So we are beating

$$i\psi_t = -\Delta\psi + |\psi|^2\,\psi + \gamma(x,y,t)$$

to death. If we are looking for periodic coefficients on a periodic box $[-L,L]\times[-L,L]$, then we are supposing that

$$\psi(x,y,t) = \sum_{n,m} \hat{\psi}_{nm}(t)e^{i\frac{\pi}{L}(mx+ny)},$$

where

$$\hat{\psi}_{nm}(t) = \frac{1}{(2L)^2}\int_{-L}^{L}\int_{-L}^{L}\psi(x,y,t)e^{-i\frac{\pi}{L}(mx+ny)}dxdy.$$

At this point, if we approximate by taking a finite number of modes in n and m, say $-K+1 \leq n \leq K$, so that the total number of modes along each direction is given by $K_T = 2K$, we can talk about two different meshes. In physical space, we have a mesh of width $\delta x = L/K$, while in frequency space, we have a mesh of width $\delta k = \pi/L$. Once we discretize in physical space, we then find, using the Trapezoid method, the approximation to the Fourier coefficients $\hat{\psi}_{nm}$ so that

$$\hat{\psi}_{nm}(t) \sim \frac{1}{K_T^2}\sum_{j,k}\psi_{jk}e^{-i\frac{\pi}{L}(mx_j+ny_k)}$$

We note though, that in Matlab, the FFT implementation places the $1/K_T^2$ on the inverse transform. So, to compare Nazarenko and Onorato, we need to multiply our Fourier coefficients by K_T^2.

Next, if are going to work in magnitudes of k-space vectors, then if we define $\mathbf{k}_{nm} = \delta k\,(n,m)$, then

$$k_{nm} = \delta k\sqrt{n^2+m^2}$$

Note, when we find

$$n(\mathbf{k}_{nm}) = \overline{\left|\hat{\psi}_{nm}\right|^2},$$

Nazarenko and Onorato assume that n is isotropic, and thus independent of angle in k space.