Factorial ANOVA

```
> ### Descriptive Statistics

> Results <- aov(Outcome~FactorA*FactorB)
> model.tables(Results,"means")

Tables of means
Grand mean
5

 FactorA
FactorA
A1  A2
 4   6

 FactorB
FactorB
 B1   B2
4.5 5.5

 FactorA:FactorB
       FactorB
FactorA B1 B2
     A1 2   6
     A2 7   5

> tapply(Outcome, list(FactorA,FactorB), sd)

        B1       B2
A1 2.44949 2.44949
A2 2.44949 2.44949

> ### Inferential Statistics

> summary(Results)

                Df Sum Sq Mean Sq F value Pr(>F)
FactorA          1     16      16   2.667 0.1284
FactorB          1      4       4   0.667 0.4301
FactorA:FactorB  1     36      36   6.000 0.0306 *
Residuals       12     72       6
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '
```

These descriptive statistics are calculated separately for each group or condition. A level (marginal) mean can be determined by taking the weighted average of the appropriate group means. For example, the marginal mean for Level 1 of Factor A:

$$M_{LEVEL} = \frac{\sum n(M_{GROUP})}{n_{LEVEL}} = \frac{4(2) + 4(7)}{8} = 4.500$$

A grand mean can be determined by taking the weighted average of all of the group means:

$$M_{TOTAL} = \frac{\sum n(M_{GROUP})}{N} = \frac{4(2) + 4(7) + 4(6) + 4(5)}{4 + 4 + 4 + 4} = 5.000$$

Overall, all of the between-group variability is a function of the group means and sample sizes:

$$SS_{MODEL} = \sum n(M_{GROUP} - M_{TOTAL})^2 = 4(2-5)^2 + 4(7-5)^2 + 4(6-5)^2 + 4(5-5)^2 = 56.000$$
$$df_{MODEL} = \#groups - 1 = 3$$

The statistics for the effects of “Factor A” and Factor B” are functions of the level (marginal) means and sample sizes. For “Factor B”:

$$SS_{FACTORB} = \sum n(M_{LEVEL} - M_{TOTAL})^2 = 8(4.5-5)^2 + 8(5.5-5)^2 = 4.000$$
$$df_{FACTORB} = \#levels - 1 = 2 - 1 = 1$$

The “Factor A * Factor B” (interaction) statistics reflect the between- group variability not accounted for by the factors:

$$SS_{INTERACTION} = SS_{MODEL} - SS_{FACTORA} - SS_{FACTORB} == 56.000 - 4.000 - 16.000 = 36.000$$
$$df_{INTERACTION} = df_A \times df_B = 1$$

“Residual” (error) statistics are a function of the within group variabilities. Because SS for each group can be determined ($SS = SD^2 \times df$):

$$SS_{ERROR} = SS_1 + SS_2 + SS_3 + SS_4 = 18.000 + 18.000 + 18.000 + 18.000 = 72.000$$
$$df_{ERROR} = df_1 + df_2 + df_3 + df_4 = 12$$

The “F” statistic is a ratio of the effect and within group (error) variance estimates. For “Factor B”:

$$F_{FACTORB} = \frac{MS_{FACTORB}}{MS_{ERROR}} = \frac{16.0}{6.0} = 2.667$$

An *F* with 1 and 12 *df* that equals 2.667 has a two-tailed probability of .128, which is not statistically significant.