Gradescope Assignment: Due 3/3/21
0 pts for no work
2 pts for attempt
4 pts for full answer

So as we discussed in class, we have the "column space" form of matrix/vector multiplication whereby if we write a 2×2 matrix A in the form

$$A = \left(\mathbf{a}_1 \mid \mathbf{a}_2\right)$$

where $\mathbf{a}_j$ is the j^{th} column of A, then for

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

we have

$$A\mathbf{x} = x_1\mathbf{a}_1 + x_2\mathbf{a}_2$$

1. (Short) Using this formula, show that for a time dependent vector $\mathbf{x}(t)$ where

$$\mathbf{x}(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix},$$

that if A is a time independent (or constant) 2×2 matrix that

$$A\frac{d}{dt}\mathbf{x} = \frac{d}{dt}\left(A\mathbf{x}\right)$$

2. (Short) Suppose A is a 2×2 matrix such that $(\text{tr}(A))^2 \neq 4\det(A)$, which guarantees that A is diagonalizable, so that

$$A = V\Lambda V^{-1},\ V = \left(\mathbf{v}_1 \mid \mathbf{v}_2\right),\ \Lambda = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix},\ A\mathbf{v}_j = \lambda_j\mathbf{v}_j,\ \lambda_1 \neq \lambda_2.$$

Using this, show that the initial value problem

$$\frac{d}{dt}\mathbf{x} = A\mathbf{x}(t),\ \mathbf{x}(0) = \mathbf{x}_0,$$

can be rewritten in the form

$$\frac{d}{dt}\mathbf{y} = \Lambda\mathbf{y}(t),\ \mathbf{y}(0) = V^{-1}\mathbf{x}_0$$

where

$$\mathbf{y}(t) = V^{-1}\mathbf{x}(t).$$

Start your solution from

$$\frac{d}{dt}\mathbf{x} = V\Lambda V^{-1}\mathbf{x}(t),\ \mathbf{x}(0) = \mathbf{x}_0,$$

so that

$$V^{-1}\frac{d}{dt}\mathbf{x} = \Lambda V^{-1}\mathbf{x}(t),\ \mathbf{x}(0) = \mathbf{x}_0.$$

Use the result of the prior problem to help you along.