Learning from Examples

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Outline

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1. Supervised Learning
   - Hypothesis Functions
   - Features
   - Evaluating Hypotheses
   - Overfitting and Underfitting
2. Supervised Learning Algorithms
   - Decision Trees
   - Random Forests
   - k-Nearest Neighbors
   - Linear and Logistic Regression
   - Support Vector Machines
   - Neural Networks

Supervised Learning

Key Ideas

- Examine pairs of inputs and outputs
- Guess a possible function mapping input to output
- Predict outputs given new inputs

$$\begin{aligned} f(1) &= 1 \\ f(2) &= 4 \\ f(3) &= 9 \\ f(4) &= 16 \\ f(5) &= ? \end{aligned}$$

$$\begin{aligned} f(\text{Romeo and Juliet}) &= \text{Shakespeare} \\ f(\text{Tom Sawyer}) &= \text{Twain} \\ f(\text{Macbeth}) &= \text{Shakespeare} \\ f(\text{Huckleberry Finn}) &= \text{Twain} \\ f(\text{Othello}) &= ? \end{aligned}$$

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\begin{aligned}
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Supervised Learning Components

Original Function

An unknown function $f : D_f \rightarrow R_f$

Training Examples

Pairs of $(x, f(x))$ where $x \in D_f$ and $f(x) \in R_f$

Hypothesis Function

Some function $h : D_f \rightarrow R_f$

Learning Goal

Pick a hypothesis h as close to f as possible

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Problem Formulation

Defining a Machine Learning Problem

Describe the function $f : D_f \to R_f$

- What is D_f?
- What is R_f?

Ex: Income Prediction

D_f = people

$R_f = \mathbb{R}$

Ex: Face Recognition

D_f = image regions

$R_f = [0, 1]$

Most algorithms work best when $R_f \subseteq \mathbb{R}$ or $|R_f| \ll 100$

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Decomposing Domain Objects

Smoker identification:

$$\begin{aligned} f(\text{John}) &= \textit{true} \\ f(\text{Mary}) &= \textit{false} \\ f(\text{Frank}) &= \textit{false} \\ f(\text{Sally}) &= ? \end{aligned}$$

Easier if we know, e.g.

- cigarette smell?
- teeth stains?
- deep cough?

Definition

Features (or attributes) are the components of a domain object believed to be important for learning the function f

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Part of Speech Tagging Example

Input	*John*	*broke*	*the*	*red*	*lamp*
Output	NOUN	VERB	DET	ADJ	NOUN

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Function Description

D_f =

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Function Description

$$D_f = \{a, aardvark, abacus, abalone, \ldots\}$$

$$R_f =$$

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$$D_f = \{a, aardvark, abacus, abalone, \ldots\}$$

$$R_f = \{\textsc{Noun}, \textsc{Verb}, \textsc{Adj}, \textsc{Adv}, \textsc{Det}, \ldots\}$$

$f(bark)$ = NOUN? VERB?

. . . *the **bark** of the tree* . . .

. . . *heard the dog **bark*** . . .

Part of Speech Tagging Example

Input	*John*	*broke*	*the*	*red*	*lamp*
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D_f = {*a*, *aardvark*, *abacus*, *abalone*, . . .} in a sentence

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f(*bark*) = Noun? Verb?

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$f(\textit{bark})$ = NOUN? VERB?

... *the* ***bark*** *of the tree* ...

... *heard the dog* ***bark*** ...

Feature Representation

$$
\begin{aligned}
f([w_0 = \textit{bark}, w_{-1} = \textit{the}]) &= \text{NOUN} \\
f([w_0 = \textit{bark}, w_{-1} = \textit{dog}]) &= \text{VERB}
\end{aligned}
$$

Named Entity Recognition Exercise

Definition

A named entity recognition program find spans of words that are people, locations, organizations, etc.

Input Bill works for Microsoft Corporation

Output [$_{\text{PER}}$ Bill] works for [$_{\text{ORG}}$ Microsoft Corporation]

Exercise

Named entity recognition as supervised learning:

- Describe the function domain
- Describe the function range (keep it small!)
- Describe the feature space

One Named Entity Recognition Approach

Function Description

$D_f = \{a, aardvark, abacus, abalone, \ldots\}$ in a sentence

$R_f = \{\text{B-Per}, \text{I-Per}, \text{B-Org}, \text{I-Org}, \ldots, \text{O}\}$

$$
\begin{aligned}
f(\mathit{Bill}) &= \text{B-Per} \\
f(\mathit{works}) &= \text{O} \\
f(\mathit{for}) &= \text{O} \\
f(\mathit{Microsoft}) &= \text{B-Org} \\
f(\mathit{Corporation}) &= \text{I-Org}
\end{aligned}
$$

Typical Features

Word itself

Preceding label

Capitalization

First in sentence?

Evaluating Learning Algorithms

1. Train learning algorithm on examples E_{train}
2. Learning algorithm produces a hypothesis h
3. Test hypothesis on new examples E_{test} - Don't peek!

Train:

x_1	x_2	$f(x)$
1	a	*true*
0	b	*false*
0	c	*false*
0	b	*false*

Test:

x_1	x_2	$f(x)$
1	a	*true*
0	b	*false*
0	c	*false*
1	c	*true*

$$h_1(x) = \begin{cases} \mathit{true} & \text{if } x_1 = 1 \\ \mathit{false} & \text{otherwise} \end{cases}$$

$$h_2(x) = \begin{cases} \mathit{true} & \text{if } x_2 = a \\ \mathit{false} & \text{otherwise} \end{cases}$$

Performance: $\frac{4}{4} = 1.0$

Performance: $\frac{3}{4} = 0.75$

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Train, Development and Test sets

Bad Idea

1. Evaluate on test set
2. Examine errors and adjust hypothesis
3. Goto 1

Tuning to your test set will overestimate model accuracy

Instead, split data into:

Train Used to train a hypothesis function

Dev Used to tune/adjust the hypothesis

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k-fold Cross Validation

Split data into k parts, then iteratively train and test:

Accuracy is average of the accuracies on each of the k folds

Warnings:

- Cross validation is not a substitution for a test set
- Cross validation can be used instead of train+dev

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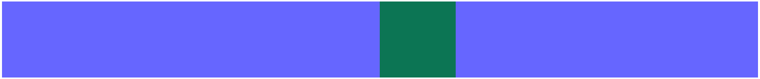

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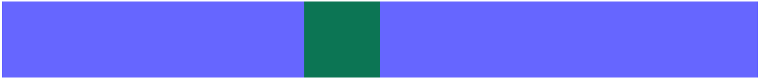

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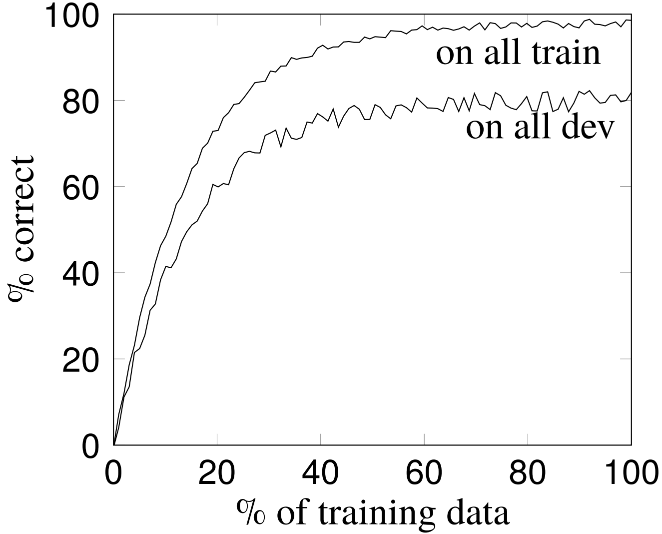
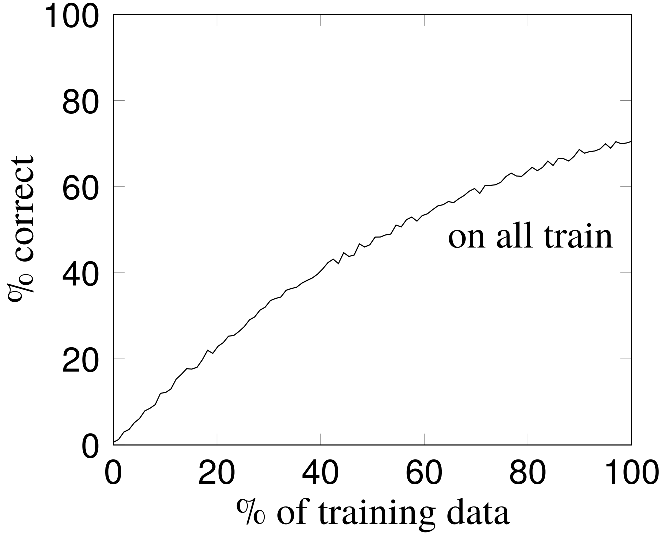
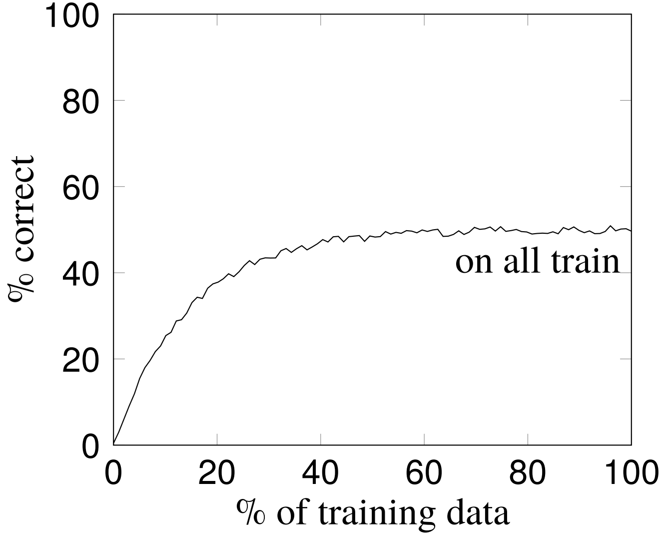
Learning Curves



Insufficient Training Data



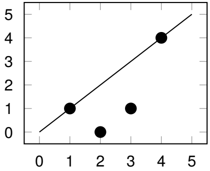
Underfitting



Addressing Underfitting

x_1	$f(x)$	$h(x)$
1	1	1
2	0	2
3	1	3
4	4	4

$h(x) = x_1$



More complex model

x_1	$f(x)$	$h(x)$
1	1	1
2	0	0
3	1	1
4	4	4

$h(x) = x_1^2 - 4x_1 + 4$

More features

x_1	x_2	$f(x)$	$h(x)$
1	1	1	1
2	4	0	0
3	9	1	1
4	16	4	4

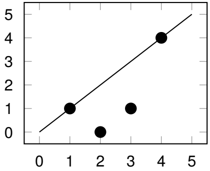
$h(x) = -4x_1 + x_2 + 4$

Addressing Underfitting

More complex model

x_1	$f(x)$	$h(x)$
1	1	1
2	0	2
3	1	3
4	4	4

$h(x) = x_1$



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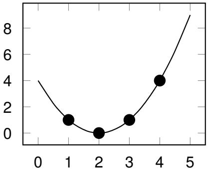

More features

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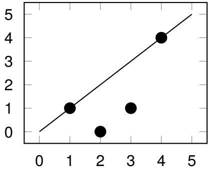

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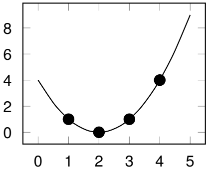

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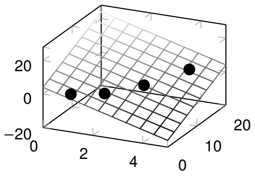
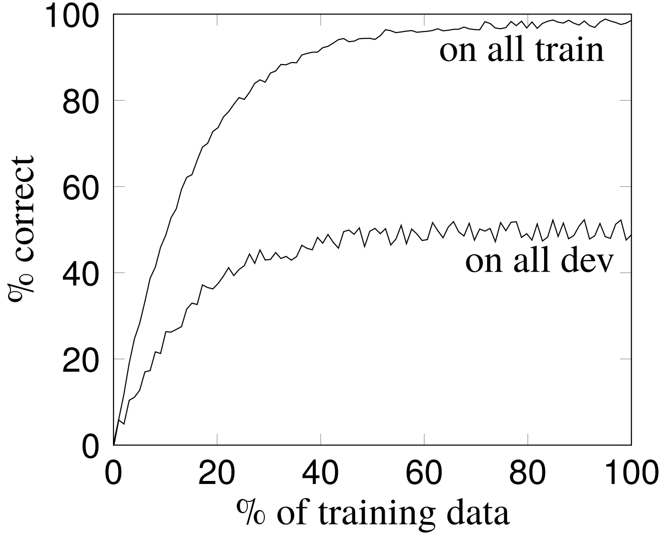
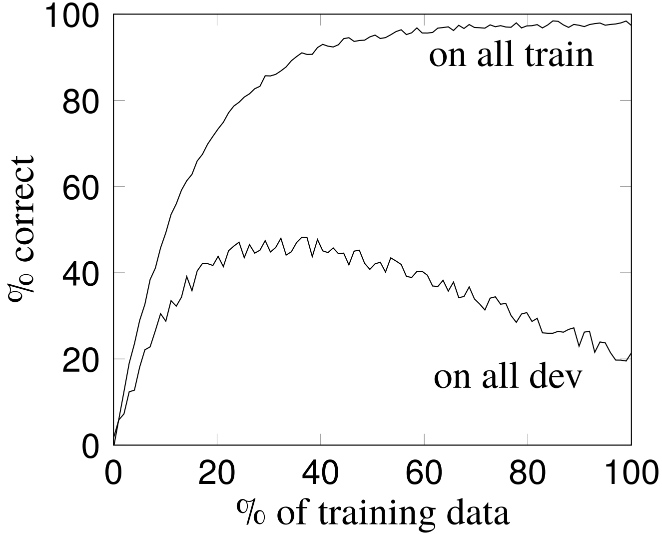


