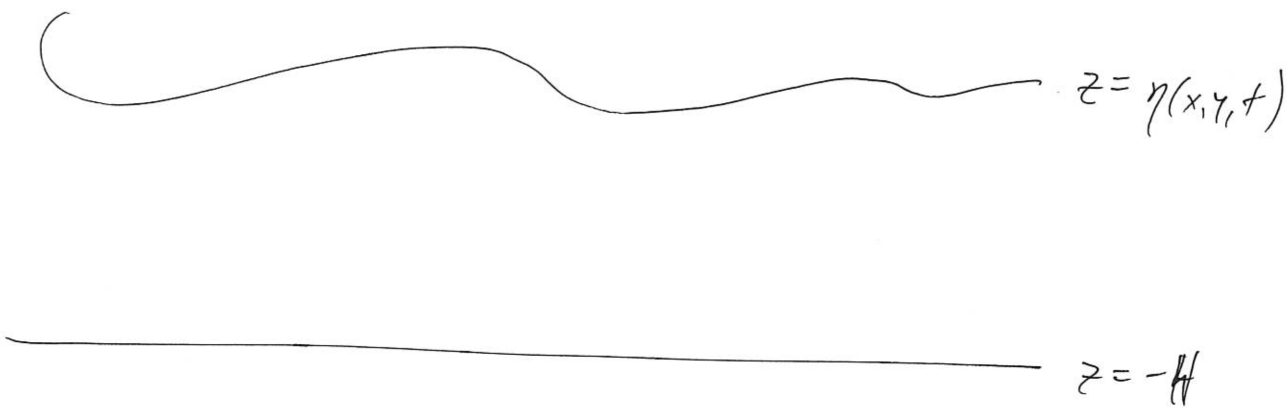
Now, as to waves:



$$\Delta\phi = 0 \qquad -H < z < \eta(x,y,t)$$

$$\phi_z = 0 \qquad z = -H$$

$$\eta_t = -\eta_x\phi_x - \eta_y\phi_y + \phi_z \quad ; \quad z = \eta(x,y,t)$$

$$\phi_t + \frac{1}{2}|\nabla\phi|^2 + g\eta = \underbrace{\frac{\sigma}{\rho}\nabla_{(x,y)}\cdot\left(\frac{\nabla_{(x,y)}\eta}{(1+\|\nabla\eta\|^2)^{1/2}}\right)}_{\substack{\text{Surface Tension} \\ \text{i.e. Meniscus effects}}} \quad ; \quad z = \eta(x,y,t)$$

So, to properly proceed, before we do anything, we need to non-dimensionalize this system.

So, there is no unique way to do this. A natural way though is to set

$$\tilde{x} = \frac{x}{L} \; ; \; \tilde{y} = \frac{y}{L} \; ; \; \tilde{z} = \frac{z}{H} \; ; \; \eta = a\tilde{\eta} \; ;$$

$$\tau = \frac{\sqrt{gH}}{H}\, t \; ; \; \phi = L\sqrt{gH}\,\tilde{\phi}$$

So for example then, the equation

$$\Delta\phi = 0 \; , \quad -H < z < \eta(x,y,t)$$

↳ using $\partial_x = \frac{1}{L}\partial_{\tilde{x}}$; $\partial_y = \frac{1}{L}\partial_{\tilde{y}}$; $\partial_z = \frac{1}{H}\partial_{\tilde{z}}$

↳ $$\frac{1}{L^2}\partial_{\tilde{x}}^2\tilde{\phi} + \frac{1}{L^2}\partial_{\tilde{y}}^2\tilde{\phi} + \frac{1}{H^2}\partial_{\tilde{z}}^2\tilde{\phi} = 0$$

↳ $$-H < H\tilde{z} < a\tilde{\eta}$$

↳ $$\partial_x^2\phi + \partial_y^2\phi + \frac{L^2}{H^2}\partial_z^2\phi = 0$$

$$-1 < z < \frac{a}{H}\eta$$

As you will note, $\frac{H}{L}$ and $\frac{a}{H}$ are both dimensionless.

