

# SOURCEBOOK

## R

## ANNOTATED OUTPUT

**Abstract:** This chapter is intended to facilitate the connection between standard introductory statistics concepts and their implementation in R. It shows the output from various types of analyses, describes how to interpret the output, and shows the link between hand calculation formulas and R output. Results derive from the examples in the other sections of this project.

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This document is part of an online statistics sourcebook.

A browser-friendly viewing platform for the sourcebook is available:

<https://cwendorf.github.io/Sourcebook>

All data, syntax, and output files are available:

<https://github.com/cwendorf/Sourcebook>

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## Frequencies

```
> ### Frequency Distribution
```

```
> FrequencyTable <- table(Outcome)
```

```
> FrequencyTable
```

```
Outcome  
0 3 4 5 7 9  
2 1 2 1 1 1
```

The first column lists all the actual scores in the entire data set. "Freq" indicates the number of times that score exists. For example, the score 4 was listed 2 times.

```
> prop.table(FrequencyTable)
```

```
Outcome  
0      3      4      5      7      9  
0.250 0.125 0.250 0.125 0.125 0.125
```

The "prop.table" provides the proportion of cases for each possible score. For example, of the 8 scores in the entire data set, the score of 4 was listed 2 times and 2/8 is .250.

```
> ### Descriptive Statistics
```

```
> length(Outcome)
```

```
[1] 8
```

```
> summary(Outcome)
```

```
Min. 1st Qu.  Median    Mean 3rd Qu.    Max.  
0.00  2.25   4.00   4.00  5.50   9.00
```

"Summary" provides the scores associated with particular percentile ranks. For example, the 50<sup>th</sup> percentile is the score in the following position:

$$Position = PR(N + 1) = .50(8 + 1) = 4.5$$

Thus, the score at the 50<sup>th</sup> percentile ("Median") is the 4.5<sup>th</sup> score in the frequency distribution – a score of 4.

## Descriptives

```
> ### Frequency Distribution
```

```
> FrequencyTable <- table(Outcome)
> FrequencyTable
```

```
Outcome
0 3 4 5 7 9
2 1 2 1 1 1
```

```
> prop.table(FrequencyTable)
```

```
Outcome
    0     3     4     5     7     9
0.250 0.125 0.250 0.125 0.125 0.125
```

```
> ### Descriptive Statistics
```

```
> mean(Outcome)
```

```
[1] 4
```

```
> var(Outcome)
```

```
[1] 9.714286
```

```
> sd(Outcome)
```

```
[1] 3.116775
```

These statistics were obtained using the command described on the previous page of this guide. Note that they are calculated separately for each variable.

The Mean and Standard Deviation are calculated as unbiased estimates of the respective population parameter. Here, the mean ("M") is determined as the average of the scores weighted by their frequencies:

$$M = \frac{\sum(fY)}{N} = \frac{(2 \times 0) + (1 \times 3) + (2 \times 4) + (1 \times 5) + (1 \times 7) + (1 \times 8)}{8} = 4$$

The Variance and Standard Deviation are both functions of the Sum of Squares (not shown in the output) of the scores in the frequency distribution:

$$SS = \sum f(Y - M)^2$$
$$SS = 2(0 - 4)^2 + 1(3 - 4)^2 + 2(4 - 4)^2 + 1(5 - 4)^2 + 1(7 - 4)^2 + 1(8 - 4)^2 = 68$$