Overfitting



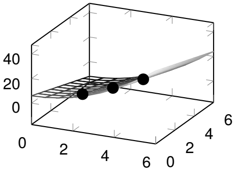
Overfitting



Addressing Overfitting

x_1	x_2	$f(x)$	$h(x)$
2	1	7	7
3	2	9	9
4	3	13	13

$h(x) = x_1^2 - 3x_2 + 6$



Simpler model

x_1	x_2	$f(x)$	$h(x)$
2	1	7	5
3	2	9	9
4	3	13	13
1	2	3	3
5	1	15	14

$h(x) = 3x_1 + x_2 - 2$

Fewer features

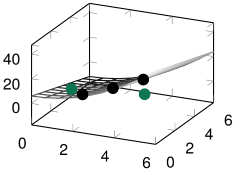
x_1	$f(x)$	$h(x)$
2	7	6
3	9	9
4	13	12
1	3	3
5	15	15

$h(x) = 3x_1$

Addressing Overfitting

x_1	x_2	$f(x)$	$h(x)$
2	1	7	7
3	2	9	9
4	3	13	13
1	2	3	1
5	1	15	28

$h(x) = x_1^2 - 3x_2 + 6$



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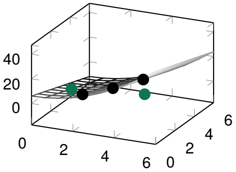

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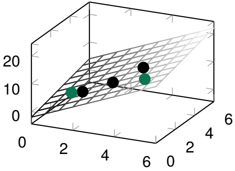

Fewer features

x_1	$f(x)$	$h(x)$
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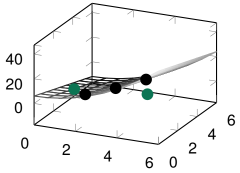

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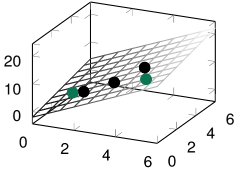

Fewer features

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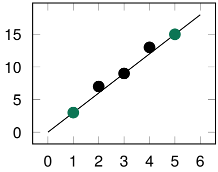
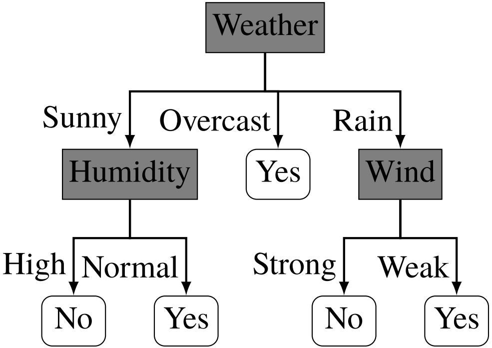


Outline

Decision Trees

Should I play golf today?



Function

D_f = days

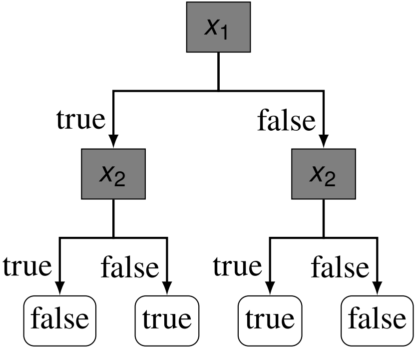
$R_f = \{Yes, No\}$

Features

- Weather
- Humidity
- Wind

Decision Trees as Functions

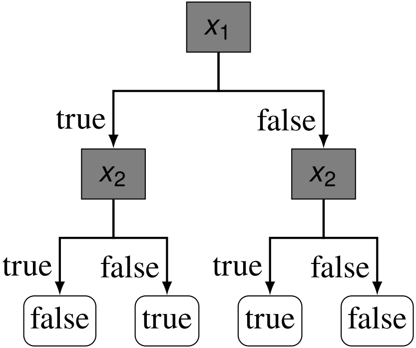
$f(x) = x_1 \; xor \; x_2$



$f(x) = (x_1 \wedge x_2) \vee \neg x_3$

Decision Trees as Functions

$f(x) = x_1 \; xor \; x_2$



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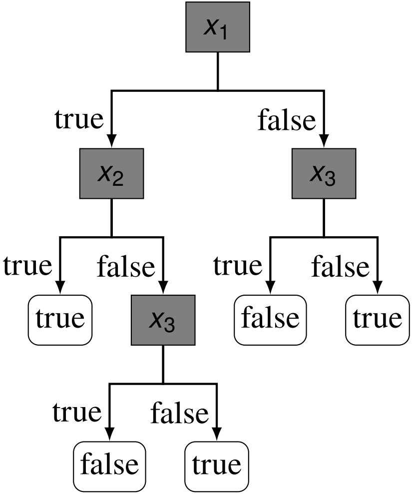

Learning Decision Trees

1. Select a feature x_i for the node
2. For each value of x_i create a child node
3. Sort training examples into child nodes
4. If examples are sorted perfectly, terminate
5. Else, repeat the process for each child node

x_1	x_2	$f(x)$
true	*true*	*true*
true	*false*	*false*
false	*true*	*true*
false	*false*	*true*

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3 true, 1 false

x_1	x_2	$f(x)$
true	*true*	*true*
true	*false*	*false*
false	*true*	*true*
false	*false*	*true*

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x_1	x_2	$f(x)$
true	*true*	*true*
true	*false*	*false*
false	*true*	*true*
false	*false*	*true*

x_2
3 true, 1 false

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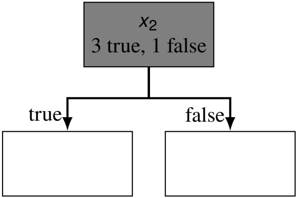
x_1	x_2	$f(x)$
true	*true*	*true*
true	*false*	*false*
false	*true*	*true*
false	*false*	*true*



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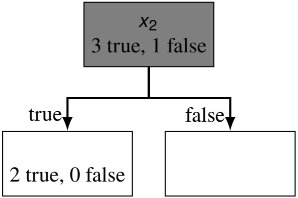

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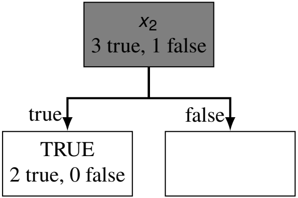

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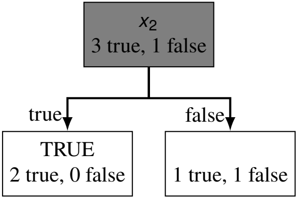

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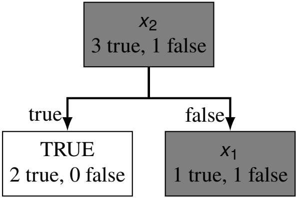

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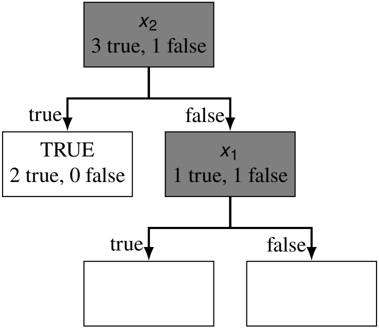

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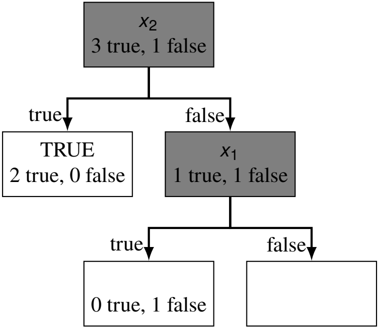

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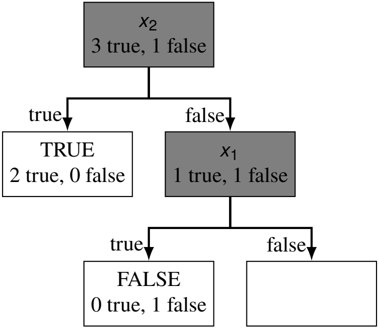

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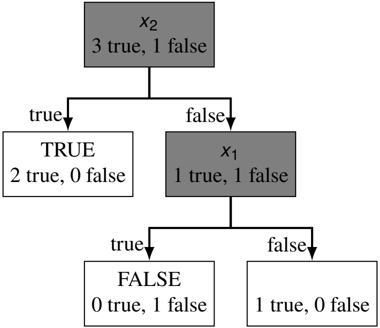

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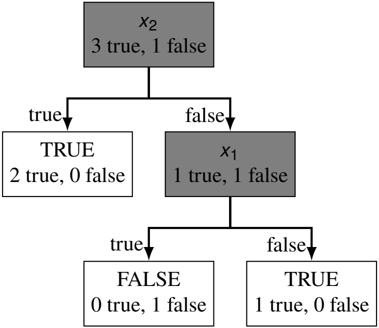

Decision Tree Exercise

Build a decision tree for:

x_1	x_2	$f(x)$
0	a	*true*
0	b	*false*
1	c	*true*
0	c	*false*
1	b	*false*
1	a	*true*

Decision Tree Exercise

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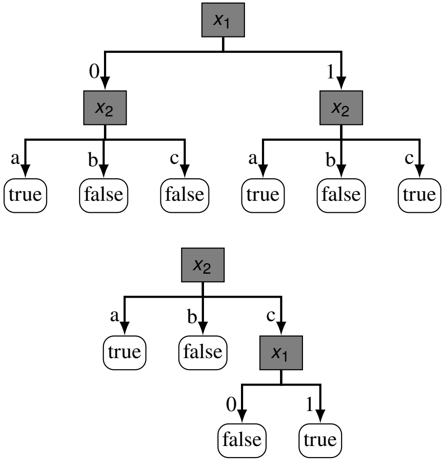
1	c	*true*
0	c	*false*
1	b	*false*
1	a	*true*

Two possible solutions:



Selecting Features for Decision Trees

Feature Selection Order

- Different orders result in different trees
- "Good" features should be used before "poor" ones

What is a "good" feature?

One whose values predict the class labels

x_1	x_2	$f(x)$
a	0	*true*
a	1	*true*
b	0	*false*
b	1	*false*

x_1 is a good feature

x_2 is a poor feature

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Information

Good features provide more information

Information can be quantified in terms of bits

Task: Encode abacabad using as few bits as possible

Simple Encoding:

a	00
b	01
c	10
d	11

abacabad → 16 bits

`0001001000010011`

Using Probability:

a	0
b	10
c	110
d	111

abacabad → 14 bits

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More likely values can be encoded in fewer bits!

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More likely values can be encoded in fewer bits!

Good features provide more information

Information can be quantified in terms of bits

Task: Encode `abacabad` using as few bits as possible

Simple Encoding:

a	00
b	01
c	10
d	11

`abacabad` $\rightarrow$ 16 bits

`0001001000010011`

Using Probability:

a	0
b	10
c	110
d	111

`abacabad` $\rightarrow$ 14 bits

`01001100100111`

More likely values can be encoded in fewer bits!

Entropy

Definition

The entropy of a random variable X is:

$$H(X) = -\sum_{x \in X} P(x) \log_2 P(x)$$

Bit-based Interpretation

Smallest number of bits that can encode a stream of values from X's distribution

Intuitions

- High entropy $\rightarrow$ unpredictable distribution
- Low entropy $\rightarrow$ predictable distribution

Entropy

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Entropy

X	Y
math	yes
history	no
cs	yes
math	no
math	no
cs	yes
history	no
math	yes

$$
\begin{aligned}
H(X) &= -\sum_{x \in X} P(x) \log_2 P(x) \\
&= -P(math) \log_2 P(math) \\
&\quad -P(history) \log_2 P(history) \\
&\quad -P(cs) \log_2 P(cs) \\
&= -\tfrac{1}{2} \log_2 \tfrac{1}{2} - \tfrac{1}{4} \log_2 \tfrac{1}{4} - \tfrac{1}{4} \log_2 \tfrac{1}{4} \\
&= -\tfrac{1}{2}(-1) - \tfrac{1}{4}(-2) - \tfrac{1}{4}(-2) \\
&= \tfrac{1}{2} + \tfrac{1}{2} + \tfrac{1}{2} \\
&= 1.5
\end{aligned}
$$

$$
H(Y) = 1.0
$$

Entropy

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$$

$$H(Y) = 1.0$$

Specific Conditional Entropy

X	Y
math	yes
history	no
cs	yes
math	no
math	no
cs	yes
history	no
math	yes

Specific Conditional Entropy

$$H(Y|X=x) = -\sum_{y \in Y} P(y|x) \log_2 P(y|x)$$

$$\begin{aligned} H(Y|X=cs) &= -P(yes|cs)\log_2 P(yes|cs) \\ &\quad -P(no|cs)\log_2 P(no|cs) \\ &= -1\log_2 1 - 0\log_2 0 \\ &= -1\cdot 0 - 0\cdot\infty \\ &= 0 \end{aligned}$$

In other words, $X = cs$ is a great predictor of Y

Specific Conditional Entropy

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math	yes
history	no
cs	yes
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math	yes
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math	yes

Conditional Entropy

$$H(Y|X) = \sum_{x \in X} P(X=x)H(Y|X=x)$$

Given:

$$\begin{aligned} H(Y|X=math) &= 1 \\ H(Y|X=history) &= 0 \\ H(Y|X=cs) &= 0 \end{aligned}$$

$$\begin{aligned} H(Y|X) &= P(X=math)H(Y|X=math) + \\ &\quad P(X=history)H(Y|X=history) + \\ &\quad P(X=cs)H(Y|X=cs) \\ &= \tfrac{1}{2}\cdot 1 + \tfrac{1}{4}\cdot 0 + \tfrac{1}{4}\cdot 0 = \tfrac{1}{2} \end{aligned}$$

In other words, X is a good predictor of Y

Conditional Entropy

X	Y
math	yes
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Information Gain

