Gradescope Assignment: Due 4/5/21
0 pts for no work
2 pts for attempt
4 pts for full answer

1. For the two-dimensional initial-value problem

$$\frac{d}{dt}\mathbf{x} = A\mathbf{x} + \begin{pmatrix} f_1(t) \\ f_2(t) \end{pmatrix}, \quad \mathbf{x}(0) = \mathbf{x}_0,$$

suppose A has the Jordan structure

$$A = V \begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix} V^{-1}.$$

- Using $\mathbf{y} = V^{-1}\mathbf{x}$, solve the transformed system. Your solution should have definite integrals from 0 to t in it.
- Return to the original coordinates. Clearly identify the homogeneous and particular parts of your solution.

2. For the system,

$$a\frac{d^2}{dt^2}x + b\frac{d}{dt}x + cx = \cos(\nu t),$$

we showed in the underdamped case, where $b^2 < 4ac$, that the particular solution is given by

$$\begin{aligned} x_p(t) =& \frac{1}{2\omega}\left(\frac{\mu}{\mu^2 + (\nu - \omega)^2}\left(\sin(\nu t) - \sin(\omega t)e^{-\mu t}\right)\right. \\ &+ \frac{(\nu - \omega)}{\mu^2 + (\nu - \omega)^2}\left(-\cos(\nu t) + \cos(\omega t)e^{-\mu t}\right) \\ &- \frac{\mu}{\mu^2 + (\nu + \omega)^2}\left(\sin(\nu t) + \sin(\omega t)e^{-\mu t}\right) \\ &\left.- \frac{(\omega + \nu)}{\mu^2 + (\nu + \omega)^2}\left(-\cos(\nu t) + \cos(\omega t)e^{-\mu t}\right)\right) \end{aligned}$$

where $\mu = b/2a$ and $\omega = \sqrt{4ac - b^2}/2a$. Show that if we take $b = 0$ so that $\mu = 0$, thereby removing friction/resistance, that when we then let $\nu \to \omega$, we get that

$$x_p(t) = t\sin(\omega t).$$

Comment on the consequences of this.