Then, the Variance (also known as Mean Squares) is calculated as:

$$MS = \frac{SS}{(N - 1)} = \frac{68}{7} = 9.714$$

Finally, the Standard Deviation ("SD") is determined by:

$$SD = \sqrt{MS} = \sqrt{9.71} = 3.117$$

## Correlations

```
> ### Descriptive Statistics
```

```
> mean(Outcome1)
```

```
[1] 2
```

```
> sd(Outcome1)
```

```
[1] 2.44949
```

```
> mean(Outcome2)
```

```
[1] 6
```

```
> sd(Outcome2)
```

```
[1] 2.44949
```

```
> cov(Outcome1, Outcome2)
```

```
[1] 3
```

```
> cor(Outcome1, Outcome2)
```

```
[1] 0.5
```

```
> ### Inferential Statistics
```

```
> cor.test(Outcome1, Outcome2)
```

```
Pearson's product-moment correlation
```

```
data: Outcome1 and Outcome2
```

```
t = 0.8165, df = 2, p-value = 0.5
```

```
alternative hypothesis: true correlation is not equal to 0
```

```
95 percent confidence interval:
```

```
-0.8876337 0.9868586
```

```
sample estimates:
```

```
cor
```

```
0.5
```

These statistics were obtained using the commands described on the previous page of this guide. Note that they are calculated separately for each variable.

These boxes represent the conjunction of both variables and therefore present the statistics relevant to the relationship between the two variables.

The Sum of Cross Products (SCP) is not easily determined from the summary statistics of the output, but rather from the data (and the calculations are therefore not shown here).

The Covariance ("COV") is a function of the Sum of Cross Products and the sample size:

$$COV = \frac{SCP}{(N - 1)} = \frac{9.000}{(4 - 1)} = 3.000$$

The Correlation coefficient ("r") is a function of the covariance and the standard deviations of both variables:

$$r = \frac{COV}{(SD_X)(SD_Y)} = \frac{3.000}{(2.449)(2.449)} = .500$$

The "t", "df", and "p" columns provide a statistical significance test of whether the correlation differs from zero:

$$t = \frac{r}{\sqrt{(1 - r^2)/(N - 2)}} = \frac{.500}{\sqrt{(1 - .500^2)/(4 - 2)}} = .816$$

The *t* statistic follows a non-normal (studentized or *t*) distribution that depends on degrees of freedom. Here, *df* = *N* - 2 = 4 - 2 = 2. A *t* with 2 *df* that equals .816 has a two-tailed probability (*p*) of .500, which is not a statistically significant finding.

## Confidence Intervals

```
> ### Descriptive Statistics
```

```
> mean(Outcome)
```

```
[1] 4
```

```
> sd(Outcome)
```

```
[1] 3.116775
```

```
> ### Inferential Statistics
```

```
> t.test(Outcome)$conf.int
```

```
[1] 1.394311 6.605689
```

```
attr(,"conf.level")
```

```
[1] 0.95
```

These values are produced by the “Descriptives” commands. See the earlier annotated output for details of how these are computed from frequency distributions.

The Standard Error of the Mean (“SE”) provides an estimate of how spread out the distribution of all possible random sample means would be. Here it’s calculated as:

$$SE_M = \frac{SD}{\sqrt{N}} = \frac{3.117}{\sqrt{8}} = 1.102$$

This provides a confidence interval around (centered on) the Mean (“M”). Calculation requires the appropriate critical value. Specifically, the  $t$  statistic (with 7  $df$ ) that has a probability of .05 equals 2.365. As a result:

$$CI_M = M \pm (t_{CRITICAL})(SE_M) = 4.000 \pm (2.365)(1.102)$$

Thus, the researcher estimates that the true population mean is somewhere between 1.394 and 6.606 (knowing that the estimate could be incorrect).

# One Sample t Test

```
> ### Descriptive Statistics
```

```
> mean(Outcome)
```

```
[1] 4
```

```
> sd(Outcome)
```

```
[1] 3.116775
```

```
> ### Inferential Statistics
```

```
> t.test(Outcome, mu=7)
```

```
One Sample t-test
```

```
data: Outcome
```

```
t = -2.7225, df = 7, p-value = 0.02966
```

```
alternative hypothesis: true mean is not equal to 7  
95 percent confidence interval:
```

```
1.394311 6.605689
```

```
sample estimates:
```

```
mean of x  
4
```

These values are produced by the “Descriptives” commands. See the earlier annotated output for details of how these are computed from frequency distributions.

