(X = x_i) = 1$$

Given: $\mathbf{P}(X|y, z) = \alpha\langle 0.1, 0.3\rangle$

Show: $\mathbf{P}(X|y, z) = \langle 0.25, 0.75\rangle$

Normalization Rules

Given: $\mathbf{P}(X|y,z) = \alpha\langle 0.1, 0.3\rangle$

Show: $\mathbf{P}(X|y,z) = \langle 0.25, 0.75\rangle$

$$P(y,z) = P(x,y,z) + P(\neg x,y,z)$$

Complements

$$P(\neg a) + P(a) = 1$$

Summing Out

$$P(a,b) + P(\neg a,b) = P(b)$$

Distributions

$$\sum_i^d P(X = x_i) = 1$$

Normalization Rules

Complements

$P(\neg a) + P(a) = 1$

Summing Out

$P(a, b) + P(\neg a, b) = P(b)$

Distributions

$$\sum_i^d P(X = x_i) = 1$$

Given: $\mathbf{P}(X|y, z) = \alpha\langle 0.1, 0.3\rangle$

Show: $\mathbf{P}(X|y, z) = \langle 0.25, 0.75\rangle$

$$P(y, z) = P(x, y, z) + P(\neg x, y, z)$$

$$1 = \frac{P(x, y, z) + P(\neg x, y, c)}{P(y, z)}$$

Normalization Rules

Complements

$P(\neg a) + P(a) = 1$

Summing Out

$P(a, b) + P(\neg a, b) = P(b)$

Distributions

$$\sum_{i}^{d} P(X = x_i) = 1$$

Given: $\mathbf{P}(X|y, z) = \alpha\langle 0.1, 0.3\rangle$

Show: $\mathbf{P}(X|y, z) = \langle 0.25, 0.75\rangle$

$$P(y, z) = P(x, y, z) + P(\neg x, y, z)$$

$$1 = \frac{P(x, y, z) + P(\neg x, y, c)}{P(y, z)}$$

$$1 = \frac{P(x, y, z)}{P(y, z)} + \frac{P(\neg x, y, z)}{P(y, z)}$$

Normalization Rules

Complements

$$P(\neg a) + P(a) = 1$$

Summing Out

$$P(a, b) + P(\neg a, b) = P(b)$$

Distributions

$$\sum_i^d P(X = x_i) = 1$$

Given: $\mathbf{P}(X|y, z) = \alpha\langle 0.1, 0.3\rangle$

Show: $\mathbf{P}(X|y, z) = \langle 0.25, 0.75\rangle$

$$P(y, z) = P(x, y, z) + P(\neg x, y, z)$$

$$1 = \frac{P(x, y, z) + P(\neg x, y, c)}{P(y, z)}$$

$$1 = \frac{P(x, y, z)}{P(y, z)} + \frac{P(\neg x, y, z)}{P(y, z)}$$

$$1 = P(x|y, z) + P(\neg x|y, z)$$

Normalization Rules

Complements

$$P(\neg a) + P(a) = 1$$

Summing Out

$$P(a, b) + P(\neg a, b) = P(b)$$

Distributions

$$\sum_i^d P(X = x_i) = 1$$

Given: $\mathbf{P}(X|y, z) = \alpha\langle 0.1, 0.3\rangle$

Show: $\mathbf{P}(X|y, z) = \langle 0.25, 0.75\rangle$

$$P(y, z) = P(x, y, z) + P(\neg x, y, z)$$

$$1 = \frac{P(x, y, z) + P(\neg x, y, c)}{P(y, z)}$$

$$1 = \frac{P(x, y, z)}{P(y, z)} + \frac{P(\neg x, y, z)}{P(y, z)}$$

$$1 = P(x|y, z) + P(\neg x|y, z)$$

$$1 = 0.1\alpha + 0.3\alpha$$

Normalization Rules

Complements

$$P(\neg a) + P(a) = 1$$

Summing Out

$$P(a, b) + P(\neg a, b) = P(b)$$

Distributions

$$\sum_i^d P(X = x_i) = 1$$

Given: $\mathbf{P}(X|y,z) = \alpha\langle 0.1, 0.3\rangle$

Show: $\mathbf{P}(X|y,z) = \langle 0.25, 0.75\rangle$

$$P(y,z) = P(x,y,z) + P(\neg x,y,z)$$

$$1 = \frac{P(x,y,z) + P(\neg x,y,c)}{P(y,z)}$$

$$1 = \frac{P(x,y,z)}{P(y,z)} + \frac{P(\neg x,y,z)}{P(y,z)}$$

$$1 = P(x|y,z) + P(\neg x|y,z)$$

$$1 = 0.1\alpha + 0.3\alpha$$

$$1 = 0.4\alpha$$

Normalization Rules

Complements

$P(\neg a) + P(a) = 1$

Summing Out

$P(a, b) + P(\neg a, b) = P(b)$

Distributions

$$\sum_i^d P(X = x_i) = 1$$

Given: $\mathbf{P}(X|y, z) = \alpha\langle 0.1, 0.3\rangle$

Show: $\mathbf{P}(X|y, z) = \langle 0.25, 0.75\rangle$

$$\begin{aligned}
P(y, z) &= P(x, y, z) + P(\neg x, y, z) \\
1 &= \frac{P(x, y, z) + P(\neg x, y, c)}{P(y, z)} \\
1 &= \frac{P(x, y, z)}{P(y, z)} + \frac{P(\neg x, y, z)}{P(y, z)} \\
1 &= P(x|y, z) + P(\neg x|y, z) \\
1 &= 0.1\alpha + 0.3\alpha \\
1 &= 0.4\alpha \\
2.5 &= \alpha
\end{aligned}$$

Normalization Rules

Complements

$P(\neg a) + P(a) = 1$

Summing Out

$P(a, b) + P(\neg a, b) = P(b)$

Distributions

$$\sum_{i}^{d} P(X = x_i) = 1$$

Given: $\mathbf{P}(X|y, z) = \alpha\langle 0.1, 0.3\rangle$

Show: $\mathbf{P}(X|y, z) = \langle 0.25, 0.75\rangle$

$$P(y, z) = P(x, y, z) + P(\neg x, y, z)$$

$$1 = \frac{P(x, y, z) + P(\neg x, y, c)}{P(y, z)}$$

$$1 = \frac{P(x, y, z)}{P(y, z)} + \frac{P(\neg x, y, z)}{P(y, z)}$$

$$1 = P(x|y, z) + P(\neg x|y, z)$$

$$1 = 0.1\alpha + 0.3\alpha$$

$$1 = 0.4\alpha$$

$$2.5 = \alpha$$

Normalization Rule Practice

Complements

$$P(\neg a) + P(a) = 1$$
$$P(a|b) + P(\neg a|b) = 1$$

Summing Out

$$P(a, b) + P(\neg a, b) = P(b)$$

Distributions

$$\sum_i^d P(X = x_i) = 1$$

Given:

$$\begin{aligned} P(y|x) &= 0.4 \\ P(y|\neg x) &= 0.9 \\ P(x) &= 0.2 \\ P(\neg x) &= 0.8 \end{aligned}$$

Show: $\mathbf{P}(X|y) = \langle 0.1, 0.9 \rangle$

Key Ideas

Probability Measures Belief

$P(\mathit{false}) = 0 \quad P(\mathit{true}) = 1 \quad \sum_x P(X = x) = 1$

Types of probabilities

- Prior, $P(X = x)$
- Joint, $P(X = x, Y = y)$
- Conditional, $P(X = x | Y = y)$

Inference

- Sums over full joint distribution
- Conditional independence + product and Bayes' rule