

Math 639: Nonlinear Waves

Spring 2018

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- Office Hours: TBA
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Text: So, if there is one book we will be pulling from the most, it is Kundu and Cohen's Fluid Mechanics, 5th edition. That being said, I'll be pulling from several sources throughout the semester such as Whitham's Linear and Nonlinear Waves, Ablowitz's Nonlinear Dispersive Waves, and anything else I think is helpful.

Prerequisites: Mathematics 531, though as much math as possible is the best answer.

Official Course Description: Introduction to the fundamentals of fluid mechanics from a mathematical perspective. Conservation laws, vorticity, potential theory, dispersion relationships, free boundary value problems, pseudo-differential operators, multiple-scales analysis, nonlinear water wave models.

Learning Outcomes:

1. Conservation Laws: We will derive the core conservation laws of fluid mechanics from both an Eulerian and Lagrangian perspective. Bernoulli's equation will be derived and basic physical intuition in fluids will be developed through its use to solve fluid flow problems. Scaling will be introduced, and you will rescale the core conservation equations in several different regimes and explore the impact of non-dimensional parameters.

2. Vorticity: The impact of vorticity on flows will be determined through the use of point-vortex approximations. Connections with differential equations will be derived as well as core theoretical results such as Kelvin and Helmholtz's Theorems. The definitions of baroclinic and barotropic fluids will also be introduced.
3. Potential Theory: The use of analytic function theory to determine inviscid, incompressible, irrotational flows will be presented in several different geometries. The formulas for finding drag and lift on a body will be derived and used to compute profiles for a given body.
4. Fourier Analysis for PDE's: We will solve linear PDE's using Fourier Series/Transforms. The concepts of group and phase velocity and dispersion will be explored and used to show how energy is distributed a dispersive PDE.
5. Free Boundary Value Problems: Techniques for succinctly deriving evolution equations in the context of oceanic-free surface flows will be developed. This will make use of core results in vector calculus, such as the Divergence Theorem.
6. Multiple Scales Analysis: The use of multiple scale ansatzes to reduce complex nonlinear models to simpler ones over smaller scales will be developed and used to solve several problems in fluids.
7. Deriving nonlinear wave equations: The Korteweg de-Vries and Non-linear Schrödinger equations will be derived. Solutions will be derived and their properties and implications for oceanic flows will be examined.

Grading Policy: Your final score will consist of homework (50%), and a final project (50%). Homework is roughly due every two weeks, though please pay attention to the schedule since there are exceptions to this (and every) rule.

Homework Policy: Any work you submit should be as professional as possible. I am not requiring that you type it, but I reserve the right to deduct credit for work that is difficult to read or follow. If you decide to type your work, please feel free to see me for help with LaTeX. You could use Word if you like, but I would not recommend it. Late work is not accepted unless you make arrangements with me in advance.

Project Policy: The project consists of three pieces: a proposal (15%), presentation (15%), and final paper (20%). Guidelines for the project are formalized in separate document which can be found on the Blackboard Course page.

Students with Disabilities: If you are a student with a disability and believe you will need accommodations for this class, it is your responsibility to contact Student Disability Services at (619) 594-6473. To avoid any delay in the receipt of your accommodations, you should contact Student Disability Services as soon as possible. Please note that accommodations are not retroactive, and that accommodations based upon disability cannot be provided until you have presented your instructor with an accommodation letter from Student Disability Services. Your cooperation is appreciated.

Week	Date	Sections
Week 1	01/18	Motivation, Review
Week 2	01/23	Review
	01/25	Review
Week 3	01/30	Conservation Laws
	02/01	Conservation Laws, Hmwk 1 Due
Week 4	02/06	Conservation Laws
	02/08	Conservation Laws
Week 5	02/13	Vorticity
	02/15	Vorticity, Hmwk 2 Due
Week 6	02/20	Potential Flow
	02/22	Potential Flow
Week 7	02/27	Potential Flow
	03/01	Potential Flow, Hmwk 3 Due
Week 8	03/06	Fourier Analysis and Linear PDE's
	03/08	Fourier Analysis and Linear PDE's
Week 9	03/13	Dispersion, Group Velocity, Stationary Phase
	03/15	Dispersion, Group Velocity, Stationary Phase, Hmwk 4 Due
Week 10	03/20	Multiple Scales Analysis
	03/22	Multiple Scales Analysis
Week 11	03/27	Spring Break
	03/29	Spring Break, Hmwk 5 Due
Week 12	04/03	Free Boundary Problems and Surface Waves
	04/05	Free Boundary Problems and Surface Waves
Week 13	04/10	The Korteweg-de Vries Equation
	04/12	The Korteweg-de Vries Equation
Week 14	04/17	The Korteweg-de Vries Equation
	04/19	The Nonlinear Schrödinger Equation
Week 15	04/24	The Nonlinear Schrödinger Equation
	04/26	The Nonlinear Schrödinger Equation
Week 16	05/01	Presentations
	05/03	Presentations
Week 17		Final Project is Due, Have a good summer.