

Comparing Models for the Prediction of Home Prices

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Abstract

Being able to accurately predict housing prices is critical to many industries. Recently, analysts have attempted to improve price prediction with enhanced statistical techniques. In this paper, we take a more comparative approach, examining 6 standard regression techniques (OLS, ridge, lasso, elastic net, random forest and gradient boosting) to assess the best performance. We used a kaggle dataset (<https://www.kaggle.com/c/house-prices-advanced-regression-techniques>) in order to test the performance of the model. We found Lasso to be the best predictor, which we speculate is because the dataset has a high number of predictors relative to the number of observations.

Introduction

In this paper we analyze housing prices by comparing three prediction methodologies: OLS, ridge, lasso, elastic net, random forest and gradient boosting. The purpose is to compare the methodologies and draw conclusions about which are most effective and why. Regression alone is not necessarily the optimal strategy for predicting housing prices.¹ However, when data sets and/or analysis resources are limited, regression can perform adequately.

Background and Literature Review

The ability to accurately predict home prices is of tremendous value to a number of industries, including investors, real estate agents, and municipalities who depend upon property tax revenue. Predictive models for home prices fall roughly into two kinds. First, there are those which predict market trends, busts, and booms. These predictions rely mainly on time series data and analysis of housing prices in the aggregate.

¹Li, 2021

The other type of prediction involves the capacity to predict individual house prices from a set of factors. These usually employ some form of regression and/or machine learning.²

For either sort of prediction, there is no consensus about the best method. Many researchers have sought to enhance the traditional models with other methodologies.³ For example, Guan et. al. propose a “data stream” approach in which past sale records are treated as an evolving datastream.⁴ Li et. al. introduce a “grey seasonal model” in which seasonal fluctuations are modeled using grey systems theory, which incorporates uncertainty.⁵ Alfiyatin, et. el. use particle swarm optimization (PSO) to select independent variables.⁶ (PSO is an optimization system in which population is initialized with random solutions and searches for optima by updating generations.) Finally, Liu et.al incorporate both spatial and temporal autocorrelation in their models by analyzing experience-based submarkets identified by real estate professionals.⁷

All of these researchers report that their innovations improve their regression models. Indeed, any real estate agent can tell you that a predictive model can be improved simply by knowing what other houses in the neighborhood sold for. The problem is, the data at the center of these enhancements is not always available. The researcher may have home sales from only a short time span, and neighborhoods that are not defined by real estate experts but by traditional boundary lines which may contain a mix of house types. Even when data is available, the complex models proposed may be computationally expensive and/or require data analysis expertise that is not generally available.

In this project we approach the question comparatively. Restricting ourselves to regression models, we compare six types of regression: OLS, ridge, lasso, elastic net, random forest and gradient boosting. The data is drawn from the Advanced Regression Techniques housing data set for Ames, Iowa. We test the accuracy of our models by submitting each to the Kaggle competition to see how they perform. We then discuss the merits of the different sorts of approaches.

Modeling

We are modeling a data set containing 1460 records of houses sold in the Ames, Iowa area between 2006 and 2010. The variables are mostly related to house features, such as square footage, the presence of a pool, etc. The response variable, “SalePrice”, is a continuous variable representing the sale price of the house in thousands of dollars.

²Journal, 2019

³Wu, 2020

⁴Guan, 2021

⁵Li, 2021

⁶Alfiyatin, 2017

⁷Liu, X. 2012

We examine the data:

1. Dataset Description

A. Summary Statistics

```
##      Id      MSSubClass      MSZoning      LotFrontage
##  Min.    : 1.0    Min.    : 20.0    C (all): 10    Min.    : 21.00
## 1st Qu.: 365.8    1st Qu.: 20.0    FV      : 65    1st Qu.: 59.00
## Median : 730.5    Median : 50.0    RH      : 16    Median : 69.00
## Mean   : 730.5    Mean    : 56.9    RL      :1151    Mean   : 70.05
## 3rd Qu.:1095.2    3rd Qu.: 70.0    RM      : 218    3rd Qu.: 80.00
## Max.    :1460.0    Max.     :190.0                      Max.    :313.00
##                                     NA's    :259

##      LotArea      Street      Alley      LotShape  LandContour  Utilities
##  Min.    : 1300    Grvl: 6    Grvl: 50    IR1:484    Bnk: 63    AllPub:1459
## 1st Qu.: 7554    Pave:1454    Pave: 41    IR2: 41    HLS: 50    NoSeWa: 1
## Median : 9478                      NA's:1369    IR3: 10    Low: 36
## Mean    : 10517                      Reg:925    Lvl:1311
## 3rd Qu.: 11602
## Max.    :215245

##
##      LotConfig      LandSlope      Neighborhood      Condition1      Condition2
##  Corner : 263    Gtl:1382    NAmes :225    Norm :1260    Norm :1445
##  CulDSac: 94    Mod: 65    CollgCr:150    Feedr : 81    Feedr : 6
##  FR2     : 47    Sev: 13    OldTown:113    Artery : 48    Artery : 2
##  FR3     : 4                      Edwards:100    RRAn : 26    PosN : 2
##  Inside :1052                      Somerst: 86    PosN : 19    RRNn : 2
##                                     Gilbert: 79    RRAe : 11    PosA : 1
##                                     (Other):707    (Other): 15    (Other): 2

##      BldgType      HouseStyle      OverallQual      OverallCond      YearBuilt
##  1Fam :1220    1Story :726    Min. : 1.000    Min. :1.000    Min. :1872
##  2fmCon: 31    2Story :445    1st Qu.: 5.000    1st Qu.:5.000    1st Qu.:1954
##  Duplex: 52    1.5Fin :154    Median : 6.000    Median :5.000    Median :1973
```

```

## Twnhs : 43 SLvl : 65 Mean : 6.099 Mean :5.575 Mean :1971
## TwnhsE: 114 SFoyer : 37 3rd Qu.: 7.000 3rd Qu.:6.000 3rd Qu.:2000
## 1.5Unf : 14 Max. :10.000 Max. :9.000 Max. :2010
## (Other): 19
## YearRemodAdd RoofStyle RoofMatl Exterior1st Exterior2nd
## Min. :1950 Flat : 13 CompShg:1434 VinylSd:515 VinylSd:504
## 1st Qu.:1967 Gable :1141 Tar&Grv: 11 HdBoard:222 MetalSd:214
## Median :1994 Gambrel: 11 WdShngl: 6 MetalSd:220 HdBoard:207
## Mean :1985 Hip : 286 WdShake: 5 Wd Sdng:206 Wd Sdng:197
## 3rd Qu.:2004 Mansard: 7 ClyTile: 1 Plywood:108 Plywood:142
## Max. :2010 Shed : 2 Membran: 1 CemntBd: 61 CmentBd: 60
## (Other): 2 (Other):128 (Other):136
## MasVnrType MasVnrArea ExterQual ExterCond Foundation BsmtQual
## BrkCmn : 15 Min. : 0.0 Ex: 52 Ex: 3 BrkTil:146 Ex :121
## BrkFace:445 1st Qu.: 0.0 Fa: 14 Fa: 28 CBlock:634 Fa : 35
## None :864 Median : 0.0 Gd:488 Gd: 146 PConc :647 Gd :618
## Stone :128 Mean : 103.7 TA:906 Po: 1 Slab : 24 TA :649
## NA's : 8 3rd Qu.: 166.0 TA:1282 Stone : 6 NA's: 37
## Max. :1600.0 Wood : 3
## NA's :8
## BsmtCond BsmtExposure BsmtFinType1 BsmtFinSF1 BsmtFinType2
## Fa : 45 Av :221 ALQ :220 Min. : 0.0 ALQ : 19
## Gd : 65 Gd :134 BLQ :148 1st Qu.: 0.0 BLQ : 33
## Po : 2 Mn :114 GLQ :418 Median : 383.5 GLQ : 14
## TA :1311 No :953 LwQ : 74 Mean : 443.6 LwQ : 46
## NA's: 37 NA's: 38 Rec :133 3rd Qu.: 712.2 Rec : 54
## Unf :430 Max. :5644.0 Unf :1256
## NA's: 37 NA's: 38
## BsmtFinSF2 BsmtUnfSF TotalBsmtSF Heating HeatingQC
## Min. : 0.00 Min. : 0.0 Min. : 0.0 Floor: 1 Ex:741
## 1st Qu.: 0.00 1st Qu.: 223.0 1st Qu.: 795.8 GasA :1428 Fa: 49
## Median : 0.00 Median : 477.5 Median : 991.5 GasW : 18 Gd:241
## Mean : 46.55 Mean : 567.2 Mean :1057.4 Grav : 7 Po: 1

```

```

## 3rd Qu.: 0.00 3rd Qu.: 808.0 3rd Qu.:1298.2 OthW : 2 TA:428
## Max. :1474.00 Max. :2336.0 Max. :6110.0 Wall : 4
##
## CentralAir Electrical X1stFlrSF X2ndFlrSF LowQualFinSF
## N: 95 FuseA: 94 Min. : 334 Min. : 0 Min. : 0.000
## Y:1365 FuseF: 27 1st Qu.: 882 1st Qu.: 0 1st Qu.: 0.000
## FuseP: 3 Median :1087 Median : 0 Median : 0.000
## Mix : 1 Mean :1163 Mean : 347 Mean : 5.845
## SBrkr:1334 3rd Qu.:1391 3rd Qu.: 728 3rd Qu.: 0.000
## NA's : 1 Max. :4692 Max. :2065 Max. :572.000
##
## GrLivArea BsmtFullBath BsmtHalfBath FullBath
## Min. : 334 Min. :0.0000 Min. :0.00000 Min. :0.000
## 1st Qu.:1130 1st Qu.:0.0000 1st Qu.:0.00000 1st Qu.:1.000
## Median :1464 Median :0.0000 Median :0.00000 Median :2.000
## Mean :1515 Mean :0.4253 Mean :0.05753 Mean :1.565
## 3rd Qu.:1777 3rd Qu.:1.0000 3rd Qu.:0.00000 3rd Qu.:2.000
## Max. :5642 Max. :3.0000 Max. :2.00000 Max. :3.000
##
## HalfBath BedroomAbvGr KitchenAbvGr KitchenQual TotRmsAbvGrd
## Min. :0.0000 Min. :0.000 Min. :0.000 Ex:100 Min. : 2.000
## 1st Qu.:0.0000 1st Qu.:2.000 1st Qu.:1.000 Fa: 39 1st Qu.: 5.000
## Median :0.0000 Median :3.000 Median :1.000 Gd:586 Median : 6.000
## Mean :0.3829 Mean :2.866 Mean :1.047 TA:735 Mean : 6.518
## 3rd Qu.:1.0000 3rd Qu.:3.000 3rd Qu.:1.000 3rd Qu.: 7.000
## Max. :2.0000 Max. :8.000 Max. :3.000 Max. :14.000
##
## Functional Fireplaces FireplaceQu GarageType GarageYrBlt
## Maj1: 14 Min. :0.000 Ex : 24 2Types : 6 Min. :1900
## Maj2: 5 1st Qu.:0.000 Fa : 33 Attchd :870 1st Qu.:1961
## Min1: 31 Median :1.000 Gd :380 Basement: 19 Median :1980
## Min2: 34 Mean :0.613 Po : 20 BuiltIn: 88 Mean :1979
## Mod : 15 3rd Qu.:1.000 TA :313 CarPort: 9 3rd Qu.:2002