X	Y
math	yes
history	no
cs	yes
math	no
math	no
cs	yes
history	no
math	yes

Information Gain

$IG(Y|X) = H(Y) - H(Y|X)$

Intuitive Explanation

How many bits would it save to know X?

$$H(Y|X) = 0.5$$
$$H(Y) = 1$$

$$IG(Y|X) = 1 - 0.5 = 0.5$$

Information Gain

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Decision Trees and Information Gain

Select the feature with the highest information gain:

X_1	X_2	Y
0	a	T
0	b	F
1	c	T
0	c	F
1	b	F
1	a	T

Decision Trees and Information Gain

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X_1	X_2	Y
0	a	T
0	b	F
1	c	T
0	c	F
1	b	F
1	a	T

$$H(Y) =$$

Decision Trees and Information Gain

Select the feature with the highest information gain:

X_1	X_2	Y
0	a	T
0	b	F
1	c	T
0	c	F
1	b	F
1	a	T

$$H(Y) = -P(\text{T})\log_2 P(\text{T}) - P(\text{F})\log_2 P(\text{F})$$

Decision Trees and Information Gain

Select the feature with the highest information gain:

X_1	X_2	Y
0	a	T
0	b	F
1	c	T
0	c	F
1	b	F
1	a	T

$$\begin{aligned} H(Y) &= -P(\text{T})\log_2 P(\text{T}) - P(\text{F})\log_2 P(\text{F}) \\ &= -\tfrac{1}{2}\log_2 \tfrac{1}{2} - \tfrac{1}{2}\log_2 \tfrac{1}{2} \end{aligned}$$

Decision Trees and Information Gain

Select the feature with the highest information gain:

X_1	X_2	Y
0	a	T
0	b	F
1	c	T
0	c	F
1	b	F
1	a	T

$$\begin{aligned} H(Y) &= -P(\text{T})\log_2 P(\text{T}) - P(\text{F})\log_2 P(\text{F}) \\ &= -\tfrac{1}{2}\log_2 \tfrac{1}{2} - \tfrac{1}{2}\log_2 \tfrac{1}{2} = 1 \end{aligned}$$

Decision Trees and Information Gain

Select the feature with the highest information gain:

X_1	X_2	Y
0	a	T
0	b	F
1	c	T
0	c	F
1	b	F
1	a	T

$$
\begin{aligned}
H(Y) &= -P(\mathrm{T})\log_2 P(\mathrm{T}) - P(\mathrm{F})\log_2 P(\mathrm{F}) \\
&= -\tfrac{1}{2}\log_2 \tfrac{1}{2} - \tfrac{1}{2}\log_2 \tfrac{1}{2} = 1 \\
H(Y|X_1{=}0) &=
\end{aligned}
$$

Decision Trees and Information Gain

Select the feature with the highest information gain:

X_1	X_2	Y
0	a	T
0	b	F
1	c	T
0	c	F
1	b	F
1	a	T

$$
\begin{aligned}
H(Y) &= -P(\textsc{t})\log_2 P(\textsc{t}) - P(\textsc{f})\log_2 P(\textsc{f}) \\
&= -\tfrac{1}{2}\log_2\tfrac{1}{2} - \tfrac{1}{2}\log_2\tfrac{1}{2} = 1 \\
H(Y|X_1{=}0) &= -P(\textsc{t}|0)\log_2 P(\textsc{t}|0) -
\end{aligned}
$$

Decision Trees and Information Gain

Select the feature with the highest information gain:

X_1	X_2	Y
0	a	T
0	b	F
1	c	T
0	c	F
1	b	F
1	a	T

$$
\begin{aligned}
H(Y) &= -P(\text{T}) \log_2 P(\text{T}) - P(\text{F}) \log_2 P(\text{F}) \\
&= -\tfrac{1}{2} \log_2 \tfrac{1}{2} - \tfrac{1}{2} \log_2 \tfrac{1}{2} = 1 \\
H(Y|X_1{=}0) &= -P(\text{T}|0) \log_2 P(\text{T}|0) - P(\text{F}|0) \log_2 P(\text{F}|0)
\end{aligned}
$$

Decision Trees and Information Gain

Select the feature with the highest information gain:

X_1	X_2	Y
0	a	T
0	b	F
1	c	T
0	c	F
1	b	F
1	a	T

$$\begin{aligned}
H(Y) &= -P(\text{T})\log_2 P(\text{T}) - P(\text{F})\log_2 P(\text{F}) \\
&= -\tfrac{1}{2}\log_2\tfrac{1}{2} - \tfrac{1}{2}\log_2\tfrac{1}{2} = 1 \\
H(Y|X_1{=}0) &= -P(\text{T}|0)\log_2 P(\text{T}|0) - P(\text{F}|0)\log_2 P(\text{F}|0) \\
&= -\tfrac{1}{3}\log_2\tfrac{1}{3} -
\end{aligned}$$

Decision Trees and Information Gain

Select the feature with the highest information gain:

X_1	X_2	Y
0	a	T
0	b	F
1	c	T
0	c	F
1	b	F
1	a	T

$$\begin{aligned}
H(Y) &= -P(\text{T})\log_2 P(\text{T}) - P(\text{F})\log_2 P(\text{F}) \\
&= -\tfrac{1}{2}\log_2\tfrac{1}{2} - \tfrac{1}{2}\log_2\tfrac{1}{2} = 1 \\
H(Y|X_1{=}0) &= -P(\text{T}|0)\log_2 P(\text{T}|0) - P(\text{F}|0)\log_2 P(\text{F}|0) \\
&= -\tfrac{1}{3}\log_2\tfrac{1}{3} - \tfrac{2}{3}\log_2\tfrac{2}{3}
\end{aligned}$$

Decision Trees and Information Gain

Select the feature with the highest information gain:

X_1	X_2	Y
0	a	T
0	b	F
1	c	T
0	c	F
1	b	F
1	a	T

$$
\begin{aligned}
H(Y) &= -P(\text{T})\log_2 P(\text{T}) - P(\text{F})\log_2 P(\text{F}) \\
&= -\tfrac{1}{2}\log_2\tfrac{1}{2} - \tfrac{1}{2}\log_2\tfrac{1}{2} = 1 \\
H(Y|X_1{=}0) &= -P(\text{T}|0)\log_2 P(\text{T}|0) - P(\text{F}|0)\log_2 P(\text{F}|0) \\
&= -\tfrac{1}{3}\log_2\tfrac{1}{3} - \tfrac{2}{3}\log_2\tfrac{2}{3} = 0.92
\end{aligned}
$$

Decision Trees and Information Gain

Select the feature with the highest information gain:

X_1	X_2	Y
0	a	T
0	b	F
1	c	T
0	c	F
1	b	F
1	a	T

$$
\begin{aligned}
H(Y) &= -P(\mathrm{T})\log_2 P(\mathrm{T}) - P(\mathrm{F})\log_2 P(\mathrm{F}) \\
&= -\tfrac{1}{2}\log_2 \tfrac{1}{2} - \tfrac{1}{2}\log_2 \tfrac{1}{2} = 1 \\
H(Y|X_1{=}0) &= -P(\mathrm{T}|0)\log_2 P(\mathrm{T}|0) - P(\mathrm{F}|0)\log_2 P(\mathrm{F}|0) \\
&= -\tfrac{1}{3}\log_2 \tfrac{1}{3} - \tfrac{2}{3}\log_2 \tfrac{2}{3} = 0.92 \\
H(Y|X_1{=}1) &=
\end{aligned}
$$

Decision Trees and Information Gain

Select the feature with the highest information gain:

X_1	X_2	Y
0	a	T
0	b	F
1	c	T
0	c	F
1	b	F
1	a	T

$$
\begin{aligned}
H(Y) &= -P(\text{T})\log_2 P(\text{T}) - P(\text{F})\log_2 P(\text{F}) \\
&= -\tfrac{1}{2}\log_2\tfrac{1}{2} - \tfrac{1}{2}\log_2\tfrac{1}{2} = 1 \\
H(Y|X_1{=}0) &= -P(\text{T}|0)\log_2 P(\text{T}|0) - P(\text{F}|0)\log_2 P(\text{F}|0) \\
&= -\tfrac{1}{3}\log_2\tfrac{1}{3} - \tfrac{2}{3}\log_2\tfrac{2}{3} = 0.92 \\
H(Y|X_1{=}1) &= \ldots = 0.92
\end{aligned}
$$

Decision Trees and Information Gain

Select the feature with the highest information gain:

X_1	X_2	Y
0	a	T
0	b	F
1	c	T
0	c	F
1	b	F
1	a	T

$$
\begin{aligned}
H(Y) &= -P(\text{T})\log_2 P(\text{T}) - P(\text{F})\log_2 P(\text{F})\\
&= -\tfrac{1}{2}\log_2\tfrac{1}{2} - \tfrac{1}{2}\log_2\tfrac{1}{2} = 1\\
H(Y|X_1{=}0) &= -P(\text{T}|0)\log_2 P(\text{T}|0) - P(\text{F}|0)\log_2 P(\text{F}|0)\\
&= -\tfrac{1}{3}\log_2\tfrac{1}{3} - \tfrac{2}{3}\log_2\tfrac{2}{3} = 0.92\\
H(Y|X_1{=}1) &= \ldots = 0.92\\
H(Y|X_1) &=
\end{aligned}
$$

Decision Trees and Information Gain

Select the feature with the highest information gain:

X_1	X_2	Y
0	a	T
0	b	F
1	c	T
0	c	F
1	b	F
1	a	T

$$
\begin{aligned}
H(Y) &= -P(\text{T})\log_2 P(\text{T}) - P(\text{F})\log_2 P(\text{F}) \\
&= -\tfrac{1}{2}\log_2\tfrac{1}{2} - \tfrac{1}{2}\log_2\tfrac{1}{2} = 1 \\
H(Y|X_1{=}0) &= -P(\text{T}|0)\log_2 P(\text{T}|0) - P(\text{F}|0)\log_2 P(\text{F}|0) \\
&= -\tfrac{1}{3}\log_2\tfrac{1}{3} - \tfrac{2}{3}\log_2\tfrac{2}{3} = 0.92 \\
H(Y|X_1{=}1) &= \ldots = 0.92 \\
H(Y|X_1) &= P(X_1{=}0)H(Y|X_1{=}0) + P(X_1{=}1)H(Y|X_1{=}1)
\end{aligned}
$$

Decision Trees and Information Gain

Select the feature with the highest information gain:

X_1	X_2	Y
0	a	T
0	b	F
1	c	T
0	c	F
1	b	F
1	a	T

$$\begin{aligned}
H(Y) &= -P(\text{T})\log_2 P(\text{T}) - P(\text{F})\log_2 P(\text{F}) \\
&= -\tfrac{1}{2}\log_2\tfrac{1}{2} - \tfrac{1}{2}\log_2\tfrac{1}{2} = 1 \\
H(Y|X_1{=}0) &= -P(\text{T}|0)\log_2 P(\text{T}|0) - P(\text{F}|0)\log_2 P(\text{F}|0) \\
&= -\tfrac{1}{3}\log_2\tfrac{1}{3} - \tfrac{2}{3}\log_2\tfrac{2}{3} = 0.92 \\
H(Y|X_1{=}1) &= \ldots = 0.92 \\
H(Y|X_1) &= P(X_1{=}0)H(Y|X_1{=}0) + P(X_1{=}1)H(Y|X_1{=}1) \\
&= 0.5 \cdot 0.92 +
\end{aligned}$$

Decision Trees and Information Gain

Select the feature with the highest information gain:

X_1	X_2	Y
0	a	T
0	b	F
1	c	T
0	c	F
1	b	F
1	a	T

$$\begin{aligned}
H(Y) &= -P(\textsc{t})\log_2 P(\textsc{t}) - P(\textsc{f})\log_2 P(\textsc{f}) \\
&= -\tfrac{1}{2}\log_2 \tfrac{1}{2} - \tfrac{1}{2}\log_2 \tfrac{1}{2} = 1 \\
H(Y|X_1{=}0) &= -P(\textsc{t}|0)\log_2 P(\textsc{t}|0) - P(\textsc{f}|0)\log_2 P(\textsc{f}|0) \\
&= -\tfrac{1}{3}\log_2 \tfrac{1}{3} - \tfrac{2}{3}\log_2 \tfrac{2}{3} = 0.92 \\
H(Y|X_1{=}1) &= \ldots = 0.92 \\
H(Y|X_1) &= P(X_1{=}0)H(Y|X_1{=}0) + P(X_1{=}1)H(Y|X_1{=}1) \\
&= 0.5 \cdot 0.92 + 0.5 \cdot 0.92
\end{aligned}$$

Decision Trees and Information Gain

Select the feature with the highest information gain:

X_1	X_2	Y
0	a	T
0	b	F
1	c	T
0	c	F
1	b	F
1	a	T

$$
\begin{aligned}
H(Y) &= -P(\textsc{t})\log_2 P(\textsc{t}) - P(\textsc{f})\log_2 P(\textsc{f}) \\
&= -\tfrac{1}{2}\log_2\tfrac{1}{2} - \tfrac{1}{2}\log_2\tfrac{1}{2} = 1 \\
H(Y|X_1{=}0) &= -P(\textsc{t}|0)\log_2 P(\textsc{t}|0) - P(\textsc{f}|0)\log_2 P(\textsc{f}|0) \\
&= -\tfrac{1}{3}\log_2\tfrac{1}{3} - \tfrac{2}{3}\log_2\tfrac{2}{3} = 0.92 \\
H(Y|X_1{=}1) &= \ldots = 0.92 \\
H(Y|X_1) &= P(X_1{=}0)H(Y|X_1{=}0) + P(X_1{=}1)H(Y|X_1{=}1) \\
&= 0.5 \cdot 0.92 + 0.5 \cdot 0.92 = 0.92
\end{aligned}
$$

Decision Trees and Information Gain

Select the feature with the highest information gain:

X_1	X_2	Y
0	a	T
0	b	F
1	c	T
0	c	F
1	b	F
1	a	T

$$
\begin{aligned}
H(Y) &= -P(\text{T})\log_2 P(\text{T}) - P(\text{F})\log_2 P(\text{F}) \\
&= -\tfrac{1}{2}\log_2\tfrac{1}{2} - \tfrac{1}{2}\log_2\tfrac{1}{2} = 1 \\
H(Y|X_1{=}0) &= -P(\text{T}|0)\log_2 P(\text{T}|0) - P(\text{F}|0)\log_2 P(\text{F}|0) \\
&= -\tfrac{1}{3}\log_2\tfrac{1}{3} - \tfrac{2}{3}\log_2\tfrac{2}{3} = 0.92 \\
H(Y|X_1{=}1) &= \ldots = 0.92 \\
H(Y|X_1) &= P(X_1{=}0)H(Y|X_1{=}0) + P(X_1{=}1)H(Y|X_1{=}1) \\
&= 0.5\cdot 0.92 + 0.5\cdot 0.92 = 0.92 \\
IG(Y|X_1) &=
\end{aligned}
$$

Decision Trees and Information Gain

Select the feature with the highest information gain:

X_1	X_2	Y
0	a	T
0	b	F
1	c	T
0	c	F
1	b	F
1	a	T

$$\begin{aligned}
H(Y) &= -P(\text{T})\log_2 P(\text{T}) - P(\text{F})\log_2 P(\text{F}) \\
&= -\tfrac{1}{2}\log_2\tfrac{1}{2} - \tfrac{1}{2}\log_2\tfrac{1}{2} = 1 \\
H(Y|X_1{=}0) &= -P(\text{T}|0)\log_2 P(\text{T}|0) - P(\text{F}|0)\log_2 P(\text{F}|0) \\
&= -\tfrac{1}{3}\log_2\tfrac{1}{3} - \tfrac{2}{3}\log_2\tfrac{2}{3} = 0.92 \\
H(Y|X_1{=}1) &= \ldots = 0.92 \\
H(Y|X_1) &= P(X_1{=}0)H(Y|X_1{=}0) + P(X_1{=}1)H(Y|X_1{=}1) \\
&= 0.5\cdot 0.92 + 0.5\cdot 0.92 = 0.92 \\
IG(Y|X_1) &= H(Y) - H(Y|X_1)
\end{aligned}$$

Decision Trees and Information Gain

Select the feature with the highest information gain:

X_1	X_2	Y
0	a	T
0	b	F
1	c	T
0	c	F
1	b	F
1	a	T

$$
\begin{aligned}
H(Y) &= -P(\text{T}) \log_2 P(\text{T}) - P(\text{F}) \log_2 P(\text{F}) \\
&= -\tfrac{1}{2} \log_2 \tfrac{1}{2} - \tfrac{1}{2} \log_2 \tfrac{1}{2} = 1 \\
H(Y|X_1{=}0) &= -P(\text{T}|0) \log_2 P(\text{T}|0) - P(\text{F}|0) \log_2 P(\text{F}|0) \\
&= -\tfrac{1}{3} \log_2 \tfrac{1}{3} - \tfrac{2}{3} \log_2 \tfrac{2}{3} = 0.92 \\
H(Y|X_1{=}1) &= \ldots = 0.92 \\
H(Y|X_1) &= P(X_1{=}0)H(Y|X_1{=}0) + P(X_1{=}1)H(Y|X_1{=}1) \\
&= 0.5 \cdot 0.92 + 0.5 \cdot 0.92 = 0.92 \\
IG(Y|X_1) &= H(Y) - H(Y|X_1) = 1 - 0.92
\end{aligned}
$$

Decision Trees and Information Gain

Select the feature with the highest information gain:

X_1	X_2	Y
0	a	T
0	b	F
1	c	T
0	c	F
1	b	F
1	a	T

$$\begin{aligned}
H(Y) &= -P(\text{T})\log_2 P(\text{T}) - P(\text{F})\log_2 P(\text{F}) \\
&= -\tfrac{1}{2}\log_2\tfrac{1}{2} - \tfrac{1}{2}\log_2\tfrac{1}{2} = 1 \\
H(Y|X_1{=}0) &= -P(\text{T}|0)\log_2 P(\text{T}|0) - P(\text{F}|0)\log_2 P(\text{F}|0) \\
&= -\tfrac{1}{3}\log_2\tfrac{1}{3} - \tfrac{2}{3}\log_2\tfrac{2}{3} = 0.92 \\
H(Y|X_1{=}1) &= \ldots = 0.92 \\
H(Y|X_1) &= P(X_1{=}0)H(Y|X_1{=}0) + P(X_1{=}1)H(Y|X_1{=}1) \\
&= 0.5 \cdot 0.92 + 0.5 \cdot 0.92 = 0.92 \\
IG(Y|X_1) &= H(Y) - H(Y|X_1) = 1 - 0.92 = 0.08
\end{aligned}$$

Decision Trees and Information Gain

Select the feature with the highest information gain:

X_1	X_2	Y
0	a	T
0	b	F
1	c	T
0	c	F
1	b	F
1	a	T

$$
\begin{aligned}
H(Y) &= -P(\text{T})\log_2 P(\text{T}) - P(\text{F})\log_2 P(\text{F}) \\
&= -\tfrac{1}{2}\log_2\tfrac{1}{2} - \tfrac{1}{2}\log_2\tfrac{1}{2} = 1 \\
H(Y|X_1{=}0) &= -P(\text{T}|0)\log_2 P(\text{T}|0) - P(\text{F}|0)\log_2 P(\text{F}|0) \\
&= -\tfrac{1}{3}\log_2\tfrac{1}{3} - \tfrac{2}{3}\log_2\tfrac{2}{3} = 0.92 \\
H(Y|X_1{=}1) &= \ldots = 0.92 \\
H(Y|X_1) &= P(X_1{=}0)H(Y|X_1{=}0) + P(X_1{=}1)H(Y|X_1{=}1) \\
&= 0.5\cdot 0.92 + 0.5\cdot 0.92 = 0.92 \\
IG(Y|X_1) &= H(Y) - H(Y|X_1) = 1 - 0.92 = 0.08 \\
IG(Y|X_2) &=
\end{aligned}
$$

Decision Trees and Information Gain

Select the feature with the highest information gain:

X_1	X_2	Y
0	a	T
0	b	F
1	c	T
0	c	F
1	b	F
1	a	T

$$
\begin{aligned}
H(Y) &= -P(\text{T})\log_2 P(\text{T}) - P(\text{F})\log_2 P(\text{F}) \\
&= -\tfrac{1}{2}\log_2\tfrac{1}{2} - \tfrac{1}{2}\log_2\tfrac{1}{2} = 1 \\
H(Y|X_1{=}0) &= -P(\text{T}|0)\log_2 P(\text{T}|0) - P(\text{F}|0)\log_2 P(\text{F}|0) \\
&= -\tfrac{1}{3}\log_2\tfrac{1}{3} - \tfrac{2}{3}\log_2\tfrac{2}{3} = 0.92 \\
H(Y|X_1{=}1) &= \ldots = 0.92 \\
H(Y|X_1) &= P(X_1{=}0)H(Y|X_1{=}0) + P(X_1{=}1)H(Y|X_1{=}1) \\
&= 0.5\cdot 0.92 + 0.5\cdot 0.92 = 0.92 \\
IG(Y|X_1) &= H(Y) - H(Y|X_1) = 1 - 0.92 = 0.08 \\
IG(Y|X_2) &= \text{Your turn!}
\end{aligned}
$$

Decision Trees and Information Gain

Select the feature with the highest information gain:

X_1	X_2	Y
0	a	T
0	b	F
1	c	T
0	c	F
1	b	F
1	a	T

$$\begin{aligned}
H(Y) &= -P(\text{T})\log_2 P(\text{T}) - P(\text{F})\log_2 P(\text{F}) \\
&= -\tfrac{1}{2}\log_2\tfrac{1}{2} - \tfrac{1}{2}\log_2\tfrac{1}{2} = 1 \\
H(Y|X_1{=}0) &= -P(\text{T}|0)\log_2 P(\text{T}|0) - P(\text{F}|0)\log_2 P(\text{F}|0) \\
&= -\tfrac{1}{3}\log_2\tfrac{1}{3} - \tfrac{2}{3}\log_2\tfrac{2}{3} = 0.92 \\
H(Y|X_1{=}1) &= \ldots = 0.92 \\
H(Y|X_1) &= P(X_1{=}0)H(Y|X_1{=}0) + P(X_1{=}1)H(Y|X_1{=}1) \\
&= 0.5\cdot 0.92 + 0.5\cdot 0.92 = 0.92 \\
IG(Y|X_1) &= H(Y) - H(Y|X_1) = 1 - 0.92 = 0.08 \\
IG(Y|X_2) &= \text{Your turn!} = 0.67
\end{aligned}$$

Decision Trees and Information Gain

Select the feature with the highest information gain:

X_1	X_2	Y
0	a	T
0	b	F
1	c	T
0	c	F
1	b	F
1	a	T

$$
\begin{aligned}
H(Y) &= -P(\textsc{t})\log_2 P(\textsc{t}) - P(\textsc{f})\log_2 P(\textsc{f}) \\
&= -\tfrac{1}{2}\log_2\tfrac{1}{2} - \tfrac{1}{2}\log_2\tfrac{1}{2} = 1 \\
H(Y|X_1{=}0) &= -P(\textsc{t}|0)\log_2 P(\textsc{t}|0) - P(\textsc{f}|0)\log_2 P(\textsc{f}|0) \\
&= -\tfrac{1}{3}\log_2\tfrac{1}{3} - \tfrac{2}{3}\log_2\tfrac{2}{3} = 0.92 \\
H(Y|X_1{=}1) &= \ldots = 0.92 \\
H(Y|X_1) &= P(X_1{=}0)H(Y|X_1{=}0) + P(X_1{=}1)H(Y|X_1{=}1) \\
&= 0.5\cdot 0.92 + 0.5\cdot 0.92 = 0.92 \\
IG(Y|X_1) &= H(Y) - H(Y|X_1) = 1 - 0.92 = 0.08 \\
IG(Y|X_2) &= \text{Your turn!} \quad = 0.67
\end{aligned}
$$

So X_2 is a much better feature to split on

Decision Trees and Information Gain

Select the feature with the highest information gain:

X_1	X_2	Y
0	a	T
0	b	F
1	c	T
0	c	F
1	b	F
1	a	T

$$
\begin{aligned}
H(Y) &= -P(\text{T})\log_2 P(\text{T}) - P(\text{F})\log_2 P(\text{F}) \\
&= -\tfrac{1}{2}\log_2\tfrac{1}{2} - \tfrac{1}{2}\log_2\tfrac{1}{2} = 1 \\
H(Y|X_1{=}0) &= -P(\text{T}|0)\log_2 P(\text{T}|0) - P(\text{F}|0)\log_2 P(\text{F}|0) \\
&= -\tfrac{1}{3}\log_2\tfrac{1}{3} - \tfrac{2}{3}\log_2\tfrac{2}{3} = 0.92 \\
H(Y|X_1{=}1) &= \ldots = 0.92 \\
H(Y|X_1) &= P(X_1{=}0)H(Y|X_1{=}0) + P(X_1{=}1)H(Y|X_1{=}1) \\
&= 0.5\cdot 0.92 + 0.5\cdot 0.92 = 0.92 \\
IG(Y|X_1) &= H(Y) - H(Y|X_1) = 1 - 0.92 = 0.08 \\
IG(Y|X_2) &= \text{Your turn!} = 0.67
\end{aligned}
$$

So X_2 is a much better feature to split on

Repeat at each node

Decision Trees and Information Gain

Select the feature with the highest information gain:

X_1	X_2	Y
0	a	T
0	b	F
1	c	T
0	c	F
1	b	F
1	a	T

$$
\begin{aligned}
H(Y) &= -P(\text{T})\log_2 P(\text{T}) - P(\text{F})\log_2 P(\text{F}) \\
&= -\tfrac{1}{2}\log_2\tfrac{1}{2} - \tfrac{1}{2}\log_2\tfrac{1}{2} = 1 \\
H(Y|X_1{=}0) &= -P(\text{T}|0)\log_2 P(\text{T}|0) - P(\text{F}|0)\log_2 P(\text{F}|0) \\
&= -\tfrac{1}{3}\log_2\tfrac{1}{3} - \tfrac{2}{3}\log_2\tfrac{2}{3} = 0.92 \\
H(Y|X_1{=}1) &= \ldots = 0.92 \\
H(Y|X_1) &= P(X_1{=}0)H(Y|X_1{=}0) + P(X_1{=}1)H(Y|X_1{=}1) \\
&= 0.5 \cdot 0.92 + 0.5 \cdot 0.92 = 0.92 \\
IG(Y|X_1) &= H(Y) - H(Y|X_1) = 1 - 0.92 = 0.08 \\
IG(Y|X_2) &= \text{Your turn!} \quad = 0.67
\end{aligned}
$$

So X_2 is a much better feature to split on

Repeat at each node; different examples⇒different entropy

Random Forests

Algorithm:

1. Generate a bootstrap sample of n examples
2. For each node, select k features at random
3. Split on the feature with the highest information gain
4. Repeat steps 1, 2 and 3 to generate n decision trees
5. Classify by taking the majority vote

Why random forests?

- Combining independent classifiers $\Rightarrow$ better model
- Partially random trees should be more independent

Random Forests

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Why random forests?

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Random Forests

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1. Generate a bootstrap sample of n examples
2. For each node, select k features at random
3. Split on the feature with the highest information gain
4. Repeat steps 1, 2 and 3 to generate n decision trees
5. Classify by taking the majority vote

Why random forests?

