

Factorial ANOVA

Data

The following data set reflects a between-subjects Factorial design with two factors (with two levels for each factor). The data are presented in a format suitable for entry into statistical software.

	FactorA	FactorB	Outcome
1	1.00	1.00	.00
2	1.00	1.00	.00
3	1.00	1.00	3.00
4	1.00	1.00	5.00
5	2.00	1.00	4.00
6	2.00	1.00	7.00
7	2.00	1.00	4.00
8	2.00	1.00	9.00
9	1.00	2.00	9.00
10	1.00	2.00	6.00
11	1.00	2.00	4.00
12	1.00	2.00	9.00
13	2.00	2.00	3.00
14	2.00	2.00	6.00
15	2.00	2.00	8.00
16	2.00	2.00	3.00

Computer Output

The following tables represent typical output from statistical software. Options, labels, and layout vary from program to program.

The table of descriptive statistics can be used to determine the inferential statistics.

FactorA	FactorB	Mean	Std. Deviation	N
Level 1	Level 1	2.000	2.449	4
	Level 2	7.000	2.449	4
Level 2	Level 1	6.000	2.449	4
	Level 2	5.000	2.449	4

The table of inferential statistics shows the key elements to be calculated.

Source	SS	df	MS	F	p	Partial Eta ²
FactorA	4.000	1	4.000	0.667	0.430	0.053
FactorB	16.000	1	16.000	2.667	0.128	0.182
Interaction	36.000	1	36.000	6.000	0.031	0.333
Within	72.000	12	6.000			
Total	128.000	15				

Calculations

Error (Within Groups) Statistics: Within-groups error statistics are a function of the within group variabilities.

$$SS_1 = (SD_1^2)(df_1) = (2.44949^2)(3) = 18.000$$

$$SS_2 = (SD_2^2)(df_2) = (2.44949^2)(3) = 18.000$$

$$SS_3 = (SD_3^2)(df_3) = (2.44949^2)(3) = 18.000$$

$$SS_4 = (SD_4^2)(df_4) = (2.44949^2)(3) = 18.000$$

$$SS_{ERROR} = SS_1 + SS_2 + SS_3 + SS_4 = 18.000 + 18.000 + 18.000 + 18.000 = 72.000$$

$$df_{ERROR} = df_1 + df_2 + df_3 + df_4 = 3 + 3 + 3 + 3 = 12$$

$$MS_{ERROR} = \frac{SS_{ERROR}}{df_{ERROR}} = \frac{72.000}{12} = 6.000$$

Grand (or Total) Mean: A grand mean can be determined by taking the weighted average of all of the group means.

$$M_{TOTAL} = \frac{\sum n(M_{GROUP})}{N} = \frac{4(2.000) + 4(7.000) + 4(6.000) + 4(5.000)}{(4 + 4 + 4 + 4)} = 5.000$$

Marginal Means: A level (marginal) mean can be determined by taking the weighted average of the appropriate group means.

For Factor A:

$$M_{A1} = \frac{\sum n(M_{GROUP})}{n_{LEVEL}} = \frac{4(2.000) + 4(7.000)}{(4 + 4)} = 4.500$$

$$M_{A2} = \frac{\sum n(M_{GROUP})}{n_{LEVEL}} = \frac{4(6.000) + 4(5.000)}{(4 + 4)} = 5.500$$

For Factor B:

$$M_{B1} = \frac{\sum n(M_{GROUP})}{n_{LEVEL}} = \frac{4(2.000) + 4(6.000)}{(4 + 4)} = 4.000$$

$$M_{B2} = \frac{\sum n(M_{GROUP})}{n_{LEVEL}} = \frac{4(7.000) + 4(5.000)}{(4 + 4)} = 6.000$$

Effect (Between Groups) Statistics: The Model statistics represent the overall differences among the groups. The Factor A and Factor B statistics are a function of the level (marginal) means and sample sizes. The interaction statistics reflect the between-groups variability not accounted for by the factors individually.