The Standard Error of the Mean (“SE”) provides an estimate of how spread out the distribution of all possible random sample means would be. Here it’s calculated as:

$$SE_M = \frac{SD}{\sqrt{N}} = \frac{3.117}{\sqrt{8}} = 1.102$$

The “t”, “df”, and “p” values provide the results of the statistical significance test. First,  $t$  provides the standardized statistic for the mean difference:

$$t = \frac{M - \mu}{SE_M} = \frac{-3.000}{1.102} = -2.722$$

The  $t$  statistic follows a non-normal (studentized or  $t$ ) distribution that depends on degrees of freedom. Here,  $df = N - 1 = 8 - 1 = 7$ . A  $t$  with 7  $df$  that equals -2.722 has a two-tailed probability ( $p$ ) of .030, a statistically significant finding.

The Mean Difference is the difference between the sample mean ( $M = 4$ ) and the user-specified test value ( $u = 7$ ). For the example, the sample had a mean one point higher than the test value. This raw effect size is important for the significance test, the confidence interval, and the effect size.

Cohen’s “d” provides a standardized effect size for the difference between the two means.

$$d = \frac{M - \mu}{SD} = \frac{-3.000}{3.117} = 0.963$$

Given Cohen’s heuristics for interpreting effect sizes, this would be considered a large effect.

## Paired Samples t Test

```
> ### Descriptive Statistics
```

```
> mean(Outcome1)
```

```
[1] 2
```

```
> sd(Outcome1)
```

```
[1] 2.44949
```

```
> mean(Outcome2)
```

```
[1] 6
```

```
> sd(Outcome2)
```

```
[1] 2.44949
```

```
> ### Inferential Statistics
```

```
> t.test(Outcome2, Outcome1, paired=TRUE)
```

Paired t-test

```
data: Outcome2 and Outcome1
```

```
t = 3.266, df = 3, p-value = 0.04692
```

```
alternative hypothesis: true mean difference is not equal to 0
```

```
95 percent confidence interval:
```

```
0.1023152 7.8976848
```

```
sample estimates:
```

```
mean difference
```

```
4
```

These statistics were obtained using the commands described on the previous page of this guide. Note that they are calculated separately for each variable.

The Mean Difference is simply the difference between the two means listed above. However, the "SE" is not determinable from the summary statistics presented here but rather the raw data.

The Std. Deviation of the differences can be determined from this information:

$$SD_D = (SE_D)(\sqrt{N})$$

$$SD_D = (1.225)(\sqrt{4}) = 2.449$$

The "t", "df", and "p" values provide the results of the statistical significance test. First,  $t$  provides the standardized statistic for the mean difference:

$$t = \frac{M_D}{SE_D} = \frac{4.000}{1.225} = 3.226$$

The  $t$  statistic follows a non-normal (studentized or  $t$ ) distribution that depends on degrees of freedom. Here,  $df = N - 1 = 4 - 1 = 3$ . A  $t$  with 3  $df$  that equals 3.226 has a two-tailed probability ( $p$ ) of .047, a statistically significant finding.

This confidence interval is centered on the Mean Difference ("Diff") of the two variables. Calculation requires the appropriate critical value. Specifically, the  $t$  statistic (with 3  $df$ ) that has a probability of .05 equals 3.182. As a result:

$$CI_D = M_D \pm (t_{CRITICAL})(SE_D)$$

$$CI_D = 4.00 \pm (3.182)(1.225)$$

Thus, the researcher estimates that the true population mean difference is somewhere between 0.102 to 7.898 (knowing that the estimate could be incorrect).

Cohen's "d" provides a standardized effect size for the difference between the two means.

$$d = \frac{M_D}{SD_D} = \frac{4.000}{2.449}$$

$$d = 1.633$$

Given Cohen's heuristics for interpreting effect sizes, this would be considered an extremely large effect.