```

```

## Sev : 1 Max. :3.000 NA's:690 Detchd :387 Max. :2010
## Typ :1360 NA's : 81 NA's :81
## GarageFinish GarageCars GarageArea GarageQual GarageCond
## Fin :352 Min. :0.000 Min. : 0.0 Ex : 3 Ex : 2
## RFn :422 1st Qu.:1.000 1st Qu.: 334.5 Fa : 48 Fa : 35
## Unf :605 Median :2.000 Median : 480.0 Gd : 14 Gd : 9
## NA's: 81 Mean :1.767 Mean : 473.0 Po : 3 Po : 7
## 3rd Qu.:2.000 3rd Qu.: 576.0 TA :1311 TA :1326
## Max. :4.000 Max. :1418.0 NA's: 81 NA's: 81
##
## PavedDrive WoodDeckSF OpenPorchSF EnclosedPorch X3SsnPorch
## N: 90 Min. : 0.00 Min. : 0.00 Min. : 0.00 Min. : 0.00
## P: 30 1st Qu.: 0.00 1st Qu.: 0.00 1st Qu.: 0.00 1st Qu.: 0.00
## Y:1340 Median : 0.00 Median : 25.00 Median : 0.00 Median : 0.00
## Mean : 94.24 Mean : 46.66 Mean : 21.95 Mean : 3.41
## 3rd Qu.:168.00 3rd Qu.: 68.00 3rd Qu.: 0.00 3rd Qu.: 0.00
## Max. :857.00 Max. :547.00 Max. :552.00 Max. :508.00
##
## ScreenPorch PoolArea PoolQC Fence MiscFeature
## Min. : 0.00 Min. : 0.000 Ex : 2 GdPrv: 59 Gar2: 2
## 1st Qu.: 0.00 1st Qu.: 0.000 Fa : 2 GdWo : 54 Othr: 2
## Median : 0.00 Median : 0.000 Gd : 3 MnPrv: 157 Shed: 49
## Mean : 15.06 Mean : 2.759 NA's:1453 MnWw : 11 TenC: 1
## 3rd Qu.: 0.00 3rd Qu.: 0.000 NA's :1179 NA's:1406
## Max. :480.00 Max. :738.000
##
## MiscVal MoSold YrSold SaleType
## Min. : 0.00 Min. : 1.000 Min. :2006 WD :1267
## 1st Qu.: 0.00 1st Qu.: 5.000 1st Qu.:2007 New : 122
## Median : 0.00 Median : 6.000 Median :2008 COD : 43
## Mean : 43.49 Mean : 6.322 Mean :2008 ConLD : 9
## 3rd Qu.: 0.00 3rd Qu.: 8.000 3rd Qu.:2009 ConLI : 5
## Max. :15500.00 Max. :12.000 Max. :2010 ConLw : 5

```

```

##                                     (Other):    9

## SaleCondition    SalePrice
## Abnorml: 101    Min.      : 34900
## AdjLand:   4    1st Qu.:129975
## Alloca :  12    Median :163000
## Family  : 20    Mean     :180921
## Normal :1198    3rd Qu.:214000
## Partial: 125    Max.      :755000
##

## 'data.frame':    1460 obs. of  81 variables:
## $ Id           : int  1 2 3 4 5 6 7 8 9 10 ...
## $ MSSubClass    : int  60 20 60 70 60 50 20 60 50 190 ...
## $ MSZoning      : Factor w/ 5 levels "C (all)","FV",...: 4 4 4 4 4 4 4 4 5 4 ...
## $ LotFrontage   : int  65 80 68 60 84 85 75 NA 51 50 ...
## $ LotArea       : int  8450 9600 11250 9550 14260 14115 10084 10382 6120 7420 ...
## $ Street        : Factor w/ 2 levels "Grvl","Pave": 2 2 2 2 2 2 2 2 2 2 ...
## $ Alley         : Factor w/ 2 levels "Grvl","Pave": NA NA NA NA NA NA NA NA NA ...
## $ LotShape      : Factor w/ 4 levels "IR1","IR2","IR3",...: 4 4 1 1 1 1 4 1 4 4 ...
## $ LandContour   : Factor w/ 4 levels "Bnk","HLS","Low",...: 4 4 4 4 4 4 4 4 4 4 ...
## $ Utilities     : Factor w/ 2 levels "AllPub","NoSeWa": 1 1 1 1 1 1 1 1 1 1 ...
## $ LotConfig     : Factor w/ 5 levels "Corner","CulDSac",...: 5 3 5 1 3 5 5 1 5 1 ...
## $ LandSlope     : Factor w/ 3 levels "Gtl","Mod","Sev": 1 1 1 1 1 1 1 1 1 1 ...
## $ Neighborhood : Factor w/ 25 levels "Blmngtn","Blueste",...: 6 25 6 7 14 12 21 17 18 4 ...
## $ Condition1    : Factor w/ 9 levels "Artery","Feedr",...: 3 2 3 3 3 3 3 5 1 1 ...
## $ Condition2    : Factor w/ 8 levels "Artery","Feedr",...: 3 3 3 3 3 3 3 3 1 ...
## $ BldgType      : Factor w/ 5 levels "1Fam","2fmCon",...: 1 1 1 1 1 1 1 1 1 2 ...
## $ HouseStyle    : Factor w/ 8 levels "1.5Fin","1.5Unf",...: 6 3 6 6 6 1 3 6 1 2 ...
## $ OverallQual   : int  7 6 7 7 8 5 8 7 7 5 ...
## $ OverallCond   : int  5 8 5 5 5 5 5 6 5 6 ...
## $ YearBuilt     : int  2003 1976 2001 1915 2000 1993 2004 1973 1931 1939 ...
## $ YearRemodAdd  : int  2003 1976 2002 1970 2000 1995 2005 1973 1950 1950 ...
## $ RoofStyle     : Factor w/ 6 levels "Flat","Gable",...: 2 2 2 2 2 2 2 2 2 2 ...

```



```

## $ RoofMatl      : Factor w/ 8 levels "ClyTile","CompShg",...: 2 2 2 2 2 2 2 2 ...
## $ Exterior1st   : Factor w/ 15 levels "AsbShng","AsphShn",...: 13 9 13 14 13 13 13 7 4 9 ...
## $ Exterior2nd   : Factor w/ 16 levels "AsbShng","AsphShn",...: 14 9 14 16 14 14 14 7 16 9 ...
## $ MasVnrType    : Factor w/ 4 levels "BrkCmn","BrkFace",...: 2 3 2 3 2 3 4 4 3 3 ...
## $ MasVnrArea    : int   196 0 162 0 350 0 186 240 0 0 ...
## $ ExterQual     : Factor w/ 4 levels "Ex","Fa","Gd",...: 3 4 3 4 3 4 3 4 4 4 ...
## $ ExterCond     : Factor w/ 5 levels "Ex","Fa","Gd",...: 5 5 5 5 5 5 5 5 5 5 ...
## $ Foundation    : Factor w/ 6 levels "BrkTil","CBlock",...: 3 2 3 1 3 6 3 2 1 1 ...
## $ BsmtQual      : Factor w/ 4 levels "Ex","Fa","Gd",...: 3 3 3 4 3 3 1 3 4 4 ...
## $ BsmtCond      : Factor w/ 4 levels "Fa","Gd","Po",...: 4 4 4 2 4 4 4 4 4 4 ...
## $ BsmtExposure  : Factor w/ 4 levels "Av","Gd","Mn",...: 4 2 3 4 1 4 1 3 4 4 ...
## $ BsmtFinType1  : Factor w/ 6 levels "ALQ","BLQ","GLQ",...: 3 1 3 1 3 3 3 1 6 3 ...
## $ BsmtFinSF1    : int   706 978 486 216 655 732 1369 859 0 851 ...
## $ BsmtFinType2  : Factor w/ 6 levels "ALQ","BLQ","GLQ",...: 6 6 6 6 6 6 6 2 6 6 ...
## $ BsmtFinSF2    : int    0 0 0 0 0 0 0 32 0 0 ...
## $ BsmtUnfSF     : int   150 284 434 540 490 64 317 216 952 140 ...
## $ TotalBsmtSF   : int   856 1262 920 756 1145 796 1686 1107 952 991 ...
## $ Heating       : Factor w/ 6 levels "Floor","GasA",...: 2 2 2 2 2 2 2 2 2 2 ...
## $ HeatingQC     : Factor w/ 5 levels "Ex","Fa","Gd",...: 1 1 1 3 1 1 1 1 1 3 ...
## $ CentralAir    : Factor w/ 2 levels "N","Y": 2 2 2 2 2 2 2 2 2 2 ...
## $ Electrical    : Factor w/ 5 levels "FuseA","FuseF",...: 5 5 5 5 5 5 5 5 2 5 ...
## $ X1stFlrSF     : int   856 1262 920 961 1145 796 1694 1107 1022 1077 ...
## $ X2ndFlrSF     : int   854 0 866 756 1053 566 0 983 752 0 ...
## $ LowQualFinSF  : int    0 0 0 0 0 0 0 0 0 0 ...
## $ GrLivArea     : int   1710 1262 1786 1717 2198 1362 1694 2090 1774 1077 ...
## $ BsmtFullBath  : int    1 0 1 1 1 1 1 1 0 1 ...
## $ BsmtHalfBath  : int    0 1 0 0 0 0 0 0 0 0 ...
## $ FullBath      : int    2 2 2 1 2 1 2 2 2 1 ...
## $ HalfBath      : int    1 0 1 0 1 1 0 1 0 0 ...
## $ BedroomAbvGr : int    3 3 3 3 4 1 3 3 2 2 ...
## $ KitchenAbvGr  : int    1 1 1 1 1 1 1 1 2 2 ...
## $ KitchenQual   : Factor w/ 4 levels "Ex","Fa","Gd",...: 3 4 3 3 3 4 3 4 4 4 ...
## $ TotRmsAbvGrd : int    8 6 6 7 9 5 7 7 8 5 ...

