Factorial ANOVA

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> ### Descriptive Statistics

> (Outcome ~ FactorA) |> describeMoments(by

Summary Statistics for the Data: B1
        N       M      SD    Skew    Kurt
A1  4.000   2.000   2.449   0.544  -2.944
A2  4.000   7.000   2.449  -0.544  -2.944

Descriptive Statistics for the Data: B2
        N       M      SD    Skew    Kurt
A1  4.000   6.000   2.449   0.544  -2.944
A2  4.000   5.000   2.449   0.544  -2.944

> ### Inferential Statistics

> (Outcome ~ FactorA) |> describeFactorial(by =

Source Table for the Model
                   SS      df      MS
Factor          4.000   1.000   4.000
Blocks         16.000   1.000  16.000
Factor:Blocks  36.000   1.000  36.000
Residual       72.000  12.000   6.000

> (Outcome ~ FactorA) |> testFactorial(by = Fact

Hypothesis Tests for the Model
                    F     df1     df2       p
Factor          0.667   1.000  12.000   0.430
Blocks          2.667   1.000  12.000   0.128
Factor:Blocks   6.000   1.000  12.000   0.031

> (Outcome ~ FactorA) |> estimateFactorial(by = FactorB)

Proportion of Variance
                  Est
Factor          0.053
Blocks          0.182
Factor:Blocks   0.333
```

These descriptive statistics are calculated separately for each group or condition. A level (marginal) mean can be determined by taking the weighted average of the appropriate group means. For example, the marginal mean for Level 1 of Factor A:

$$M_{LEVEL} = \frac{\sum n(M_{GROUP})}{n_{LEVEL}} = \frac{4(2) + 4(7)}{8} = 4.500$$

A grand mean can be determined by taking the weighted average of all of the group means:

$$M_{TOTAL} = \frac{\sum n(M_{GROUP})}{N} = \frac{4(2) + 4(7) + 4(6) + 4(5)}{4 + 4 + 4 + 4} = 5.000$$

Overall, all of the between-group variability is a function of the group means and sample sizes:

$$SS_{MODEL} = \sum n(M_{GROUP} - M_{TOTAL})^2 = 4(2-5)^2 + 4(7-5)^2 + 4(6-5)^2 + 4(5-5)^2 = 56.000$$
$$df_{MODEL} = \#groups - 1 = 3$$

The statistics for the effects of “Factor A” and Factor B” are functions of the level (marginal) means and sample sizes. For “Factor B”:

$$SS_{FACTORB} = \sum n(M_{LEVEL} - M_{TOTAL})^2 = 8(4.5-5)^2 + 8(5.5-5)^2 = 4.000$$
$$df_{FACTORB} = \#levels - 1 = 2 - 1 = 1$$

The “Factor A * Factor B” (interaction) statistics reflect the between- group variability not accounted for by the factors:

$$SS_{INTERACTION} = SS_{MODEL} - SS_{FACTORA} - SS_{FACTORB} == 56.000 - 4.000 - 16.000 = 36.000$$
$$df_{INTERACTION} = df_A \times df_B = 1$$

“Residual” (error) statistics are a function of the within group variabilities. Because SS for each group can be determined ($SS = SD^2 \times df$):

$$SS_{ERROR} = SS_1 + SS_2 + SS_3 + SS_4 = 18.000 + 18.000 + 18.000 + 18.000 = 72.000$$
$$df_{ERROR} = df_1 + df_2 + df_3 + df_4 = 12$$

The “$\eta^2 p$” statistic is a ratio of the effect and the effect plus residual variability. For “Factor B”:

$$\eta^2 p = \frac{SS_{FACTOR}}{SS_{FACTOR} + SS_{ERROR}} = \frac{16.000}{16.000 + 72.000} = 0.182$$

Thus, 18.2% of the variability among the scores is accounted for by Factor B.

The “F” statistic is a ratio of the effect and within group (error) variance estimates. For “Factor B”:

$$F_{FACTORB} = \frac{MS_{FACTORB}}{MS_{ERROR}} = \frac{16.0}{6.0} = 2.667$$

An F with 1 and 12 df that equals 2.667 has a two-tailed probability of .128, which is not statistically significant.