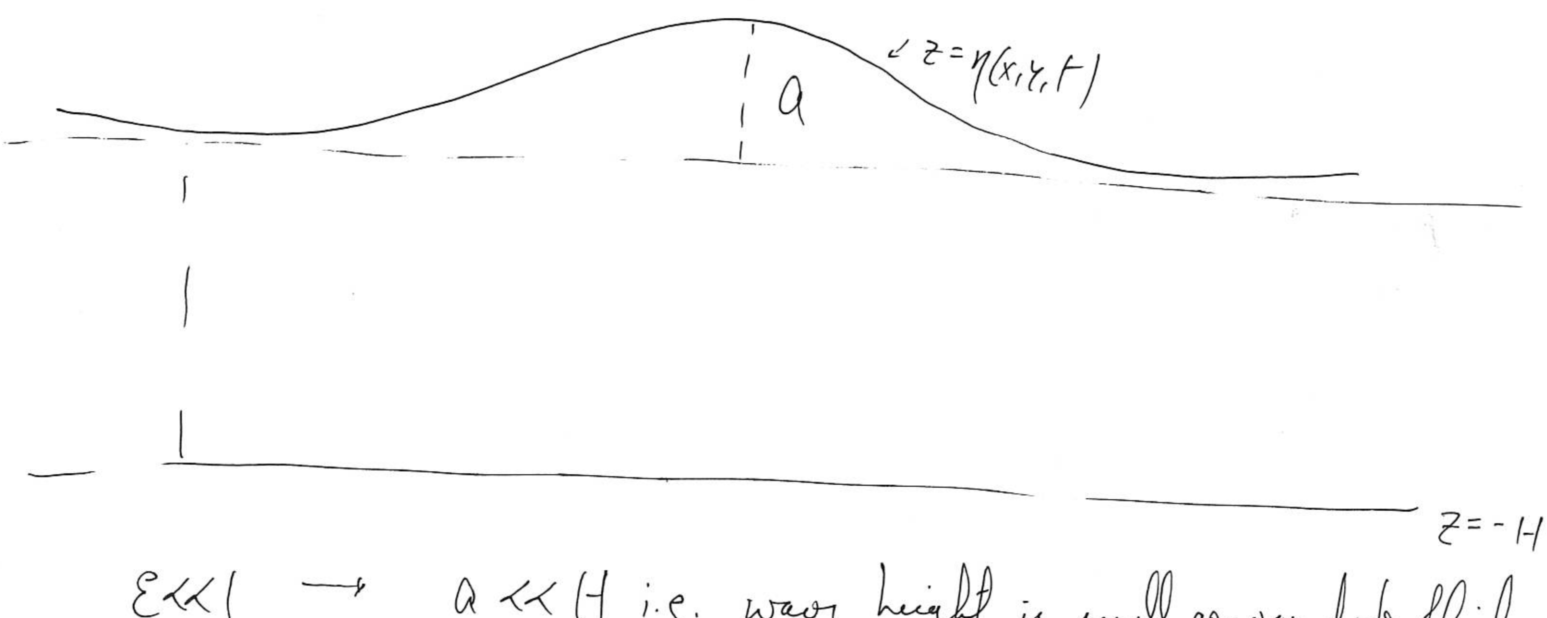
We therefore introduce the labels:

$$\varepsilon = a/H$$

$$\gamma = H/L$$

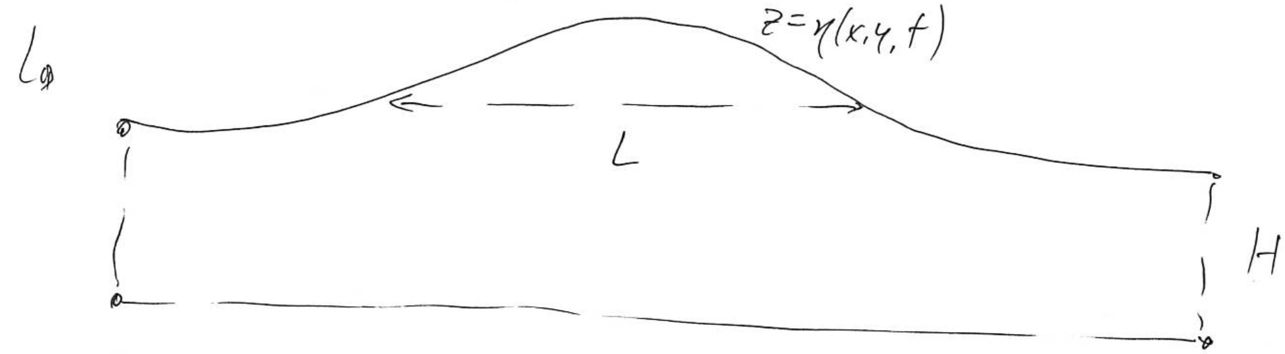
Les $\partial_x^2 \phi + \partial_y^2 \phi + \frac{1}{\gamma^2}\partial_z^2 \phi = 0 \quad ; \quad -1 < z < \varepsilon\eta(x,y,t)$