```

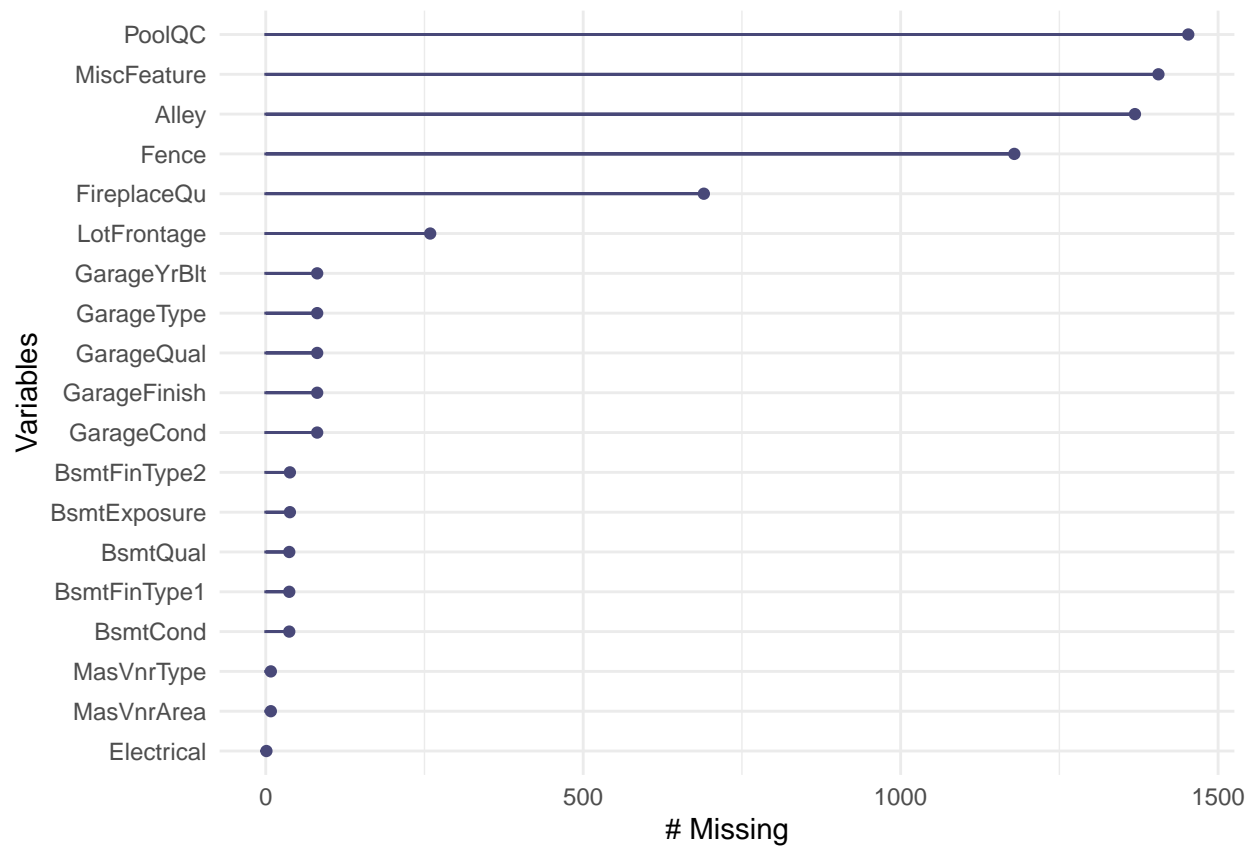
```

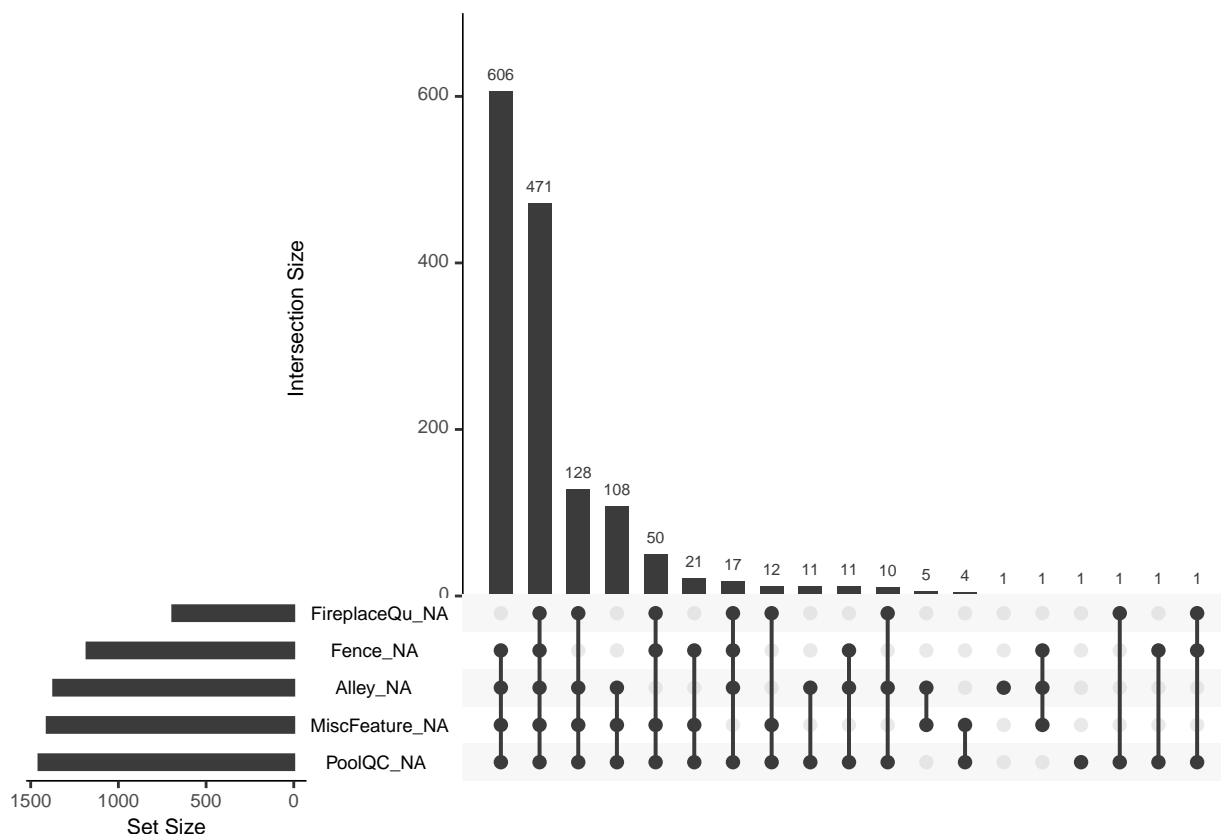
## $ Functional      : Factor w/ 7 levels "Maj1","Maj2",...: 7 7 7 7 7 7 7 3 7 ...
## $ Fireplaces      : int   0 1 1 1 1 0 1 2 2 2 ...
## $ FireplaceQu     : Factor w/ 5 levels "Ex","Fa","Gd",...: NA 5 5 3 5 NA 3 5 5 5 ...
## $ GarageType      : Factor w/ 6 levels "2Types","Attchd",...: 2 2 2 6 2 2 2 2 6 2 ...
## $ GarageYrBltd   : int   2003 1976 2001 1998 2000 1993 2004 1973 1931 1939 ...
## $ GarageFinish    : Factor w/ 3 levels "Fin","RFn","Unf": 2 2 2 3 2 3 2 2 3 2 ...
## $ GarageCars      : int   2 2 2 3 3 2 2 2 2 1 ...
## $ GarageArea      : int   548 460 608 642 836 480 636 484 468 205 ...
## $ GarageQual      : Factor w/ 5 levels "Ex","Fa","Gd",...: 5 5 5 5 5 5 5 5 2 3 ...
## $ GarageCond      : Factor w/ 5 levels "Ex","Fa","Gd",...: 5 5 5 5 5 5 5 5 5 5 ...
## $ PavedDrive      : Factor w/ 3 levels "N","P","Y": 3 3 3 3 3 3 3 3 3 3 ...
## $ WoodDeckSF      : int   0 298 0 0 192 40 255 235 90 0 ...
## $ OpenPorchSF     : int   61 0 42 35 84 30 57 204 0 4 ...
## $ EnclosedPorch   : int   0 0 0 272 0 0 0 228 205 0 ...
## $ X3SsnPorch      : int   0 0 0 0 0 320 0 0 0 0 ...
## $ ScreenPorch     : int   0 0 0 0 0 0 0 0 0 0 ...
## $ PoolArea        : int   0 0 0 0 0 0 0 0 0 0 ...
## $ PoolQC          : Factor w/ 3 levels "Ex","Fa","Gd": NA NA NA NA NA NA NA NA NA NA ...
## $ Fence           : Factor w/ 4 levels "GdPrv","GdWo",...: NA NA NA NA NA 3 NA NA NA NA ...
## $ MiscFeature     : Factor w/ 4 levels "Gar2","Othr",...: NA NA NA NA NA 3 NA 3 NA NA ...
## $ MiscVal         : int   0 0 0 0 0 700 0 350 0 0 ...
## $ MoSold          : int   2 5 9 2 12 10 8 11 4 1 ...
## $ YrSold           : int   2008 2007 2008 2006 2008 2009 2007 2009 2008 2008 ...
## $ SaleType        : Factor w/ 9 levels "COD","Con","ConLD",...: 9 9 9 9 9 9 9 9 9 9 ...
## $ SaleCondition   : Factor w/ 6 levels "Abnorml","AdjLand",...: 5 5 5 1 5 5 5 5 1 5 ...
## $ SalePrice       : int   208500 181500 223500 140000 250000 143000 307000 200000 129900 118000 ...

```

The dataset consists of 1460 observations and 81 variables, some numeric and some categorical. The target variable has a minimum of 34,950 and a maximum of 7,550,000. The low median compared to the mean suggests some skew.

B. Missing values There are missing values scattered throughout the dataset. We analyse them:





A few categorical features like fireplace, fence, etc. take up the bulk of missings. They do not appear to be important enough to retain so we delete them (FireplaceQu, Fence, Alley, MiscFeature, PoolQC, and LotFrontage). We impute the mean for the rest.

C. Create dummy variables Now we create dummy variables for all of the character variables. Categorical NA's will be handled by adding a dummy variable for NA.

D. Reconcile training and test sets We check if the dataset is missing columns from the test dataset and if so, drop them from the training set. This way we don't risk making predictions on training set variables not found in the test set.

E. Multicollinearity We examine multicollinearity in the dataset. We look at all of the pairs of correlations over .8 There are 24 pairs.

```
##           col1           col2 correlation
## 1      TotalBsmtSF      X1stFlrSF    0.8195300
```