- Combining independent classifiers $\Rightarrow$ better model
- Partially random trees should be more independent

Random Forest Example

x_0	x_1	x_2	$f(x)$
0	0	0	F
0	0	1	F
0	1	1	F
1	0	1	F
1	0	0	T
1	1	1	T

Bootstrap sample:

$[0, 5, 0, 1, 5, 4]$

Random feature subsets:

$[1, 2]$, $[2, 0]$

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000F
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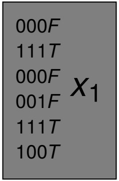

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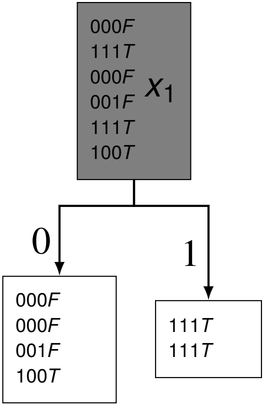

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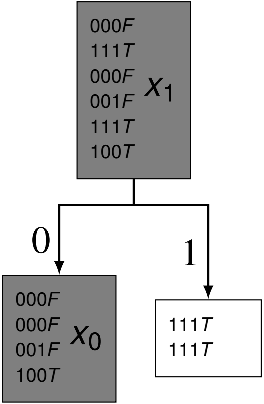

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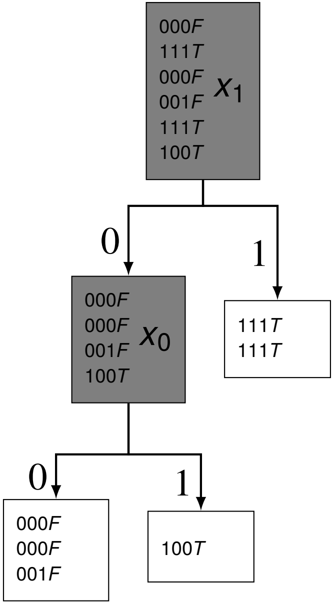

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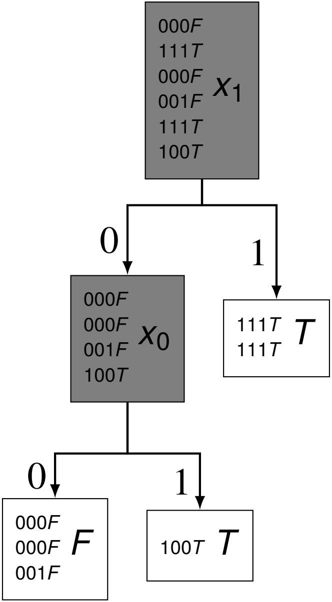

Random Forest Properties

Disadvantages

- Hard to estimate complexity due to random factor
- k (# of features) must be determined experimentally

Advantages

- State of the art performance on many datasets
- Relatively simple to implement
- Expected error can be determined while training
  - Bootstrap sample leaves out about $\frac{1}{3}$ of examples
  - Use these out-of-bag examples to test individual trees
  - Collect votes from all trees to get overall accuracy

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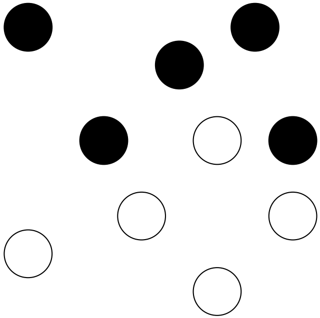
k-Nearest Neighbor Classifiers

To classify a new example, p

- Find the k training examples closest to p
- Classify p with the most common $f(x)$



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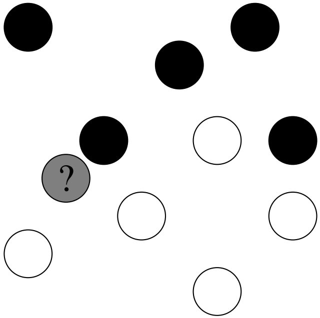

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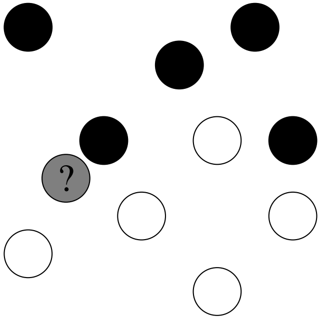

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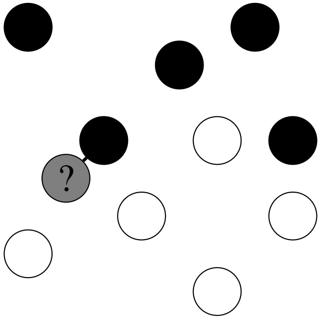

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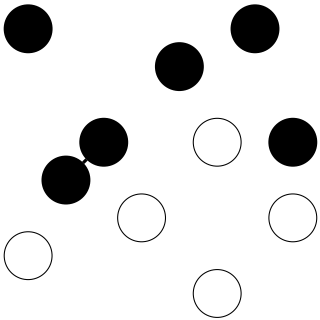

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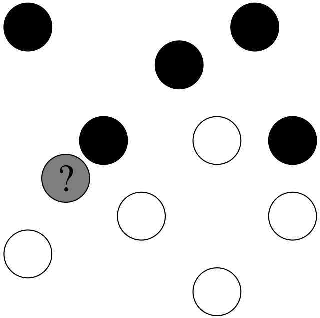

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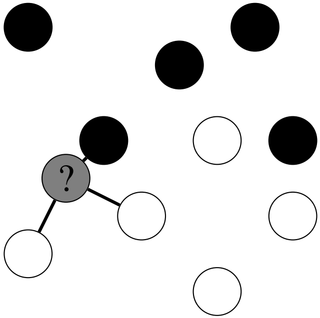

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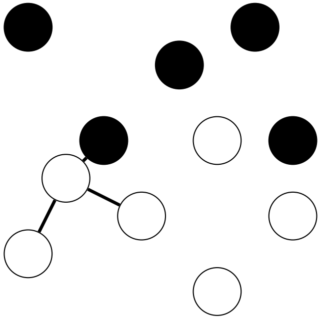

Distance Metrics

Euclidean Distance

$$\sqrt{\sum_{i=1}^{n}(a_i - b_i)^2}$$

close 0

far ∞

Cosine Similarity

$$\frac{\sum_{i=1}^{n} a_i b_i}{\sqrt{\sum_{i=1}^{n} a_i^2}\sqrt{\sum_{i=1}^{n} b_i^2}}$$

close 1

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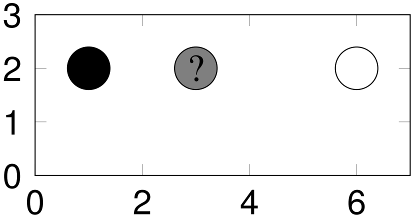

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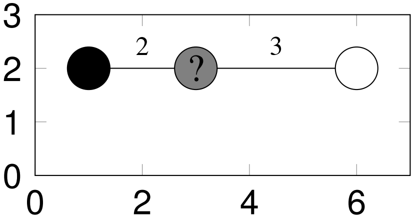

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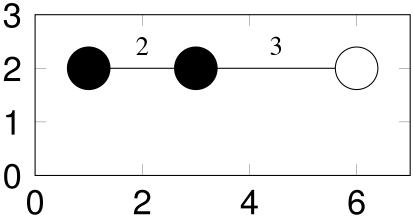

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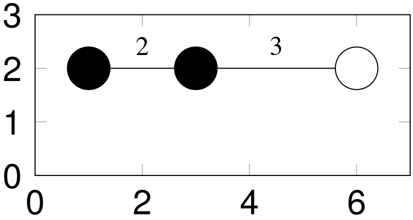

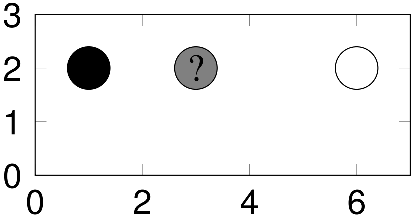

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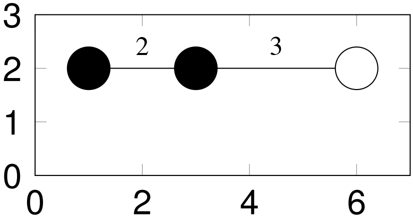

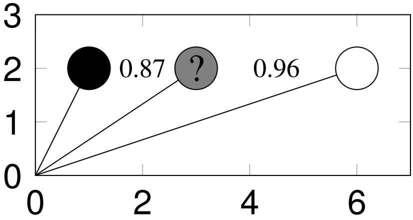

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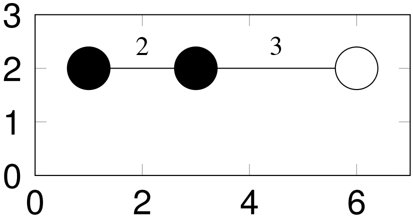

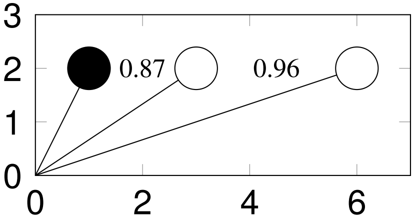

k-Nearest Neighbor Exercise

Training data:

x_1	x_2	$f(x)$
-1	1	A
0	1	A
0	2	A
2	2	B
3	2	B
3	3	B

Classify $x = [1, 1]$ with 3-NN

Euclidean? Cosine?

Euclidean Distance

$$\sqrt{\sum_{i=1}^{n} (a_i - b_i)^2}$$

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Euclidean? A

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Cosine? B

2	2	B
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Euclidean Distance

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close 0

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Cosine Similarity

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k-Nearest Neighbor Properties

Theoretical Properties

- Given enough data and the right k, minimizes error
- Able to approximate many kinds of functions

Empirical Issues

- Simple to implement given a distance function
- Large training data means long search times
- Not always clear what distance function to use

Linear Models

$$h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \ldots$$

Where:

- $x_1, x_2, x_3, \ldots$ are the features of x
- θ are feature weights

Learning linear models:

- Select parameters (weights) to minimize *loss*

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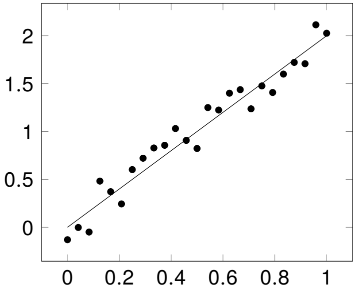
Loss

How "wrong" is this $h(x)$?



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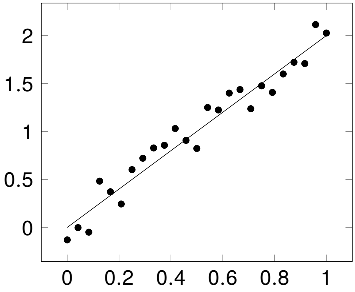

Loss at one point

$$L_{0/1}(\theta, x) = \begin{cases} 0 & \text{if } h_\theta(x) = f(x) \\ 1 & \text{otherwise} \end{cases}$$

Loss

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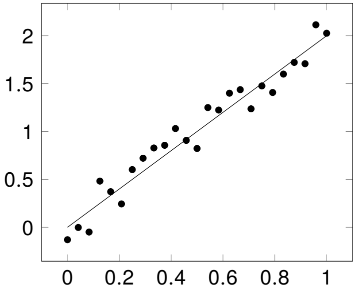

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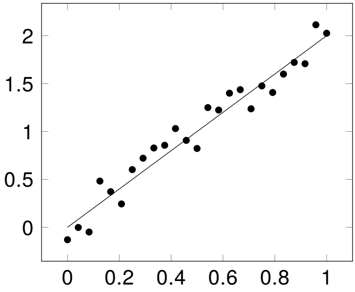

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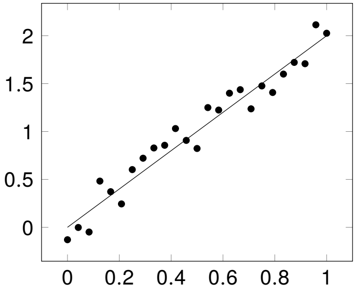

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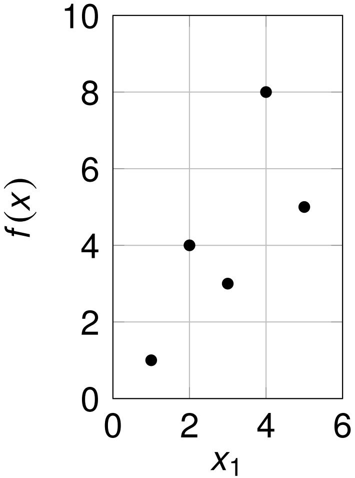
Empirical Loss

$$L(\theta) = \frac{1}{|X|} \sum_{x \in X} L(\theta, x)$$

Loss Exercise

Recall that $h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \ldots$ and consider:

1. $\boldsymbol{\theta}_a = [0, 1]$
2. $\boldsymbol{\theta}_b = [0.6, 1.2]$



Which θ is better by:

- $L_{0/1}(\theta) = \frac{1}{|X|}\sum_{x\in X} h_\theta(x) \neq f(x)$?
- $L_1(\theta) = \frac{1}{|X|}\sum_{x\in X} |f(x) - h_\theta(x)|$?
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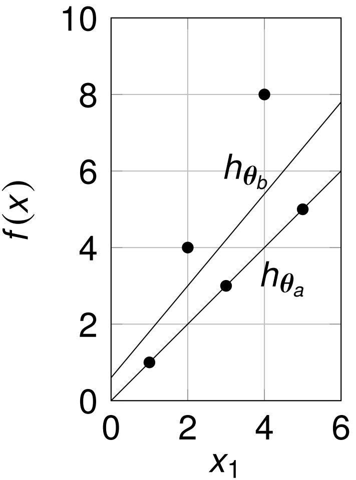
  $\boldsymbol{\theta}_a \left(\frac{2}{5} \text{ vs. } \frac{5}{5} \right)$
- $L_1(\theta) = \dfrac{1}{|X|} \sum_{x \in X} |f(x) - h_\theta(x)|$?
  $\boldsymbol{\theta}_a \left(\frac{6}{5} \text{ vs. } \frac{7.2}{5} \right)$
- $L_2(\theta) = \dfrac{1}{|X|} \sum_{x \in X} (f(x) - h_\theta(x))^2$?
  $\boldsymbol{\theta}_b \left(\frac{20}{5} \text{ vs. } \frac{12.4}{5} \right)$



