Letting the MNLS equation be given in the form

$$\partial_\tau \eta - i\alpha_d \partial_\xi^2 \eta - \epsilon\alpha_{d,m}\partial_\xi^3 \eta - i\alpha_{nl}\left|\eta\right|^2 \eta + \epsilon\left(c_1 \left|\eta\right|^2 \partial_\xi \eta + c_2 \eta^2 \partial_\xi \eta^* + ic_3 \eta \mathcal{H}\partial_\xi \left|\eta\right|^2\right) = 0,$$

letting

$$\eta(x,t) = \sum_m \hat{\eta}_m(t) e^{ik_m x}, \ k_m = \frac{2\pi m}{L},$$

we get that

$$\frac{d}{dt}\hat{\eta}_m + i\omega_m \hat{\eta}_m + i \sum_{m_1,m_2,m_3} \left(-\alpha_{nl} + \epsilon\alpha_c\right) \hat{\eta}_{m_1}\hat{\eta}_{m_2}\hat{\eta}^*_{m_3}\delta^{m_3 m}_{m_1 m_2} = 0,$$

where

$$\omega_m = \alpha_d k_m^2 + \epsilon\alpha_{d,m} k_m^3,$$

and

$$\alpha_c\left(k_{m_1}, k_{m_2}, k_{m_3}\right) = c_1 k_{m_1} - c_2 k_{m_3} - c_3 \left|k_{m_2} - k_{m_3}\right|.$$

Moving to an "interaction" picture via the transformation

$$\tilde{\eta}_m = e^{i\omega_m t}\frac{\hat{\eta}_m}{\sqrt{\epsilon}},$$

we then get the dynamical system

$$\frac{d}{dt}\tilde{\eta}_m + i\epsilon \sum_{m_1,m_2,m_3} \left(-\alpha_{nl} + \epsilon\alpha_c\right) e^{if(m_1,m_2,m_3,m)t}\tilde{\eta}_{m_1}\tilde{\eta}_{m_2}\tilde{\eta}^*_{m_3}\delta^{m_3 m}_{m_1 m_2} = 0,$$

where

$$f(m_1,m_2,m_3,m) = \omega_m + \omega_{m_3} - \omega_{m_1} - \omega_{m_2}.$$

Following in the usual approach, we then expand $\tilde{\eta}_m$ as

$$\tilde{\eta}_m = \tilde{\eta}^{(0)}_m + \epsilon\tilde{\eta}^{(1)}_m + \epsilon^2\tilde{\eta}^{(2)}_m + \cdots,$$

so that we get to leading order in the fast time that $\tilde{\eta}^0_m$ is constant, $\tilde{\eta}^1_m(t)$ is

$$\tilde{\eta}^{(1)}_m(t) = i\alpha_{nl}\sum_{m_1,m_2,m_3} \Delta(f,t)\tilde{\eta}^{(0)}_{m_1}\tilde{\eta}^{(0)}_{m_2}\tilde{\eta}^{(0),*}_{m_3}\delta^{m_3 m}_{m_1 m_2},$$

where

$$\Delta(f,t) = \frac{e^{ift}-1}{if}.$$

Lastly, we have that

$$\begin{aligned}
\tilde{\eta}_m^{(2)}(t) =& i\alpha_{nl} \sum_{m_1,m_2,m_3} \delta_{m_1 m_2}^{m_3 m} \tilde{\eta}_{m_1}^{(0)} \tilde{\eta}_{m_2}^{(0)} \int_0^t e^{if\tau} \tilde{\eta}_{m_3}^{(1),*}(\tau) d\tau \\
&+ i\alpha_{nl} \sum_{m_1,m_2,m_3} \delta_{m_1 m_2}^{m_3 m} \tilde{\eta}_{m_2}^{(0)} \tilde{\eta}_{m_3}^{(0),*} \int_0^t e^{if\tau} \tilde{\eta}_{m_1}^{(1)}(\tau) d\tau \\
&+ i\alpha_{nl} \sum_{m_1,m_2,m_3} \delta_{m_1 m_2}^{m_3 m} \tilde{\eta}_{m_1}^{(0)} \tilde{\eta}_{m_3}^{(0),*} \int_0^t e^{if\tau} \tilde{\eta}_{m_2}^{(1)}(\tau) d\tau \\
&- i \sum_{m_1,m_2,m_3} \alpha_c \Delta(f,t) \tilde{\eta}_{m_1}^{(0)} \tilde{\eta}_{m_2}^{(0)} \tilde{\eta}_{m_3}^{(0),*} \delta_{m_1 m_2}^{m_3 m}
\end{aligned}$$

Introducing the generating function

$$\mathcal{L}_m(\lambda, t) = \left\langle e^{\lambda |\tilde{\eta}_m|^2} \right\rangle,$$

we get that

$$\begin{aligned}
\mathcal{L}_m(\lambda, t) =& \left\langle e^{\lambda \left|\tilde{\eta}_m^{(0)}\right|^2} \left(1 + 2\epsilon\lambda \mathrm{Re}\left\{\tilde{\eta}_m^{(0)} \tilde{\eta}_m^{(1),*}\right\} \right.\right. \\
&\left.\left. + \epsilon^2 \left(2\lambda \mathrm{Re}\left\{\tilde{\eta}_m^{(0)} \tilde{\eta}_m^{(2),*}\right\} + \lambda \left|\tilde{\eta}_m^{(1)}\right|^2 + 2\lambda^2 \mathrm{Re}^2\left\{\tilde{\eta}_m^{(0)} \tilde{\eta}_m^{(1),*}\right\}\right) + \cdots \right) \right\rangle
\end{aligned}$$