

# Standardized Scores

## Data

The following data set reflects one sample of eight individuals measured on one variable. The data are presented in a format suitable for entry into statistical software.

|   | Outcome |
|---|---------|
| 1 | .00     |
| 2 | .00     |
| 3 | 3.00    |
| 4 | 5.00    |
| 5 | 4.00    |
| 6 | 7.00    |
| 7 | 4.00    |
| 8 | 9.00    |

## Computer Output

The following tables represent typical output from statistical software. Options, labels, and layout vary from program to program.

The frequency distribution can be used to determine the descriptive statistics.

|       |       | Frequency | Percent | Valid Percent | Cumulative Percent |
|-------|-------|-----------|---------|---------------|--------------------|
| Valid | 0.00  | 2         | 25.0    | 25.0          | 25.0               |
|       | 3.00  | 1         | 12.5    | 12.5          | 37.5               |
|       | 4.00  | 2         | 25.0    | 25.0          | 62.5               |
|       | 5.00  | 1         | 12.5    | 12.5          | 75.0               |
|       | 7.00  | 1         | 12.5    | 12.5          | 87.5               |
|       | 9.00  | 1         | 12.5    | 12.5          | 100.0              |
|       | Total | 8         | 100.0   | 100.0         |                    |

The table of descriptive statistics can be used to assist in calculating the standardized scores.

|         | N | Mean  | Std. Deviation |
|---------|---|-------|----------------|
| Outcome | 8 | 4.000 | 3.117          |

## Calculations

**Standardized (z) Score:** A standardized score is a deviation score divided by the standard deviation. For the first score in the distribution:

$$z = \frac{(Y - M)}{SD} = \frac{(0 - 4.000)}{3.117} = \frac{-4.000}{3.117} = -1.283$$

This is repeated for each score in the distribution. In software programs, these would be calculated and presented back in the data set:

|   | Outcome | zOutcome |
|---|---------|----------|
| 1 | .00     | -1.283   |
| 2 | .00     | -1.283   |
| 3 | 3.00    | -0.321   |
| 4 | 5.00    | 0.321    |
| 5 | 4.00    | 0        |
| 6 | 7.00    | 0.963    |
| 7 | 4.00    | 0        |
| 8 | 9.00    | 1.604    |

**Percentile Rank:** The 50<sup>th</sup> percentile (the Median) and the 25<sup>th</sup> and 75<sup>th</sup> percentiles (collectively known as the Interquartile Range) are the most commonly calculated. Assuming a perfectly normal distribution:

From the Standard Normal Distribution table: for  $PR = .250$ ,  $z = -0.680$

From the Standard Normal Distribution table: for  $PR = .500$ ,  $z = 0.000$

From the Standard Normal Distribution table: for  $PR = .750$ ,  $z = +0.680$

**Percentile Point:** Percentiles provide the scores associated with particular percentile ranks. These can be estimated using the z score formula and the descriptive information from the original variable:

$$\text{For } PR = .250: -.680 = \frac{(Y - 4.000)}{3.117}; Y = 1.880$$

$$\text{For } PR = .500: 0.000 = \frac{(Y - 4.000)}{3.117}; Y = 4.000$$

$$\text{For } PR = .750: 0.680 = \frac{(Y - 4.000)}{3.117}; Y = 6.120$$

Thus, we estimate the scores at the 25<sup>th</sup>, 50<sup>th</sup>, and 75<sup>th</sup> percentiles to be approximately 1.880, 4.000, and 6.120 respectively. These are accurate estimates only if the distribution is perfectly normal.

**Mean (of the z Scores):** The mean (or arithmetic average) is calculated as an unbiased estimate of the population mean. Here, the mean is determined as the average of the scores weighted by their frequencies:

$$M = \frac{\sum(fY)}{N} = \frac{(2 \times -1.283) + (1 \times -.321) + (2 \times 0) + (1 \times .321) + (1 \times .963) + (1 \times 1.604)}{8} = 0.000$$

Sum of Squares (of the z Scores): The Sum of Squares is the basic measure of the variability of the scores. Formally, it is the sum of the weighted deviations of the scores about the mean.

$$SS = \sum f(Y - M)^2 = 2(-1.283 - 0.000)^2 + 1(-.321 - 0.000)^2 + 2(0.000 - 0.000)^2 + 1(.321 - 0.000)^2 + 1(.963 - 0.000)^2 + 1(1.604 - 0.000)^2 = 7.000$$

Mean Squares (of the z Scores): Mean Squares (also known as Variance) is a function of the Sum of Squares. It is calculated as an unbiased estimate of the population variance.

$$MS = \frac{SS}{(N - 1)} = \frac{7.000}{7} = 1.000$$

Standard Deviation (of the z Scores): Standard Deviation is a function of Mean Squares. It is also calculated as an unbiased estimate of the population standard deviation.

$$SD = \sqrt{MS} = \sqrt{1.000} = 1.000$$

## APA Style

Standardized scores are typically NOT presented in the summary of the data. Rather, they are often a first step in the calculations. As such, APA style is not presented here.