Learning Parameters as Minimizing Loss

Key Ideas

- Best model parameters $\theta = \underset{\theta'}{\operatorname{argmin}}\, Loss(h_{\theta'})$
- Minimized function has partial derivatives = 0

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Example: $x = [x_1]$ with L_2 loss

$$Loss(h_\theta) = \frac{1}{|X|} \sum_{x \in X} (f(x) - (\theta_0 + \theta_1 x_1))^2$$

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$$\begin{aligned} \mathit{Loss}(h_\theta) &= \frac{1}{|X|} \sum_{x \in X} (f(x) - (\theta_0 + \theta_1 x_1))^2 \\ &= \frac{1}{|X|} \sum_{x \in X} f(x)^2 - 2\theta_0 f(x) - 2\theta_1 x_1 f(x) + \theta_0^2 + 2\theta_0\theta_1 x_1 + \theta_1^2 x_1^2 \end{aligned}$$

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\theta_0 &= \frac{1}{|X|}\sum_{x\in X} f(x) - \theta_1 x_1
\end{aligned}
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Loss(h_\theta) &= \frac{1}{|X|}\sum_{x\in X}\left(f(x) - (\theta_0 + \theta_1 x_1)\right)^2 \\
&= \frac{1}{|X|}\sum_{x\in X} f(x)^2 - 2\theta_0 f(x) - 2\theta_1 x_1 f(x) + \theta_0^2 + 2\theta_0\theta_1 x_1 + \theta_1^2 x_1^2 \\
\frac{\delta}{\delta\theta_0} Loss(h_\theta) &= \frac{1}{|X|}\sum_{x\in X} -2f(x) + 2\theta_0 + 2\theta_1 x_1 = 0 \\
\theta_0 &= \frac{1}{|X|}\sum_{x\in X} f(x) - \theta_1 x_1 \\
\frac{\delta}{\delta\theta_1} Loss(h_\theta) &= \ldots
\end{aligned}
$$

Learning Parameters as Minimizing Loss

Formal approach:

- Given: function h_θ and function $Loss(\theta)$
- Derive $\nabla Loss(\theta) = [\frac{\delta}{\delta\theta_0}Loss(\theta), \frac{\delta}{\delta\theta_1}Loss(\theta), \ldots]$
- Solve for θ in $\nabla Loss(\theta) = 0$

But there may be no closed form solution!

Gradient Descent

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θ = any setting of all parameters
while θ has not converged:
  for i in 0...|θ|:
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Gradient Descent Properties

For convex functions:

- Given small enough α, converges to global minimum
- May be slow: scans entire training data every step

Alternative: stochastic gradient descent:

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Avoiding Overfitting with Regularization

Recall overfitting solutions:

- Use simpler model
- Use fewer features

Learning as optimization allows another: *Regularization*

- Instead of minimizing $\mathit{Loss}(\theta)$
- Minimize $\mathit{Loss}(\theta) + \lambda\, \mathit{Complexity}(\theta)$
- Where λ can be tuned

Common choices for $\mathit{Complexity}(\theta)$

- $L_1(\theta) = \sum_i |\theta_i|$, encourages weights of 0 (sparsity)
- $L_2(\theta) = \sum_i |\theta_i|^2$, often makes math easier

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Regularization Exercise

x_1	x_2	$f(x)$
2	1	4
3	2	9
4	3	13

Given L_2 loss, which is better:

1. $\boldsymbol{\theta}_a = [-2, 3, 1]$
2. $\boldsymbol{\theta}_b = [0, 3, 0]$

for the regularization:

- None?
- $L_1,\ \lambda = \frac{1}{3}$?
- $L_2,\ \lambda = \frac{1}{3}$?

Loss:

$$L_2(\theta) = \frac{1}{|X|}\sum_{x\in X}(f(x) - h_\theta(x))^2$$

Regularizers:

$$L_1(\theta) = \sum_i |\theta_i| \quad L_2(\theta) = \sum_i |\theta_i|^2$$

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- None? $\boldsymbol{\theta}_a$ $\left(\frac{1}{3} \text{ vs. } \frac{5}{3}\right)$
- L_1, $\lambda = \frac{1}{3}$? $\boldsymbol{\theta}_a$ $\left(\frac{7}{3} \text{ vs. } \frac{8}{3}\right)$
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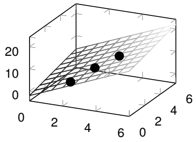
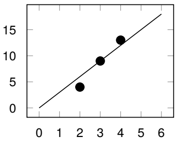
$h(x) = -2 + 3x_1 + x_2$

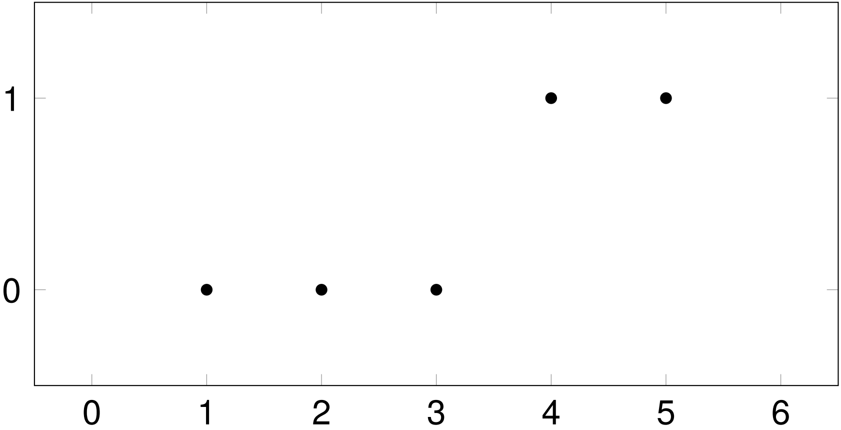


$h(x) = 3x_1$



Linear vs. Logistic Regression

Linear regression is bad for classification:



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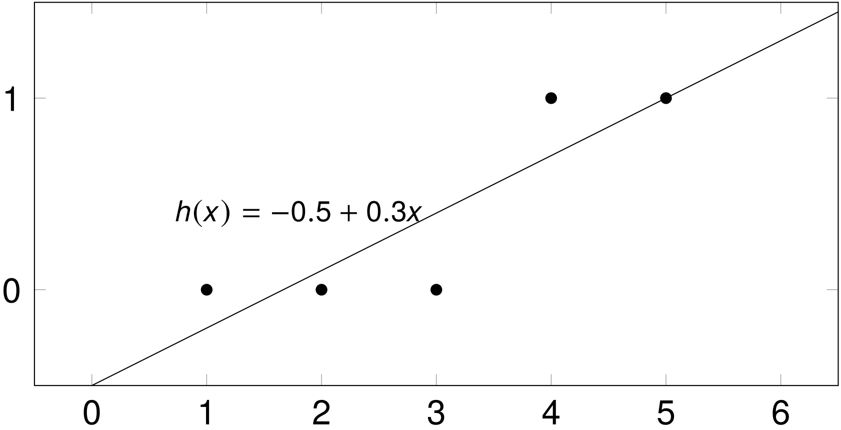

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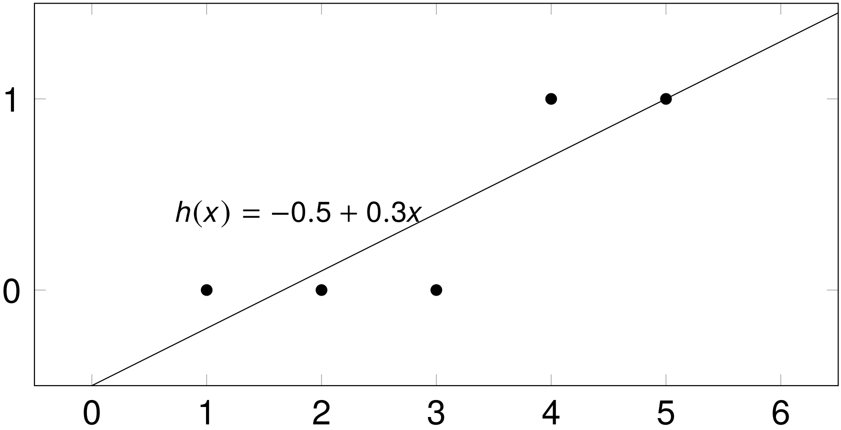

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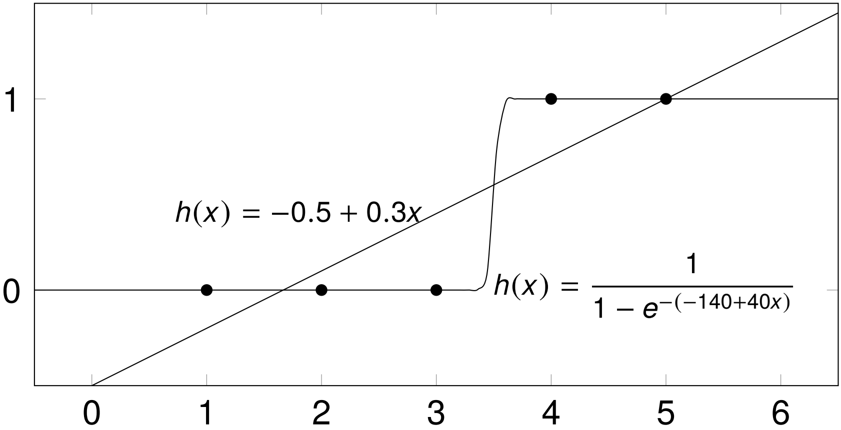

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Linear regression:

- $h(x) : \mathbb{R}^n \Rightarrow \mathbb{R}$
- $h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \ldots$
- $L_2(h_\theta) = \frac{1}{|X|} \sum_{x \in X} (f(x) - (\theta_0 + \theta_1 x_1 + \theta_2 x_2 + \ldots))^2$

Logistic regression:

- $h(x) : \mathbb{R}^n \Rightarrow [0, 1]$
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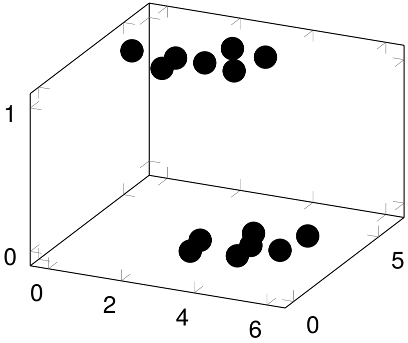
Logistic Regression Properties

- $h_\theta(x)$ can be interpreted as $P(f(x) = 1)$
- Can be generalized for multi-class classification
- With regularization, state-of-the-art on many problems

Max Margin Classification

Classification problem:



Classification hyperplanes:

- $x_2 = 3.7 \quad \Rightarrow \frac{13}{14}$
- $2x_1 - x_2 = 3 \quad \Rightarrow \frac{12}{14}$
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Maximizing margin:

$$2x_1 - x_2 = 3 \Rightarrow 0.35$$

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Data points at margin are called support vectors

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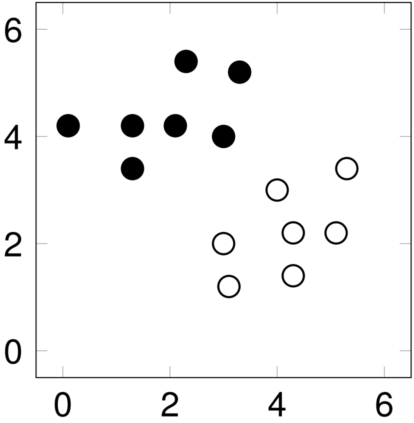

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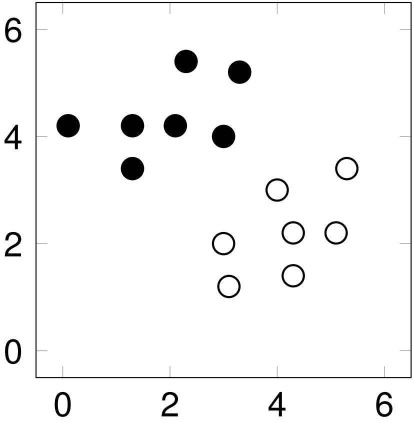

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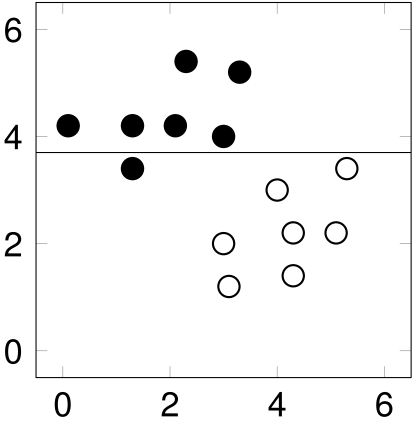

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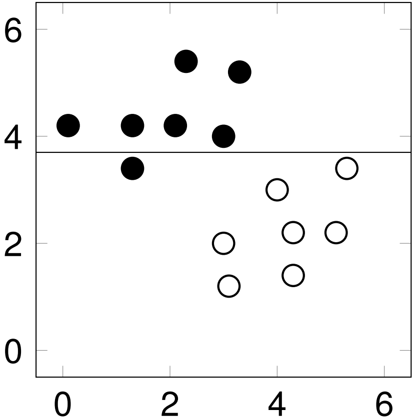

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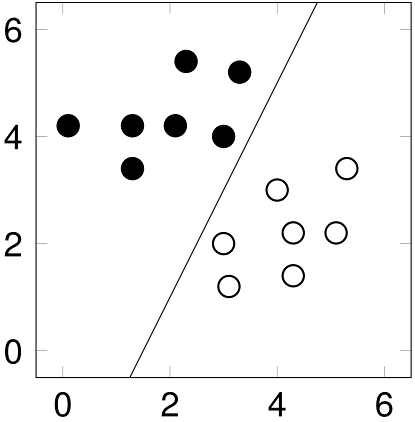