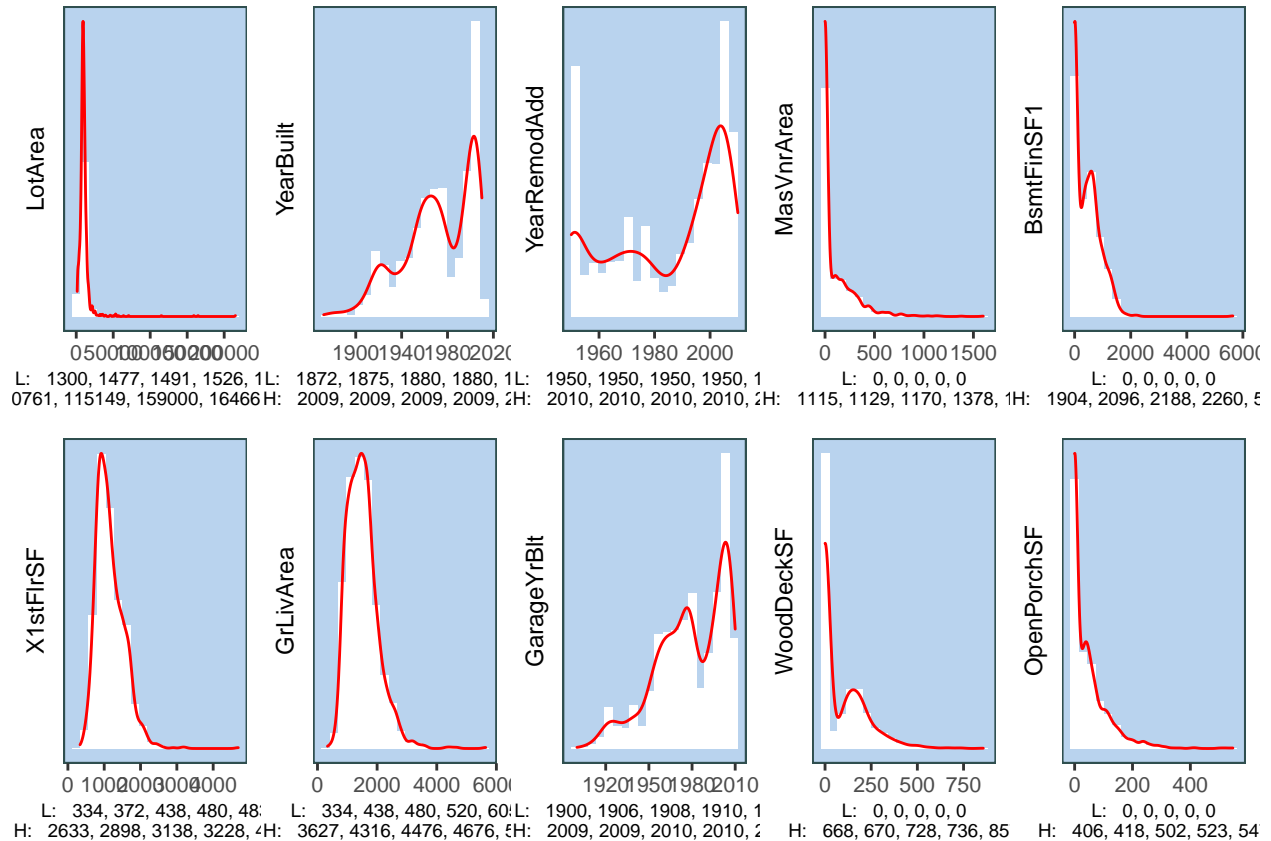
## 3	GrLivArea	TotRmsAbvGrd	0.8254894
## 5	GarageCars	GarageArea	0.8824754
## 7	MSZoning_FV	Neighborhood_Somerst	0.8628071
## 9	RoofStyle_Flat	RoofMatl_Tar.Grv	0.8349139
## 11	Exterior1st_AsbShng	Exterior2nd_AsbShng	0.8479167
## 12	Exterior1st_CemntBd	Exterior2nd_CmentBd	0.9741711
## 13	Exterior1st_HdBoard	Exterior2nd_HdBoard	0.8832714
## 14	Exterior1st_MetalSd	Exterior2nd_MetalSd	0.9730652
## 15	Exterior1st_Wd.Sdng	Exterior2nd_Wd.Sdng	0.8592439
## 21	Foundation_Slab	BsmtQual_NA	0.8017334
## 22	Foundation_Slab	BsmtCond_NA	0.8017334
## 23	Foundation_Slab	BsmtFinType1_NA	0.8017334
## 25	BsmtQual_NA	BsmtCond_NA	1.0000000
## 26	BsmtQual_NA	BsmtExposure_NA	0.9864076
## 27	BsmtQual_NA	BsmtFinType1_NA	1.0000000
## 28	BsmtQual_NA	BsmtFinType2_NA	0.9864076
## 31	BsmtCond_NA	BsmtExposure_NA	0.9864076
## 32	BsmtCond_NA	BsmtFinType1_NA	1.0000000
## 33	BsmtCond_NA	BsmtFinType2_NA	0.9864076
## 36	BsmtExposure_NA	BsmtFinType1_NA	0.9864076
## 37	BsmtExposure_NA	BsmtFinType2_NA	0.9729810
## 42	BsmtFinType1_NA	BsmtFinType2_NA	0.9864076
## 47	SaleType_New	SaleCondition_Partial	0.9868190

Most of the pairs make sense - siding on the first floor will match siding on the second floor, the number of cars a garage can hold will be related to its area. We will address the multicollinearity more closely when we run the analysis.

2. Transformations

A. Log of SalePrice The skew in the dependent variable suggests a log transformation.

B. Other transformations A number of histograms suggest issues with some of the independent variables.



We can see some transformations might be useful. We: 1. Add a dummy variable to mark YearBuilt before and after 1920 2. We set YearRemodAdd = 1950 to 0, and create a dummy variable YearRemodUnknown to track it 3. We add dummies for NoFinBsmt, HasDeck, and HasPorch 4. We eliminate outliers by setting GrLivArea<4000

3. Model and Predict:

A. Base Model We run an OLS regression using the stepAIC algorithm to minimize AIC.

```
##
## Call:
## lm(formula = SalePrice ~ Id + LotArea + OverallQual + OverallCond +
##     YearBuilt + YearRemodAdd + BsmtFinSF1 + BsmtFinSF2 + BsmtUnfSF +
##     X1stFlrSF + X2ndFlrSF + LowQualFinSF + BsmtFullBath + FullBath +
##     HalfBath + KitchenAbvGr + Fireplaces + GarageCars + GarageArea +
##     WoodDeckSF + OpenPorchSF + EnclosedPorch + X3SsnPorch + ScreenPorch +
```

```

## PoolArea + MSZoning_C..all. + MSZoning_RM + Street_Grvl +
## LotConfig_CulDSac + LandSlope_Sev + Neighborhood_BrkSide +
## Neighborhood_ClearCr + Neighborhood_Crawfor + Neighborhood_Edwards +
## Neighborhood_IDOTRR + Neighborhood_MeadowV + Neighborhood_Mitchel +
## Neighborhood_NWAmes + Neighborhood_NoRidge + Neighborhood_NridgHt +
## Neighborhood_Somerst + Neighborhood_StoneBr + Condition1_Artery +
## Condition1_RRAe + Condition1_RRAn + BldgType_Duplex + BldgType_Twnhs +
## BldgType_TwnhsE + RoofStyle_Flat + Exterior1st_BrkComm +
## Exterior1st_BrkFace + Exterior1st_CemntBd + Exterior1st_HdBoard +
## Exterior1st_Plywood + Exterior1st_Wd.Sdng + Exterior2nd_CmentBd +
## Exterior2nd_Wd.Sdng + MasVnrType_BrkCmn + MasVnrType_Stone +
## ExterCond_Ex + ExterCond_Fa + ExterCond_Gd + Foundation_Stone +
## Foundation_Wood + BsmtQual_Ex + BsmtCond_Fa + BsmtCond_Po +
## BsmtExposure_Gd + BsmtFinType2_BLQ + BsmtFinType2_GLQ + Heating_GasW +
## Heating_Grav + Heating_Wall + CentralAir_N + KitchenQual_Ex +
## Functional_Maj1 + Functional_Maj2 + Functional_Min1 + Functional_Min2 +
## Functional_Mod + Functional_Sev + GarageType_2Types + GarageType_NA +
## GarageQual_Fa + GarageQual_Po + GarageCond_Fa + GarageCond_Po +
## PavedDrive_N + SaleType_CWD + SaleType_Con + SaleType_ConLD +
## SaleType_ConLw + SaleType_New + SaleType_Oth + SaleCondition_Abnorml +
## SaleCondition_Family + SaleCondition_Partial + BuiltAfter1920 +
## YearRemodUnknown, data = df)
##
## Residuals:
##      Min        1Q    Median        3Q        Max
## -0.67671 -0.04919  0.00285  0.05111  0.46324
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    4.497e+00  5.680e-01   7.918 4.97e-15 ***
## Id             -9.095e-06  6.429e-06  -1.415 0.157355
## LotArea         2.600e-06  3.859e-07   6.738 2.37e-11 ***
## OverallQual     4.738e-02  3.865e-03  12.258 < 2e-16 ***

```

## OverallCond	3.739e-02	3.501e-03	10.681	< 2e-16	***
## YearBuilt	2.426e-03	2.618e-04	9.267	< 2e-16	***
## YearRemodAdd	8.239e-04	2.532e-04	3.255	0.001164	**
## BsmtFinSF1	1.614e-04	1.476e-05	10.936	< 2e-16	***
## BsmtFinSF2	1.153e-04	2.312e-05	4.986	6.98e-07	***
## BsmtUnfSF	9.846e-05	1.305e-05	7.548	8.09e-14	***
## X1stFlrSF	2.669e-04	1.638e-05	16.293	< 2e-16	***
## X2ndFlrSF	2.570e-04	1.274e-05	20.169	< 2e-16	***
## LowQualFinSF	2.394e-04	5.854e-05	4.090	4.58e-05	***
## BsmtFullBath	2.752e-02	7.453e-03	3.693	0.000231	***
## FullBath	1.898e-02	8.570e-03	2.215	0.026912	*
## HalfBath	1.867e-02	8.196e-03	2.277	0.022926	*
## KitchenAbvGr	-5.083e-02	1.895e-02	-2.683	0.007388	**
## Fireplaces	1.974e-02	5.418e-03	3.643	0.000279	***
## GarageCars	3.053e-02	9.111e-03	3.351	0.000827	***
## GarageArea	6.558e-05	3.021e-05	2.171	0.030083	*
## WoodDeckSF	8.737e-05	2.401e-05	3.639	0.000284	***
## OpenPorchSF	1.055e-04	4.660e-05	2.264	0.023702	*
## EnclosedPorch	1.094e-04	5.013e-05	2.182	0.029263	*
## X3SsnPorch	1.885e-04	9.255e-05	2.037	0.041878	*
## ScreenPorch	2.295e-04	5.078e-05	4.518	6.78e-06	***
## PoolArea	1.175e-04	7.820e-05	1.503	0.133101	
## MSZoning_C..all.	-4.260e-01	4.027e-02	-10.580	< 2e-16	***
## MSZoning_RM	-6.101e-02	1.060e-02	-5.756	1.06e-08	***
## Street_Grvl	-8.150e-02	4.677e-02	-1.742	0.081684	.
## LotConfig_CulDSac	3.232e-02	1.157e-02	2.793	0.005289	**
## LandSlope_Sev	-1.274e-01	3.958e-02	-3.218	0.001323	**
## Neighborhood_BrkSide	5.767e-02	1.588e-02	3.631	0.000293	***
## Neighborhood_ClearCr	5.267e-02	2.233e-02	2.358	0.018517	*
## Neighborhood_Crawfor	1.453e-01	1.659e-02	8.760	< 2e-16	***
## Neighborhood_Edwards	-3.461e-02	1.193e-02	-2.901	0.003782	**
## Neighborhood_IDOTRR	3.522e-02	2.210e-02	1.593	0.111325	
## Neighborhood_MeadowV	-1.180e-01	3.263e-02	-3.617	0.000309	***

