Fitting : Data $\{x_j, y_j\}_{j=1}^{n}$

$$\text{Loss}\; E(a,b) = \sum_{j=1}^{n} \Big(y_j - f(x_j, \underbrace{a, b}_{\text{2 unknowns}})\Big)^2$$

$$\left.\begin{aligned} \frac{\partial E}{\partial a} &= 0 \\ \frac{\partial E}{\partial b} &= 0 \end{aligned}\right\} \text{2 equations}$$

$$\hookrightarrow f(x_j, a, b) = \boxed{a x_j + b}$$

$$\hookrightarrow -2 \sum_{j=1}^{n} \big(y_j - (a x_j + b)\big) x_j = 0$$

$$\frac{\partial E}{\partial a} = 0 : \hookrightarrow \sum_{j=1}^{n} y_j x_j - a \sum_{j=1}^{n} x_j^2 - b \sum_{j=1}^{n} x_j = 0 .$$

$$\frac{\partial E}{\partial b} = 0 \rightarrow -2 \sum_{j=1}^{n} \left(y_j - (a x_j + b) \right) = 0$$

$$\Leftrightarrow \sum_{j=1}^{n} y_j - a \sum_{j=1}^{n} x_j - bn = 0$$

$$\Leftrightarrow \begin{pmatrix} \sum_{j=1}^{n} y_j x_j \\ \sum_{j=1}^{n} y_j \end{pmatrix} = \begin{pmatrix} \sum_{j=1}^{n} x_j^2 & \sum_{j=1}^{n} x_j \\ \sum_{j=1}^{n} x_j & n \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} a \\ b \end{pmatrix} = \frac{1}{n \sum_{j=1}^{n} x_j^2 - \left(\sum_{j=1}^{n} x_j \right)^2} \begin{pmatrix} n & -\sum_{j=1}^{n} x_j \\ -\sum_{j=1}^{n} x_j & \sum_{j=1}^{n} x_j^2 \end{pmatrix} \begin{pmatrix} \sum_j y_j x_j \\ \sum_j y_j \end{pmatrix}$$

$$\cancel{\left(\sum_{j=1}^{n} x_j \right)^2 \text{"}=\text{"} \sum_{j=1}^{n} x_j^2}$$

TEMPTING

$$\left(\sum_{j=1}^{n} x_j\right)^2 = \left(\sum_{j=1}^{n} x_j\right)\left(\sum_{k=1}^{n} x_k\right)$$

$$= \sum_{j=1}^{n}\sum_{k=1}^{n} x_j x_k$$

$$(x_1 + x_2)^2 = x_1^2 + x_1x_2 + x_2x_1 + x_2^2$$

$$W(t) = W_*^k\left(1 - e^{-bt}\right)^k$$

$$\frac{dH}{dt} = kW(t) = kW_*\left(1 - e^{-bt}\right)^k$$

$$\hookrightarrow H(t) = kW_*^k \int_0^t \left(1 - e^{-bs}\right)^k ds$$

$$= kW_*^k \int_0^t \sum_{l=0}^{k} \binom{k}{l}(1)^{k-l}\left(-e^{-bs}\right)^l ds$$

$$= kW_*^k \sum_{l=0}^{k} \binom{k}{l}(-1)^l \int_0^t e^{-lbs}\, ds$$

$$\binom{k}{0} = 1$$

$$H(t) = \kappa \omega_*^k \left(t + \sum_{l=1}^{k} \binom{k}{l} (-1)^l \int_0^t e^{-lbs}\, ds \right)$$

$$= \kappa \omega_*^k \left(t + \sum_{l=1}^{k} \binom{k}{l} \frac{(-1)^l}{lb} \left(1 - e^{-lbt}\right) \right)$$

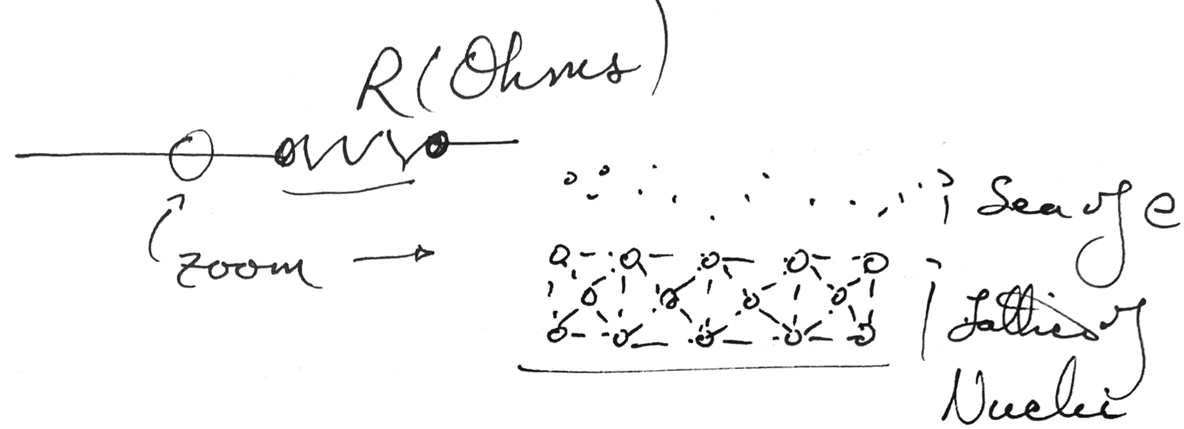
TeX:

\left(\right) (1st)

\begin{pmatrix} = \end{pmatrix}

Kirchoff's Circuit Laws:

"Wire"



So $Q(t)$ measures amount of charge in motion at a given time.