Now $\varepsilon \ll 1$ is relatively clear in meaning:



$\varepsilon \ll 1 \rightarrow a \ll H$ i.e. wave height is small compared to fluid depth.

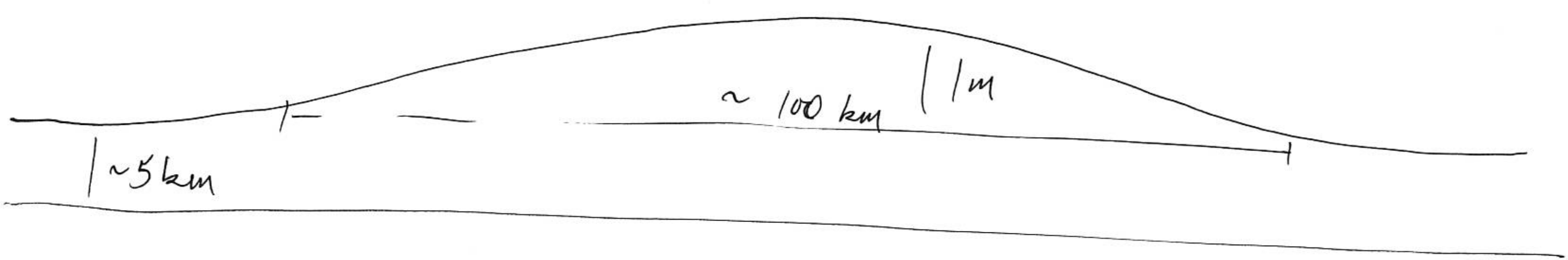
$\gamma \ll 1$ is a little funnier. So this would mean $H \ll L$



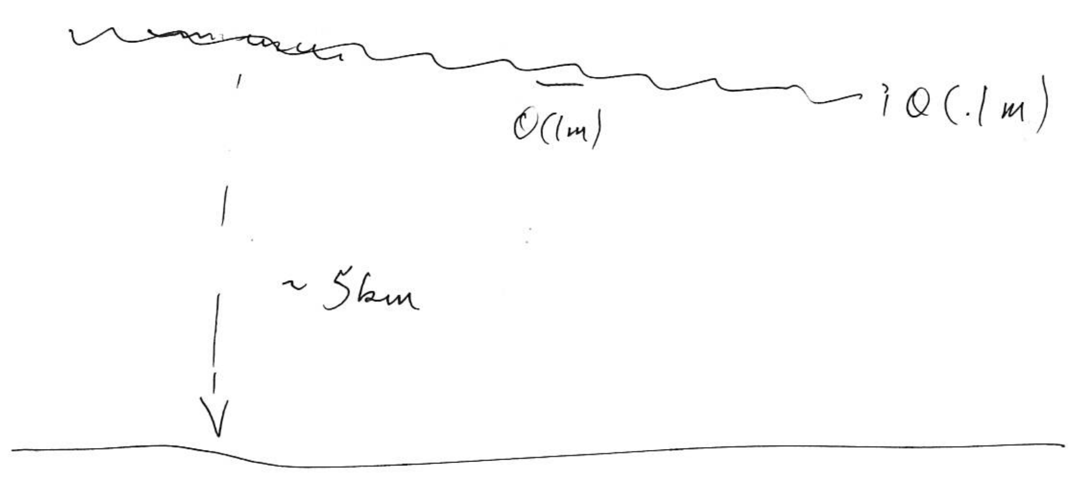
So $\gamma \ll 1$ means we are supposing waves are long compared to the depth.

or $\gamma \gg 1$ means we suppose waves are short compared to the depth.