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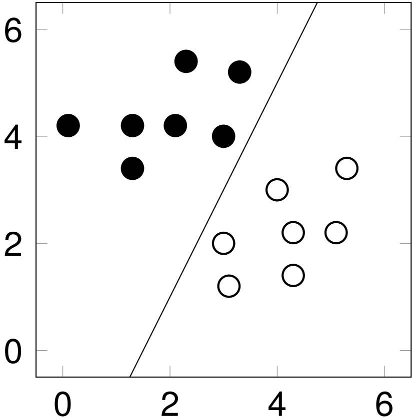

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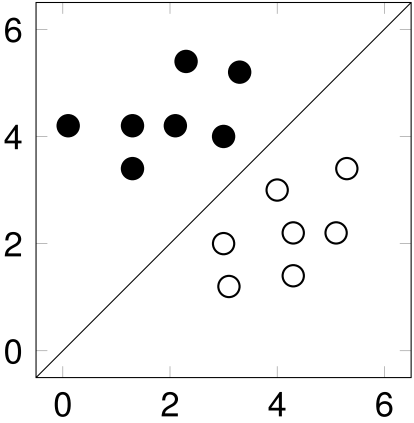

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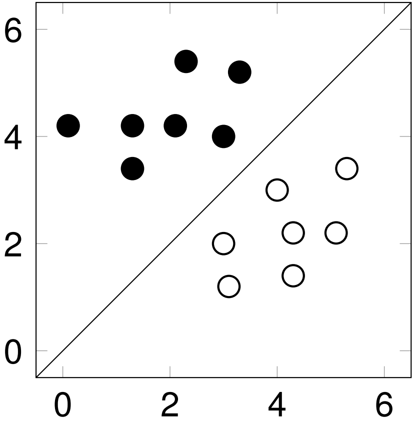

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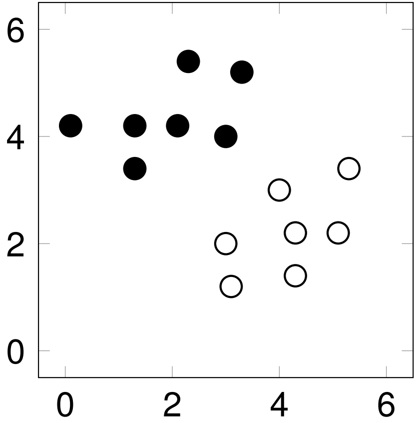

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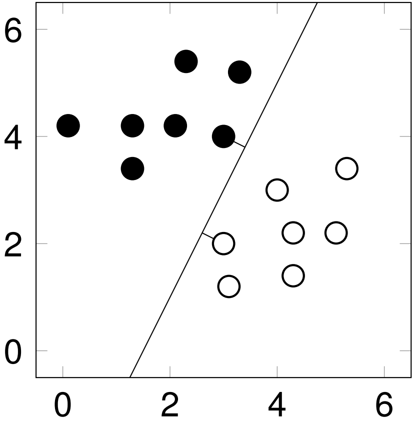

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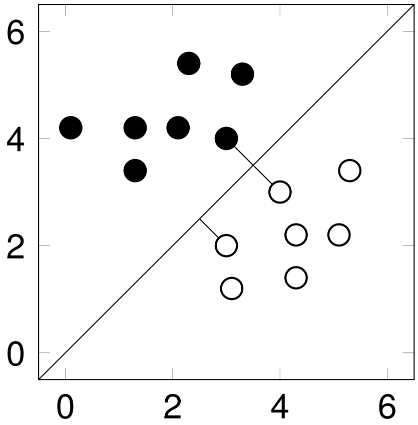

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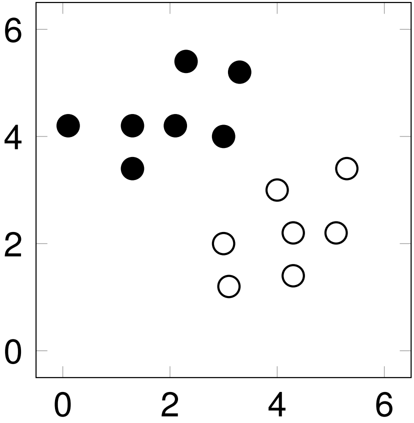

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Support Vector Machine Classifiers

Support vector machine loss, with $f(x) : \mathbb{R} \Rightarrow \{-1, +1\}$:

$$L_2(\theta) = \underbrace{\frac{1}{2} \sum_i \theta_i^2}_{\text{regularizer}} + \underbrace{C}_{\substack{\text{misclassify} \\ \text{cost}}} \sum_{x \in X} \max\left(0, 1 - f(x) \underbrace{\sum_i \theta_i x_i}_{\text{linear model}}\right)$$

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Kernel Methods

Support vector machine dual form:

$$L(\alpha) = \sum_{x \in X} \alpha_x - \frac{1}{2} \sum_{x \in X,\, x' \in X} \alpha_x \alpha_{x'} f(x) f(x') \sum_i x_i x_i'$$

Common kernels $k(x, x')$:

- Linear – $(\sum_i x_i x_i')$
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Demo: kernels allow non-linear classification boundaries

- http://www.csie.ntu.edu.tw/~cjlin/libsvm/index.html?js=1

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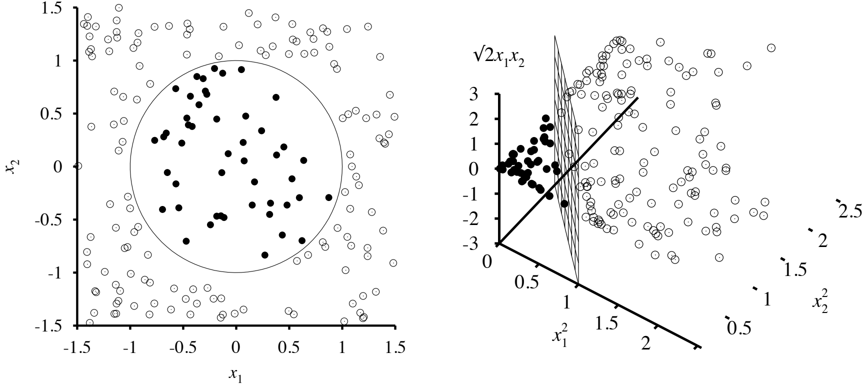

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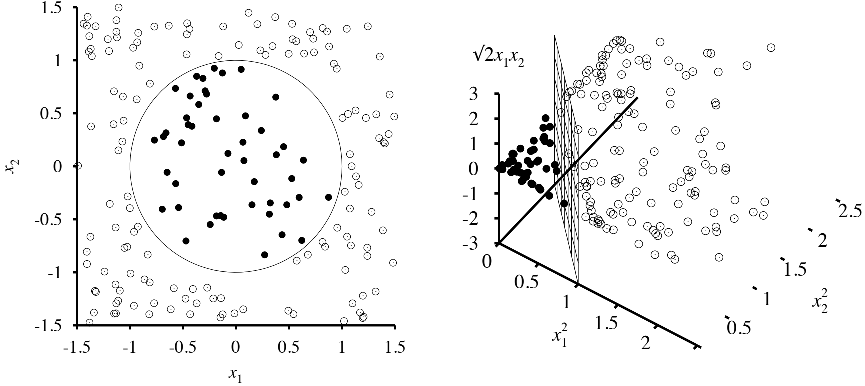

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Support Vector Machine Properties

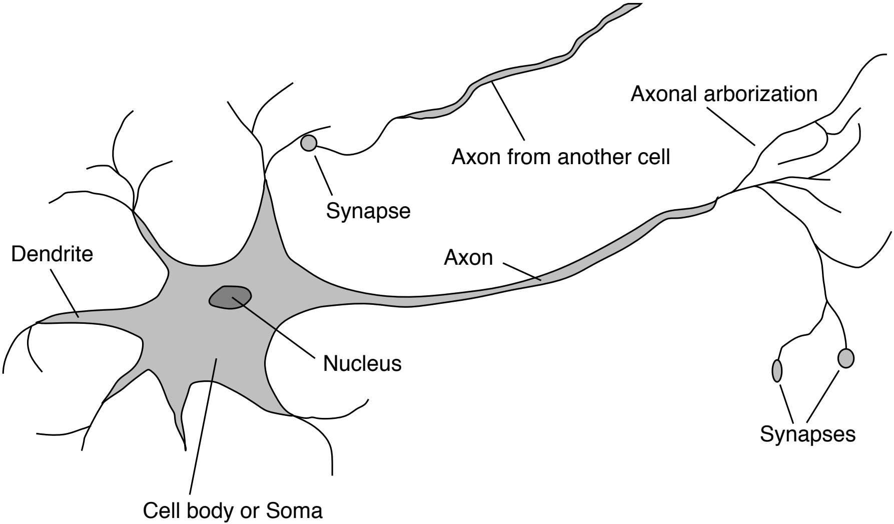
Theoretical Properties

- Efficient optimal separators in huge feature spaces
- Can approximate essentially any function

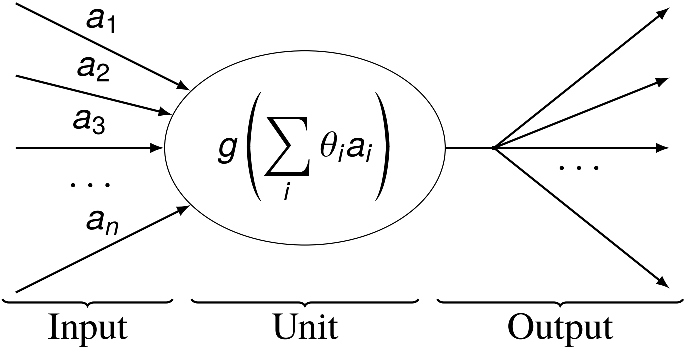
Empirical Issues

- Classification usually fast, but training often slow
- Kernel functions (and parameters) chosen empirically

Neurons in the Brain



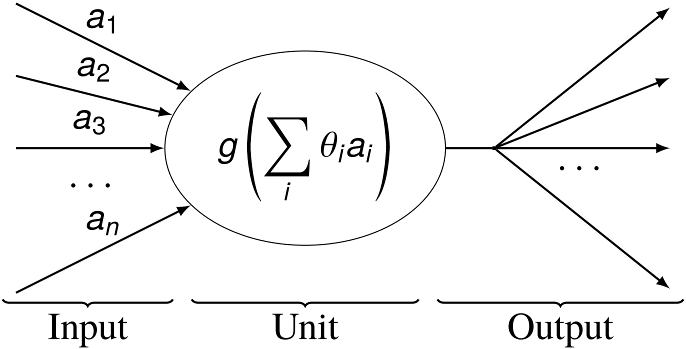
Neurons in a Neural Network



Common choices for *activation function* g:

$$threshold(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases} \qquad sigmoid(x) = \frac{1}{1 + e^{-x}}$$

Neurons in a Neural Network



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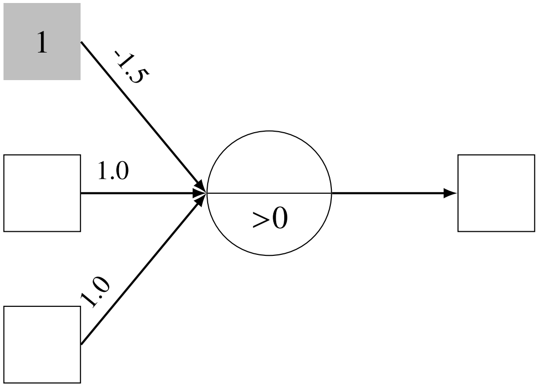

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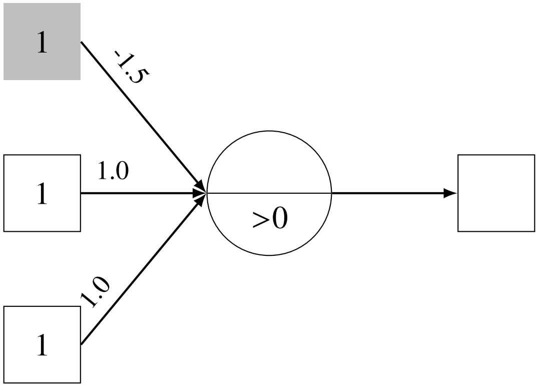

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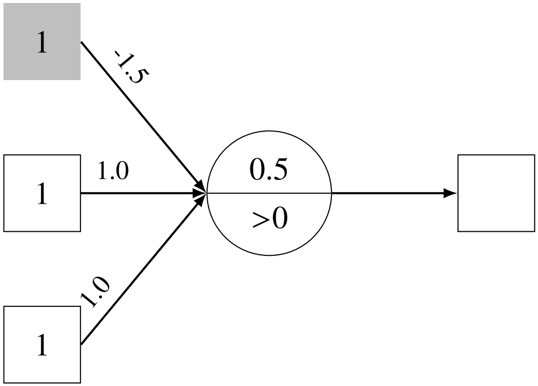

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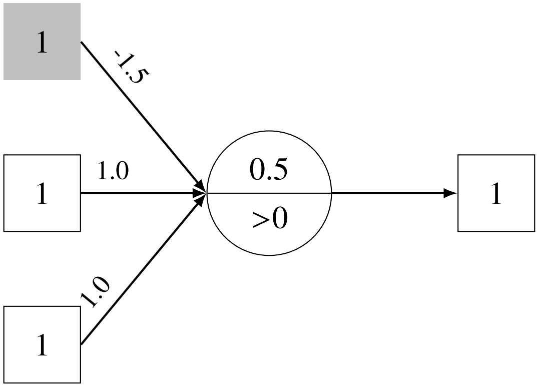