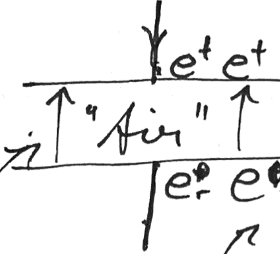
Current - $I(t) = \dfrac{dQ}{dt}$

Voltage - $V(t) \equiv$ the energy needed to make charges move.

Resistance - $R \equiv$ any substance through which electrons cannot move as freely. Energy is lost, which usually manifests as heat.

↳ $\boxed{\text{Ohm's Law: } V = IR}$

energy lost

Capacitance - $C = Q/V \equiv$ 

charge piles

Metal Plates

(2)

$V = -Q/C$

Farads

field resisting current forms.

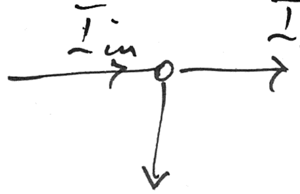
Opposite charge piles up at other side in response

Inductor - $L = -V/\frac{dI}{dt}$ or $V = -L\frac{dI}{dt} = -L\frac{d^2Q}{dt^2}$

Henry's

Time changing current induces field changes which manifest as voltage rises and falls.

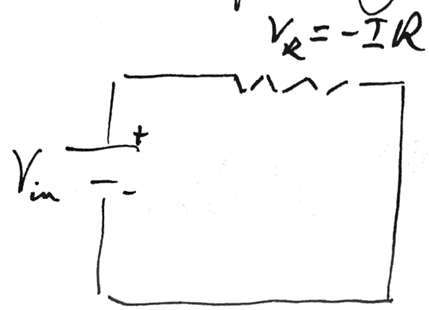
Kirchoff's Current Law: Charge is neither spontaneously created nor destroyed!



$\rightarrow$ $I_{in} = I_{out,1} + I_{out,2}$

Kirchoff's Voltage Law: The change in voltage around a closed loop is zero.

So:

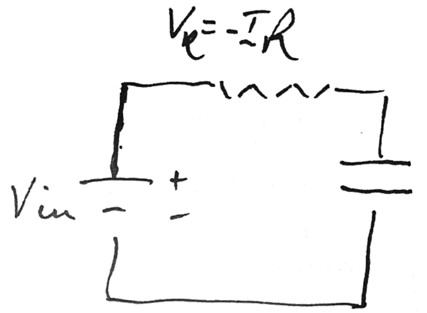


↳ $V_{in} + V_R = 0$

↳ $V_{in} - IR = 0$

or $V_{in} = IR$

So $I = V_{in}/R$



$C = Q/V_c$ or $V_c = -Q/C$

i.e. voltage drop

$$V_{in} + V_R + V_c = 0 \rightarrow V_{in} - IR - \frac{Q}{C} = 0$$

↳ using $I = \frac{dQ}{dt}$

↳ $R\frac{dQ}{dt} + \frac{1}{C}Q = V_{in}(t)$.

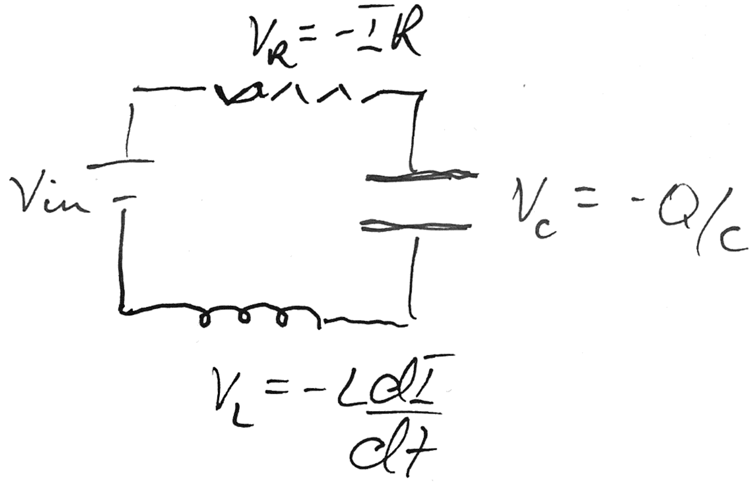
$$\to \frac{dQ}{dt} + \frac{1}{RC} Q = \frac{1}{R} V_{in}(t)$$

$$\to \frac{d}{dt}\left(Q e^{t/RC}\right) = \frac{1}{R} V_{in}(t) e^{t/RC}$$

$$\to Q(t) e^{t/RC} - Q_0 = \frac{1}{R}\int_0^t V_{in}(s) e^{s/RC} ds$$

$$\to Q(t) = Q_0 e^{-t/RC} + \frac{1}{R}\int_0^t V_{in}(s) e^{-(t-s)/RC} ds.$$

note convolution



$$\to V_{in} = IR + \frac{1}{C} Q + L\frac{dI}{dt}$$

$$\to \frac{dV_{in}}{dt} = R\frac{dI}{dt} + \frac{1}{C} I + L\frac{d^2 I}{dt^2}$$

So, compared to mass-spring:

$$m\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + kx = F(t)$$

Resist. Friction "Hooke's" Law.

vs.

$$L\frac{d^2I}{dt^2} + R\frac{dI}{dt} + \frac{1}{C}I = \frac{dV_{in}}{dt}$$