## Neighborhood_Mitchel	-2.182e-02	1.573e-02	-1.387	0.165621	
## Neighborhood_NWAmes	-2.698e-02	1.351e-02	-1.997	0.046026	*
## Neighborhood_NoRidge	5.057e-02	1.853e-02	2.729	0.006443	**
## Neighborhood_NridgHt	6.080e-02	1.605e-02	3.789	0.000158	***
## Neighborhood_Somerst	4.679e-02	1.377e-02	3.399	0.000696	***
## Neighborhood_StoneBr	1.244e-01	2.282e-02	5.449	6.00e-08	***
## Condition1_Artery	-6.968e-02	1.609e-02	-4.331	1.60e-05	***
## Condition1_RRAe	-1.178e-01	3.122e-02	-3.774	0.000168	***
## Condition1_RRAn	-3.755e-02	2.077e-02	-1.808	0.070808	.
## BldgType_Duplex	-4.643e-02	2.187e-02	-2.123	0.033904	*
## BldgType_Twnhs	-9.935e-02	1.853e-02	-5.362	9.68e-08	***
## BldgType_TwnhsE	-3.713e-02	1.224e-02	-3.034	0.002460	**
## RoofStyle_Flat	6.984e-02	3.551e-02	1.967	0.049422	*
## Exterior1st_BrkComm	-2.137e-01	7.879e-02	-2.712	0.006775	**
## Exterior1st_BrkFace	3.442e-02	1.680e-02	2.049	0.040636	*
## Exterior1st_CemntBd	-9.097e-02	6.111e-02	-1.489	0.136825	
## Exterior1st_HdBoard	-3.707e-02	8.732e-03	-4.246	2.33e-05	***
## Exterior1st_Plywood	-3.344e-02	1.206e-02	-2.772	0.005649	**
## Exterior1st_Wd.Sdng	-6.159e-02	1.661e-02	-3.707	0.000218	***
## Exterior2nd_CmentBd	8.753e-02	6.156e-02	1.422	0.155343	
## Exterior2nd_Wd.Sdng	4.423e-02	1.637e-02	2.702	0.006970	**
## MasVnrType_BrkCmn	-5.277e-02	2.780e-02	-1.898	0.057862	.
## MasVnrType_Stone	2.179e-02	1.121e-02	1.944	0.052089	.
## ExterCond_Ex	1.013e-01	6.055e-02	1.672	0.094696	.
## ExterCond_Fa	-3.659e-02	2.264e-02	-1.616	0.106303	
## ExterCond_Gd	-1.809e-02	9.700e-03	-1.864	0.062477	.
## Foundation_Stone	1.026e-01	4.444e-02	2.309	0.021089	*
## Foundation_Wood	-1.391e-01	5.955e-02	-2.336	0.019641	*
## BsmtQual_Ex	2.848e-02	1.331e-02	2.140	0.032529	*
## BsmtCond_Fa	-3.251e-02	1.692e-02	-1.922	0.054858	.
## BsmtCond_Po	1.691e-01	9.088e-02	1.861	0.062987	.
## BsmtExposure_Gd	5.052e-02	1.093e-02	4.621	4.19e-06	***
## BsmtFinType2_BLQ	-3.235e-02	1.922e-02	-1.683	0.092602	.

## BsmtFinType2_GLQ	5.425e-02	3.079e-02	1.762	0.078250	.
## Heating_GasW	5.224e-02	2.727e-02	1.916	0.055622	.
## Heating_Grav	-1.477e-01	4.379e-02	-3.372	0.000766	***
## Heating_Wall	8.911e-02	5.559e-02	1.603	0.109163	
## CentralAir_N	-6.317e-02	1.427e-02	-4.426	1.04e-05	***
## KitchenQual_Ex	6.301e-02	1.374e-02	4.587	4.92e-06	***
## Functional_Maj1	-9.671e-02	2.996e-02	-3.228	0.001278	**
## Functional_Maj2	-3.104e-01	4.978e-02	-6.237	5.96e-10	***
## Functional_Min1	-4.847e-02	1.949e-02	-2.487	0.013004	*
## Functional_Min2	-3.440e-02	1.879e-02	-1.830	0.067441	.
## Functional_Mod	-1.175e-01	2.820e-02	-4.167	3.28e-05	***
## Functional_Sev	-4.125e-01	1.119e-01	-3.685	0.000238	***
## GarageType_2Types	-8.141e-02	4.455e-02	-1.827	0.067892	.
## GarageType_NA	-4.205e-02	1.710e-02	-2.459	0.014066	*
## GarageQual_Fa	-3.572e-02	1.914e-02	-1.866	0.062228	.
## GarageQual_Po	-1.359e-01	8.849e-02	-1.536	0.124782	
## GarageCond_Fa	-3.201e-02	2.103e-02	-1.522	0.128259	
## GarageCond_Po	1.079e-01	5.623e-02	1.919	0.055189	.
## PavedDrive_N	-1.862e-02	1.352e-02	-1.378	0.168529	
## SaleType_CWD	8.151e-02	5.230e-02	1.559	0.119343	
## SaleType_Con	1.233e-01	7.211e-02	1.710	0.087502	.
## SaleType_ConLD	1.333e-01	3.657e-02	3.645	0.000277	***
## SaleType_ConLw	6.869e-02	4.704e-02	1.460	0.144458	
## SaleType_New	1.772e-01	6.258e-02	2.832	0.004696	**
## SaleType_Oth	8.899e-02	5.994e-02	1.485	0.137864	
## SaleCondition_Abnorml	-6.940e-02	1.128e-02	-6.150	1.02e-09	***
## SaleCondition_Family	-4.724e-02	2.339e-02	-2.020	0.043582	*
## SaleCondition_Partial	-1.311e-01	6.197e-02	-2.116	0.034563	*
## BuiltAfter1920	-2.748e-02	1.554e-02	-1.768	0.077271	.
## YearRemodUnknown	1.598e+00	5.011e-01	3.188	0.001464	**
## ---					
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1					
##					

Residual standard error: 0.09995 on 1356 degrees of freedom

Multiple R-squared: 0.9406, Adjusted R-squared: 0.9363

F-statistic: 217.1 on 99 and 1356 DF, p-value: < 2.2e-16

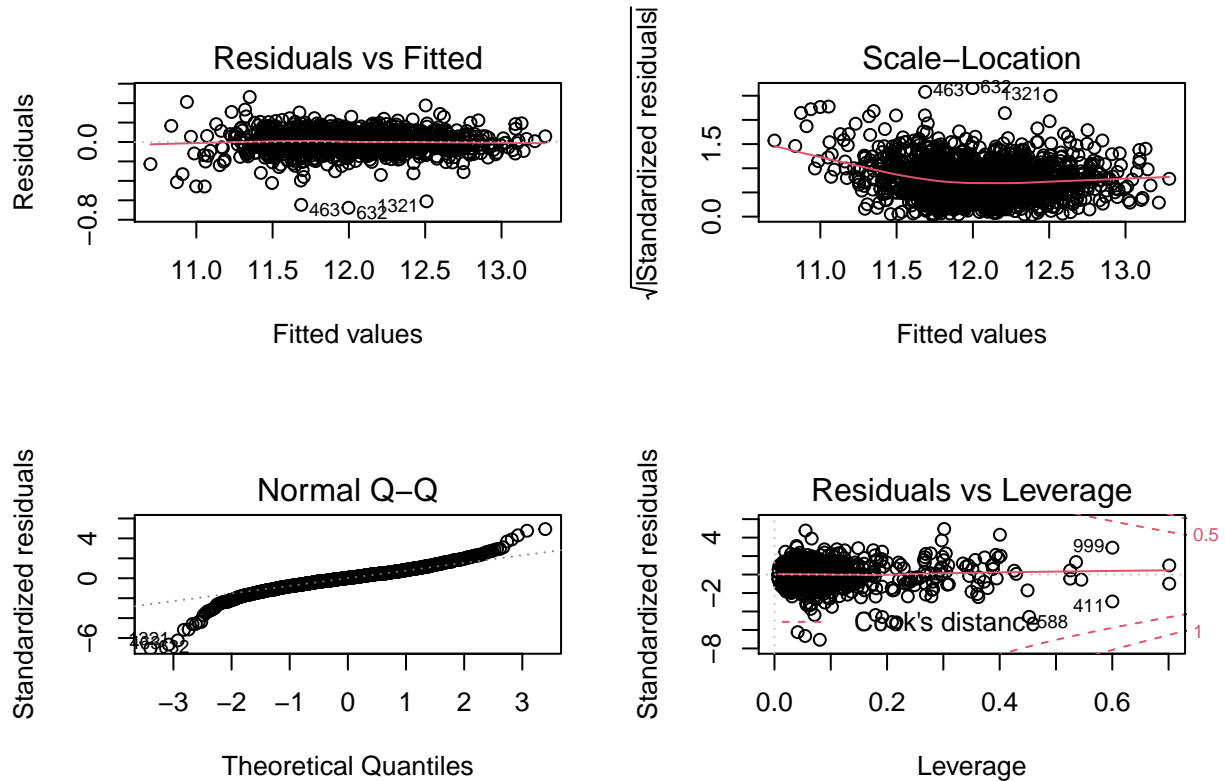
##

[1] "VIF Analysis"

##	Id	LotArea	OverallQual
##	1.070446	2.109227	4.082242
##	OverallCond	YearBuilt	YearRemodAdd
##	2.214804	9.101482	3969.919813
##	BsmtFinSF1	BsmtFinSF2	BsmtUnfSF
##	5.870162	2.030635	4.846297
##	X1stFlrSF	X2ndFlrSF	LowQualFinSF
##	5.329108	4.402165	1.183029
##	BsmtFullBath	FullBath	HalfBath
##	2.165471	3.208325	2.470764
##	KitchenAbvGr	Fireplaces	GarageCars
##	2.544832	1.752755	6.732297
##	GarageArea	WoodDeckSF	OpenPorchSF
##	5.970826	1.315971	1.350534
##	EnclosedPorch	X3SsnPorch	ScreenPorch
##	1.370222	1.074979	1.170604
##	PoolArea	MSZoning_C..all.	MSZoning_RM
##	1.114980	1.611695	2.083850
##	Street_Grvl	LotConfig_CulDSac	LandSlope_Sev
##	1.308506	1.178051	2.020225
##	Neighborhood_BrkSide	Neighborhood_ClearCr	Neighborhood_Crawfor
##	1.406034	1.371166	1.355105
##	Neighborhood_Edwards	Neighborhood_IDOTRR	Neighborhood_MeadowV
##	1.302697	1.763348	1.790537
##	Neighborhood_Mitchel	Neighborhood_NWAmes	Neighborhood_NoRidge
##	1.172639	1.266834	1.305002
##	Neighborhood_NridgHt	Neighborhood_Somerst	Neighborhood_StoneBr
##	1.879891	1.535288	1.281182