# Independent Samples t Test

> ### Descriptive Statistics

> by(Outcome, Factor, mean)

Factor: 1  
[1] 2

Factor: 2  
[1] 6

> by(Outcome, Factor, sd)

Factor: 1  
[1] 2.44949

Factor: 2  
[1] 2.44949

> ### Inferential Statistics

> t.test(Outcome~Factor, var.equal=T)

Two Sample t-test

data: Outcome by Factor  
t = -2.3094, df = 6, p-value = 0.06032  
alternative hypothesis: true difference in means between  
95 percent confidence interval:  
-8.2381756 0.2381756  
sample estimates:  
mean in group 1 mean in group 2

This section provides a confidence interval around (centered on) the Mean Difference. Calculation requires the appropriate critical value. Specifically, the  $t$  statistic (with 6  $df$ ) that has a probability of .05 equals 2.447. As a result:

$$CI_{DIFF} = M_{DIFF} \pm (t_{CRITICAL})(SE_{DIFF})$$

$$CI_{DIFF} = 4 \pm (2.447)(1.732)$$

Thus, the researcher estimates that the true population mean difference is somewhere between - .238 and 8.238 (knowing that the estimate could be incorrect).

These values of the group statistics are calculated separately for each level or condition. They are not identical to the values obtained from analyzing the variable as a whole.

The standard errors for each condition can be calculated separately but note that both groups have the same standard deviation and sample size.:

$$SE_M = \frac{SD}{\sqrt{N}} = \frac{2.449}{\sqrt{4}} = 1.225$$

The "SE" is a function of the two groups' individual standard errors. When sample sizes are equal:

$$SE_{DIFF} = \sqrt{SE_1^2 + SE_2^2} = \sqrt{1.225^2 + 1.225^2} = 1.732$$

This value is important for both the significance test and the confidence interval. [Importantly, the computation of the standard error of the difference is more complex for unequal sample sizes.]

The "t", "df", and "p" values provide the results of the statistical significance test. First,  $t$  provides the standardized statistic for the mean difference:

$$t = \frac{M_{DIFF}}{SE_{DIFF}} = \frac{4.000}{1.732} = 2.309$$

The  $t$  statistic follows a non-normal (studentized or  $t$ ) distribution that depends on degrees of freedom. Here,  $df = N - 2 = 8 - 2 = 6$ . A  $t$  with 6  $df$  that equals 2.309 has a two-tailed probability ( $p$ ) of .060, a finding that is not statistically significant.

The pooled (or weighted average) Std. Deviation of the groups can be determined from the group descriptive statistics:

$$SD_{WITHIN} = \sqrt{\frac{(SD_1^2)(df_1) + (SD_2^2)(df_2)}{df_1 + df_2}} = \sqrt{\frac{(2.449^2)(3) + (2.449^2)(3)}{3 + 3}}$$

$$= 2.449$$

Cohen's "d" provides a standardized effect size for the difference between the two means:

$$d = \frac{M_{DIFF}}{SD_{WITHIN}} = \frac{4.000}{2.449} = 1.633$$

# One Way ANOVA

> ### Descriptive Statistics

> by (Outcome, Factor, sd)

Factor: 1  
[1] 2.44949

Factor: 2  
[1] 2.44949

Factor: 3  
[1] 2.44949

> Results <- aov(Outcome~Factor)  
> model.tables(Results, "means")

Tables of means  
Grand mean

5

Factor  
Factor  
1 2 3  
2 6 7

> ### Inferential Statistics

> summary(Results)

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
Factor	2	56	28	4.667	0.0407
Residuals	9	54	6		

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05

These values of the group statistics are calculated separately for each group. They are not identical to the values obtained from analyzing the variable as a whole.

