

So if we need more games to play with Airy's equation, which is

$$y'' - xy = 0,$$

then letting

$$y = \sum_{m=0}^{\infty} c_m x^m,$$

we get the other linearly independent solution to be

$$y(x) = x + \sum_{m=1}^{\infty} d_m x^{3m+1}, \quad d_m = \prod_{l=1}^m \frac{1}{(3l+1)(3l)}$$

So having them play with this is a good idea.

If for some reason there is still time after working through this, then we can move on to looking at parabolic cylinder function solutions which come from the ODE

$$y'' - (1 + x^2)y = 0,$$

so that

$$y(x) = \sum_{m=0}^{\infty} c_m x^{2m},$$

where the  $c_m$  satisfy the multi-step recurrence relationship

$$c_{m+1} = \frac{1}{(2m+2)(2m+1)} (c_m + c_{m-1}), \quad m \geq 1,$$

with  $c_0 = 2$  and  $c_1 = 1$ . That should keep them busy.