##	Condition1_Artery	Condition1_RRAe	Condition1_RRAe
##	1.202671	1.065123	1.102368
##	BldgType_Duplex	BldgType_Twnhs	BldgType_TwnhsE
##	2.399890	1.434197	1.575527
##	RoofStyle_Flat	Exterior1st_BrkComm	Exterior1st_BrkFace
##	1.626001	1.240871	1.363344
##	Exterior1st_CemntBd	Exterior1st_HdBoard	Exterior1st_Plywood
##	21.501494	1.430582	1.456569
##	Exterior1st_Wd.Sdng	Exterior2nd_CmentBd	Exterior2nd_Wd.Sdng
##	4.865587	21.475610	4.567847
##	MasVnrType_BrkCmn	MasVnrType_Stone	ExterCond_Ex
##	1.148006	1.447534	1.098791
##	ExterCond_Fa	ExterCond_Gd	Foundation_Stone
##	1.408762	1.237170	1.181241
##	Foundation_Wood	BsmtQual_Ex	BsmtCond_Fa
##	1.062645	1.907796	1.249316
##	BsmtCond_Po	BsmtExposure_Gd	BsmtFinType2_BLQ
##	1.651124	1.426619	1.192674
##	BsmtFinType2_GLQ	Heating_GasW	Heating_Grav
##	1.315372	1.323281	1.336804
##	Heating_Wall	CentralAir_N	KitchenQual_Ex
##	1.233841	1.811117	1.693583
##	Functional_Maj1	Functional_Maj2	Functional_Min1
##	1.245837	1.235746	1.153368
##	Functional_Min2	Functional_Mod	Functional_Sev
##	1.173951	1.181688	1.253489
##	GarageType_2Types	GarageType_NA	GarageQual_Fa
##	1.187210	2.238917	1.702411
##	GarageQual_Po	GarageCond_Fa	GarageCond_Po
##	2.346550	1.512048	2.204323
##	PavedDrive_N	SaleType_CWD	SaleType_Con
##	1.543691	1.092156	1.039413
##	SaleType_ConLD	SaleType_ConLw	SaleType_New

```
##          1.197037          1.103716          43.157779
##          SaleType_0th SaleCondition_Abnorml SaleCondition_Family
##          1.076456          1.186853          1.079773
## SaleCondition_Partial      BuiltAfter1920      YearRemodUnknown
##          43.285823          2.622103          3926.926004
```



```
## NULL
##
## studentized Breusch-Pagan test
##
## data: step3
## BP = 270.14, df = 99, p-value < 2.2e-16
##
##
## Shapiro-Wilk normality test
##
```

```

## data:  step3$residuals
## W = 0.93874, p-value < 2.2e-16
##
## [1] "AIC:  -2476.10190734742"

##
## Call:
## lm(formula = SalePrice ~ Id + LotArea + OverallQual + OverallCond +
##      YearBuilt + YearRemodAdd + BsmtFinSF1 + BsmtFinSF2 + BsmtUnfSF +
##      X1stFlrSF + X2ndFlrSF + LowQualFinSF + BsmtFullBath + FullBath +
##      HalfBath + KitchenAbvGr + Fireplaces + GarageCars + GarageArea +
##      WoodDeckSF + OpenPorchSF + EnclosedPorch + X3SsnPorch + ScreenPorch +
##      PoolArea + MSZoning_C..all. + MSZoning_RM + Street_Grvl +
##      LotConfig_CulDSac + LandSlope_Sev + Neighborhood_BrkSide +
##      Neighborhood_ClearCr + Neighborhood_Crawfor + Neighborhood_Edwards +
##      Neighborhood_IDOTRR + Neighborhood_MeadowV + Neighborhood_Mitchel +
##      Neighborhood_NWAmes + Neighborhood_NoRidge + Neighborhood_NridgHt +
##      Neighborhood_Somerst + Neighborhood_StoneBr + Condition1_Artery +
##      Condition1_RRAe + Condition1_RRAn + BldgType_Duplex + BldgType_Twnhs +
##      BldgType_TwnhsE + RoofStyle_Flat + Exterior1st_BrkComm +
##      Exterior1st_BrkFace + Exterior1st_CemntBd + Exterior1st_HdBoard +
##      Exterior1st_Plywood + Exterior1st_Wd.Sdng + Exterior2nd_CmentBd +
##      Exterior2nd_Wd.Sdng + MasVnrType_BrkCmn + MasVnrType_Stone +
##      ExterCond_Ex + ExterCond_Fa + ExterCond_Gd + Foundation_Stone +
##      Foundation_Wood + BsmtQual_Ex + BsmtCond_Fa + BsmtCond_Po +
##      BsmtExposure_Gd + BsmtFinType2_BLQ + BsmtFinType2_GLQ + Heating_GasW +
##      Heating_Grav + Heating_Wall + CentralAir_N + KitchenQual_Ex +
##      Functional_Maj1 + Functional_Maj2 + Functional_Min1 + Functional_Min2 +
##      Functional_Mod + Functional_Sev + GarageType_2Types + GarageType_NA +
##      GarageQual_Fa + GarageQual_Po + GarageCond_Fa + GarageCond_Po +
##      PavedDrive_N + SaleType_CWD + SaleType_Con + SaleType_ConLD +
##      SaleType_ConLw + SaleType_New + SaleType_Oth + SaleCondition_Abnorml +
##      SaleCondition_Family + SaleCondition_Partial + BuiltAfter1920 +

```

```
##      YearRemodUnknown, data = df)
##
## Residuals:
##      Min        1Q    Median        3Q        Max
## -0.67671 -0.04919  0.00285  0.05111  0.46324
##
## Coefficients:
##
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)      4.497e+00  5.680e-01   7.918 4.97e-15 ***
## Id              -9.095e-06  6.429e-06  -1.415 0.157355
## LotArea          2.600e-06  3.859e-07   6.738 2.37e-11 ***
## OverallQual      4.738e-02  3.865e-03  12.258 < 2e-16 ***
## OverallCond      3.739e-02  3.501e-03  10.681 < 2e-16 ***
## YearBuilt        2.426e-03  2.618e-04   9.267 < 2e-16 ***
## YearRemodAdd      8.239e-04  2.532e-04   3.255 0.001164 **
## BsmtFinSF1       1.614e-04  1.476e-05  10.936 < 2e-16 ***
## BsmtFinSF2       1.153e-04  2.312e-05   4.986 6.98e-07 ***
## BsmtUnfSF        9.846e-05  1.305e-05   7.548 8.09e-14 ***
## X1stFlrSF        2.669e-04  1.638e-05  16.293 < 2e-16 ***
## X2ndFlrSF        2.570e-04  1.274e-05  20.169 < 2e-16 ***
## LowQualFinSF     2.394e-04  5.854e-05   4.090 4.58e-05 ***
## BsmtFullBath     2.752e-02  7.453e-03   3.693 0.000231 ***
## FullBath         1.898e-02  8.570e-03   2.215 0.026912 *
## HalfBath         1.867e-02  8.196e-03   2.277 0.022926 *
## KitchenAbvGr     -5.083e-02  1.895e-02  -2.683 0.007388 **
## Fireplaces       1.974e-02  5.418e-03   3.643 0.000279 ***
## GarageCars       3.053e-02  9.111e-03   3.351 0.000827 ***
## GarageArea       6.558e-05  3.021e-05   2.171 0.030083 *
## WoodDeckSF       8.737e-05  2.401e-05   3.639 0.000284 ***
## OpenPorchSF      1.055e-04  4.660e-05   2.264 0.023702 *
## EnclosedPorch    1.094e-04  5.013e-05   2.182 0.029263 *
## X3SsnPorch       1.885e-04  9.255e-05   2.037 0.041878 *
## ScreenPorch      2.295e-04  5.078e-05   4.518 6.78e-06 ***
```

## PoolArea	1.175e-04	7.820e-05	1.503	0.133101	
## MSZoning_C..all.	-4.260e-01	4.027e-02	-10.580	< 2e-16	***
## MSZoning_RM	-6.101e-02	1.060e-02	-5.756	1.06e-08	***
## Street_Grvl	-8.150e-02	4.677e-02	-1.742	0.081684	.
## LotConfig_CulDSac	3.232e-02	1.157e-02	2.793	0.005289	**
## LandSlope_Sev	-1.274e-01	3.958e-02	-3.218	0.001323	**
## Neighborhood_BrkSide	5.767e-02	1.588e-02	3.631	0.000293	***
## Neighborhood_ClearCr	5.267e-02	2.233e-02	2.358	0.018517	*
## Neighborhood_Crawfor	1.453e-01	1.659e-02	8.760	< 2e-16	***
## Neighborhood_Edwards	-3.461e-02	1.193e-02	-2.901	0.003782	**
## Neighborhood_IDOTRR	3.522e-02	2.210e-02	1.593	0.111325	
## Neighborhood_MeadowV	-1.180e-01	3.263e-02	-3.617	0.000309	***
## Neighborhood_Mitchel	-2.182e-02	1.573e-02	-1.387	0.165621	
## Neighborhood_NWAmes	-2.698e-02	1.351e-02	-1.997	0.046026	*
## Neighborhood_NoRidge	5.057e-02	1.853e-02	2.729	0.006443	**
## Neighborhood_NridgHt	6.080e-02	1.605e-02	3.789	0.000158	***
## Neighborhood_Somerst	4.679e-02	1.377e-02	3.399	0.000696	***
## Neighborhood_StoneBr	1.244e-01	2.282e-02	5.449	6.00e-08	***
## Condition1_Artery	-6.968e-02	1.609e-02	-4.331	1.60e-05	***
## Condition1_RRAe	-1.178e-01	3.122e-02	-3.774	0.000168	***
## Condition1_RRAn	-3.755e-02	2.077e-02	-1.808	0.070808	.
## BldgType_Duplex	-4.643e-02	2.187e-02	-2.123	0.033904	*
## BldgType_Twnhs	-9.935e-02	1.853e-02	-5.362	9.68e-08	***
## BldgType_TwnhsE	-3.713e-02	1.224e-02	-3.034	0.002460	**
## RoofStyle_Flat	6.984e-02	3.551e-02	1.967	0.049422	*
## Exterior1st_BrkComm	-2.137e-01	7.879e-02	-2.712	0.006775	**
## Exterior1st_BrkFace	3.442e-02	1.680e-02	2.049	0.040636	*
## Exterior1st_CemntBd	-9.097e-02	6.111e-02	-1.489	0.136825	
## Exterior1st_HdBoard	-3.707e-02	8.732e-03	-4.246	2.33e-05	***
## Exterior1st_Plywood	-3.344e-02	1.206e-02	-2.772	0.005649	**
## Exterior1st_Wd.Sdng	-6.159e-02	1.661e-02	-3.707	0.000218	***
## Exterior2nd_CmentBd	8.753e-02	6.156e-02	1.422	0.155343	
## Exterior2nd_Wd.Sdng	4.423e-02	1.637e-02	2.702	0.006970	**