A grand mean can be determined by taking the weighted average of all of the group means:

$$M_{TOTAL} = \frac{\sum n(M_{GROUP})}{N} = \frac{4(2) + 4(6) + 4(7)}{4 + 4 + 4} = 5.000$$

"Factor" statistics are a function of the differences among the groups:

$$SS_{BETWEEN} = \sum n(M_{GROUP} - M_{TOTAL})^2$$

$$SS_{BETWEEN} = 4(2 - 5)^2 + 4(6 - 5)^2 + 4(7 - 5)^2 = 56.000$$

The degrees of freedom ("df") are a function of the number of groups:

$$df_{BETWEEN} = \#groups - 1 = 2$$

The "Mean Square" is the ratio of the "Sum of Squares" to the "df":

$$MS_{BETWEEN} = \frac{SS_{BETWEEN}}{df_{BETWEEN}} = \frac{56.000}{2} = 28.000$$

"Residual" statistics are a function of the within group variabilities. Because SS for each group equals 2.00 ( $SS = SD^2 \times df$ ):

$$SS_{WITHIN} = SS_1 + SS_2 + SS_3 = 18.000 + 18.000 + 18.000 = 54.000$$

The degrees of freedom ("df") are a function of the number of people in each group:

$$df_{WITHIN} = df_1 + df_2 + df_3 = 9$$

The "Mean Square" is the ratio of the "Sum of Squares" to the "df":

$$MS_{WITHIN} = \frac{SS_{WITHIN}}{df_{WITHIN}} = \frac{54.000}{9} = 6.000$$

The "F" statistic is a ratio of the between and within group variance estimates:

$$F = \frac{MS_{BETWEEN}}{MS_{WITHIN}} = \frac{28.000}{6.000} = 4.667$$

An F with 2 and 9 df that equals 4.667 has a two-tailed probability (p) of .041, a statistically significant finding.

The " $\eta^2$ " statistic is a ratio of the between group and the total group variability ("Sum of Squares") estimates:

$$\eta^2 = \frac{SS_{BETWEEN}}{SS_{TOTAL}} = \frac{SS_{BETWEEN}}{SS_{BETWEEN} + SS_{WITHIN}} = \frac{56.000}{56.000 + 54.000} = 0.509$$

Thus, 50.9% of the total variability among all of the scores in the study is accounted for by group membership.

## Post Hoc Tests

```
> ### Descriptive Statistics
```

```
> by(Outcome, Factor, sd)
```

```
Factor: 1
[1] 2.44949
```

```
Factor: 2
[1] 2.44949
```

```
Factor: 3
[1] 2.44949
```

```
> Results <- aov(Outcome~Factor)
> model.tables(Results, "means")
```

```
Tables of means
Grand mean
```

```
5
Factor
Factor
1 2 3
2 6 7
```

```
> ### Inferential Statistics
```

```
> TukeyHSD(Results)
```

```
Tukey multiple comparisons of means
95% family-wise confidence level
```

```
Fit: aov(formula = Outcome ~ Factor)
```

```
$Factor
      diff      lwr      upr    p adj
2-1      4 -0.8358956  8.835896 0.1055254
3-1      5  0.1641044  9.835896 0.0431300
3-2      1 -3.8358956  5.835896 0.8352889
```

These values of the group statistics are calculated separately for each group. They are not identical to the values obtained from analyzing the variable as a whole.

A grand mean can be determined by taking the weighted average of all of the group means:

$$M_{TOTAL} = \frac{\sum n(M_{GROUP})}{N} = \frac{4(2) + 4(6) + 4(7)}{4 + 4 + 4} = 5.000$$

The "Standard Errors" are for the difference between the two group means. The values are a function of the  $MS_{WITHIN}$  (from the ANOVA) and the sample sizes:

$$SE_{DIFF} = \sqrt{\left(\frac{MS_{WITHIN}}{n_{GROUP}}\right) + \left(\frac{MS_{WITHIN}}{n_{GROUP}}\right)} = \sqrt{\left(\frac{6}{4}\right) + \left(\frac{6}{4}\right)} = 1.732$$

In this case, because all groups are of the same size, the standard error for each comparison is the same.