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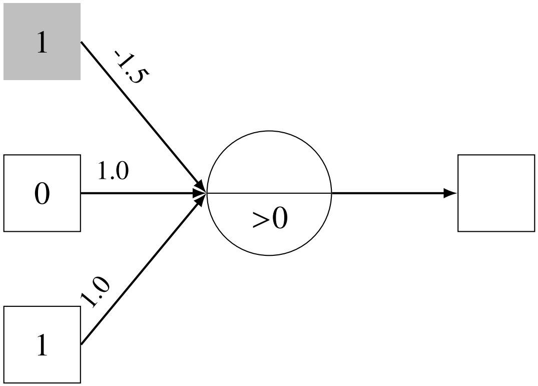

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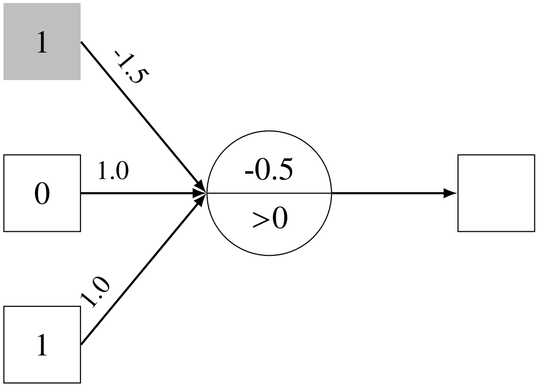

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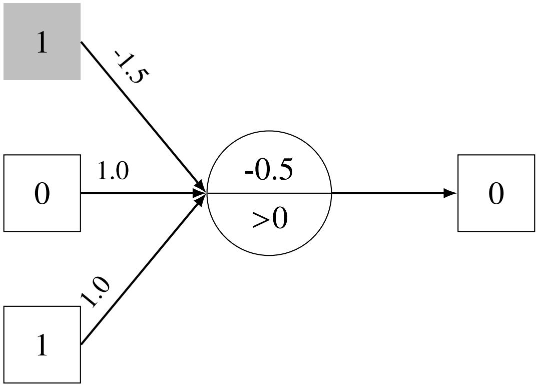
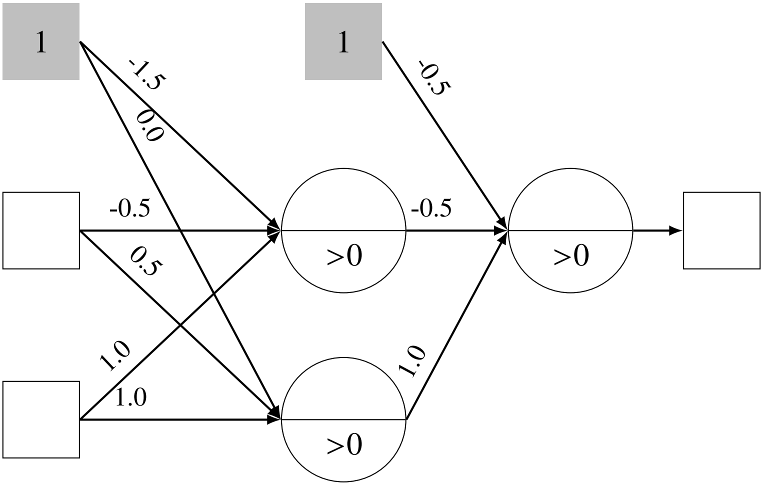


Neural Network Classification

Sum inputs, apply activation function, repeat



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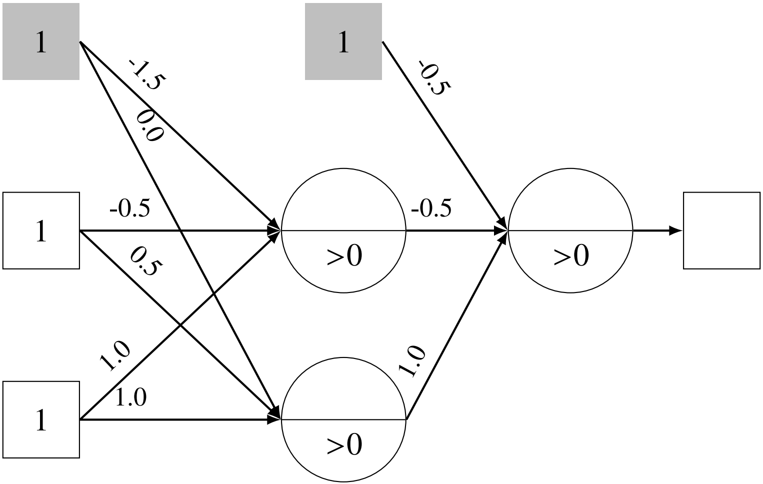

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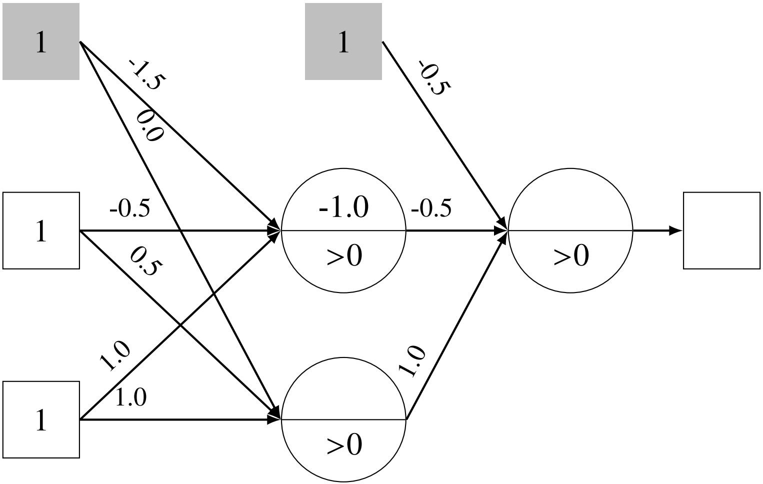

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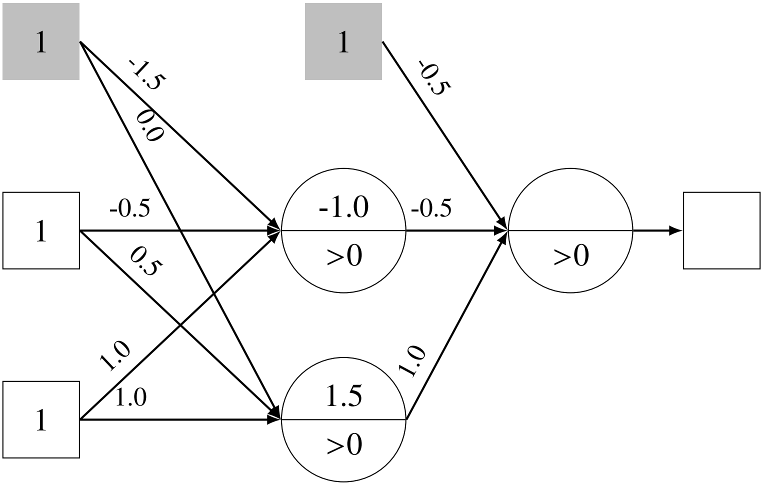

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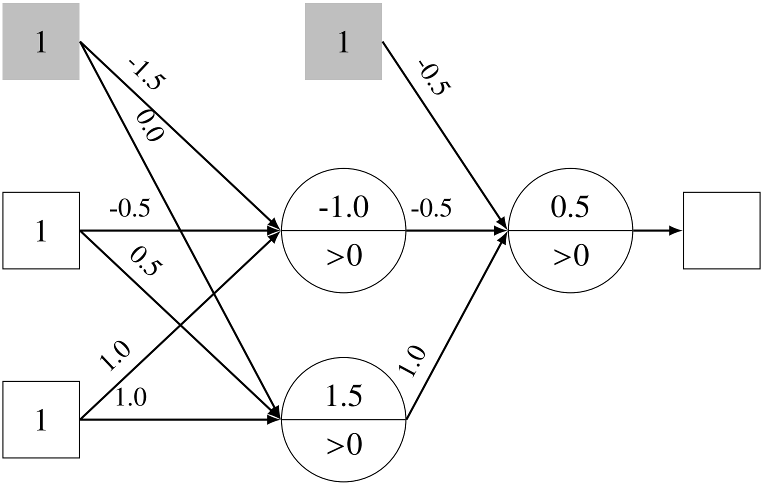

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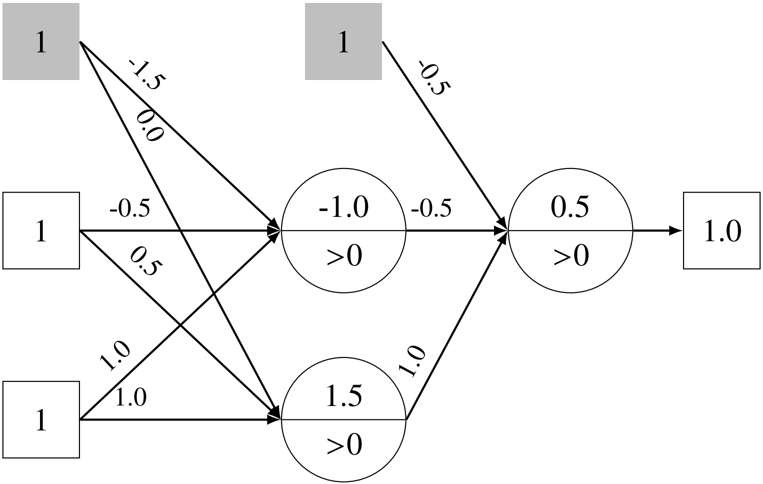
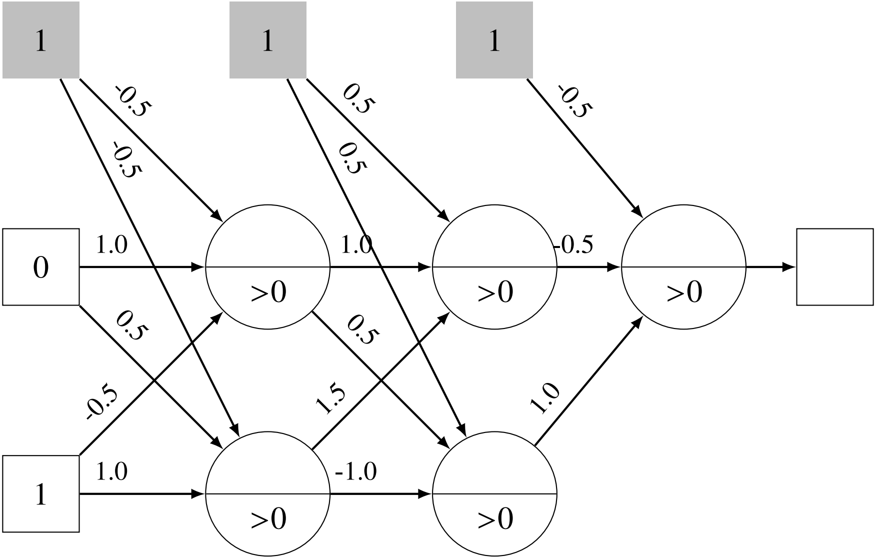


Neural Network Exercise



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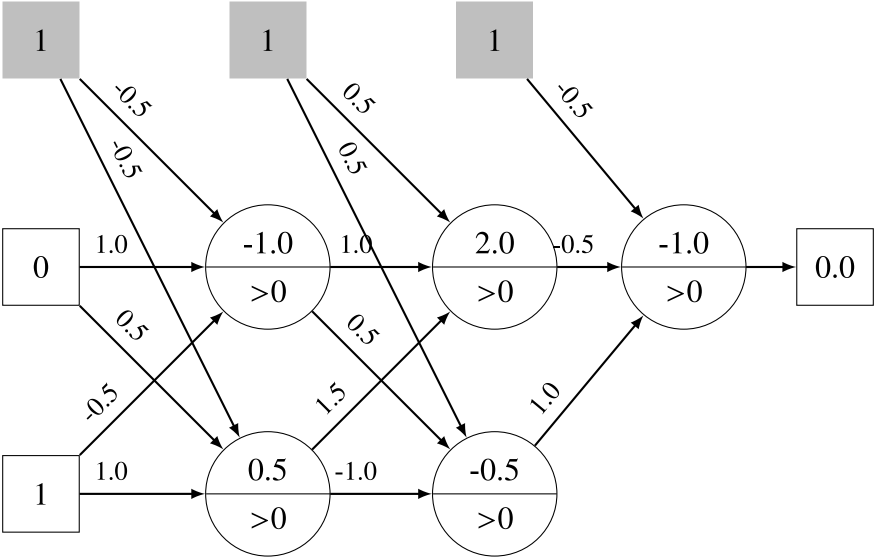

Learning Neural Networks

Gradient descent over the θs in all nodes:

1. Forward-propagate activation, get output a_o
2. $L(\theta) = \frac{1}{|X|} \sum_{x \in X} -f(x) \log(a_o) - (1 - f(x)) \log(1 - a_o)$
3. Calculate $\nabla L(\theta)$ by *back propagation*:
    1. Measure how much each unit was "responsible" for errors
    2. For output unit, $(a_o - f(x))$
    3. For hidden layer k, weighted average of layer $k + 1$
4. Use $L(\theta)$ and $\nabla L(\theta)$ to take a gradient descent step
5. Goto 1

Neural Net Properties

Expressive Power

- Single layer networks: linearly separable functions
- Multi-layer networks: essentially any function

Empirical Issues

- How many hidden layers?
- How many nodes in each layer?
- How to initialize weights?
- Has gradient descent gotten stuck at local minimum?

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Key Ideas

Supervised Learning:

- Input: $(x, f(x))$ examples; Output: h, a guess at f
- Representation: x decomposed into features
- Evaluation: train, development, test
- Learning curves: reveal underfitting, overfitting

Supervised Learning Algorithms:

- Decision trees and random forests
- Linear and logistic regression
- Support vector machines
- k-nearest neighbors
- Neural networks