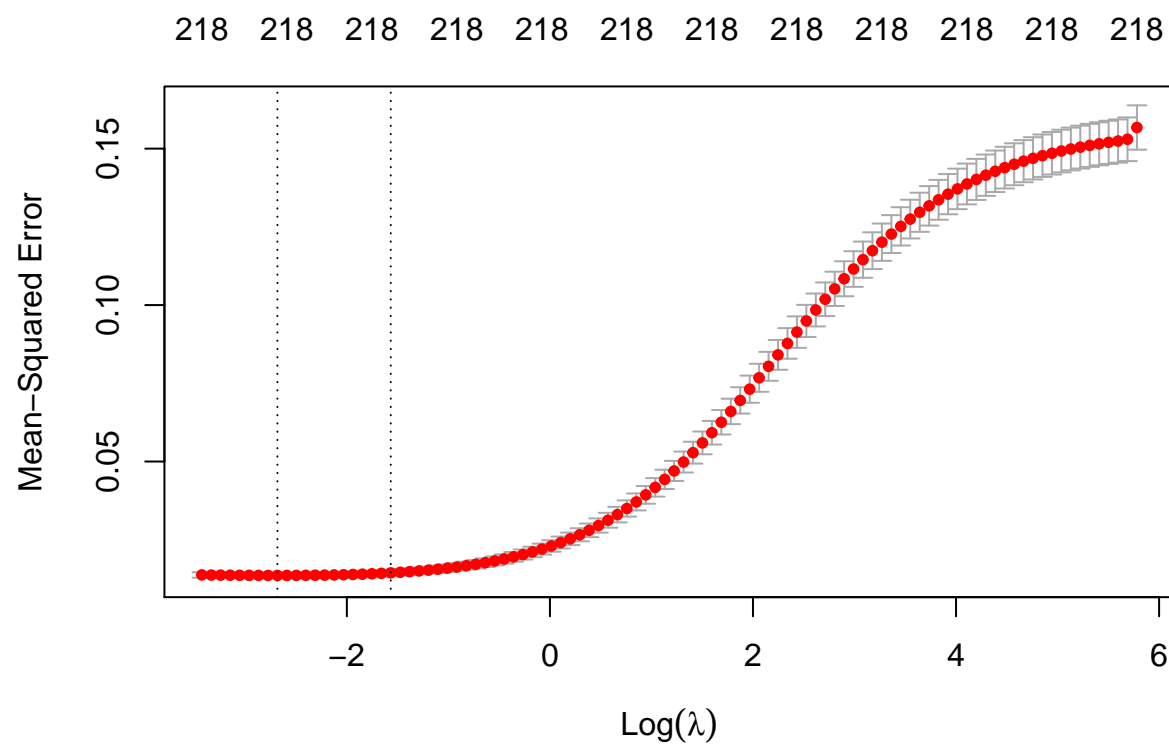
## MasVnrType_BrkCmn	-5.277e-02	2.780e-02	-1.898	0.057862	.
## MasVnrType_Stone	2.179e-02	1.121e-02	1.944	0.052089	.
## ExterCond_Ex	1.013e-01	6.055e-02	1.672	0.094696	.
## ExterCond_Fa	-3.659e-02	2.264e-02	-1.616	0.106303	
## ExterCond_Gd	-1.809e-02	9.700e-03	-1.864	0.062477	.
## Foundation_Stone	1.026e-01	4.444e-02	2.309	0.021089	*
## Foundation_Wood	-1.391e-01	5.955e-02	-2.336	0.019641	*
## BsmtQual_Ex	2.848e-02	1.331e-02	2.140	0.032529	*
## BsmtCond_Fa	-3.251e-02	1.692e-02	-1.922	0.054858	.
## BsmtCond_Po	1.691e-01	9.088e-02	1.861	0.062987	.
## BsmtExposure_Gd	5.052e-02	1.093e-02	4.621	4.19e-06	***
## BsmtFinType2_BLQ	-3.235e-02	1.922e-02	-1.683	0.092602	.
## BsmtFinType2_GLQ	5.425e-02	3.079e-02	1.762	0.078250	.
## Heating_GasW	5.224e-02	2.727e-02	1.916	0.055622	.
## Heating_Grav	-1.477e-01	4.379e-02	-3.372	0.000766	***
## Heating_Wall	8.911e-02	5.559e-02	1.603	0.109163	
## CentralAir_N	-6.317e-02	1.427e-02	-4.426	1.04e-05	***
## KitchenQual_Ex	6.301e-02	1.374e-02	4.587	4.92e-06	***
## Functional_Maj1	-9.671e-02	2.996e-02	-3.228	0.001278	**
## Functional_Maj2	-3.104e-01	4.978e-02	-6.237	5.96e-10	***
## Functional_Min1	-4.847e-02	1.949e-02	-2.487	0.013004	*
## Functional_Min2	-3.440e-02	1.879e-02	-1.830	0.067441	.
## Functional_Mod	-1.175e-01	2.820e-02	-4.167	3.28e-05	***
## Functional_Sev	-4.125e-01	1.119e-01	-3.685	0.000238	***
## GarageType_2Types	-8.141e-02	4.455e-02	-1.827	0.067892	.
## GarageType_NA	-4.205e-02	1.710e-02	-2.459	0.014066	*
## GarageQual_Fa	-3.572e-02	1.914e-02	-1.866	0.062228	.
## GarageQual_Po	-1.359e-01	8.849e-02	-1.536	0.124782	
## GarageCond_Fa	-3.201e-02	2.103e-02	-1.522	0.128259	
## GarageCond_Po	1.079e-01	5.623e-02	1.919	0.055189	.
## PavedDrive_N	-1.862e-02	1.352e-02	-1.378	0.168529	
## SaleType_CWD	8.151e-02	5.230e-02	1.559	0.119343	
## SaleType_Con	1.233e-01	7.211e-02	1.710	0.087502	.

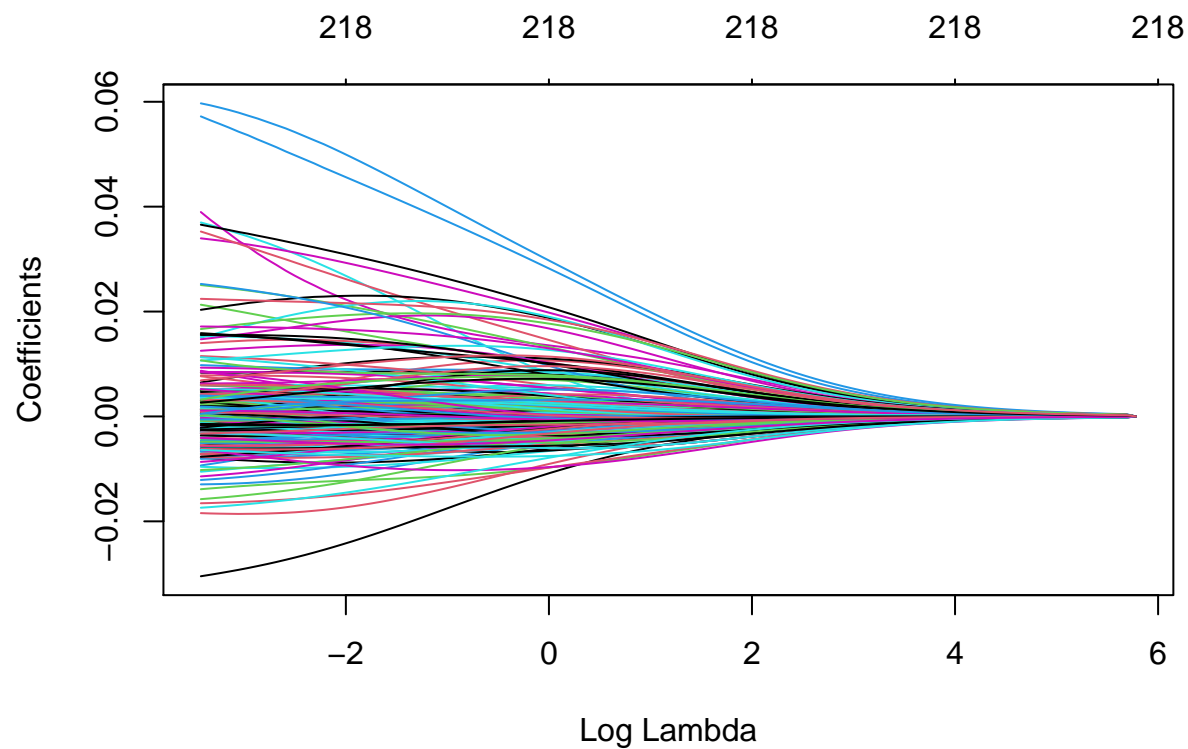
```
## SaleType_ConLD      1.333e-01  3.657e-02   3.645 0.000277 ***
## SaleType_ConLw      6.869e-02  4.704e-02   1.460 0.144458
## SaleType_New        1.772e-01  6.258e-02   2.832 0.004696 **
## SaleType_Oth        8.899e-02  5.994e-02   1.485 0.137864
## SaleCondition_Abnorml -6.940e-02  1.128e-02  -6.150 1.02e-09 ***
## SaleCondition_Family -4.724e-02  2.339e-02  -2.020 0.043582 *
## SaleCondition_Partial -1.311e-01  6.197e-02  -2.116 0.034563 *
## BuiltAfter1920      -2.748e-02  1.554e-02  -1.768 0.077271 .
## YearRemodUnknown     1.598e+00  5.011e-01   3.188 0.001464 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.09995 on 1356 degrees of freedom
## Multiple R-squared:  0.9406, Adjusted R-squared:  0.9363
## F-statistic: 217.1 on 99 and 1356 DF,  p-value: < 2.2e-16
```

Now we make predictions

We achieve a score of .14586 on kaggle. This puts us in the 60th percentile.

B. Ridge regression: R makes it easy to find the best lambda by using kfold validation. Below are the results of our ridge regression analysis. Unlike stepAIC, ridge regression will retain all of the variables.





```
## 219 x 1 sparse Matrix of class "dgCMatrix"
```

```
##                                s0
```

```
## (Intercept)          1.202194e+01
```

```
## Id                  -3.309836e-03
```

```
## MSSubClass          -1.225691e-03
```

```
## LotArea              1.859563e-02
```

```
## OverallQual          5.522861e-02
```

```
## OverallCond          3.240408e-02
```

```
## YearBuilt            2.808566e-02
```

```
## YearRemodAdd         8.337800e-03
```

```
## MasVnrArea           4.715705e-03
```

```
## BsmtFinSF1           2.293512e-02
```

```
## BsmtFinSF2           3.291850e-03
```

```
## BsmtUnfSF            5.856714e-03
```

```
## TotalBsmtSF          3.170250e-02
```

## X1stFlrSF	3.362422e-02
## X2ndFlrSF	3.014979e-02
## LowQualFinSF	1.458749e-03
## GrLivArea	5.140704e-02
## BsmtFullBath	1.494973e-02
## BsmtHalfBath	7.479945e-04
## FullBath	2.232489e-02
## HalfBath	1.462309e-02
## BedroomAbvGr	2.095325e-03
## KitchenAbvGr	-1.222422e-02
## TotRmsAbvGrd	1.874505e-02
## Fireplaces	1.659798e-02
## GarageYrBlt	8.644594e-03
## GarageCars	2.200921e-02
## GarageArea	1.835350e-02
## WoodDeckSF	9.434325e-03
## OpenPorchSF	5.485457e-03
## EnclosedPorch	5.310886e-03
## X3SsnPorch	5.883013e-03
## ScreenPorch	1.077103e-02
## PoolArea	5.210768e-03
## MiscVal	-1.826869e-03
## MoSold	-6.844923e-04
## YrSold	-1.838384e-03
## MSZoning_C..all.	-2.778368e-02
## MSZoning_FV	7.754895e-03
## MSZoning_RM	-1.287264e-02
## Street_Grvl	-5.485240e-03
## LotShape_IR1	1.275073e-03
## LotShape_IR2	4.693476e-03
## LotShape_IR3	1.459738e-03
## LandContour_Bnk	-1.316576e-03
## LandContour_HLS	2.998724e-03

## LandContour_Low	-9.807642e-04
## LotConfig_Corner	2.805178e-03
## LotConfig_CulDSac	9.005300e-03
## LotConfig_FR2	-5.134474e-03
## LotConfig_FR3	-1.527726e-03
## LandSlope_Mod	2.217261e-03
## LandSlope_Sev	-6.980931e-03
## Neighborhood_Blmngtn	-1.033306e-04
## Neighborhood_Blueste	-2.913935e-03
## Neighborhood_BrDale	-8.644756e-03
## Neighborhood_BrkSide	5.954637e-03
## Neighborhood_ClearCr	5.654935e-03
## Neighborhood_Crawfor	2.324794e-02
## Neighborhood_Edwards	-9.885241e-03
## Neighborhood_Gilbert	6.603171e-04
## Neighborhood_IDOTRR	-2.848547e-03
## Neighborhood_MeadowV	-1.854802e-02
## Neighborhood_Mitchel	-4.519131e-03
## Neighborhood_NPkVill	-1.751129e-03
## Neighborhood_NWAmes	-5.169143e-03
## Neighborhood_NoRidge	1.348037e-02
## Neighborhood_NridgHt	1.491112e-02
## Neighborhood_OldTown	-7.038924e-03
## Neighborhood_SWISU	2.132987e-03
## Neighborhood_Sawyer	-4.115669e-03
## Neighborhood_SawyerW	3.315733e-03
## Neighborhood_Somerst	8.475634e-03
## Neighborhood_StoneBr	1.498198e-02
## Neighborhood_Timber	2.702160e-03
## Neighborhood_Veenker	4.905399e-03
## Condition1_Artery	-1.093675e-02
## Condition1_PosA	-1.392710e-03
## Condition1_PosN	-1.567494e-03