For the Model:

$$SS_{MODEL} = \sum n_{GROUP} (M_{GROUP} - M_{TOTAL})^2$$

$$SS_{MODEL} = 4(2.000 - 5.000)^2 + 4(7.000 - 5.000)^2 + 4(6.000 - 5.000)^2 + 4(5.000 - 5.000)^2 = 56.000$$

$$df_{MODEL} = \#groups - 1 = 3$$

For Factor A:

$$SS_{FACTORA} = \sum n_{LEVEL} (M_{LEVEL} - M_{TOTAL})^2 = 8(4.500 - 5.000)^2 + 8(5.500 - 5.000)^2 = 4.000$$

$$df_{FACTORA} = \#levels - 1 = 2 - 1 = 1$$

$$MS_{FACTORA} = \frac{SS_{FACTORA}}{df_{FACTORA}} = \frac{4.000}{1} = 4.000$$

For Factor B:

$$SS_{FACTORB} = \sum n_{LEVEL} (M_{LEVEL} - M_{TOTAL})^2 = 8(4.000 - 5.000)^2 + 8(6.000 - 5.000)^2 = 16.000$$

$$df_{FACTORB} = \#levels - 1 = 2 - 1 = 1$$

$$MS_{FACTORB} = \frac{SS_{FACTORB}}{df_{FACTORB}} = \frac{16.000}{1} = 16.000$$

For the Interaction:

$$SS_{INTER} = SS_{MODEL} - SS_{FACTORA} - SS_{FACTORB} = 56.000 - 4.000 - 16.000 = 36.000$$

$$df_{INTER} = df_{MODEL} - df_{FACTORA} - df_{FACTORB} = 3 - 1 - 1 = 1$$

$$MS_{INTER} = \frac{SS_{INTER}}{df_{INTER}} = \frac{36.000}{1} = 36.000$$

Statistical Significance: The F statistic is the ratio of the between- and within-group variance estimates.

For the Factor A Main Effect:

$$F_{FACTORA} = \frac{MS_{FACTORA}}{MS_{ERROR}} = \frac{4.000}{6.000} = 0.667$$

With $df_{FACTORA} = 1$ and $df_{ERROR} = 12$, $F_{CRITICAL} = 4.747$

Because $F_{FACTORA} < F_{CRITICAL}$, $p > .05$

For the Factor B Main Effect:

$$F_{FACTORB} = \frac{MS_{FACTORB}}{MS_{ERROR}} = \frac{16.000}{6.000} = 2.667$$

With $df_{FACTORB} = 1$ and $df_{ERROR} = 12$, $F_{CRITICAL} = 4.747$

Because $F_{FACTORB} < F_{CRITICAL}$, $p > .05$

For the Interaction:

$$F_{INTER} = \frac{MS_{INTER}}{MS_{ERROR}} = \frac{36.000}{6.000} = 6.000$$

With $df_{INTER} = 1$ and $df_{ERROR} = 12$, $F_{CRITICAL} = 4.747$

Because $F_{INTER} > F_{CRITICAL}$, $p < .05$

Effect Size: The partial eta-squared statistic is a ratio of the between-subjects effect and the remaining variability (Sum of Squares) estimates after within-subjects error has been partialled out.

For the Factor A Main Effect:

$$Partial \eta^2_{FACTORA} = \frac{SS_{FACTORA}}{SS_{FACTORA} + SS_{ERROR}} = \frac{4.000}{4.000 + 72.000} = 0.053$$

For the Factor B Main Effect:

$$Partial \eta^2_{FACTORB} = \frac{SS_{FACTORB}}{SS_{FACTORB} + SS_{ERROR}} = \frac{16.000}{16.000 + 72.000} = 0.182$$

For the Interaction:

$$Partial \eta^2_{INTER} = \frac{SS_{INTER}}{SS_{INTER} + SS_{ERROR}} = \frac{36.000}{36.000 + 72.000} = 0.333$$

Confidence Intervals: For Factorial ANOVA, calculate the confidence intervals around (centered on) each mean separately (not shown here).

APA Style

The Factorial ANOVA provides statistics for the main effects and interactions in a factorial design. Each effect would be summarized in a style analogous to a One Way ANOVA. The first example focuses on statistical significance testing, whereas the second version includes and emphasizes interpretation of the effect size.

A 2 (Factor A) x 2 (Factor B) ANOVA was conducted on the Outcome scores. Neither Factor A, $F(1,12) = 0.67$, $p = .430$, nor Factor B, $F(1,12) = 2.67$, $p = .128$, had a statistically significant impact on the Outcome. However, the interaction was statistically significant, $F(1,12) = 6.00$, $p = .031$.

Analyses revealed that neither Factor A, partial $\eta^2 = .05$, $F(1,12) = 0.67$, $p = .430$, nor Factor B, partial $\eta^2 = .18$, $F(1,12) = 2.67$, $p = .128$, had an appreciable impact on the Outcome. However, the interaction had a large impact on the Outcome, partial $\eta^2 = .33$, $F(1,12) = 6.00$, $p = .031$.

Typically, the means, standard deviations, and confidence intervals would be presented in a table or figure associated with this text.