Tsunami:



Open Ocean:



Okay, $\phi_z = 0$ at $z = -H$ $\longrightarrow$ $\phi_z = 0$ at $z = -1$.

But what of:

$$\eta_t = -\eta_x \phi_x - \eta_y \phi_y + \phi_z \quad , \quad z = \eta$$

$\leadsto$ $$\frac{a}{H}\sqrt{gH}\,\tilde{\eta}_{\tilde{t}} = \frac{a\sqrt{gH}}{L}\left(-\tilde{\eta}_{\tilde{x}}\tilde{\phi}_{\tilde{x}} - \tilde{\eta}_{\tilde{y}}\tilde{\phi}_{\tilde{y}}\right) + \frac{L\sqrt{gH}}{H}\tilde{\phi}_{\tilde{z}}$$

$\leadsto$ $$\tilde{\eta}_{\tilde{t}} = \gamma\left(-\tilde{\eta}_{\tilde{x}}\tilde{\phi}_{\tilde{x}} - \tilde{\eta}_{\tilde{y}}\tilde{\phi}_{\tilde{y}}\right) + \frac{1}{\varepsilon\gamma}\tilde{\phi}_{\tilde{z}}$$

or: $$\eta_t = \gamma\left(-\eta_x\phi_x - \eta_y\phi_y\right) + \frac{1}{\varepsilon\gamma}\phi_z$$

Likewise, we have:

$$L\sqrt{gH}\left(\frac{\sqrt{gH}}{H}\right)\phi_t + \frac{1}{2}\frac{L^2(\sqrt{gH})^2}{L^2}\left\{|\nabla_{x,y}\phi|^2 + \frac{1}{\gamma^2}(\partial_z\phi)^2\right\} + ag\eta = \frac{a\sigma}{L^2\rho}\nabla\cdot\left(\frac{\nabla\eta}{(1+\varepsilon^2\gamma^2\|\nabla\eta\|^2)^{1/2}}\right)$$

$\leadsto$ $$\phi_t + \frac{\gamma}{2}\left\{|\nabla_{x,y}\phi|^2 + \frac{1}{\gamma^2}(\partial_z\phi)^2\right\} + \varepsilon\gamma\eta = \frac{a\sigma}{gL^3\rho}\nabla\cdot\left(\frac{\nabla\eta}{(1+\varepsilon^2\gamma^2\|\nabla\eta\|^2)^{1/2}}\right)$$

So, okay we have an answer. But is it especially useful? Well, that can very much be in the eye of the beholder, so what would be useful?

So clearly, there is a linear problem lurking about here...

We try $\phi = \varepsilon\gamma\tilde{\phi}$

Why? we want $\phi_t \sim \varepsilon\gamma\eta$

↳ So what do we get if we follow this line of thought?

$$\eta_t = -\varepsilon\gamma^2(\eta_x\phi_x + \eta_y\phi_y) + \phi_z$$

Note asymmetry!

$$\phi_t + \frac{\varepsilon\gamma^2}{2}\left(|\nabla_{x,y}\phi|^2 + \frac{1}{\gamma^2}(\partial_z\phi)^2\right) + \eta = \tilde{\sigma}\,\nabla\cdot\left(\frac{\nabla\eta}{(1+\varepsilon^2\gamma^2\|\nabla\eta\|^2)^{1/2}}\right)$$

But also could have tried $\phi = \varepsilon\tilde{\phi}$, $\tilde{t} = \gamma t$

$$\eta_t = -\varepsilon(\eta_x\phi_x + \eta_y\phi_y) + \frac{1}{\gamma^2}\phi_z$$

$$\phi_t + \frac{\varepsilon}{2}\left(|\nabla_{x,y}\phi|^2 + \frac{1}{\gamma^2}(\partial_z\phi)^2\right) + \eta = \tilde{\sigma}\,\nabla\cdot\left(\frac{\nabla\eta}{(1+\varepsilon^2\gamma^2\|\nabla\eta\|^2)^{1/2}}\right)$$