## Condition1_RRAe	-6.939742e-03
## Condition1_RRAn	-4.167803e-03
## Condition1_RRNe	-1.043826e-03
## Condition1_RRNn	2.685757e-04
## Condition2_Artery	-2.792394e-03
## Condition2_Feedr	8.845921e-04
## Condition2_PosA	1.826738e-03
## Condition2_PosN	-1.959860e-03
## BldgType_2fmCon	2.461652e-04
## BldgType_Duplex	-7.720083e-03
## BldgType_Twnhs	-9.246957e-03
## BldgType_TwnhsE	-5.664514e-03
## HouseStyle_1.5Fin	4.589653e-03
## HouseStyle_1.5Unf	2.622439e-03
## HouseStyle_2.5Unf	4.133334e-03
## HouseStyle_SFoyer	-2.142000e-04
## HouseStyle_SLvl	-8.149125e-05
## RoofStyle_Flat	6.344150e-03
## RoofStyle_Gambrel	1.399573e-03
## RoofStyle_Hip	7.558624e-04
## RoofStyle_Mansard	3.267699e-03
## RoofStyle_Shed	3.464159e-03
## RoofMatl_Tar.Grv	-3.488009e-03
## RoofMatl_WdShake	1.005160e-03
## RoofMatl_WdShngl	5.107074e-03
## Exterior1st_AsbShng	8.842911e-05
## Exterior1st_AsphShn	-3.490164e-05
## Exterior1st_BrkComm	-7.098625e-03
## Exterior1st_BrkFace	1.018103e-02
## Exterior1st_CBlock	-2.627458e-04
## Exterior1st_CemntBd	-1.138571e-03
## Exterior1st_HdBoard	-7.955485e-03
## Exterior1st_MetalSd	-2.137415e-03

## Exterior1st_Plywood	-4.950223e-03
## Exterior1st_Stucco	1.443392e-03
## Exterior1st_Wd.Sdng	-1.016206e-02
## Exterior1st_WdShng	-3.626782e-03
## Exterior2nd_AsbShng	-3.167782e-03
## Exterior2nd_AsphShn	8.851272e-04
## Exterior2nd_Brk.Cmn	-1.993930e-03
## Exterior2nd_BrkFace	-5.682713e-03
## Exterior2nd_CBlock	-2.608360e-04
## Exterior2nd_CmentBd	1.602779e-03
## Exterior2nd_HdBoard	-6.957516e-03
## Exterior2nd_ImStucc	-7.791633e-04
## Exterior2nd_MetalSd	-2.250036e-03
## Exterior2nd_Plywood	-6.785435e-03
## Exterior2nd_Stone	-1.520937e-03
## Exterior2nd_Stucco	-1.067672e-03
## Exterior2nd_Wd.Sdng	-6.634241e-04
## Exterior2nd_Wd.Shng	-3.531450e-03
## MasVnrType_BrkCmn	-6.477507e-03
## MasVnrType_NA	-1.730478e-03
## MasVnrType_Stone	6.498912e-03
## ExterQual_Ex	2.473912e-03
## ExterQual_Fa	-1.669856e-03
## ExterCond_Ex	2.757060e-03
## ExterCond_Fa	-5.982590e-03
## ExterCond_Gd	-2.977297e-03
## ExterCond_Po	-3.011582e-03
## Foundation_BrkTil	-3.914878e-03
## Foundation_Slab	-1.726069e-03
## Foundation_Stone	4.252869e-03
## Foundation_Wood	-3.933622e-03
## BsmtQual_Ex	1.183683e-02
## BsmtQual_Fa	5.283737e-05

## BsmtQual_NA	-9.837304e-04
## BsmtCond_Fa	-5.905817e-03
## BsmtCond_Gd	1.816026e-03
## BsmtCond_NA	-7.426450e-04
## BsmtCond_Po	2.188656e-03
## BsmtExposure_Av	5.212810e-03
## BsmtExposure_Gd	1.555693e-02
## BsmtExposure_Mn	4.363811e-03
## BsmtExposure_NA	-8.392964e-04
## BsmtFinType1_ALQ	-3.424010e-03
## BsmtFinType1_BLQ	-6.691301e-03
## BsmtFinType1_LwQ	-5.434083e-03
## BsmtFinType1_NA	-3.909525e-04
## BsmtFinType1_Unf	-4.714283e-03
## BsmtFinType2_ALQ	1.302431e-03
## BsmtFinType2_BLQ	-6.453946e-03
## BsmtFinType2_GLQ	4.933001e-03
## BsmtFinType2_NA	-9.686522e-04
## BsmtFinType2_Rec	-2.638284e-03
## Heating_GasW	5.962168e-03
## Heating_Grav	-9.322110e-03
## Heating_Wall	2.798426e-03
## HeatingQC_Fa	-2.257284e-03
## HeatingQC_Gd	-3.261854e-03
## HeatingQC_Po	-2.072154e-03
## CentralAir_N	-1.604088e-02
## Electrical_FuseA	-1.297850e-05
## Electrical_FuseF	1.003718e-03
## Electrical_FuseP	-1.783995e-03
## KitchenQual_Ex	1.702968e-02
## KitchenQual_Fa	6.755406e-06
## Functional_Maj1	-6.420311e-03
## Functional_Maj2	-1.438031e-02

## Functional_Min1	-4.484481e-03
## Functional_Min2	-5.998710e-03
## Functional_Mod	-7.415710e-03
## Functional_Sev	-6.676408e-03
## GarageType_2Types	-5.547047e-03
## GarageType_Basment	-1.726614e-03
## GarageType_BuiltIn	1.516632e-03
## GarageType_CarPort	-9.557306e-04
## GarageType_Detchd	-8.238035e-03
## GarageType_NA	-3.676013e-03
## GarageFinish_Fin	4.768176e-03
## GarageFinish_NA	-3.551420e-03
## GarageQual_Fa	-4.143374e-03
## GarageQual_Gd	3.645263e-03
## GarageQual_NA	-3.429499e-03
## GarageQual_Po	-7.935856e-04
## GarageCond_Ex	4.909539e-04
## GarageCond_Fa	-4.899035e-03
## GarageCond_Gd	-6.189334e-04
## GarageCond_NA	-3.434050e-03
## GarageCond_Po	3.985927e-03
## PavedDrive_N	-6.120871e-03
## PavedDrive_P	-2.901632e-03
## SaleType_COD	-7.296118e-04
## SaleType_CWD	3.847008e-03
## SaleType_Con	3.452244e-03
## SaleType_ConLD	7.080162e-03
## SaleType_ConLI	-1.548098e-03
## SaleType_ConLw	2.888350e-03
## SaleType_New	9.174225e-03
## SaleType_Oth	3.321495e-03
## SaleCondition_Abnorml	-1.619954e-02
## SaleCondition_AdjLand	9.557434e-04

```
## SaleCondition_Alloca -1.721635e-03
## SaleCondition_Family -6.311124e-03
## SaleCondition_Partial 5.237688e-03
## BuiltAfter1920 2.239660e-03
## YearRemodUnknown -6.932021e-03
## NoFinBsmt -4.500193e-03
## HasDeck 3.888406e-03
## HasPorch 8.441819e-03
```

We predict values based on our Ridge regressions.

Despite the large number of independent variables, ridge regression performs better, with a score of .14047. This puts us at 1690 out of 4216 individuals.

C. Lasso Regression To perform Lasso regression, first we define the predictor and response variables for the training dataset. Similarly to the Ridge model, we'll use the `glmnet` library, which makes it easy to use k-fold cross-validation to find the optimal value for lambda. Next, we find the coefficients for the Lasso model using our optimized lambda. Lastly, we predict new values using our optimized Lasso model. Here is our lambda:

```
## [1] 0.003096298
```

We try Lasso with both scaled and unscaled data. Because lasso incorporates a penalty based on the size of the coefficients, we expect the scaled data to perform better, and it does. Our lasso regression gives us a .1375, which outperforms ridge.

D. Elastic Net Regression In order to form elastic net, first, build a control model. Next, train the elastic net regression model. Then we optimize the elastic net model based on tuning parameters selected from model training.

Our elastic net result falls between ridge and lasso.

E. Tree based models: Random Forest and Gradient Boosting Our final models are Random Forest and Gradient Boosting.

Below is the result from our Random Forest model:

Our Random Forest model does not improve our score, but Gradient Boosting does. We land on .12786, which puts us in the 80th percentile.

Discussion and Conclusions

Ordinary Least Squares is a regression technique with a long history of use as a predictive model. However, standard measures of fit (like R^2) will always increase (or stay the same) as you add independent variables. This can result in models which incorporate noise - in other words, overfit the data so that idiosyncrasies in the training set affect predictions in the test set. Other methods of measuring fit, such as adjusted R^2 and AIC, help mitigate the overfitting effect by penalizing the addition of factors.

More recently, other techniques which employ regularization have been introduced to deal with overfit. For example, in ridge regression, we reduce the sum of our coefficients, not the number of variables. We do this by introducing a penalty in the loss function represented by the squared sum of the coefficients themselves, multiplied by a factor (designated as λ) which allows us to control the degree to which the size of the coefficients matters. If λ is zero, there is no difference between ridge regression and OLS.

Ridge regression will keep all the variables but may significantly reduce the coefficients for some. Lasso regression is similar in that it employs a constraint where the sum of the absolute value of the coefficients is less than a fixed value. Lasso regression may drop coefficients altogether to stay under the constraint.

Elastic Net regression is a hybrid approach that blends both of the penalizations of lasso and ridge methods. An α parameter weights which penalty to emphasize - lasso or ridge.

Random Forest (RF) and Gradient Boosting (GB) both rely on decision trees. Decision trees incorporate sequential choice-point steps, providing different outcomes for each choice-point. Decision trees have the advantage that they do not make assumptions that the dependent variable is linearly related to the independent variables. In addition, RF and GB both combine multiple trees so the results are averages of many samples, which improves their predictability. However, the results may be difficult to interpret.

RF and BG handle the combination of trees differently. RF builds each tree independently and averages at the end. BG proceeds in a stage-wise manner, improving the performance of weak learners as you go. This can result in better performance.

Our dataset has features that lend to overfitting. Most significant of these is the high number of potential independent variables (over 200 once the dummy variables are created.) Multicollinearity is also a problem, though less than we might have expected.

We used stepAIC to fit our OLS model. StepAIC uses backward substitution to find the best model with the lowest AIC. With an adjusted R^2 of over 90% overfitting was expected. However, even with an overfit model our predictions performed at the 60th percentile on the Kaggle.

Because of the large number of potential predictors, ridge (and by extension elastic net) were not as good candidates as Lasso - however, potential issues with collinearity actually favored ridge. We found that Lasso improved our score the most of the regression-based models, followed by elastic net (which is a compromise between lasso and ridge), followed by ridge. All were improvements over OLS - however, the improvements were not dramatic.

Gradient boosting had the most success. Because gradient boosting is a machine-learning technique in which the model receives direct feedback with each iteration, it can often do a better job of predicting than the other models. However, if we were looking for insight into the data, GB is something of a “black box” which makes interpretation difficult.

In conclusion, it is important to keep in mind that while regularization improved our model, the base OLS model also performed adequately, so regularization, while important, may in some cases improve models at the margin. It is also important to recognize the strengths of each of the techniques and use the appropriate one for the situation.

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