An HSD value is conceptually similar to a  $t$  statistic in that it is a function of the "Diff" and the "SE". For the first comparison in the example:

$$HSD = \frac{M_2 - M_1}{SE_{DIFF}} = \frac{4.000}{1.732} = 2.309$$

The "p adj" column provides the probability of the HSD statistic. An HSD of -2.309 (with 2  $df_{BETWEEN}$  and 9  $df_{WITHIN}$  like in the ANOVA source table) has a two-tailed probability ( $p$ ) of .106, a finding that is not statistically significant.

This section provides confidence intervals around (centered on) the Mean Differences. Calculation requires the appropriate critical value. Specifically, the HSD statistic (with 2  $df_{BETWEEN}$  and 9  $df_{WITHIN}$ ) that has a probability of .05 equals 3.068. For the first comparison in the example:

$$CI_{DIFF} = M_{DIFF} \pm (HSD_{CRITICAL})(SE_{DIFF})$$

$$CI_{DIFF} = 4.000 \pm (2.792)(1.732)$$

Thus, the estimates that the true population mean difference is somewhere between -8.836 and 0.836 (knowing that the estimate could be incorrect).

## Repeated Measures ANOVA

```
> ### Descriptive Statistics
```

```
> sd(Outcome1)
```

```
[1] 2.44949
```

```
> sd(Outcome2)
```

```
[1] 2.44949
```

```
> StackData=reshape(RepeatedData, varying = c("Outcome1", "Outcome2"), times = 2,
> attach(StackData)
> Results=aov(Outcome~factor(Factor))
> model.tables(Results, "means")
```

```
Tables of means
Grand mean
```

```
4
```

```
factor(Factor)
factor(Factor)
1 2
2 6
```

```
> ### Inferential Statistics
```

```
> summary(Results)
```

```
Error: factor(Subject)
      Df Sum Sq Mean Sq F value Pr(>F)
Residuals  3      27      9
```

```
Error: Within
```

```
      Df Sum Sq Mean Sq F value Pr(>F)
factor(Factor)  1      32      32    10.67 0.0469 *
Residuals      3       9       3
```

```
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

These descriptive statistics are calculated separately for each level or condition. Because sample sizes are equal, a grand mean can be determined by averaging these two level means:

$$M_{TOTAL} = (M_{LEVEL} + M_{LEVEL})/2 = (2.000 + 6.000)/2 = 4.000$$

Between-subjects error refers to the average differences across the participants of the study. This Sum of Squares is not easily determined from the summary statistics output, but rather from the data (and the calculations are therefore not shown here). However:

$$df_{SUBJECTS} = \#subjects - 1 = 3$$

The "Mean Square" is the usual ratio of the Sum of Squares to the degrees of freedom.

The within-subjects "Error" is a function of variabilities of the separate levels or conditions of the factor and the "between-subjects error" given above. Because SS for each level can be determined ( $SS = SD^2 \times df$ , which equals 18.000 for each of the two outcomes):

$$SS_{ERROR} = SS_1 + SS_2 - SS_{SUBJECTS}$$

$$SS_{ERROR} = 18.000 + 18.000 - 27.000 = 9.000$$

$$df_{ERROR} = df_1 + df_2 - df_{SUBJECTS} = 3 + 3 - 3 = 3.000$$

The "Mean Square" is the usual ratio of the Sum of Squares to the degrees of freedom.

The statistics for the effect ("Factor") are functions of the means of the levels or conditions and the sample sizes:

$$SS_{EFFECT} = \sum n_{LEVEL} (M_{LEVEL} - M_{TOTAL})^2$$

$$SS_{EFFECT} = 4(2.0 - 4.0)^2 + 4(6.0 - 4.0)^2 = 32.000$$

$$df_{EFFECT} = \#levels - 1 = 1$$

The "Mean Square" is the usual ratio of the Sum of Squares to the degrees of freedom.

$$MS_{EFFECT} = \frac{SS_{EFFECT}}{df_{EFFECT}} = \frac{32.000}{1} = 32.000$$

The "F" statistic is a ratio of the effect and within-subjects error variance estimates:

$$F = \frac{MS_{EFFECT}}{MS_{ERROR}} = \frac{32.000}{3.000} = 10.667$$

An *F* with 1 and 3 *df* that equals 10.667 has a two-tailed probability of .047, a statistically significant finding.

# Factorial ANOVA

> ### Descriptive Statistics

> Results <- aov(Outcome~FactorA\*FactorB)

> model.tables(Results,"means")

Tables of means

Grand mean

5

FactorA

FactorA

A1 A2

4 6

FactorB

FactorB

B1 B2

4.5 5.5

FactorA:FactorB

FactorB

FactorA B1 B2

A1 2 6

A2 7 5

> ### Inferential Statistics

> summary(Results)

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
FactorA	1	16	16	2.667	0.1284
FactorB	1	4	4	0.667	0.4301
FactorA:FactorB	1	36	36	6.000	0.0306
Residuals	12	72	6		

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05

These descriptive statistics are calculated separately for each group or condition. A level (marginal) mean can be determined by taking the weighted average of the appropriate group means. For example, the marginal mean for Level 1 of Factor A:

$$M_{LEVEL} = \frac{\sum n(M_{GROUP})}{n_{LEVEL}} = \frac{4(2) + 4(7)}{8} = 4.500$$

A grand mean can be determined by taking the weighted average of all of the group means:

$$M_{TOTAL} = \frac{\sum n(M_{GROUP})}{N} = \frac{4(2) + 4(7) + 4(6) + 4(5)}{4 + 4 + 4 + 4} = 5.000$$

Overall, all of the between-group variability is a function of the group means and sample sizes:

$$SS_{MODEL} = \sum n(M_{GROUP} - M_{TOTAL})^2 = 4(2 - 5)^2 + 4(7 - 5)^2 + 4(6 - 5)^2 + 4(5 - 5)^2 = 56.000$$

$$df_{MODEL} = \#groups - 1 = 3$$

The statistics for the effects of "Factor A" and Factor B" are functions of the level (marginal) means and sample sizes. For "Factor B":

$$SS_{FACTORB} = \sum n(M_{LEVEL} - M_{TOTAL})^2 = 8(4.5 - 5)^2 + 8(5.5 - 5)^2 = 4.000$$

$$df_{FACTORB} = \#levels - 1 = 2 - 1 = 1$$

The "Factor A \* Factor B" (interaction) statistics reflect the between- group variability not accounted for by the factors:

$$SS_{INTERACTION} = SS_{MODEL} - SS_{FACTORA} - SS_{FACTORB} = 56.000 - 4.000 - 16.000 = 36.000$$

$$df_{INTERACTION} = df_A \times df_B = 1$$

"Residual" (error) statistics are a function of the within group variabilities. Because SS for each group can be determined ( $SS = SD^2 \times df$ ):

$$SS_{ERROR} = SS_1 + SS_2 + SS_3 + SS_4 = 18.000 + 18.000 + 18.000 + 18.000 = 72.000$$

$$df_{ERROR} = df_1 + df_2 + df_3 + df_4 = 12$$

The "F" statistic is a ratio of the effect and within group (error) variance estimates. For "Factor B":

$$F_{FACTORB} = \frac{MS_{FACTORB}}{MS_{ERROR}} = \frac{16.0}{6.0} = 2.667$$

An F with 1 and 12 df that equals 2.667 has a two-tailed probability of .128, which is not statistically significant.